Sound vibration branches of a gas interacting resonantly with light

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We study the effect of light pressure, and the effect of light-induced drift (LID) and related effects on sound vibrations in a gas. We predict a new branch of transverse sound vibrations caused by the recoil effect. The LID effect causes the occurrence of sound vibrations of the absorbing gas relative to the buffer gas. In this case the gas mixture as a whole remains at rest provided the masses of the absorbing and the buffer particles are the same. The pressure tensor and the heat flux induced by the radiation give rise to two additional vibrational branches.

It is well known that light acts on the translational degrees of freedom of a particle. Striking examples of this action are the light pressure effect,[1] the light-induced drift effect,[2] and effects related to it.[3] The aim of the present paper is to study how these effects influence the sound vibration branches of a gas in a radiation field.[4]

1. GAS DYNAMICS EQUATIONS

One can determine the spectrum of the sound vibrations by solving the eigenvalue problem for the kinetic equations or for the hydrodynamical equations which are obtained from the kinetic equations.[5] We choose the second method. The state of the gas which is interacting with light is completely determined if we know the density matrix $\rho_{ij}(\mathbf{v})$ of the gas which is absorbing the light. In the simplest model of two-level absorbing particles which interact with the field of a travelling monochromatic wave the following kinetic equations[6] hold for $\rho_{ij}(\mathbf{v})$ ($\rho_i \equiv \rho_{ii}$):

$$\begin{aligned}
&(\partial/\partial t+\mathbf{v}\nabla+\Gamma_1)\rho_1(\mathbf{v})=m_aS_1(\mathbf{v})-2\,\mathrm{Re}\,[iG^*\rho_{10}(\mathbf{v}-\mathbf{v}_0/2)],\\
&(\partial/\partial t+\mathbf{v}\nabla)\rho_0(\mathbf{v})\\
&\quad=m_aS_0(\mathbf{v})+\hat{\Gamma}_1\rho_1(\mathbf{v})+2\,\mathrm{Re}\,(iG^*\rho_{10}(\mathbf{v}+\mathbf{v}_0/2)),\qquad (1.1)\\
&(\partial/\partial t+\mathbf{v}\nabla+\Gamma-i(\Omega-\mathbf{kv}))\rho_{10}(\mathbf{v})\\
&\quad=iG(\rho_0(\mathbf{v}-\mathbf{v}_0/2)-\rho_1(\mathbf{v}+\mathbf{v}_0/2)).
\end{aligned}$$

The indices 0, 1, a indicate, respectively, quantities referring to unexcited and excited particles and to the absorbing gas as a whole; $G=Ed_{10}/2\hbar$; E and $\mathbf{k}$ are the amplitude and wave vector of the electromagnetic field; d_{10} is the dipole moment matrix element for the 0–1 transition; $\mathbf{v}_0=\hbar\mathbf{k}/m_a$ is the recoil velocity; Γ_1 is the radiative decay constant of the excited state; $\hat{\Gamma}_1$ is an integral operator which takes into account the recoil effect in spontaneous emission;[6] Ω is the detuning from the resonance which equals the difference between the frequency of the radiation and the frequency of the 0–1 transition; and S_α is the Boltzmann collision integral for particles of type α and mass m_α. We shall also assume that a buffer gas is present which does not interact with the radiation and the state of which is described by the usual Boltzmann equation

$$(\partial/\partial t+\mathbf{v}\nabla)\rho_b(\mathbf{v})=m_bS_b(\mathbf{v}).\qquad (1.2)$$

As in classical gas dynamics the transition from the kinetic Eqs. (1.1) and (1.2) to the closed gas dynamics equations is strictly justified only under conditions close to equilibrium. The departure from equilibrium in the velocity distribution of the absorbing particles depends on the ratio of the homogeneous half-width Γ and the Doppler half-width $k\bar{v}_a$ of the absorption line, where $\bar{v}_\alpha=(2T_\alpha/m_\alpha)^{1/2}$ is the mean thermal velocity and T_α the temperature of particles of type α (we set the Boltzmann constant equal to unity in what follows). In order that the departure from equilibrium be weak, we shall assume that

$$\Gamma\gg k\bar{v}_a.\qquad (1.3)$$

We shall use Grad's 13-moment method[7,8] to solve Eqs. (1.1) and (1.2) under this condition. In accordance with this method we write each distribution function $\rho_\alpha(\mathbf{v})$ ($\alpha=0, 1, a, b$) in the form

$$\rho_\alpha(\mathbf{v})=\rho_\alpha{}^0(\mathbf{v})\Big[1+\frac{2\mathbf{w}_\alpha\mathbf{c}}{\bar{v}_\alpha{}^2}+\sum_{rs}\frac{\pi_{\alpha rs}}{p_\alpha\bar{v}_\alpha{}^2}\Big(c_rc_s-\frac{1}{3}\delta_{rs}c^2\Big)+\frac{4\mathbf{h}_\alpha\mathbf{c}}{5p_\alpha\bar{v}_\alpha{}^4}\Big(c^2-\frac{5}{2}\bar{v}_\alpha{}^2\Big)\Big],\qquad (1.4)$$

$\rho^0_\alpha(\mathbf{v})=\rho_\alpha(\pi\bar{v}^2_\alpha)^{-3/2}\exp(-c^2/\bar{v}^2_\alpha)$, $\mathbf{c}=\mathbf{v}-\mathbf{u}$, $\mathbf{w}_\alpha=\mathbf{u}_\alpha-\mathbf{u}$, $\mathbf{q}_\alpha=\mathbf{h}_\alpha+\frac{5}{2}p_\alpha\mathbf{w}_\alpha$. The transport equations for the mass density $\rho_\alpha=m_\alpha n_\alpha$, the average velocity $\mathbf{u}_\alpha$, the pressure p_α, the pressure tensor $\pi_{\alpha rs}$, and the heat flux $\mathbf{q}_\alpha$ are obtained from the kinetic equations (1.1) and (1.2) by multiplying them by the functions $\psi^{(i)}_\alpha\ldots$;

$$\psi_\alpha^{(1)}=1,\quad \psi_\alpha^{(2)}=\mathbf{c},\quad \psi_\alpha^{(3)}={}^1/_3(c^2-{}^3/_2\bar{v}_\alpha{}^2),$$
$$\psi_{\alpha rs}^{(4)}=c_rc_s-{}^1/_3\delta_{rs}c^2,\quad \psi_\alpha^{(5)}={}^1/_2\mathbf{c}(c^2-{}^5/_2\bar{v}_\alpha{}^2)$$

and then integrating over the velocities. To reduce the unwieldiness of the notation we neglect here the light pressure which we shall consider separately in the next section. The excited particles are described by the following transport equations

$$(\Gamma_1+\partial/\partial t)\rho_1+\nabla\rho_1\mathbf{u}_1=\Pi^{(1)},$$
$$(\Gamma_1+d/dt)\rho_1\mathbf{w}_1+\nabla p_1+\nabla\cdot\boldsymbol{\pi}_1+\rho_1\,d\mathbf{u}/dt=\mathbf{R}_1^{(2)}+\mathbf{\Pi}^{(2)},$$

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$$\frac{3}{2}\frac{d}{dt}p_1+\frac{5}{2}p_1\nabla\mathbf{u}+\nabla\mathbf{q}_1+(\boldsymbol{\pi}_1\cdot\nabla)\mathbf{u}=R_1^{(3)}+\Pi^{(3)},$$
$$(\Gamma_1+d/dt)\pi_{1rs}=R_{1rs}^{(4)}+\Pi_{rs}^{(4)},\qquad(\Gamma_1+d/dt)\mathbf{h}_1=\mathbf{R}_1^{(5)}+\mathbf{\Pi}^{(5)}. \tag{1.5}$$

Here

$$d/dt=\partial/\partial t+\mathbf{u}\nabla,\quad R_{\alpha\ldots}^{(i)}=m_\alpha\int\psi_{\alpha\ldots}^{(i)}\,S_\alpha(\mathbf{v})\,d\mathbf{v},$$
$$\Pi_{\ldots}^{(i)}=-\int\psi_{1\ldots}^{(i)}\,2\,\mathrm{Re}\,iG^*\rho_{10}(\mathbf{v})\,d\mathbf{v},$$

and the vector

$$\mathbf{a}\cdot\boldsymbol{\pi}=\sum_{rs}\mathbf{e}_r a_s\pi_{sr}$$

is the contraction of the tensor product of the vector **a** and the tensor $\boldsymbol{\pi}$; $\mathbf{e}_r$ is a unit vector along the x_r axis; the macroscopic variables without index α refer to the mixture as a whole. One can also write down equations for the particles in the ground state which are similar to (1.5). However, for the equations which supplement (1.5) it is convenient to use the equations for the absorbing gas as a whole

$$\frac{\partial}{\partial t}\rho_\alpha+\nabla\rho_\alpha\mathbf{u}_\alpha=0,\quad\frac{d}{dt}\rho_\alpha\mathbf{w}_\alpha+\nabla p_\alpha+\nabla\cdot\boldsymbol{\pi}_\alpha+\rho_\alpha\frac{d\mathbf{u}}{dt}=\mathbf{R}_\alpha^{(2)},$$
$$\frac{3}{2}\frac{d}{dt}p_\alpha+\frac{5}{2}p_\alpha\nabla\mathbf{u}+\nabla q_\alpha+(\boldsymbol{\pi}_\alpha\cdot\nabla)\mathbf{u}=R_\alpha^{(3)},\tag{1.6}$$
$$\frac{d}{dt}\pi_{\alpha rs}=R_{\alpha rs}^{(4)},\quad\frac{d}{dt}\mathbf{h}_\alpha=\mathbf{R}_\alpha^{(5)}.$$

Equations (1.5) and (1.6) are closed by the gas-dynamic equations for the gas as a whole:

$$\frac{\partial}{\partial t}\rho+\nabla\rho\mathbf{u}=0,\quad\rho\frac{d\mathbf{u}}{dt}+\nabla p+\nabla\cdot\boldsymbol{\pi}=0,$$
$$\frac{3}{2}\frac{d}{dt}p+\frac{5}{2}p\nabla\mathbf{u}+\nabla\mathbf{q}+(\boldsymbol{\pi}\cdot\nabla)\mathbf{u}=0,$$
$$\frac{d}{dt}\pi_{rs}=R_{rs}^{(4)},\quad\frac{d}{dt}\mathbf{h}=\mathbf{R}^{(5)},\tag{1.7}$$
$$R_{\ldots}^{(i)}=R_{a\ldots}^{(i)}+R_{b\ldots}^{(i)},\quad R_{a\ldots}^{(i)}=R_{0\ldots}^{(i)}+R_{1\ldots}^{(i)}.$$

On the left-hand side of Eqs. (1.5) to (1.7) we have dropped terms which are unimportant for the present paper (e.g., terms due to viscosity and thermal conductivity). The complete gas-dynamic equations in a radiation field are given in Refs. 9, 10. The linearized moments $R_{\alpha\ldots}^{(i)}$ of the Boltzmann collision integral have the form[7,8]

$$\mathbf{R}_\alpha^{(2)}=\sum_\beta n_\alpha n_\beta\left[\chi_{\alpha\beta}^{(1)}(\mathbf{u}_\beta-\mathbf{u}_\alpha)+\chi_{\alpha\beta}^{(2)}\frac{\mu_{\alpha\beta}}{T}\left(\frac{\mathbf{h}_\beta}{\rho_\beta}-\frac{\mathbf{h}_\alpha}{\rho_\alpha}\right)\right],$$

$$R_\alpha^{(3)}=\sum_\beta n_\alpha n_\beta\frac{3\chi_{\alpha\beta}^{(1)}}{m_\alpha+m_\beta}(T_\beta-T_\alpha),$$

$$R_{\alpha rs}^{(4)}=-\sum_\beta\frac{n_\alpha n_\beta}{m_\alpha+m_\beta}\left[\chi_{\alpha\beta}^{(3)}\frac{\pi_{\alpha rs}}{n_\alpha}+\chi_{\alpha\beta}^{(4)}\frac{\pi_{\beta rs}}{n_\beta}\right],$$

$$\mathbf{R}_\alpha^{(5)}=\sum_\beta\frac{n_\alpha n_\beta}{m_\alpha}\left[-\chi_{\alpha\beta}^{(5)}\frac{\mathbf{h}_\alpha}{n_\alpha}-\chi_{\alpha\beta}^{(6)}\frac{\mathbf{h}_\beta}{n_\beta}+\frac{5}{2}\frac{m_\beta T\chi_{\alpha\beta}^{(2)}}{m_\alpha+m_\beta}(\mathbf{u}_\beta-\mathbf{u}_\alpha)\right],\tag{1.8}$$

where $\beta=0,1,b$ and the parameters $\chi_{\alpha\beta}^{(l)}$ can[9,10] be expressed in terms of Chapman and Cowling's Ω integrals $\Omega_{\alpha\beta}^{lr}$ of the elastic cross sections $\sigma_{\alpha\beta}$ for the scattering of a particle α by a particle β; $\mu_{\alpha\beta}=m_\alpha m_\beta/(m_\alpha+m_\beta)$.[8,11]

It is clear from Eqs. (1.8) for $\mathbf{R}_\alpha^{(2)}$ and $\mathbf{R}_\alpha^{(5)}$ that there is a collisional interaction between the particle flux $n_\alpha\mathbf{u}_\alpha$ and the heat flux $\mathbf{h}_\alpha$ which is the cause of thermodiffusion and demonstrates the general effect of a coupling of the moments of the distribution function through the collision integral. However, in accordance with Kihara's observations[11] this coupling is weak. One verifies this easily for the example of the interaction between $\mathbf{u}_\alpha$, and $\mathbf{h}_\alpha$, by calculating

$$\chi^{(2)}/\chi^{(1)}={}^6/_5C^*-1,\qquad|\chi^{(2)}/\chi^{(1)}|\ll1,\tag{1.9}$$

where $C^*=\Omega^{*12}/\Omega^{*11}$, $\Omega^{*lr}=\Omega^{lr}/\Omega_0^{lr}$ is the reduced Ω integral,[11] Ω_0^{lr} is the Ω integral for the hard sphere model. Results from numerical calculations show[11] that the ratio $\chi^{(2)}/\chi^{(1)}$ is indeed small, of order 0.2 to -0.2. the coupling of $\mathbf{u}_\alpha$ with higher order moments is even weaker. The weak coupling of the distribution function moments with one another in the limit (1.3) is the basis of the 13-moment method we have used.

The departure from equilibrium in the velocities produced by the electromagnetic field is described by the moments $\Pi_{\ldots}^{(i)}$ of the field term $-2\,\mathrm{Re}\,iG^*\rho_{10}(\mathbf{v})$ which up to and including terms of order $(k\bar{v}_\alpha/\Gamma)^2$ have the form

$$\Pi^{(1)}=\frac{\varkappa\Gamma_1}{2}\left[(\rho_a-2\rho_1)\left(1+\frac{(3\Omega^2-\Gamma^2)(k\bar{v}_a)^2}{2(\Gamma^2+\Omega^2)^2}\right)+\frac{2\Omega\mathbf{k}}{(\Gamma^2+\Omega^2)}(\rho_a\mathbf{u}_a-2\rho_1\mathbf{u}_1)\right],$$

$$\mathbf{\Pi}^{(2)}=\frac{\varkappa\Gamma_1}{2}\left[\frac{\mathbf{k}\bar{v}_a{}^2\Omega}{\Gamma^2+\Omega^2}(\rho_a-2\rho_1)+\rho_a\mathbf{w}_a-2\rho_1\mathbf{w}_1\right],$$

$$\Pi^{(3)}=\frac{\varkappa\Gamma_1\bar{v}_a{}^2}{8}\left[\frac{3\Omega^2-\Gamma^2}{(\Gamma^2+\Omega^2)^2}(k\bar{v}_a)^2(\rho_a-2\rho_1)+\frac{3}{T}(\rho_a(T_a-T)-2\rho_1(T_1-T))\right],$$

$$\Pi_{rs}^{(4)}=\frac{\varkappa\Gamma_1}{2}\left[(\rho_a-2\rho_1)\frac{(3\Omega^2-\Gamma^2)}{2(\Gamma^2+\Omega^2)^2}k^2\bar{v}_a{}^2\delta_{rs}\left(\delta_{rz}-\frac{1}{3}\right)+\pi_{ars}-2\pi_{1rs}\right],$$

$$\mathbf{\Pi}^{(5)}=\frac{\varkappa\Gamma_1}{3}(\mathbf{h}_a-2\mathbf{h}_1),\qquad\varkappa=\frac{4|G|^2\Gamma}{\Gamma_1(\Gamma^2+\Omega^2)}.\tag{1.10}$$

Here $\varkappa$ is the saturation parameter; the z axis is parallel to **k**

2. SOUND AND LIGHT PRESSURE

In order to consider in its "pure form" the role of the light pressure in the formation of the sound vibration spectrum we shall digress from the LID effect[2] and the effects related to it,[3] i.e., we shall assume that $\sigma_{0\alpha} = \sigma_{1\alpha}$. Assuming that $v_0/\bar{v} \ll 1$ and following Grad's method we get from (1.1) the following linearized equations for a one-component gas:

$$\frac{\partial}{\partial t}\rho+\nabla\rho\mathbf{u}=0, \quad \rho\frac{\partial\mathbf{u}}{\partial t}+\nabla p=\mathbf{F}, \quad \frac{\partial}{\partial t}p+\mathbf{u}\nabla p+c^2\rho\nabla\mathbf{u}=Q, \tag{2.1}$$

which generalize Eqs. (1.7) with the recoil effect. Here

$$\mathbf{F}=-2\mathbf{v}_0\,\mathrm{Re}\,iG^*\int\rho_{10}(\mathbf{v})\,d\mathbf{v}\approx\mathbf{v}_0\Gamma_1\varkappa\rho/2(1+\varkappa), \qquad Q=-{}^4/_3\mathbf{v}_0\,\mathrm{Re}\,iG^*\int\mathbf{v}\rho_{10}(\mathbf{v})\,d\mathbf{v}, \tag{2.2}$$

$c^2 = (\partial p/\partial\rho)_s = 5p/3\rho$ is the usual sound velocity; $\mathbf{F}$ is the force per unit volume due to the light pressure.[1] One notes easily that the rate of change of the gas energy density $3Q/2$ is an antisymmetric function of the mismatch Ω, ($Q \propto \Omega$). We assume for the sake of simplicity that $\Omega = 0$, i.e., $Q = 0$. In an absorbing cell with closed ends the stationary state of the gas has a spatially nonuniform pressure distribution,

$$\nabla p=\mathbf{F}. \tag{2.3}$$

We consider a weak deviation of the gas from the stationary state (2.3). The deviations from the stationary density and pressure values ($\delta\rho$ and δp) satisfy according to (2.1) the following set of equations;

$$-\Delta\delta p+[\partial^2/\partial t^2+(\nabla\mathbf{f})+\mathbf{f}\nabla]\delta\rho=0,$$

$$\left[\frac{\partial^2}{\partial t^2}-\frac{1}{\rho}(\mathbf{F}-c^2(\nabla\rho))\nabla\right]\delta p - \left[c^2\frac{\partial^2}{\partial t^2}+\frac{\mathbf{f}}{\rho}(\mathbf{F}-c^2(\nabla\rho))\right]\delta\rho=0, \tag{2.4}$$

where $\mathbf{f} = \partial\mathbf{F}/\partial\rho$. It is difficult to find an exact solution of Eqs. (2.4) which take into account the $\mathbf{r}$-dependence of $\mathbf{f}$. We restrict ourselves to the case when $\mathbf{f}$ is a constant which is valid for a constant half-width Γ on $\varkappa \gg 1$. We look for a solution of Eqs. (2.4) in the form $\rho^{1/2}\exp(i\omega t - i\mathbf{g}\cdot\mathbf{r})$. This solution yields the following dispersion equation:

$$\omega^4-\omega^2c^2(g^2+(\nabla\rho/2\rho)^2)+(\omega_0 cg\sin\theta)^2=0. \tag{2.5}$$

Here

$$\omega_0=\left[\mathbf{f}\left(\frac{2\mathbf{f}}{3c^2}-\frac{\nabla T}{T}\right)\right]^{1/2}\sim\Gamma_1\frac{v_0\varkappa}{c(1+\varkappa)}, \tag{2.6}$$

θ is the angle between the light and the sound wavevectors $\mathbf{k}$ and $\mathbf{g}$. It follows from (2.6) that the stationary state of the gas is unstable if

$$(\mathbf{k}/k)\nabla\ln T>2f/3c^2.$$

In the limit $gc \gg \omega_0$ the dispersion Eq. (2.5) gives two vibration branches, viz., the weakly anisotropic ordinary sound

$$\omega^2=g^2c^2+(c\nabla\rho/2\rho)^2-\omega_0^2\sin^2\theta \tag{2.7}$$

and a new sound mode

$$\omega^2=\omega_0^2\sin^2\theta. \tag{2.8}$$

One verifies easily that in contrast to ordinary sound the new sound mode is transverse in the limit $gc \gg \omega_0$. We note the close analogy between the branch (2.8) and the intrinsic gravitational waves.[12]

3. LIGHT-INDUCED DRIFT AND SOUND

We consider the LID effect on sound, forgetting for the moment the recoil effect. One notes easily from Eqs. (1.5) and (1.10) that in the limit (1.3) and (1.9) the inequality $\rho_1|\mathbf{u}_1| \gg |\mathbf{h}_1|/c^2, |\pi_{1rs}|/c$ is satisfied. Hence in the analysis of the sound vibrations connected with LID we can neglect the pressure tensor $\boldsymbol{\pi}$ and the heat flux $\mathbf{h}$ induced by the radiation. We shall show in the next section that the light-induced pressure tensor and heat flux also lead to new vibration branches, but in the limit (1.3) those vibrations lie in the low-frequency region of the spectrum and therefore interact weakly with the branches caused by the drift. Taking what we have said into account we get from the first three of Eqs. (1.7) the following equations for the nonequilibrium corrections to the density and the pressure:

$$\frac{\partial^2}{\partial t^2}\delta\rho-\Delta\delta p=0,$$

$$\left[\frac{\partial^2}{\partial t^2}-c^2\Delta+c^2\left(\frac{\nabla\rho}{\rho}\right)\nabla\right]\delta p + (m_b-m_a)\nabla\frac{\partial}{\partial t}\left[(\mathbf{u}_a-\mathbf{u}_b)\frac{n_a n_b}{n}c^2\right]=0. \tag{3.1}$$

As in the preceding section we consider a gas in a cell with closed ends. In such a case the various components of the gas are in the stationary state nonuniformly distributed along the length of the cell thanks to the LID effect (for constant total pressure, $\nabla p = 0$). If the masses of the absorbing and the buffer particles are different, the total mass density $\rho = m_a n_a + m_b n_b$ is also spatially nonuniform and when the temperature is constant, equals 2 (for $n_b \gg n_a$)

$$\nabla\rho=(m_a-m_b)\nabla n_a = \mathbf{k}\left(\frac{m_a-m_b}{m_a}\right)\frac{\rho_a(\nu_0^{(1)}-\nu_1^{(1)})\varkappa\Gamma_1\Omega/(\Gamma^2+\Omega^2)}{\nu_1^{(1)}+\frac{1}{2}(\nu_0^{(1)}+\nu_1^{(1)})\varkappa+\Gamma_1(1+\varkappa)^2}, \tag{3.2}$$

where $\nu_\alpha^{(1)} = \chi_{\alpha\beta}^{(1)} n_\beta/m_\alpha$ is the collision frequency. We solve Eqs. (3.1) in the limit $|\nu_1^{(1)} - \nu_0^{(1)}|/\nu_0^{(1)} \ll 1$. One checks easily that the last term in the second equation of (3.1) is smaller than the other terms by a factor $\omega/\nu^{(1)}$ and we can thus neglect it. Changing to a new variable $\delta p = \rho^{1/2}\eta$ in the resulting equations, we have the wave equation

$$\left[-\frac{\partial^2}{\partial t^2}+c^2\Delta-\tilde{\omega}_0^2(\mathbf{r})\right]\eta=0, \tag{3.3}$$

$$\tilde{\omega}_0(\mathbf{r})=c\left[\left(\frac{\nabla\rho}{2\rho}\right)^2-\frac{1}{2}\nabla\left(\frac{\nabla\rho}{\rho}\right)\right]^{1/2} \sim (m_a-m_b)(\nu_0^{(1)}-\nu_1^{(1)})\frac{\Gamma_1\Omega k\bar{v}_a\varkappa}{m_a\nu_0^{(1)}\Gamma^2}. \tag{3.4}$$

In the limit $gc \gg \tilde{\omega}_0$ we can neglect the $\mathbf{r}$ dependence of $\tilde{\omega}_0$ and look for a solution in the form $\exp(i\omega t - i\mathbf{gr})$; as a result we get

$$\omega = \pm (gc + \tilde{\omega}_0^2/2gc). \tag{3.5}$$

To obtain the dispersion relation in the region $gc \lesssim \tilde{\omega}_0$ it is necessary to solve the stationary Schroedinger equation with a potential $\tilde{\omega}_0^2(\mathbf{r})$.

Apart from a change in the dispersion law (3.5) for the ordinary sound the LID effect also produces a new branch of oscillations. For simplicity we consider the case where the masses of the absorbing and the buffer particles are the same: $m_a = m_b = m$. As $m_a = m_b$, when $\omega^2 \neq g^2c^2$ the only solution of Eqs. (3.1) is

$$\delta\rho = \delta p = \delta\mathbf{u} = 0. \tag{3.6}$$

We find a solution of Eqs. (1.6) for $\delta\rho_a$ and $\delta\mathbf{j}_a = \delta(\rho_a \mathbf{u}_a)$ using the fact (3.6) that the mixture as a whole does not move:

$$\begin{gathered}\frac{\partial}{\partial t}\delta\rho_a + \nabla\delta\mathbf{j}_a = 0,\\ \left(\bar{\nu}^{(1)} + \frac{\partial}{\partial t}\right)\delta\mathbf{j}_a + \left[\frac{\bar{v}_a^2}{2}\nabla + (\nu_1^{(1)} - \nu_0^{(1)})\mathbf{u}^{(0)}\right]\delta\rho_a = 0,\end{gathered} \tag{3.7}$$

where

$$\mathbf{u}^{(0)} = \left(\frac{\partial}{\partial\rho_a} - \frac{\partial}{\partial\rho_b} - \frac{1}{\rho_b}\right)\mathbf{j}_1, \quad \bar{\nu}^{(1)} = \frac{1}{\rho}[\nu_0^{(1)}\rho + (\nu_1^{(1)} - \nu_0^{(1)})\rho_a],$$

$\mathbf{j}_1$ is the stationary mass flux of the excited particles ($n_b \gg n_a$):

$$\mathbf{j}_1 = \frac{\varkappa\bar{v}_a^2\rho_a}{2(1+\varkappa)}\left(\frac{\Gamma_1\mathbf{k}\Omega}{\Gamma^2+\Omega^2} - \frac{\nabla n_a}{n_a}\right)\frac{1}{\Gamma_1(1+\varkappa)+\nu_1^{(1)}}.$$

Using the substitution $\exp(i\omega t - i\mathbf{gr})$ we get from (3.7)

$$(\omega - i\bar{\nu}^{(1)})\left[\omega + \left(\frac{\nu_1^{(1)} - \nu_0^{(1)}}{\bar{\nu}^{(1)}}\right)\mathbf{gu}^{(0)} - i\frac{\bar{v}_a^2 g^2}{2\bar{\nu}^{(1)}}\right] = 0. \tag{3.8}$$

The first root of the dispersion relation Eq. (3.8) $\omega = i\bar{\nu}^{(1)}$ corresponds to a dissipative process caused by the friction between the absorbing and the buffer particles. The second root

$$\omega = -\left(\frac{\nu_1^{(1)} - \nu_0^{(1)}}{\bar{\nu}^{(1)}}\right)gu^{(0)}\cos\theta + i\frac{\bar{v}_a^2 g^2}{2\bar{\nu}^{(1)}} \tag{3.9}$$

describes weakly damped oscillations of the absorbing gas relative to the buffer gas[4] when the mixture as a whole is at rest. One notes easily that the speed of this sound is of the same order of magnitude as the speed of the light-induced drift u_{LID}.[2]

4. EFFECT OF PRESSURE AND HEAT FLUX ANISOTROPY ON SOUND

From the last two equations in (1.5), (1.7), and (1.10) it is clear that the radiation produces a pressure tensor $\boldsymbol{\pi}$ and a heat flux $\mathbf{h}$ both in a one-component gas[3] and in a multi-component gas. We study the effect of these quantities on the sound vibrations in the simplest system, a one-component gas. The deviation of the gas from the spatially uniform stationary state is described according to (1.7) by the following linearized equations:

$$\begin{gathered}\frac{\partial}{\partial t}\delta\rho + \rho\nabla\delta\mathbf{u} = 0, \quad \rho\frac{\partial}{\partial t}\delta\mathbf{u} + \nabla\delta p + \sum_r \mathbf{e}_r\frac{\partial}{\partial x_r}\delta\pi_{rr} = 0,\\ \frac{\partial}{\partial t}\delta p + c^2\rho\nabla\delta\mathbf{u} + \frac{2}{3}\nabla\delta\mathbf{h} + \frac{2}{3}\sum_r \pi_{rr}\frac{\partial}{\partial x_r}\delta u_r = 0.\end{gathered} \tag{4.1}$$

In the limit $\omega/\nu \ll 1$, $|\nu_1 - \nu_0|/\nu_0 \ll 1$ the condition that Eqs. (4.1), the solution of which we look for in the form $\exp(i\omega t - i\mathbf{gr})$, are compatible is the dispersion relation

$$\begin{gathered}\omega^4 - \omega^2 g^2[c^2 + s_0^2(3\cos^2\theta - 1)]\\ -\omega g^3 s^3\cos\theta + (g^2 c s_1\sin\theta\cos\theta)^2 = 0.\end{gathered} \tag{4.2}$$

In that equation, obtained using (1.5) and (1.7) we have introduced the notation

$$s_0^2 = \frac{\partial\pi}{\partial\rho} + \frac{2}{3}\frac{\pi}{\rho}, \quad s_1^2 = -\frac{6\pi}{\rho c^2}\frac{\partial\pi}{\partial\rho} > 0, \quad s^3 = \frac{2}{3}\frac{\partial h}{\partial\rho},$$

where the pressure tensor and heat flux in the approximation which is linear in $\varkappa$ have the form

$$\Pi = \frac{1}{2}\Pi_{zz} = -\Pi_{xx} = \frac{(\nu_0 - \nu_1)}{\nu_0}\frac{\rho\bar{v}^2\varkappa(k\bar{v})^2(3\Omega^2 - \Gamma^2)\Gamma_1}{12(\Gamma^2+\Omega^2)^2\left(\Gamma_1 + \frac{1}{2}\nu^{(3)}\right)},$$

$$h = \frac{5(\nu_1 - \nu_0)\nu_1^{(2)}\Gamma_1\Omega\bar{v}^4 k\varkappa\rho}{16\nu_0(\Gamma_1 + \nu_1^{(5)})(\Gamma_1 + \nu_1^{(1)})(\Gamma^2+\Omega^2)},$$

$$\nu_\alpha^{(i)} = \chi_{\alpha 0}^{(i)} n/m, \quad \nu_\alpha = \nu_\alpha^{(3)} + \nu_\alpha^{(4)}.$$

Using the fact that the "velocities" s_0, s_1, and s are small compared to the usual sound speed we get from (4.2)

$$\begin{gathered}\omega_{1,2} = \pm gc \mp \frac{gs_0^2}{2c}(3\cos^2\theta - 1) + \frac{gs^3}{2c^2}\cos\theta,\\ \omega_{3,4} = \frac{1}{2}g\cos\theta\left[-\frac{s^3}{c^2} \pm \left(\left(\frac{s^3}{c^2}\right)^2 + 4s_1^2\sin^2\theta\right)^{1/2}\right].\end{gathered} \tag{4.3}$$

The first two roots ω_1 and ω_2 describe a weak splitting and shift of the usual branch $\omega = \pm gc$. The roots ω_3 and ω_4 correspond to new low-frequency vibration branches.

If $|s^3/c^2 s_1| \gg 1$, we have[4]

$$\omega_3 = -\frac{gs^3}{c^2}\cos\theta, \quad \omega_4 = g\frac{s_1^2 c^2}{s^3}\sin^2\theta\cos\theta. \tag{4.4}$$

In the opposite case, $|c^2 s_1\sin\theta/s^3| \gg 1$, we have

$$\omega_{3,4} = \pm gs_1\sin\theta\cos\theta. \tag{4.5}$$

In these two limiting cases the branches ω_4 from (4.4) and $\omega_{3,4}$ from (4.5) correspond to transverse vibrations.

CONCLUSION

We estimate the frequencies and propagation speeds of the sound vibrations. Taking into account that $v_0/c \sim 10^{-4}$ and taking $\Gamma_1 \sim 10^7\ s^{-1}$, which is characteristic for atomic transitions in the visual range, we find that the frequency (2.6) of the sound vibrations caused by the recoil effect for $\varkappa \sim 1$ is of order $\omega_0 \sim \Gamma_1 v_0/c \sim 10^3\ s^{-1}$. The value $\varkappa \sim 1$ is typical in present-day experiments with laser radiation.[13] The second characteristic frequency (3.4) is connected with the LID effect and has the following order of magnitude: $\omega \sim \nu u_{\rm LID}/c$. In a recent experiment with sodium vapors[13] a value of the ratio of the LID velocity to the normal sound speed was reached; $u_{\rm LID}/c \sim 10^{-2}$, corresponding to $\tilde{\omega}_0 \sim 10^5\ s^{-1}$ for $\nu \sim 10^7\ s^{-1}$. The remaining vibration branches are characterized by linear dispersion laws $\omega = c'g$, where c' is the sound speed for the branch in question. We therefore estimate the ratio c'/c. For the LID branch (3.9) we find $c'/c \sim u_{\rm LID}/c \sim 10^{-2}$. For the branches $\omega_{3,4}$ of (4.3), when $k\bar{v}/\Gamma \sim \chi^{(2)}/\chi^{(1)} \sim 10^{-1}$, we have $c'/c \sim \chi^{(2)} u_{\rm LID}/\chi^{(1)} c \sim 10^{-3}$.

[1]A. P. Kazantsev, Usp. Fiz. Nauk **124**, 113 (1978) [Sov. Phys. Usp. **21**, 58 (1978)].

[2]F. Kh. Gel'mukhanov and A. M. Shalagin, Pis'ma Zh. Eksp. Teor. Fiz. **29**, 773 (1979); Zh. Eksp. Teor. Fiz. **78**, 1674 (1980) [JETP Lett. **29**, 711 (1979); Sov. Phys. JETP **51**, 839 (1980)].

[3]G. A. Levin and K. G. Folin, Pis'ma Zh. Eksp. Teor. Fiz. **32**, 160 (1980) [JETP Lett. **32**, 148 (1980)].

[4]F. Kh. Gel'mukhanov and L. V. Il'ichev, Preprint Inst. Automat. Electrometry, Sib. Div. Acad. Sc. USSR, Novosibirsk, No. 203, 1983; Phys. Lett. **103A**, 61 (1984).

[5]R. Balescu, Equilibrium and Non-equilibrium Statistical Mechanics, Wiley, NY, 1975.

[6]S. G. Rautian, G. I. Smirnov, and A. M. Shalagin, Nelineĭnye rezonansy v spektrakh atomov i molekul (Nonlinear resonances in atomic and molecular spectra) Nauka, Novosibirsk, 1979.

[7]H. Grad, Comm. Pure and Appl. Math. **2**, 331 (1949).

[8]V. M. Zhdanov, Yavleniya perenosa v mnogokomponentnoĭ plazme (Transport effects in a one-component plasma) Energoizdat, Moscow, 1982.

[9]F. Kh. Gel'mukhanov, Kvantovaya Elektron. (Moscow) **11**, 510 (1984) [Sov. J. Quantum Electron. **14**, 347 (1984)].

[10]F. Kh. Gel'mukhanov, Avtometriya No. 1, p. 49 (1985).

[11]J. H. Ferziger and H. G. Kaper, Mathematical Theory of Transport Processes in Gases, North Holland Publ. Cy, Amsterdam, 1972.

[12]G. B. Whitham, Linear and Non-linear Waves, Wiley, NY, 1974.

[13]H. G. C. Wery, J. P. Woerdman, J. J. M. Beenakker, and I. Kuscer, Phys. Rev. Lett. **52**, 2237 (1984).

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