Orientational phase transitions in disordered magnetic materials

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The effect of a small amount of random anisotropy on second-order orientational phase transitions is analyzed. Randomly directed anisotropy is shown to have the same effect on the order parameter that a randomly directed field has on the magnetic moment in the Ising model. Even a slight random anisotropy thus has a qualitative effect on an orientational transition. Random anisotropy may terminate the phase transition or substantially change the properties of the system near the transition. It may convert a second-order phase transition into a first-order transition, accompanied by a large jump in the order parameter. Some new possibilities thus arise for an experimental study of the effect of a random field on long-range order.

1. INTRODUCTION

The magnetic anisotropy in a real ferromagnet always fluctuates in space to some extent or other. Fluctuations of the anisotropy are caused by, for example, structural imperfections of the ferromagnet, which cause a scatter in the crystal fields at the magnetic ions. If the arrangement of magnetic atoms is disordered, there will be fluctuations in the energy of the dipole interaction, which will ultimately also give rise to fluctuations of the anisotropy.[1] Fluctuations of the anisotropy energy are particularly important in concentrated solid solutions or amorphous magnetic materials containing rare earth ions.

In an anisotropic Ising ferromagnet, any arbitrarily weak random anisotropy will disrupt the long-range order.[2] In most cases, in both crystals and amorphous materials, anisotropy of constant direction (uniaxial, cubic, etc.) is present, which stabilizes the magnetic order, so that random anisotropy has only a weak effect on the long-range order and on the phase transition to the ordered state.

In this paper we show that random anisotropy has a qualitative effect on the nature of an orientational phase transition even if this anisotropy is small in comparison with the constant anisotropy. Random anisotropy can terminate a second-order phase transition or substantially change the behavior of the system near the transition. It is possible that reorientation of the moment will occur, not smoothly, as in an ideal crystal, but abruptly, with a large jump in magnetization and with a rotation of the entire moment through an angle of 90°.

The reason for the anomalously strong effect of random anisotropy on an orientational phase transition is that random anisotropy acts on the order parameter of the orientational transition in the same way that a random field acts on a magnetic moment in the Ising model. Random anisotropy may act in a similar way on transitions of other types. For example, it turns out that random anisotropy can terminate a phase transition in the Ising model with a transverse magnetic field. This circumstance opens up some new possibilities for experimentally studying the properties of an Ising magnetic material in a random field.

We wish to stress that phenomena analogous to those which are the subject of the present paper can occur not only in ferromagnets but also in other systems in which orientational phase transitions are observed.

2. TRANSITION AT ABSOLUTE ZERO

2.1 *Dimensional estimates.* We consider the simple model of an orientational phase transition in a uniaxial ferromagnet with an anisotropy easy axis. We assume that an external magnetic field H is applied along the z axis, perpendicular to the easy axis (the x axis). In an ideal ferromagnet, an orientational phase transition of second order in the magnetic field would of course occur under such conditions: In a strong field the moment would be directed along the z axis, and at a certain critical field a component M_x would arise and serve as an order parameter.

In a real crystal, structural defects unavoidably give rise to fluctuations of the directions of the anisotropy axes. Such fluctuations are particularly numerous in amorphous magnetic materials, solid solutions, and so forth. Fluctuations of the dipole-dipole energy in disordered magnetic materials also effectively give rise to fluctuations of the anisotropy axes.[1]

The energy density (E) of a system at $T=0$, corrected for a random anisotropy, is

$$E={}^1/_2V(\nabla\mathbf{m})^2-Hm_z(\mathbf{r})-{}^1/_2Km_x{}^2(\mathbf{r})-D(\mathbf{m}(\mathbf{r})\boldsymbol{\xi}(\mathbf{r}))^2, \tag{1}$$

$$\mathbf{m}(\mathbf{r})=\mathbf{M}(\mathbf{r})/|\mathbf{M}(\mathbf{r})|.$$

The energy of the magnetic anisotropy has been broken up into two components here: one corresponding to the easy axis (the x axis), with a constant $(1/2)K$, and a fluctuating component, with a constant $D\ll K$. The unit vectors $\xi(\mathbf{r})$ are oriented randomly in space. For simplicity we ignore fluctuations in K and D. Distances are conveniently expressed in units of the average distance between magnetic atoms. The exchange parameter V is then on the order of the Curie temperature T_C.

If we ignore the term with the random anisotropy, we see that expression (1) describes a second-order orientational phase transition at $K=H$. The magnetization distribution is determined by the condition for a minimum of the energy, which takes the following form near the transition, i.e., at small values of $m_x(\mathbf{r})$:

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$$-V\nabla^2 m_x(\mathbf{r})+(H-K)m_x(\mathbf{r})+{}^1/_2 H m_x^3(\mathbf{r})-2D\xi_x(\mathbf{r})\xi_z(\mathbf{r})=0. \tag{2}$$

The fluctuating term $1/2Hm_x^2(\mathbf{r})\,m_y^2(\mathbf{r})$ has been omitted from Eq. (2). From the equation for $m_y(\mathbf{r})$ which is analogous to (2) we easily see that we have $m_y^2(\mathbf{r})\sim D^2/V^{3/2}H^{1/2}$. We will see below that the condition $m_y^2 \ll m_x^2$ holds, so that this term is in fact unimportant.

The last term in (2) is the effective random external field $h(\mathbf{r})$ which is acting on the order parameter $m_x(\mathbf{r})$. Its expectation value $\langle h \rangle$ is zero. We assume that its autocorrelation is a δ-function:

$$\langle h(\mathbf{r})h(\mathbf{r}_1)\rangle=h^2\delta(\mathbf{r}-\mathbf{r}_1),\quad h^2={}^4/_9 D^2. \tag{3}$$

The angle bracket denote a configurational average. The problem here is thus equivalent to the problem of a phase transition in the Ising model with a random field in the self-consistent-field approximation (without thermodynamic fluctuations).

We first consider the paramagnetic phase. The term with m_x^3 can be ignored if we are not extremely close to the transition point. We then find

$$m_x(\mathbf{r})=\int h(\mathbf{r}')G(\mathbf{r}-\mathbf{r}')\,d^3r', \tag{4}$$

$$R=\left[\frac{V}{(H-K)}\right]^{1/2},\quad G(\mathbf{r}-\mathbf{r}')=\frac{1}{4\pi}\frac{e^{-|\mathbf{r}-\mathbf{r}'|/R}}{|\mathbf{r}-\mathbf{r}'|}.$$

The expectation value of $m_x(\mathbf{r})$ is zero, while its variance is

$$\langle m_x^2\rangle=\frac{h^2}{8\pi V^2}\left(\frac{V}{H-K}\right)^{1/2}. \tag{5}$$

Perturbation theory is valid under the condition $\langle m_x^2\rangle \ll t = (H-K)/K$, i.e.,

$$t\gg h^{4/3}/VK^{1/3}. \tag{6}$$

We now consider a region which is so close to the transition point that the opposite inequality holds. We can then ignore the second term in Eq. (2). In the equation found as a result, we impose the scale transformation

$$m_x(\mathbf{r})=\mu\Phi(\mathbf{r}),\quad \mathbf{r}=\rho L, \tag{7}$$

where

$$\mu=\frac{h^{2/3}}{V^{1/2}K^{1/6}},\quad L=\frac{V}{K^{1/3}h^{2/3}}\gg 1. \tag{8}$$

Equation (2) reduces to the equation

$${}^1/_2\Phi^3(\rho)-\nabla^2\Phi=\tilde{h}(\rho), \tag{9}$$

where $\tilde{h}(\rho)$ is a random function with the correlation function

$$\langle\tilde{h}(\rho)h(\rho_1)\rangle=\delta(\rho-\rho_1). \tag{10}$$

There are no parameters in Eqs. (9) and (10), so that the variance of $\Phi(\rho)$ is on the order of unity. We thus have

$$\langle m_x^2\rangle=\mu^2\langle\Phi^2(\rho)\rangle\approx h^{4/3}/VK^{1/3}. \tag{11}$$

The numerical coefficient of $\langle\Phi^2(\rho)\rangle$ in (11) cannot be found.

The quantity $L \gg 1$ determines the scale length for changes in m_x and in the correlation function $\langle m_x(\mathbf{r})m_x(\mathbf{r}_1)\rangle$. In fields satisfying inequality (6), the susceptibility is evidently $\chi=(2Kt)^{-1}$. Substituting in the boundary condition $t=h^{4/3}/VK^{1/3}$, and noting that at values of t which satisfy the inequality opposite (6) the susceptibility is independent of t, we find an estimate of the susceptibility in this region:

$$\chi\sim L^2/V. \tag{12}$$

2.2 *Scaling relations.* We now transform Eq. (2) to an integral form:

$$m_x(\mathbf{r})=\frac{1}{V}\int\left(h(\mathbf{r}_1)-\frac{H}{2}m_x^3(\mathbf{r}_1)\right)G(\mathbf{r}-\mathbf{r}_1)\,d^3\mathbf{r}_1. \tag{13}$$

Integrating (13), we find a perturbation-theory series in powers of $h(\mathbf{r})$, which can conveniently be represented graphically.

In first order in h we have

$$m_{x1}=\frac{1}{V}\int h(\mathbf{r}_1)G(\mathbf{r}-\mathbf{r}_1)\,d^3\mathbf{r}_1. \tag{14}$$

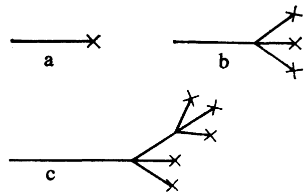

FIG. 1.

Following Ref. 3, we associate the integral on the right side with Fig. 1a. The straight line corresponds to the Green's function $G(\mathbf{r}-\mathbf{r}_1)$, while the cross corresponds to the random field $h(\mathbf{r}_1)/V$. The diagrams in Figs. 1b and 1c arise in the subsequent orders. An integration is carried out over the coordinates of the vertices and of the crosses.

To calculate the macroscopic susceptibility, we include a static external field h_0 in the right side of (12). The part of the moment m_{h_0} which is proportional to h_0 is determined by the same diagrams, in which one of the crosses is replaced by a dot to correspond to a static field. The macroscopic susceptibility is found from m_{h_0} by taking an average over the distribution of random fields and by dividing by h_0. Since we are interested in large regions, with dimensions on the order of R, we may assume that the distribution of random fields is Gaussian. Using (3), we then find that taking the configurational average reduces to equating the coordinates of the crosses in pairs. We thus find diagrams of the type in Fig. 2 for the susceptibility χ. The filled squares correspond to h^2/V^2, while vertices correspond to $3H/2V$. For the susceptibility we thus find the usual perturbation-theory series for the problem of a phase transition with an interaction φ^4, except that the line along whose momenta the integration is to be carried out has some squares, whose total number is equal to the number of vertices. The structure of the resulting series is the same as in the case of a temperature-induced phase transition in a magnetic material with a random field.[4] The only

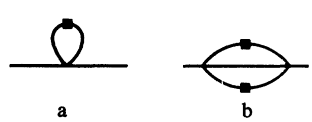

FIG. 2.

difference is in the expressions for the vertices. In a temperature-induced phase transition, on the other hand, thermodynamic fluctuations cause a renormalization of the vertices; for a phase transition at absolute zero, the vertices are not renormalized.

It is easy to see that the perturbation-theory series for the susceptibility goes in powers of the parameter $Kh^2V^{-3}\varkappa^{d-6}$ (d is the dimensionality of the space), so that we can write the following scaling formula for the susceptibility:

$$\chi=\frac{1}{\varkappa^2 V}f\left(\frac{h^2K}{V^3\varkappa^{6-d}}\right),\quad \varkappa=R^{-1}. \tag{15}$$

In the limit $\varkappa\to 0$ the susceptibility should remain finite, since the random field shifts the transition. We thus find the estimate

$$\chi(\varkappa\to 0)\approx V^{d/(6-d)}/h^{4/(6-d)}K^{2/(6-d)}, \tag{16}$$

which agrees with (12), as it should.

The first Hartree diagram (Fig. 2a) makes a contribution of order $(h^2/V^2)(K/V)(1/\varkappa^{4-d})$ to the eigenenergy part; correspondingly, and anomalous point in the susceptibility (the nature of this anomaly will be discussed in the following section of this paper) shifts by an amount of order

$$\Delta H_c/H_c\sim -h^{4/(6-d)}/K^{(4-d)/(6-d)}V^{d/(6-d)}\approx -\langle m_x^2\rangle. \tag{17}$$

The same estimate for ΔH_c can be found by following Ref. 5, where a disordered antiferromagnet in a magnetic field was studied, and determining H_c by equating the argument of the function $f(x)$ in (15) to a constant of order unity.

Figure 3 shows the simplest diagrams for the moment correlation function $Q(\mathbf{r})=\langle m_x(0)m_x(\mathbf{r})\rangle$. We thus find the following expression for the Fourier transform of the function $Q(\mathbf{r})$:

$$Q(\mathbf{k})|_{\mathbf{k}=0}\sim\frac{h^2}{V^2\varkappa^4}\varphi\left(\frac{h^2K}{\varkappa^{6-d}V^3}\right). \tag{18}$$

By analogy with (16) we find

$$Q(\mathbf{k})|_{\mathbf{k}=0,\,\varkappa=0}\sim V^{2d/(6-2d)}/K^{4/(6-d)}h^{(2d-4)/(6-d)}. \tag{19}$$

From the dimensional estimates we easily find a generalization of Eq. (8) to a space of arbitrary dimensionality:

$$L\approx V^{3/(6-d)}K^{-1/(6-d)}h^{-2/(6-d)}. \tag{20}$$

Comparing (19) with (17) and (20), we note that, at small values of $\varkappa$, we would have

$$Q(k=0)\approx\langle m_x^2\rangle L^d.$$

It follows that $Q(\mathbf{r})$ is approximately constant at $r<L$ and falls off rapidly (probably exponentially) at $r\gg L$.

The quantity $Q(k=0)$ could be measured directly in experiments on diffuse neutron scattering.

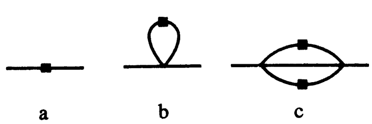

FIG. 3.

3. DOES AN ORDERED PHASE EXIST?

As we have already mentioned, the orientational phase transition model in a disordered ferromagnet which we are considering here is equivalent to the Ising model in a random field. It has been shown elsewhere[4,6–8] that the critical behavior of such a system in a space of dimensionality d is the same as the critical behavior of an ordered system of dimensionality $d-2$. It was shown in Ref. 9 for the case of an Ising model with a transverse field that quantum fluctuations do not alter this conclusion. It follows that a random field disrupts the order in both two-dimensional and three-dimensional Ising ferromagnets.

However, simple qualitative arguments[2] show that the lower critical dimensionality for the Ising model with a random field is $d_c=2$. At $d\leqslant 2$, the spins tend to break up into domains oriented in opposite directions. An attempt[10–12] to show that domain walls of complex shape cause d_c to increase to three was criticized in Ref. 13. The numerical simulation of Ref. 14 again leads to the conclusion $d_c=2$. So far, extensive experimental studies[15–19] have failed to definitely resolve which dimensionality is the critical one. Most of the experimentalists accept $d_c=3$.

It follows that in the case of an orientational phase transition random anisotropy terminates the transition if $d\leqslant 2$ and may do so even if $d=3$. At any rate, we should expect experiments to reveal a shift of the phase-transition point (or shifts of structural features in physical quantities) and a rounding of maxima, as has been seen for an Ising system in a random field.[15–19]

Regardless of what the lower critical dimensionality is, there must be a transition region (crossover) between two regimes: that in which the field $h(\mathbf{r})$ has little effect [inequality (6) holds] and that near $H_c^*=H_c+\Delta H_c$. This transition region has a width of order $K<\langle m_x^2\rangle$, and it is to this region that the expressions derived for the physical quantities in the case $\varkappa=0$ in the preceding section apply.

We will now show that this model of an orientational transition is equivalent to a model with a random field even if the condition $H<K$ holds far from the critical point. In the absence of a random anisotropy under the condition $H<K$ there are two equilibrium orientations of the moment, corresponding to the following angles from the x axis: $\alpha_1=\arcsin(H/K)$ and $\alpha_2=\pi-\alpha_1$. Fluctuations of the angles around α_1 and α_2 caused by the random anisotropy are of order $D/K\ll 1$ and can be ignored. The energy of the magnetic system can then be written

$$E=-D\sum_i S^2\cos^2(\varphi_i-\psi_i)-{}^1/_2\sum_{i,j}V_{ij}\cos(\varphi_i-\varphi_j)S^2, \tag{21}$$

where the angles φ_i can take on the values α_1 and α_2, and ψ_i is a random angle. The exchange term cannot be written as the square of a gradient here, since the angle α_1 is generally not small.

We now introduce the pseudospin σ_i, which has the value 1 if $\varphi_i=\alpha_2$ or -1 if $\psi_i=\alpha_1$. It is then easy to see that expression (21) takes the following form, within an inconsequential constant:

$$E=-{}^1/_2 S^2\cos^2\alpha_1\sum_{i,j}V_{ij}\sigma_i\sigma_j-\sum_i h_i\sigma_i,$$
$$h_i=-{}^1/_2 DS^2\sin 2\psi_i\sin 2\alpha_1. \qquad (22)$$

We have obtained the Hamiltonian of the Ising model with a random field. The termination of the orientational phase transition would mean that under the condition $H<K$ the system breaks up into domains in which the spins are oriented at angles α_1 and α_2, so that the average moment along the x axis is zero.

There is yet another important consequence here. We know that in a uniaxial ferromagnet with random anisotropy a long-range order will exist if $K>h\,(h/V)^{d/(4-d)}$. It follows from the discussion above that an arbitrarily weak transverse field will give rise to an effective random field along the anisotropy axis, so that the long-range order will definitely be disrupted in a two-dimensional ferromagnet and possibly in a three-dimensional ferromagnet.

Fishman and Aharony[5] have proposed a method for implementing the Ising model with a random field. Their suggestion stimulated many of the experimental studies which we have already cited. We see that a uniaxial ferromagnet with random anisotropy to which a transverse field is applied is another implementation of this model.

Analogous phenomena may occur in a cubic crystal. This question will be studied separately.

4. ORIENTATIONAL TRANSITION AT A NONZERO TEMPERATURE

The model of Ref. 1 also describes an orientational transition induced by a temperature change in a constant magnetic field if the anisotropy constant K depends on the temperature. As before, we may ignore thermodynamic fluctuations if the Ginzburg criterion

$$\tau=(T-T_c)/T_c\gg K^2T_c^2/BV^3$$

holds, where T_c is the temperature of the orientational phase transition, and $B=(dK/d\tau)_{T=T_c}$. On the other hand, the criterion for the applicability of a perturbation theory in a random anisotropy, (6), is rewritten in this case as

$$\tau\gg h^{4/3}K^{2/3}/VB. \qquad (23)$$

Two situations are possible.

1) The random fields are so weak that a perturbation theory for them becomes inapplicable only in the critical region near T_c, where the Ginzburg criterion no longer holds:

$$h\ll K(T_c/V)^{3/4}. \qquad (24)$$

The effect of random fields in the critical region can then be taken into account by the approach of Ref. 5.

2) The random fields are so strong that the limit opposite (24) holds. In this case there is a broad region along the τ scale in which the results of Section 1 are valid. In a narrow region near the renormalized transition temperature T_c, where the Ginzburg criterion is violated, a decrease in τ is accompanied by a transition to a critical region in a highly inhomogeneous system (highly inhomogeneous because of the random field).

A temperature-induced phase transition may also occur in the absence of a magnetic field, if a crystal has two special axes. In the simplest model for such a transition,[20] the free-energy density is written in the form

$$F(\mathbf{r})=V(\nabla m_x)^2+K_1m_x^2+K_2m_x^4-D(\xi(\mathbf{r})\mathbf{m})^2. \qquad (25)$$

An orientational phase transition occurs in the XZ plane. As in the preceding case, the component m_y is always small and has no effect on the orientational transition; it has accordingly been omitted from (25).

We assume $K_2>0$ and that the sign of K_1 depends on T. In the absence of the slight anisotropy of the free energy, expression (25) corresponds to two phase transitions: one in which K_1 changes sign from positive to negative, and a component m_x arises; and one in which K_1+2K_2 changes sign from positive to negative, and m_z vanishes. Obviously, at both small values of m_x and small values of m_z we can rewrite the random anisotropy in (25) in the form $-hm$, where m is the order parameter of the orientational transition. The results derived previously regarding the behavior of the system near the phase-transition point therefore apply to this case also.

5. FIRST-ORDER PHASE TRANSITION INDUCED BY A RANDOM FIELD

In this section we show that the random anisotropy may cause a second-order orientational phase transition to become a first-order phase transition.

We consider a biaxial ferromagnet with $K_1<0$ and $K_1+2K_2>0$. If $D\ll K_1, K_2$, the spins are oriented primarily along the favored axes, which are determined by a minimum of the crystalline-anisotropy energy (Fig. 4):

$$\theta_1=\arccos(|K_1|/2K_2)^{1/2},\quad \theta_2=\pi-\theta_1. \qquad (26)$$

We introduce two Ising variables: s_i and σ_i. We have $s_i=1$ if the spin S_i is directed along axis 1, or we have $s_i=-1$ if it is directed along axis 2; we have $\sigma_i=1$ if the spin is directed in the upper half-plane, or $\sigma_i=-1$ if it is in the lower half-plane. The energy of the system, which is the sum of the exchange energy and the random-anisotropy energy, can then be written

$$E=-{}^1/_2 S^2\sum_{i,j}V_{ij}\sigma_i\sigma_j(\sin^2\theta_1+s_is_j\cos^2\theta_1)-\sum_i h_is_i,$$
$$h_i=-{}^1/_2 DS^2\sin 2\theta_1\sin 2\psi_i, \qquad (27)$$

where the angle ψ_i is the angle made by the axis of the random anisotropy and the x axis.

If $\theta_i>45°$, the expression in parentheses in the first term in (27) is positive for arbitrary s_i and s_j, so that a mini-

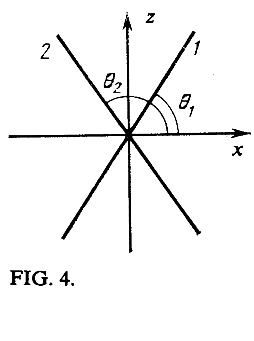

FIG. 4.

mum of the energy is reached in the case $\sigma_i\ \sigma_j = 1$. In terms of the pseudospins s_i, expression (27) represents the energy of an Ising ferromagnet in a random external field. Let us assume that the dimensionality of the space is such that the random field disrupts the long-range order in the Ising magnetic material. All the spins are then directed in the upper (or lower) half-plane, but they are scattered in a random way along axes 1 and 2, so that we have $\langle m_x \rangle = 0$ and $\langle m_z \rangle \neq 0$.

If $\theta < 45°$, it is convenient to introduce pseudospins σ_i such that $\sigma_i = 1$ if the spin lies in the right-hand half-plane in Fig. 4 or $\sigma_i = -1$ if the spin S_i lies in the left-hand half-plane. Arguments analogous to those above lead to the conclusion that in this case the spins are oriented in the right-hand (or left-hand) half-plane, so that we have $m_z = 0$, $\langle m_x \rangle \neq 0$.

Thus, for values of the anisotropy constants corresponding to $\theta = 45°$ the magnetization rotates abruptly from the z axis to the x axis. Thermodynamic fluctuations smear out this abrupt change. However, the interval $(\Delta\theta)$ in which the transition from $m_z = 0$ to $m_x = 0$ occurs is very narrow. It is not difficult to see that the condition $\Delta\theta \sim T/Vl^{d-1}$ holds, where l is the size of a domain. This quantity is smaller than the parameter h/V, in terms of which the actual Hamiltonian is transformed into Ising Hamiltonian (27), so we will not discuss the behavior of the order parameter in the interval $\Delta\theta$.

6. ORIENTATIONAL TRANSITION WITH AN EASY-PLANE ANISOTROPY

If an external magnetic field is directed perpendicular to the easy plane, the moments in an ordered ferromagnet may undergo a reorientation from an axis to a plane. A random anisotropy, like a random field, disrupts the long-range order in the plane at $d < 4$ (Ref. 2), thereby terminating the orientational transition.

In the equation of motion for the order parameter $\mathbf{m}_\perp = (m_x, m_y)$ at $T = 0$, a random anisotropy is, as in Section 1, equivalent to a random field $h_\alpha(\mathbf{r})$:

$$-V\nabla^2 m_\alpha + (H-K)\,m_\alpha + \frac{H}{2}\,m_\perp{}^2 m_\alpha - h_\alpha(\mathbf{r}),$$
$$h_\alpha(\mathbf{r}) = 2D\xi_\alpha(\mathbf{r})\,\xi_z(\mathbf{r}), \qquad m_\perp{}^2 = \sum_\alpha m_\alpha{}^2. \tag{28}$$

Here $\alpha = x, y$.

Equation (28) can be solved exactly for a spherical model, in which the number (n) of components of the vector $\mathbf{m}_\perp$ goes to infinity, and the correlation function of the random fields becomes

$$\langle h_\alpha(\mathbf{r})\, h_\beta(\mathbf{r}) \rangle = (h^2/n)\,\delta_{\alpha\beta}\delta(\mathbf{r}-\mathbf{r}_1). \tag{29}$$

In this case $m_\perp^2$ is the sum of an infinite number of terms m_α^2, so that the fluctuations of $m_\perp^2$ are infinitesimal. Replacing $m_\perp^2$ in (28) by the expectation value $\langle m_\perp^2 \rangle$, we find the following expression for the Fourier transform $m_\alpha(\mathbf{k})$:

$$m_\alpha(\mathbf{k}) = h_\alpha(\mathbf{k})\,[Vk^2 + H - K + {}^1/_2 H\langle m_\perp{}^2\rangle]^{-1}, \tag{30}$$

where $\langle m_\perp^2 \rangle$ is found from the self-consistency condition

$$\langle m_\perp{}^2 \rangle = \sum_{\alpha=1}^{n} \langle m_\alpha{}^2 \rangle = \frac{h^2}{8\pi V^2}\left(\frac{V}{H-K+{}^1/_2 H\,\langle m_\perp{}^2\rangle}\right)^{1/2}. \tag{31}$$

The correlation function of the transverse components of the moment is

$$Q_{\alpha\beta}(\mathbf{r}) = \langle m_\alpha(0)\, m_\beta(\mathbf{r}) \rangle = \delta_{\alpha\beta}\frac{h^2\Lambda_0}{8\pi V^2 n} e^{-r/\Lambda_0}, \tag{32}$$

where

$$\Lambda_0 = V\left[H - K + \frac{H}{2}\langle m_\perp{}^2\rangle\right]^{-1}. \tag{33}$$

Finally, the homogeneous susceptibility is $\chi = \Lambda_0^2/V$.

It follows from (31) and (33) that the length Λ_0 and thus the susceptibility are finite at any value of $H-K$ and increase monotonically with decreasing H. The dependence of $m_\perp^2$, Λ_0 and χ on $H-K$, as in Section 1, is determined by the relation between $(H-K)/K$ and $h^{4/3}/VH^{1/3}$.

a) $1 \gg \dfrac{H-K}{K} \gg \dfrac{h^{4/3}}{VH^{1/3}}$.

In this region we can use perturbation theory, so that $\langle m_\perp^2 \rangle$ and Λ_0 are determined by (5) and (4).

b) $\dfrac{|H-K|}{K} \ll \dfrac{h^{4/3}}{VH^{1/3}}$.

In this case we can discard $H-K$ in comparison with $Hm_\perp^2$ in (31), so that we have

$$\langle m_\perp{}^2 \rangle = \frac{1}{4\pi^{2/3}}\frac{h^{4/3}}{VH^{1/3}}, \qquad \Lambda_0 = \frac{2^{4/3}\pi^{1/3}V}{h^{2/3}H^{1/3}}. \tag{34}$$

c) $1 \gg \dfrac{K-H}{K} \gg \dfrac{h^{4/3}}{VH^{1/3}}$.

In this region, which corresponds in the case $h = 0$ to an ordered phase, $m_\perp^2$, the length Λ_0, and χ continue to increase with increasing $K-H$ in the following way:

$$\langle m_\perp{}^2 \rangle = 2(K-H)/H + h^4H/[2(8\pi)^2(K-H)^2V^3],$$
$$\Lambda_0 = 16\pi(K-H)\,V^2/h^2H. \tag{35}$$

Extrapolating (35) to $(K-H)/K \approx 1$, we find, in order of magnitude, the scale length $\Lambda_0 = V^2/h^2$ found by Imry and Ma.[2]

To see how the magnetization distribution changes at finite values of n, we seek a solution of Eq. (28) with an accuracy of $1/n$. Rewriting (28) as

$$-V\nabla^2 m_\alpha + (H-K)\,m_\alpha + {}^1/_2 H m_\perp{}^2 m_\alpha$$
$$= h_\alpha(r) + {}^1/_2 H(\langle m_\perp{}^2\rangle - m_\perp{}^2)\,m_\alpha \tag{36}$$

and iterating on the second term on the right side, we find, in place of (31),

$$\langle m_\perp{}^2 \rangle = \frac{h^2}{8\pi V^2}\left(\frac{V}{H-K+{}^1/_2 H\,\langle m_\perp{}^2\rangle}\right)\left(1 + \frac{Hh^2\Lambda_0{}^3}{16\pi n V^3}\right). \tag{37}$$

In case a) the coefficient of $1/n$ in parentheses is small in comparison with unity, so that the results of the spherical model agree with the perturbation-theory results and hold for arbitrary n. In case b) the coefficient of $1/n$ is 1, and we have

$$\langle m_\perp{}^2 \rangle = \frac{2^{1/3}}{4\pi^{2/3}}\frac{h^{4/3}}{H^{1/3}V}\left(1 - \frac{2}{3n}\right), \quad \Lambda = \Lambda_0\left(1 - \frac{2}{3n}\right). \tag{38}$$

There is an exponential dependence of $Q_{\alpha\beta}$ on the distance at $r < \Lambda_0 n$.

In case c), the coefficient of $1/n$ is large, so that the expansion in $1/n$ is valid only at sufficiently large values of n.

We note in conclusion that the correlation function $Q_{\alpha\beta}(r)$ can also be found in the spherical model for a Heisenberg magnetic material with a weak random field without an external magnetic field. The correlation function falls off exponentially over distance with a correlation radius of $8\pi V^2/h^2$.

Note added in proof (13 February 1985). The experimental and theoretical results recently found for Ising magnetic materials in a random field [see, for example, J. Villain, Phys. Rev. Lett. **52**, 1543 (1983) and the bibliography there] indicate that the termination of the orientational transition discussed in the present paper should also be observed in three-dimensional magnetic materials if the random fields are "turned on" while the system is still in the disordered phase.

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