Gap distortion and collective excitations in the He^3 B phase

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Dipole interaction, magnetic and electric fields, superfluid flows, and rotational effects (vortices and gyromagnetism) can distort the gap in the Fermi spectrum of the B phase of He^3. The path-integral technique is used to analyze how this distortion affects the collective modes of the order parameter. Several novel collective mode effects are found, such as splitting of the collective excitation spectrum by the perturbations intersection of different branches of the squashing and real squashing modes with different J_z at nonzero momenta **k**, a dependence of the NMR frequency on the angular velocity, and others.

1. INTRODUCTION

The order parameter in the He^3 B-phase is given by

$$A_{ij}=\Delta(T)R_{ij}(\hat{n},\ \theta)e^{i\varphi}, \tag{1}$$

where $\Delta(T)$ is the energy gap in the Fermi spectrum and the matrix elements R_{ij} describe a rotation of the spin coordinate system relative to the orbital system by an angle θ about the $\hat{n}$ axis.

If there is no dipole interaction or external perturbation, the gap $\Delta(T)$ is isotropic and the rotation axis $\hat{n}$ and angle θ are arbitrary. If a dipole interaction is present, or if electromagnetic fields, temperature gradients, rotations, or vessel walls perturb the system, the Fermi gap will be distorted and $\hat{n}$ and θ will no longer be arbitrary. The action of the external perturbations on the $\hat{n}$ axis produces textures which are now under active study in NMR and ultrasound experiments.[1]

In the A phase, the Fermi gap $\Delta=\Delta_{\max}\sin\psi$ vanishes along the common direction $\hat{l}$ of the orbital angular momenta of the Cooper pairs; however, dipole interactions or external perturbations can make Δ finite even along $\hat{l}$.

In spite of its small magnitude, the gap distortion has some important consequences. For example, the longitudinal NMR in the A and B phases and the frequency shift of the transverse NMR signal in the A phase are both due to gap distortion by dipole interactions[2]; the nonlinear Zeeman effect recently observed in Ref. 4 for the real squashing (rsq) mode[3] in the B phase is caused by gap distortion induced by a magnetic field; hydrodynamic shifts in the longitudinal NMR frequency in the A and B phases are due to gap distortion by superfluid flows.[5,6]

In this paper we use the path integral technique to study how the gap distortion induced by the dipole interaction, by the magnetic and electric fields, by the superfluid flow, and by rotation (virtoces and gyromagnetic effect) alters the collective modes in the He^3 B phase. Several novel effects are found, including a perturbation-induced splitting of the collective mode spectrum, crossing of the branches of the *rsq* and squashing-mode (*sq* mode) corresponding to different values of J_z for nonzero excitation momenta **k**, and a dependence of the nonlinear Zeeman splitting on the angular velocity of rotation.

In Secs. 2–4 we calculate the gap distortion induced by different types of perturbations. In Sec. 5 we use the distorted order parameter to calculate the collective excitations in the system; finally, in Sec. 6 we analyze the consequences of gap distortion.

2. DIPOLE INTERACTION, MAGNETIC AND ELECTRIC FIELDS

The dipole interaction

$$F_D=g_D\left(A_{ii}A_{jj}^*+A_{ij}A_{ji}^*-\frac{2}{3}A_{ij}A_{ij}^*\right) \tag{2}$$

makes the energy gap anisotropic, so that the gap widths Δ_2 and Δ_1 parallel and normal to the direction $\hat{n}$ (which remains arbitrary) become unequal[2]:

$$\Delta_1^2-\Delta_2^2={}^5/_2\Omega_B^2.$$

The angle has then the definite value

$$\theta=\arccos[-{}^1/_4(\Delta_2/\Delta_1)]\approx\arccos(-{}^1/_4).$$

Here g_D is the dipole constant and Ω_B is the longitudinal NMR frequency.

In a moderately strong magnetic field of energy F_z satisfying

$$F_{cond}\gg F_z=g_zH_i(AA^+)_{ij}H_j\gg F_D,$$

where F_D and F_{cond} are the dipole and condensate energies, the gap also becomes anisotropic[5]:

$$\begin{aligned}\Delta_1^2&=\Delta^2+{}^1/_2(3\beta_{12}+\beta_{345})^{-1}\beta_{345}^{-1}\beta_{12}g_zH^2,\\ \Delta_2^2&=\Delta^2-{}^1/_2(3\beta_{12}+\beta_{345})^{-1}\beta_{345}^{-1}(2\beta_{12}+\beta_{345})g_zH^2,\end{aligned} \tag{3}$$

where

$$\Delta^2=(6\beta_{12}+2\beta_{345})^{-1}\alpha$$

is the gap for $H=0$ and $\beta_{ijk}=\beta_i+\beta_j+\beta_k$ (the β_i are the coefficients in the expansion of F_{cond}). In addition, the corrections (3) to the gap width also specify $\hat{n}$, which must be parallel to the field H.

Similar effects are produced by an electric field $\mathbf{E}$ of energy

$$F_E = -g_E (E_i A_{ki} A_{kj}^* E_j - {}^1/_3 |E|^2 A_{ki} A_{kj}^*) \quad (4)$$

(we take $\mathbf{E}$ to be parallel to the z axis).[7] However, unlike the case of dipole interaction or magnetic fields, the electric field makes the gap larger along the direction of the field. For strong fields ($F_E \gg F_D$) we have

$$\Delta_1^2 = \Delta^2 - (6\beta_{345})^{-1} g_E E^2, \quad \Delta_2^2 = \Delta^2 + (3\beta_{345})^{-1} g_E E^2. \quad (5)$$

The field E requires that $\hat{n}$ be perpendicular to $\mathbf{E}$, and the dipole interaction specifies the angle

$$\theta = \arccos\,[-\Delta_1/2(\Delta_1+\Delta_2)] \approx \arccos(-{}^1/_4).$$

3. SUPERFLUID FLOWS

Vollhardt and Maki[8] observed that even small temperature gradients can induce appreciable flows in He^3, and this is responsible for the practical interest in the study of homogeneous superfluid flows. In addition to causing a hydrodynamic shift of the longitudinal NMR frequency in the A and B phases,[5,6] these flows couple the zero-sound mode to at least one of the four branches of the *rsq*-mode with $J_z = \pm 1, \pm 2$ (Ref. 9). This results in a threefold splitting of the spectrum for absorption of zero sound by the *rsq* mode when the excitation momentum $\mathbf{k}$ is nonzero (this splitting has been observed experimentally[10,11]).

A superfluid flow of energy[12]

$$F_{flow} = ({}^1/_5 \rho_s / |\Delta|^2)\,(v_{si} A_{ij}^* v_{sl} A_{lj}) \quad (6)$$

depends only on the orbital coordinates, and (6) is independent of the direction $\hat{n}$. However, the flow slightly distorts the order parameter because of the pair-breaking effect; the magnitude of the distortion is of the order of $[1/2 \cdot \rho_s v_s^2]/F_{\rm cond}$. Because of this distortion, a flow of energy[12,13]

$$F_{flow} = -a(\mathbf{n}\mathbf{v}_s)^2 \quad (7)$$

will cause $\hat{n}$ to be parallel to $\mathbf{v}_s$. (This situation is similar to the orienting effects of magnetic fields.[12–14]) In addition to orienting $\hat{n}$, the flow also causes the gap widths Δ_2 and Δ_1 parallel and normal to $\mathbf{v}_s$ to be unequal. In what follows we will examine how the order parameter is altered by gap distortion ($\Delta_1 \neq \Delta_2$) with $\hat{n} \| \mathbf{v}_s$.

Kleinert has shown[15] that for $T \approx T_c$, gap distortion by a superfluid flow splits each of the sq- and the rsq-modes into three groups of branches. We will analyze how a homogeneous flow alters the collective modes at temperatures $0 \leqslant T \leqslant T_c/2$.

4. ROTATIONAL EFFECTS

Nuclear magnetic resonance experiments have recently revealed vortices[16] and the gyromagnetic effect[17] in rotating superfluid He^3. The rotation generates a vortex lattice of density

$$n_\Omega = 4m_3\Omega/h,$$

which alters the order parameter. In addition to orienting the $\hat{n}$ vector,[13,16] the vortices also distort the gap in a manner similar to that described above, and the same is true of the gyromagnetic effect. We will consider these two effects together when an external magnetic field is present.

The gap is distorted both by the local flow around the vortices[13] and by the vortex cores themselves[18] (in the latter case because the susceptibility is anisotropic inside the cores, where the order parameter differs from its value outside the cores). If the distance between the vortices is less than the characteristic magnetic length ξ_H (as was the case in the Helsinki experiments, where the angular velocity of rotation was $\Omega \sim 1$ rad/s), we can find the contribution of the local flow to the free energy by averaging the local $1/r$ $\mathbf{v}_s$-field near the vortices[13,16]:

$$F_{vx}^{(1)} = {}^2/_5 a\lambda^{(1)} (\hat{\Omega}_i R_{ik} H_k)^2, \quad (8)$$

where

$$a \approx (\xi/\xi_D)^2 (\chi_\perp - \chi_\|), \quad \lambda^{(1)} = \pi(\xi_D)^2 A_{cell}^{-1} \ln(\xi^{-1} A_{cell}^{1/2}).$$

Since the area of an elementary cell is

$$A_{cell} = h/4m_3\Omega,$$

$\lambda^{(1)}$ is proportional to Ω. Here ξ is the coherence length, ξ_D is the dipole length, and $\chi_\perp$ and $\chi_\|$ are the susceptibilities normal and parallel to $\hat{n}$.

If the vortices are axisymmetric, the cores contribute an amount[16]

$$F_{vx}^{(2)} = {}^2/_5 a\lambda^{(2)} (\hat{\Omega}_i R_{ik} H_k)^2 \quad (9)$$

to the free energy. Here

$$a\lambda^{(2)} \approx \Delta\chi (A_{core}/A_{cell})$$

(like $\lambda^{(1)}$) is proportional to the angular velocity Ω.

The gyromagnetic energy is given by[19]

$$F_{gm} = {}^4/_5 a\tilde{\lambda} (\hat{\Omega}_i R_{ik} H_k), \quad (10)$$

where $\tilde{\lambda}$ is proportional to the spontaneous magnetization and to the number of vortices (or to Ω).

All three of the contributions (8)–(10) have been measured as functions of the temperature and pressure and were found to be proportional to Ω (Refs. 17 and 20).

Once the gyromagnetic and vortex contributions to the free energy are known, we can calculate also the ensuing gap distortion. If we also include the contribution from the magnetic field H, we find that the resulting total free energy

$$F_z+F_{vx}^{(1)}+F_{vx}^{(2)}+F_{gm}$$

causes the longitudinal and transverse gap widths Δ_2 and Δ_1 to differ:

$$\Delta_1^2=\Delta^2+\Omega_0^2,\quad \Delta_2^2=\Delta^2+\alpha_0\Omega_0^2,$$

$$\alpha_0=-4,$$

$$\Omega_0^2=(10\beta_0)^{-1}\left\{H^2\left[g_z+\frac{2}{5}\frac{a}{\Delta^2}(\lambda^{(1)}+\lambda^{(2)})\pm\frac{4}{5}\frac{a}{\Delta^2}\tilde{\lambda}H\right\}.\tag{11}$$

5. CALCULATION OF THE COLLECTIVE MODE SPECTRUM

The order parameter has the same form

$$A_{ij}=[\Lambda^{1/2}R(\hat{n},\theta)]_{ij}e^{i\varphi}\tag{12}$$

for all of the cases considered above. Here Λ is a diagonal matrix with elements λ_1, λ_2, λ_3 and

$\lambda_1=\Delta_1^2=\Delta^2+\Omega_0^2$,

For dipole interactions,

$\Omega_0^2={}^5/_6\Omega_B^2,\quad \alpha_0=-2,\quad \hat{n}\|\hat{z};$

For magnetic fields H,

$\Omega_0^2=(g_z/10\beta_0)H^2,\ \alpha_0=-4,\ \hat{n}\|\hat{z};$

for electric fields E,

$\Omega_0^2=-(g_E/6\beta_0)E^2,\quad \alpha_0=-2,\quad \hat{n}\perp\hat{z};$

for superfluid flows, Ω_0^2 and α_0 depend on T and v_s^2, while $\hat{n}$ and $\hat{v}_s$ are both parallel to the z axis;

for rotations,

$$\Omega_0^2=(10\beta_0)^{-1}\left\{H^2\left[g_z+\frac{2}{5}\frac{a}{\Delta^2}(\lambda^{(1)}+\lambda^{(2)})\right]\pm\frac{4}{5}\frac{a}{\Delta^2}\tilde{\lambda}H\right\},$$
$$\alpha_0=-4,\quad \hat{n}\|\hat{H}\|\hat{z}.\tag{13}$$

We set θ arccos $(-1/4)$ in all of the above cases, except for superfluid flows, where as in Ref. 15 we take $\theta=0$ for simplicity.

We will use the path-integral technique developed by the author and V. N. Popov[21] to calculate the collective mode spectrum for the B phase with a distorted order parameter. In this method the initial fermions are described by Fermi fields, while the Bose fields $c_{ia}(\mathbf{x},\tau)$ are used to describe the Cooper pairs. In terms of these fields we have the hydrodynamic-action functional

$$S_h=g^{-1}\sum_{p,i,a}c_{ia}^+(p)c_{ia}(p)+\frac{1}{2}\ln\det\frac{\hat{M}(c,c^+)}{\hat{M}(0,0)}.\tag{14}$$

Here the $c_{ia}(p)$ are the Fourier components of the $c_{ia}(\mathbf{x},\tau)$, g is a negative constant proportional to the pair scattering amplitude for the quasifermions, and the operator $\hat{M}$ depends on the Bose fields and on the parameters of the quasifermions.[21] The functional S_h determines all of the physical properties of the system and, in particular, the collective-excitation spectrum.

For $T_c-T\sim T_c$ we expand the logarithm of the determinant in S_h in powers of the deviation of $c_{ia}(p)$ from its value $c_{ia}^{(0)}$ for the condensate, which for dipole interactions, magnetic fields, superfluid flows, and rotations is given by

$$c_{ia}^{(0)}(p)\sim\begin{pmatrix}\Delta_1\cos\theta & -\Delta_1\sin\theta & 0\\ \Delta_1\sin\theta & \Delta_1\cos\theta & 0\\ 0 & 0 & \Delta_2\end{pmatrix},\tag{15}$$

while for electric fields

$$c_{ia}^{(0)}(p)\sim\begin{pmatrix}\Delta_1 & 0 & 0\\ 0 & \Delta_1\cos\theta & -\Delta_1\sin\theta\\ 0 & \Delta_2\sin\theta & \Delta_2\cos\theta\end{pmatrix}.\tag{16}$$

To first order the Bose spectrum is given by the quadratic part of S_h,

$$\sum_p A_{ijab}(p)c_{ia}^+(p)c_{jb}(p)$$
$$+\frac{1}{2}B_{ijab}(p)\left(c_{ia}(p)c_{jb}(-p)+c_{ia}^+(p)c_{jb}^+(-p)\right),\tag{17}$$

which is obtained by replacing $c_{ia}(p)$ by $c_{ia}(p)+c_{ia}^{(0)}(p)$. The spectrum is given by the equation $\det Q=0$, where Q is the matrix corresponding to the quadratic form in (17).

The tensor coefficients A_{ijab} and B_{ijab} are proportional to integrals (sums) of the products of the Green functions for the quasifermions and are given by

$$A_{ijab}=\delta_{ab}\left\{\frac{\delta_{ij}}{g}+\frac{4Z^2}{\beta V}\times\sum_{p_1+p_2=p}n_{1i}n_{1j}\frac{(i\omega_1+\xi_1)(i\omega_2+\xi_2)}{[\omega_1^2+\xi_1^2+\Delta^2(\theta')][\omega_2^2+\xi_2^2+\Delta^2(\theta')]}\right\},$$

$$B_{ijab}=-\frac{4Z^2}{\beta V}\times\sum_{p_1+p_2=p}n_{1i}n_{1j}\frac{(2f_af_b-f_i^2\delta_{ab})}{[\omega_1^2+\xi_1^2+\Delta^2(\theta')][\omega_2^2+\xi_2^2+\Delta^2(\theta')]}.\tag{18}$$

Here

$$\Delta^2(\theta')=\Delta_1^2(n_1^2+n_2^2)+\Delta_2^2n_3^2$$
$$=\Delta^2+\Omega_0^2[\alpha_0+(n_1^2+n_2^2)(1-\alpha_0)],$$

$$n_1^2+n_2^2=\sin^2\theta',$$

$$\mathbf{f}=\{n_1\Delta_1;\ n_2\Delta_1\cos\theta'+n_3\Delta_2\sin\theta';\ -n_2\Delta_1\sin\theta'+n_3\Delta_2\cos\theta'\}$$

for an electric field $\mathbf{E}$, and

in the remaining cases; $\beta = T^{-1}$, V is the volume of the system, $\xi_i = c_F(k_i - k_F)$, $n_i = k_i/k_F$, Z is a normalization constant, and $\omega = (2n+1)\pi T$ are the Fermi frequencies.

6. RESULTS

In what follows we denote the Goldstone modes by *gd*, the 2Δ modes by *pb*, the squashing modes by *sq*, and the real squashing modes by *rsq*.

a) Dipole interaction

In this case the squared energies ε^2 for the various modes are given by

$$gd:\quad \frac{\Omega_B^2}{3}(1;\pm1;r),\qquad pb:\quad 4\Delta^2-\frac{\Omega_B^2}{3}\ (1;\pm1;i),$$

$$\frac{2}{3}\Omega_B^2\ (1;0;r)\qquad 4\Delta^2-\frac{2}{3}\Omega_B^2(1;0;i),$$

$$0(0;0;i)\qquad\qquad 4\Delta^2\qquad (0;0;r);$$

$$rsq:\quad \frac{8\Delta^2}{5}-\frac{\Omega_B^2}{3}\ (2;\pm1;r);\qquad \frac{8\Delta^2}{5}+\frac{2\Omega_B^2}{3}\ (2;\pm2,0;r);\quad (19)$$

$$sq:\quad \frac{12\Delta^2}{5}+\frac{\Omega_B^2}{3}\ (2;\pm1;i);\qquad \frac{12\Delta^2}{5}-\frac{2\Omega_B^2}{3}\ (2;\pm2,0;i).$$

The first two numbers in parentheses give the values of J and J_z, where J is the total moment and J_z is its projection on the z axis; r and i correspond to the real and imaginary parts of the order parameter.

A gap of order Ω_B is present in the spin wave spectrum, which suggests that the collective-mode formalism can be used for a microscopic description of the longitudinal NMR.[1)] The longitudinal NMR can excite one of these modes, the longitudinal spin waves of type (l; 0; r). For $T/T_c \approx 0.27$, $\Omega_B{}^2$ varies from $9.69\cdot10^{10}$ Hz^2 at $P = 32$ bar to $6.51\cdot10^{10}$ Hz^2 at $P = 19$ bar (Ref. 23). There is no gap in the spectrum of the (0; 0; i) Bogolyubov-Anderson acoustic mode, and the dipole interaction does not alter the mode velocity.[22]

The *pb* modes with $\varepsilon = 2\Delta$ split into three groups. In all cases the mode energies lie between

$$2\Delta_{max}=2(\Delta^2+{}^5/_6\Omega_B{}^2)^{1/2},\ \text{and}\ \ 2\Delta_{min}=2(\Delta^2-{}^5/_3\Omega_B{}^2)^{1/2}.$$

As in the case of the A phase,[24] where the gap $\Delta = \Delta_{\max}\sin\psi$ is also anisotropic and perturbations of energy $< 2\Delta_{\max}$ are only moderately damped and can be treated as resonances, these bounds imply that all the *pb* modes can be regarded as resonances. The (0; 0; r) mode is associated with density (sound waves)[25] and an additional peak is experimentally observed in the ultrasound absorption spectrum at $\varepsilon = 2\Delta$. All the *pb*-modes with $J = 1$ are coupled with the spin density (electromagnetic waves) owing to the violation of particle–hole symmetry.[26] This implies that two peaks should be observable for $\varepsilon \approx 2\Delta$ in NMR experiments; these may coalesce into a single peak, because the relative distance $\Omega_B^2/6\omega_0^2 \sim 3\cdot10^{-6}$ between the peaks is less than the resolution 10^{-5} of the NMR measurements (see Refs. 16 and 17).

We recall that if the dipole interaction is neglected, all the *pb* modes have the same energy and the dispersion coefficients γ are complex.[21] Physically, this is a consequence of the fact that the Bose excitations can decay back into their constituent fermions.

The dipole interaction splits both the *sq* and the *rsq* modes. There are two groups of modes for each type: two modes with $J_z = \pm1$ and three modes with $J_z = 0$ and ±2. The relative difference between the mode energies in both groups is $\sim\Omega_B^2/2\omega_0^2 \approx 1.3\cdot10^{-5}$, which is less than the current sensitivity (10^{-4}) in ultrasound experiments.[11] One may hope that as ultrasound techniques become more accurate, it will become possible to observe the dipole splitting of the *sq* and *rsq* modes.

The splitting of the *rsq* and *sq* modes also gives rise to intersections of their branches for nonvanishing momenta **k**. The dispersion law for these modes is of the form $\varepsilon_2 = \Omega_1^2 + \gamma k^2$ for $\mathbf{k}\neq0$. Since $\gamma(J_{z1}) > \gamma(J_{z2})$ for $J_{z1} < J_{z2}$ (see Ref. 21), the $J_z = 0$ branch of the *sq* mode crosses the $J_z = \pm1$ branches and the $J_z = \pm1$ branches of the *rsq* mode cross the $J_z = \pm2$ branches for nonvanishing **k**.

b) Magnetic fields

In this case the gap distortion introduces corrections $\propto H^2$ to the collective-mode spectrum. For $J = 2$, ε^2 is given by:

$$rsq:\quad \frac{8\Delta^2}{5}-\frac{2}{5}\tilde{H}^2\ (2;\pm1;r);\qquad \frac{8\Delta^2}{5}+\frac{8}{5}\tilde{H}^2\ (2;\pm2;r);$$

$$\frac{8\Delta^2}{5}+\frac{16}{15}\tilde{H}^2\ (2;0;r).$$

$$sq:\quad \frac{12\Delta^2}{5}+\frac{2}{5}\tilde{H}^2(2;\pm1;i);\qquad \frac{12\Delta^2}{5}-\frac{8}{5}\tilde{H}^2\ (2;\pm2;i);$$

$$\frac{12\Delta^2}{5}-\frac{16}{15}\tilde{H}^2\ (2;0;i).\qquad (20)$$

Here $\tilde{H}^2 = (g_z/10\beta_0)H^2$, and the coefficient $g_z/10\beta_0$ is a few GHz/T^2 (Refs. 3 and 4). We see that the field splits the *rsq* and *sq* modes into three groups each. The field-induced splitting produces two types of branch crossings. If we also include the linear splitting of the *rsq* and *sq* modes[27] in the field, we find that the $J_z = -1$, 0 and $J_z = 2$, 1 branches of the *rsq* modes and the $J_z = 0$, 1 and $J_z = -1$, -2 branches of the *sq* modes will cross at sufficiently strong H. We thus have a nonlinear Zeeman effect for the *rsq* and *sq* modes. Crossing of the $J_z = 2$ and $J_z = 1$ branches of the *rsq* mode has been observed experimentally in fields of ~ 0.15 T (Ref. 4).

The other type of branch intersection occurs for nonvanishing momenta **k**. In this case the $J_z = \pm2$ branches intersect the $J_z = 0$, ±1 branches of the *rsq* mode, while for the *sq* mode the branch $J_z = 0$ crosses the branches with $J_z = \pm1$.

The field H also splits the *pb* modes and is responsible for their resonant behavior, because the mode energies lie between

$$2\Delta_{max}=2(\Delta^2+\tilde{H}^2)^{1/2} \text{ and } 2\Delta_{min}=2(\Delta^2-4\tilde{H}^2)^{1/2}.$$

For the *pb* modes, ε^2 is given by

$$4\Delta^2+\frac{4}{3}\tilde{H}^2(0;0;r); \quad 4\Delta^2(1;0;i); \quad 4\Delta^2-2\tilde{H}^2 \ (1;\pm1;i).$$

The field H produces a gap ($\varepsilon^2 = 2\tilde{H}^2$) in the transverse spin wave spectrum ($\tilde{H}\sim 15$ MHz for $H\sim 0.15$ T) but leaves the longitudinal spin wave spectrum unchanged.

c) Electric fields

Electric fields E produce similar changes in the spectrum—resonant *pb*-modes can exist (we have $\varepsilon^2 = 4\Delta^2 - 2/5\cdot\tilde{E}^2$ for one of the *pb* modes, where $\tilde{E} = (g_E/6\beta_0)E^2$), the *gd* mode spectrum has a gap ($\varepsilon^2 = (2/5)\tilde{E}^2$ for one of the spin waves), and various corrections of order[2)] $\tilde{E}^2$ split the *rsq* and *sq* modes.

We will now estimate the field E required for the splitting of the *sq* and *rsq* modes to be observable. The relation $g_C \approx g_E E^2$ for the field energy to be comparable to the dipole energy implies a field strength $E\sim 15$ kV/cm (Ref. 7). Fields E_c of order 50 kV/cm are thus required in order to observe the field-induced effects in acoustic experiments. It was shown in Ref. 29 that if corrections for Fermi liquid effects are included, the required fields are an order of magnitude higher than E_c for experiments at zero pressure and two orders of magnitude higher on the melting curve, i.e., fields between $5\cdot 10^5$ and $5\cdot 10^6$ V/cm are needed, depending on the pressure. However, such strong fields shift the frequency of the NMR signal by an amount comparable to the longitudinal NMR frequency Ω_B and therefore make it impossible to observe the frequency shift in the NMR signal.

d) Superfluid flows

In this case ε^2 is given by

$$gd: \begin{cases} -\frac{2}{5}(2\alpha_0+3)\Omega_0^2 & (1;\pm1;r) \\ -\frac{2}{5}(\alpha_0+4)\Omega_0^2 & (1;0;r), \\ -\frac{2}{3}(\alpha_0-2)\Omega_0^2 & (0;0;i) \end{cases}$$

$$pb: \begin{cases} 4\Delta^2+\frac{2}{5}(2\alpha_0+3)\Omega_0^2 & (1;\pm1;i), \\ 4\Delta^2+\frac{2}{5}(\alpha_0+4)\Omega_0^2 & (1;0;i), \\ 4\Delta^2+\frac{2}{3}(\alpha_0-2)\Omega_0^2 & (0;0;r); \end{cases}$$

$$sq: \begin{cases} \frac{12\Delta^2}{5}+\frac{2}{5}\Omega_0^2 & (2;\pm1;i), \\ \frac{12\Delta^2}{5}-\frac{2}{15}(\alpha_0-4)\Omega_0^2 & (2;0;i). \\ \frac{12\Delta^2}{5}+\frac{2}{5}\alpha_0\Omega_0 & (2;\pm2;i) \end{cases}$$

$$rsq: \begin{cases} \frac{8\Delta^2}{5}-\frac{2}{5}\Omega_0^2 & (2;\pm1;r), \\ \frac{8\Delta^2}{5}+\frac{2}{15}(\alpha_0-4)\Omega_0^2 & (2;0;r), \\ \frac{8\Delta^2}{5}-\frac{2}{5}\alpha_0\Omega_0^2 & (2;\pm2;r). \end{cases} \quad (22)$$

Kleinert[30] has calculated Δ_1^2 and Δ_2^2 (equivalently, α_0 and Ω_0^2) as functions of $(v_s/v_0)^2$, where $v_0\equiv 1/m^*\xi = 12$ cm/s for $P=0$ and m^* is the effective mass. Using his data, we find that

$$\begin{aligned} &\text{for } (v_s/v_0)^2(1-T/T_c)^{-1}=0.04 \quad (\Delta=\Delta_{\text{BCS}}=1.76T_c); \\ &\quad\text{for } T\leqslant 0.1T_c \quad \alpha_0\approx 0, \quad \Omega_0^2\approx 0; \\ &\quad\text{for } T=0.3T_c \quad \alpha_0=-4.2, \quad \Omega_0^2=0.04\Delta_0^2; \\ &\quad\text{for } T=0.5T_c \quad \alpha_0=-8.4, \quad \Omega_0^2=0.04\Delta_0^2; \\ &\text{and for } (v_s/v_0)^2(1-T/T_c)^{-1}=0.02 \\ &\quad\text{for } T\leqslant 0.1T_c \quad \alpha_0\approx 0, \quad \Omega_0^2\approx 0; \\ &\quad\text{for } T=0.3T_c \quad \alpha_0=-3.3, \quad \Omega_0^2=0.02\Delta_0^2; \\ &\quad\text{for } T=0.5T_c \quad \alpha_0=-6.3, \quad \Omega_0^2=0.02\Delta_0^2. \end{aligned} \quad (23)$$

We conclude from (22) and (23) that the collective excitation spectrum remains almost unchanged for $T\leqslant 0.1\,T_c$ but that several important changes occur for higher temperatures $0.1 < T/T_c < 0.5$:

1) Gaps appear in the spectrum of all of the *gd* modes. One of these modes, the longitudinal spin waves of type (1; 0; *r*), can be excited in longitudinal NMR. The gaps in the spectrum depend on the flow and indicate that the frequency of the longitudinal NMR signal is hydrodynamically shifted.

2) The four *pb* modes with $\varepsilon = 2\Delta$ are split into three groups (the modes with $J_z = \pm 1$ remain degenerate). Because the mode energies lie between $2\Delta_{\max} = 2\Delta_1$ and $2\Delta_{\min} = 2\Delta_2$, they can be excited as resonances in ultrasound and NMR experiments [the (0; 0; *r*) mode and the modes with $J=1$, respectively].

3) The modes with $J=2$ (i.e., the *rsq* and *sq* modes) are each split into three groups in accordance with Kleinert's results[15] for $T\approx T_c$. The relative position

$$\varepsilon(J_z=\pm2)>\varepsilon(J_z=\pm1)>\varepsilon(J_z=0)$$

of the *rsq* mode branches is the same as in Ref. 15; however, the energies of the *sq* mode branches are opposite to the results there:

$$\varepsilon(J_z=\pm2)<\varepsilon(J_z=\pm1)<\varepsilon(J_z=0).$$

We note that the squares of the *gd* mode frequencies (for vanishing momentum k) are $\sim 10^{-1}\Delta_0^2$. This value is comparable to the splitting of the *pb* modes (measured in terms of the squared frequencies) and is roughly ten times greater than the *sq* and *rsq* mode splittings.

The *rsq*- and *sq*-mode splitting induced by the superfluid flow can be observed in zero-sound experiments, which reveal three peaks in the ultrasound absorption spectrum for each mode (this is similar to the *rsq* mode splitting for non-vanishing **k**, see Ref. 11). For the *rsq* mode this splitting also causes the different mode branches to intersect for $\mathbf{k}\neq 0$. Thus, if we allow for $\mathbf{k}\neq 0$ (Ref. 21) we find that the *rsq*-mode branches with $J_z = \pm 2$ cross the branches with $J_z = \pm 1$ and that the latter cross the $J_z = 0$ branch, because $\gamma(|J_{z1}|) > \gamma(J_{z2}|)$ for $|J_{z1}| < |J_{z2}|$. The *sq*-mode branches do not cross. We note that these flow-induced effects increase with temperature

e) Rotational effects (vortices and gyromagnetism)

We can express the results for the *pb*, *rsq*, and *sq* modes in the form

$$\varepsilon^2 = A\Delta^2 + Bg(\Omega)H^2 + Ca'\tilde{\lambda}H, \tag{24}$$

where

$$g(\Omega) = (25\beta_0)^{-1}\left[g_z + \frac{2}{5}\frac{a^2}{\Delta^2}(\lambda^{(1)}+\lambda^{(2)})\right], \quad a' = \frac{2a}{25\Delta^2\beta_0},$$

and $A = 4$, 12/5, and 8/5 for the *pb*, *sq*, and *rsq* modes, respectively. The magnitude and sign of the coefficients B and C differ for the different modes; $|B|, |C| \leqslant 5$.

We can use the data in Refs. 17 and 20 to estimate the magnitude of the gap distortion by vortices and gyromagnetic effects at the frequencies of the non-phonon modes excited in ultrasound experiments.

References 17 and 20 imply that $\lambda/\Omega \sim (1-3)$ s/rad (depending on the pressure) for $T \approx 0.5T_c$; λ/Ω increases rapidly with T and P, and $\tilde{\lambda}/H\Omega \sim 0.04$–$0.07$ s/rad. If we also use the estimate

$$g_z = 7\zeta(3)\chi/24\pi^2\Delta^2 \approx 35\cdot 10^{-3}\chi/\Delta^2,$$

we find that the vortex and gyromagnetic contributions are equal to 10^{-4} and $2\cdot 10^{-6}$ times the contribution from the term g_zH^2, respectively. For the fields ~ 1600 G employed in the nonlinear Zeeman effect experiments,[4] the term g_zH^2 shifts the *rsq*-mode frequency by $\sim 2\cdot 10^{-2}$ for the $J_z = 0$ branch, so that the vortex- and gyromagnetic-induced shifts are $\sim 2\cdot 10^{-6}$ and $2\cdot 10^{-8}$, respectively. Because the resolution in ultrasound experiments is currently limited to 10^{-4} of the main signal frequency,[11] these shifts (associated with gap distortion) are completely negligible. However, they may become appreciable for low pressures and $T \approx 0$.

Rotation does not alter the spectrum of the longitudinal spin waves. For the transverse spin wave spectrum we have

$$\varepsilon^2 = 5g(\Omega)H^2 \pm a'\tilde{\lambda}H \tag{25}$$

for $\mathbf{k} = 0$.

Estimates show that for $H \sim 300$ G, the relative gyromagnetic, vortex, and g_zH^2 contributions are of order $2\cdot 10^{-7}$, 10^{-5}, and 10^{-1} at the Larmor precession frequency $\sim$MHz. Since the resolution of NMR experiments is currently limited to one part in 10^5 (Ref. 17), the gyromagnetic gap distortion is clearly completely undetectable, although in principle the gap distortion caused by vortices may be observable in NMR experiments. However, the vortex distortion may be masked by gyromagnetic and vortex orientation effects, which alter the vector **n** and give contributions (relative to the principal peak) of $\sim 5\cdot 10^{-5}$ and 10^{-4}, respectively.[17]

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Note Added in Proof (March 6, 1985). Resonant absorption of zero sound by the *pb* mode in a magnetic field has been observed experimentally by M. E. Daniels *et al.* [Phys. Rev. *B* 27, 6988, (1983)]. Their results were at first interpreted as indicating the excitation of a mode with $J = 4$ and $E \approx 1.82$. However, comparison of the coupling between the zero sound and the $J = 4$ modes with the absorption actually observed indicates that the absorption by the *pb* mode was resonant in nature [Proc. LT-17 (1984), p. 763]. Schopohl and Tewordt have shown (*ibid.*, p. 777) that the magnetic-field induced distortion of the gap in the Fermi spectrum, can couple the zero sound with the (1, −1, *i*) *pb*-mode (such a coupling is otherwise not present, even if particle-hole symmetry is violated). We have shown in the present paper that electric fields, dipole interaction, and superfluid flows can also give rise to *pb*-mode resonances and lead to resonant absorption of zero sound near the absorption edge (the resonant absorption mechanisms are in addition to the absorption associated with the decay of Cooper pairs).

1) We note that the spin wave velocities will merely be rescaled if the dipole interaction is treated solely in terms of its contribution to the free energy (i.e., if the gap distortion is neglected)—in this case no gaps are present in the Goldstone mode spectrum and there is no change in the frequencies of the nophonon modes.[22]

2) As in the case of the dipole interaction, analyses of the electric field effects that ignore the gap deformation lead only to a renormalization of the speed of sound—there are no field-dependent gaps in the Goldstone and nophonon mode spectra.[28]

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