Threshold interaction of extraordinary light waves with nematics

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A theoretical study is made of the threshold interaction of extraordinary light waves with nematic liquid crystals. Various geometries in which this interaction may be manifested are considered. The threshold intensities are calculated for two types of instability and above-threshold distributions of the director are found. A hysteresis is predicted for the dependence of the degree of reorientation of the director on the intensity of light. Optical manifestations of these effects are considered.

INTRODUCTION

Recently there have been many theoretical and experimental investigations of light-induced deformations of the director in liquid crystals. Both the threshold-free effects (see, for example, Refs. 1–7) and the effects with a threshold (see, for example, Refs. 3, 6, and 8) have been investigated thoroughly. It is always understood that the threshold effect representing a light-induced Fréedericksz transition (LIFT) occurs either during propagation of an extraordinary light wave along the director of a nematic liquid crystal or in the field of an ordinary wave. An extraordinary wave traveling at an angle to the director is regarded as responsible for the threshold-free effect.

We shall consider theoretically the possibility of threshold reorientation effects in the field of extraordinary light waves traveling at an angle to the director, i.e., in the geometry in which the orientational optical nonlinearity of the mesophase of a nematic liquid crystal has been discovered.[1] There are two types of such effect. In one of them a light wave causing a threshold-free reorientation of the director in the plane of incidence induces a tilt of the director out of this plane when the intensity of light exceeds a certain threshold value. In the other effect the director deflected in a threshold manner in the plane of incidence under the action of two extraordinary waves which balance out the threshold-free effect. We shall use the designation LIFT-II for the threshold reorientation of the director in the field of extraordinary waves traveling at an angle to the director.

The occurrence of two types of instability in the LIFT-II case and the variety of the geometries in which the effect can be manifested enrich greatly the range of threshold phenomena and of the associated effects.

In § 1 we shall consider the possibility of compensation of the threshold-free director reorientation effects. In §§ 2 and 3 we shall find the threshold intensities for the planar and transverse LIFT-II cases, respectively. We shall show that there are ranges of angles of incidence of the waves on a cell for which one or the other type of LIFT-II is possible. In § 4 we shall determine the above-threshold steady-state structures of the director field.

We shall show that the degree of reorientation of the director may be controlled not only by the intensity, but also by a change in the angle of incidence of the waves.

In § 5 we shall discuss optical manifestations of these effects. We shall show that in the planar LIFT-II case we can expect either self-focusing or self-defocusing from one experiment to another. The transverse LIFT-II may be manifested not only by self-focusing, but also by a change in the state of polarization of a beam or by a nonlinear optical activity.

§1. COMPENSATION OF THRESHOLD-FREE REORIENTATION

We shall consider a cell containing a planar nematic liquid crystal filling the space $0\leqslant z\leqslant L$. The x axis of a Cartesian coordinate system is selected along the director $\mathbf{n}^{(0)}=\mathbf{e}_x$. We shall assume that a light wave is incident on the cell at an angle α and that the polarization and wave vector of the wave lie in the (x, z) plane. In this geometry a director rotation that lowers the energy of the interaction between a nematic liquid crystal and the light field has no intensity threshold and it occurs in such a way as to reduce the angle between the director and the electric field of the wave. Since the degree and direction of reorientation of the director of a nematic liquid crystal are governed by the direction of propagation of light, we can find a second wave with a suitable intensity and direction of propagation which has the same, but reversed effect on the director as the first wave. The simultaneous action of such waves should result in zero net reorientation.

We can find the compensation condition by writing down variational equations for the angle of rotation of the director under the action of light fields. The familiar procedure for finding these equations[8] gives

$$\begin{aligned}&(K_1\sin^2\theta+K_3\cos^2\theta)\frac{d^2\Omega}{dz^2}-(K_3-K_1)\sin\theta\cos\theta\left(\frac{d\theta}{dz}\right)^2\\&-\sin\theta\cos\theta[K_3-2(K_3-K_2)\sin^2\theta]\left(\frac{d\varphi}{dz}\right)^2\\&+\frac{\varepsilon_a}{16\pi}\{\sin 2\theta[\cos^2\varphi|E_x|^2\\&\qquad+\sin^2\varphi|E_y|^2-|E_z|^2+\sin\varphi\cos\varphi(E_xE_y^*+E_x^*E_y)]\\&+\cos 2\theta[\cos\varphi(E_xE_z^*+E_x^*E_z)+\sin\varphi(E_yE_z^*+E_y^*E_z)]\}=\gamma\frac{\partial\theta}{\partial t},\end{aligned}\tag{1a}$$

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$$\sin^2\theta\,(K_2\sin^2\theta+K_3\cos^2\theta)\,\frac{d^2\varphi}{dz^2}$$

$$+\sin 2\theta[K_3-2(K_3-K_2)\sin^2\theta]\,\frac{d\theta}{dz}\frac{d\varphi}{dz}$$

$$+\frac{\varepsilon_a}{16\pi}\,\{\sin^2\theta[\sin 2\varphi(|E_y|^2-|E_x|^2+\cos 2\varphi(E_xE_y^*+E_x^*E_y)]$$

$$+\sin\theta\cos\theta[\cos\varphi(E_yE_z^*+E_y^*E_z)-\sin\varphi(E_xE_z^*+E_x^*E_z)]\}$$

$$=\gamma\frac{\partial\varphi}{\partial t}, \tag{1b}$$

where the angles θ and φ define the orientation of the director as

$$\mathbf{n}=\{n_x,\ n_y,\ n_z\}=(\mathbf{e}_x\cos\varphi+\mathbf{e}_y\sin\varphi)\sin\theta+\mathbf{e}_z\cos\theta;$$

K_i are the Frank constants (in dynes); $\varepsilon_a=\varepsilon_\parallel-\varepsilon_\perp$ is the anisotropy of the permittivity of a nematic liquid crystal at the frequency ω of the incident light; E_i are the components of the complex amplitude of the electric fields of the light wave defined in such a way that the real intensity vector is $e_{i.r}=0.5(E_i+E_i^*)$; γ is the viscosity constant (in poises); $\mathbf{e}_x$, $\mathbf{e}_y$, and $\mathbf{e}_z$ are the unit vectors in a Cartesian coordinate system. We shall assume that perturbations of the director are inhomogeneous only along the z axis. This is naturally valid for the beams of transverse dimensions greater than the cell thickness. The conditions for the absence of the threshold-free effects are the same as the conditions for the absence of terms of zeroth order in respect of the director perturbations in the system of equations (1). Since the waves are extraordinary and since they travel in a plane (x, z), the appearance of the E_y component can only be due to the appearance of an n_y component of the perturbation: $E_y\propto\varepsilon_a n_y$. In the selected geometry, we have $\theta^{(0)}=\pi/2$ and $n_y\approx\varphi$. The function $\varphi\equiv 0$ is a function of Fig. 1b. This means that perturbations characterized by $\varphi\not\equiv 0$ can only have a threshold. Substituting $\varphi=0$ in Eq. (1a), we can see that sums of zeroth order in respect of the perturbations $\delta\theta=\theta-\theta^{(0)}$ are absent if

$$(E_xE_z^*+E_x^*E_z)_{\delta\theta=0}=0. \tag{2}$$

For one wave, this condition is satisfied in the trivial cases when $E_x=0$ or $E_z=0$. In the field of two waves, the condition (2) can generally be satisfied for mutually inclined orientation of the vectors $\mathbf{E}$ and $\mathbf{n}^{(0)}$.

Clearly, compensation occurs in the geometries in which beams of the same intensity are distributed in accordance with a mirror symmetry relative to the plane of the cell (Fig. 1, case *a*) or relative to the equatorial plane (case *b*).

If we use the results of Ref. 8, where the geometric-optics expressions are given for the light fields, Eq. (2) can be reduced to

$$S_{1z}\sin\alpha_1-S_{2z}\sin\alpha_2-(S_{1z}S_{2z})^{1/2}\left\{\sin\alpha_1\left(\frac{\varepsilon_\perp-\varepsilon\sin^2\alpha_2}{\varepsilon_\perp-\varepsilon\sin^2\alpha_1}\right)^{1/4}\right.$$

$$\left.-\sin\alpha_2\left(\frac{\varepsilon_\perp-\varepsilon\sin^2\alpha_1}{\varepsilon_\perp-\varepsilon\sin^2\alpha_2}\right)^{1/4}\right\}\cos\left[\left(\frac{\omega}{c}\right)(\sin\alpha_1-\sin\alpha_2)x\right.$$

$$\left.+\left(\frac{\varepsilon_\perp}{\varepsilon_\parallel}\right)^{1/2}((\varepsilon_\parallel-\varepsilon\sin^2\alpha_1)^{1/2}-(\varepsilon_\parallel-\varepsilon\sin^2\alpha_2)^{1/2})z+\Delta\varphi\right]=0, \tag{3}$$

where S_{1z} and S_{2z} are the z components of the energy flux density of the waves in the medium (in units of ergs per square centimeter per second); α_1 and α_2 are the angles of incidence; $\Delta\varphi$ is the difference between the phases of the waves. If the waves are noncoherent, the interference term is absent from Eq. (13) and the compensation condition can be satisfied also for an asymmetric orientation of the beams. The beam intensities and their angles of incidence must then satisfy

$$S_{1z}\sin\alpha_1=S_{2z}\sin\alpha_2. \tag{4}$$

This more general condition applies not only in the case of noncoherent, but also coherent beams if the period of the interference is much less than the cell thickness L. This is due to the smallness of the amplitudes of the threshold-free but rapidly oscillating perturbations of the director in space.[2,9-11] We have mentioned a cell with an initial planar orientation, but all the main conclusions remain valid also in the case of a homeotropic cell. Naturally, it is possible to achieve compensation of the threshold-free effects also in the case of a noncoplanar interaction of waves.

We thus have a number of geometries in which compensation of the threshold-free effects is possible. We shall consider the specific case when all the vectors are initially in an (x, z) plane. We shall assume that the conditions of Eqs. (3) or (4) remain satisfied for all the intensities and angles.

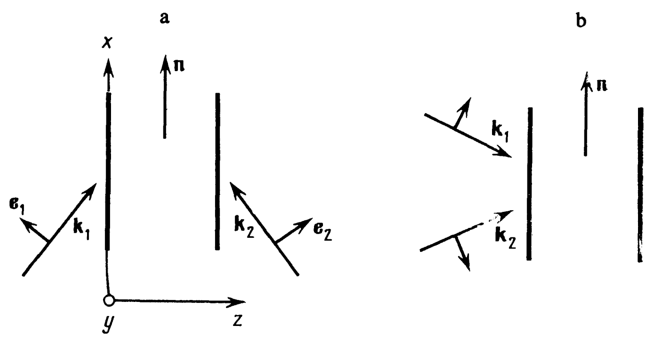

FIG. 1. Possible geometries of the threshold interaction between light waves and nematic liquid crystal. The wave vectors $\mathbf{k}_1$ and $\mathbf{k}_2$, the polarization vectors $\mathbf{e}_1$ and $\mathbf{e}_2$, and the director $\mathbf{n}^{(0)}$ are all in an (x, z) plane.

§2. PLANAR INSTABILITY THRESHOLD

We shall assume that when the total intensity is increased to a certain critical value, the director becomes reoriented in the (x, z) plane (planar LIFT-II).

We can find the threshold energy flux density by linearizing Eq. (1a) with respect to $\delta\theta=\theta-\theta^{(0)}$ followed by the substitution of $\varphi=0$; we shall use the actual expressions for the fields derived in Ref. 8. When the geometry is characterized by a mirror symmetry relative to the plane of the cell (case *a* in Fig. 1), so that the intensities and angles of incidence are equal, Eq. (1a) can be reduced to

$$\frac{d^2}{d\zeta^2}\delta\theta+(a-2q\cos 2\zeta)\delta\theta=0, \tag{5}$$

$$a=\frac{2\varepsilon_a\varepsilon_\parallel^{1/2}(2\varepsilon\sin^2\alpha-\varepsilon_\perp)S_z}{c\varepsilon_\perp^3(\varepsilon_\perp-\varepsilon\sin^2\alpha)^{1/2}K_1\varkappa^2},\quad 2q=\frac{2\varepsilon_a\varepsilon_\parallel^{1/2}S_z}{c\varepsilon_\perp^{1/2}(\varepsilon_\perp-\varepsilon\sin^2\alpha)^{1/2}K_1\varkappa^2}, \tag{6}$$

where

$$\zeta=\varkappa z+0{,}5\Delta,\quad \varkappa=(\omega/c)(\varepsilon_\parallel/\varepsilon_\perp)^{1/2}(\varepsilon_\perp-\varepsilon\sin^2\alpha)^{1/2},$$

$$\Delta=\arg E_1E_2^*;$$

Δ is the difference between the wave phases; ε is the permittivity of the medium from which the waves are incident; α is the angle of incidence; S_z is the modulus of the z component of the Poynting vector inside the medium for each of the waves.

Since $\omega/c\sim 10^5\ \text{cm}^{-1}$ and $L\sim 10^{-2}$ cm, it follows that for practically all the angles α (right up to $\varepsilon_\perp-\varepsilon\sin^2\alpha\sim 10^{-6}$) we have $\varkappa L\gg 1$. If a solution of Eq. (5) is expressed in terms of the Floquet functions $F_\nu(z)$ (Ref. 10), it follows from the boundary conditions $\theta(\Delta/2)=0$ and $\theta(\varkappa L+\Delta/2)=0$ that $\nu=\pi/\varkappa L\ll 1$. We shall also make the assumption that $q\ll 1$. Then, as is known, we find that $a\approx\nu^2-0.5q^2$. Substituting here the quantity a expressed in terms of q, we obtain from Eq. (6)

$$q=-\xi+(\xi^2+2\nu^2)^{1/2},\quad \xi=(2\varepsilon\sin^2\alpha-\varepsilon_\perp)/\varepsilon_\perp. \tag{7}$$

Hence, it is clear that in the case of small values of ν the assumption $q\ll 1$ is satisfied for all positive values of ξ and for all negative values of ξ. The following expression is obtained for the threshold from Eqs. (6) and (7)

$$S_{z.\text{th}}=\frac{c\varepsilon_\perp^{1/2}(\varepsilon_\perp-\varepsilon\sin^2\alpha)^{1/2}K_1\varkappa^2}{2\varepsilon_a\varepsilon_\parallel^{1/2}}\left\{-\xi+\left[\xi^2+2\left(\frac{\pi}{\varkappa L}\right)^2\right]^{1/2}\right\}. \tag{8}$$

If $S_{z.\text{th}}\propto L^{-2}$ when $\xi^2\gg\nu^2$, it follows that for $\xi\lesssim 0$ the threshold $S_{z.\text{th}}$ begins to rise rapidly with decreasing $\varkappa$ in accordance with the law $S_{z.\text{th}}\propto|\xi|\varkappa^2$.

Therefore, the angle α at which ξ changes sign,

$$\sin^2\alpha=\varepsilon_\perp/2\varepsilon, \tag{9}$$

is in fact the limiting angle above which the reorientation is possible. In the case of MBBA we have $\varepsilon_\perp=2.37$ and for light incident on a nematic liquid crystal from a medium with $\varepsilon=2.25$ (light crown glass) it follows from Eq. (9) that $\alpha_{\text{lim}}\approx 38°$.

If the light beams interacting with a nematic liquid crystal are noncoherent, the interference term in Eq. (5) is absent and instead we have in Eq. (8)

$$S_{z.\text{th}}=\left(\frac{\pi}{L}\right)^2\frac{c\varepsilon_\perp^{3/2}(\varepsilon_\perp-\varepsilon\sin^2\alpha)^{1/2}K_1}{2\varepsilon_a\varepsilon_\parallel^{1/2}(2\varepsilon\sin^2\alpha-\varepsilon_\perp)}. \tag{10}$$

The case when a nematic liquid crystal has the homeotropic orientation can be investigated in a similar manner and, if the beams are noncoherent, we find that

$$S_{z.\text{th}}=\left(\frac{\pi}{L}\right)^2\frac{c\varepsilon_\parallel^{3/2}(\varepsilon_\parallel-\varepsilon\sin^2\alpha)^{1/2}K_3}{2\varepsilon_a\varepsilon_\parallel^{1/2}(\varepsilon_\parallel-2\varepsilon\sin^2\alpha)}. \tag{11}$$

In a planar cell the LIFT-II can occur beginning from sufficiently large angles, whereas in a homeotropic cell the angle of incidence should be less than a certain limiting value $\sin^2\alpha_{\text{lim}}=\varepsilon_\parallel/2\varepsilon$. This is easily understood if we bear in mind that the field component perpendicular to the director has a destabilizing effect on it.

The value of S_z is defined inside a medium. This is why the thresholds given by Eq. (8) or (10) tend to zero on approach to the angle of total internal reflection $\sin^2\alpha=\varepsilon_\perp/\varepsilon$. The threshold intensity of an incident wave has a physical meaning which can be deduced using the formulas relating the intensities inside and outside the medium, which are given in the Appendix.

For example, using Eq. (A16) from the Appendix we can see that the threshold intensity for the incident beam remains finite near the angle of total internal reflection. The numerical value of the threshold radiation intensity incident on a cell containing MBBA with planar orientation amounts to $S=3\times 10^2\ \text{W/cm}^2$ for $\alpha=60°$, $L=10^{-2}$ cm, $K_1=6\times 10^{-7}$ dyn, $\varepsilon_\perp=2.37$, $\varepsilon_\parallel=3.06$, and $\varepsilon=2.25$.

§3. TRANSVERSE INSTABILITY THRESHOLD

We shall first consider the geometry shown in Fig. 1a. We can find the threshold intensity of the transverse LIFT-II by linearizing Eq. (1a) with respect to φ, assuming that $\theta=\pi/2$, and using Eqs. (A3) and (A6) for the fields. This yields the equation

$$\frac{d^2\varphi}{dz^2}+\frac{2\varepsilon\varepsilon_a\sin^2\alpha S_z\varphi}{c(\varepsilon_\parallel\varepsilon_\perp)^{1/2}(\varepsilon_\perp-\varepsilon\sin^2\alpha)^{1/2}K_2}=0. \tag{12}$$

Application of Eq. (A6) means that Eq. (12) is valid when the angle of incidence is less than the angle of total internal reflection:

$$(\varepsilon_\perp-\varepsilon\sin^2\alpha)^{1/2}\gg(\lambda/L)(\varepsilon_\parallel^{1/2}\varepsilon_\perp^{1/2}/\varepsilon_a),$$

where λ is the wavelength in vacuum. It follows from Eq. (12) and from $\varphi\propto\sin(\pi z/L)$ that

$$S_{z.\text{th}}=\left(\frac{\pi}{L}\right)^2\frac{c(\varepsilon_\parallel\varepsilon_\perp)^{1/2}(\varepsilon_\perp-\varepsilon\sin^2\alpha)^{1/2}K_2}{2\varepsilon\varepsilon_a\sin^2\alpha}. \tag{13}$$

For a homeotropic cell, we find that Eq. (13) becomes

$$S_{z.\text{th}}=\left(\frac{\pi}{L}\right)^2\frac{c\varepsilon_\parallel^{3/2}K_3}{2\varepsilon_a\varepsilon_\perp^{1/2}(\varepsilon_\parallel-\varepsilon\sin^2\alpha)^{1/2}}, \tag{14}$$

which is valid in the case of relatively large angles of incidence $\alpha\gg\alpha_k$ such that

$$\alpha_k\approx\arcsin(\lambda\varepsilon_\parallel\varepsilon_\perp^{1/2}/\varepsilon\varepsilon_a L).$$

Expression (13) or (14) defines effectively the threshold in the adiabatic approximation, i.e., when the polarization of a wave follows the rotation of the director as the wave propagates through a nematic liquid crystal.

We shall consider the role of the nonadiabatic effects in the case of a homeotropic cell. Applying Eqs. (A12) and (A15), we obtain from Eq. (1a)

$$\frac{d^2\theta}{dz^2}-\chi^2\left\{\theta+\rho\int_0^z\theta(z')\sin\mu(z-z')\,dz'\right\}=\frac{\gamma\Gamma}{K_3}\theta, \tag{15}$$

where μ is defined by Eq. (A13) and we have

$$\chi^2=\frac{2\varepsilon_a\varepsilon_\perp^{1/2}\varepsilon\sin^2\alpha S_z}{c\varepsilon_\parallel^{3/2}(\varepsilon_\parallel-\varepsilon\sin^2\alpha)^{1/2}K_3},\quad \rho=\frac{\pi\varepsilon_a\varepsilon_\parallel^{1/2}}{\lambda\varepsilon_\perp^{1/2}(\varepsilon_\parallel-\varepsilon\sin^2\alpha)^{1/2}}.$$

In writing down Eq. (15) we have omitted the rapidly oscillating terms proportional to $\sim\cos 2k_z z$ and allowed for the time dependence $\theta(z,t)$ in the form $\theta(z,t) = \theta(z)\exp(\Gamma t)$. An equation of the (15) type was first obtained in Ref. 8 where, in particular, it was pointed out that the integral operator is nonself-adjoint and, consequently, the growth of perturbations with time is more complex than simply exponential. The threshold must now be determined from the condition that the real part of the eigenvalue Γ vanishes. We shall find Γ by differentiating Eq. (12) twice with respect to z and eliminating from the resultant expression the integral term by means of Eq. (15). (This procedure is used in Ref. 12 on the assumption that $\Gamma = \Gamma' + i\Gamma'' = 0$.) In this way we obtain a homogeneous fourth-order differential equation with constant coefficients. If we assume that the boundary conditions $\theta(z=0, L) = 0$ and the initial equation (15) are satisfied, we obtain the following transcendental equation for the determination of

$$(\nu_1^2-\mu^2)\frac{\sin\nu_1 L}{\nu_1} = (\nu_2^2-\mu^2)\frac{\sin\nu_2 L}{\nu_2},$$
$$\nu_{1,2} = \frac{1}{\sqrt{2}}\left\{\mu^2-\chi^2-\frac{\gamma\Gamma}{K_3} \pm\left[\left(\mu^2-\chi^2-\frac{\gamma\Gamma}{K_3}\right)^2+4\chi^2\mu(\mu+\chi)+4\mu^2\frac{\gamma\Gamma}{K_3}\right]^{1/2}\right\}. \quad (16)$$

We then have

$$\theta(z,\ t) = \text{const}\ (\text{sh}\,\nu_1 L\ \text{sh}\,\nu_2 z - \text{sh}\,\nu_2 L\ \text{sh}\,\nu_1 z)e^{\Gamma t}. \quad (17)$$

A comparison of the threshold intensities in the range of angles in which the nonadiabatic effects are unimportant shows that the threshold of the transverse LIFT-II is always less than the threshold of the planar LIFT-II. In particular, the transverse LIFT-II can occur even for incidence angles when the planar LIFT-II is absent.

§4. ABOVE-THRESHOLD STEADY-STATE DISTRIBUTION OF THE DIRECTOR FOR LIFT-II

We shall assume that the wave intensities exceed the threshold values only by a small amount and in the variational equation system (1) we shall include nonlinear terms of the third order in respect of perturbations of the director. We shall consider a cell with a nematic liquid crystal characterized by a planar orientation. We shall assume that the intensity of light is higher than the threshold for the transverse LIFT-II but less than the threshold for the planar LIFT-II. Then, we can substitute $\theta = \pi/2$ in Eq. (1b). Substituting also the expressions for the fields from Eqs. (A3), (A6), (A8), and (A9) and retaining $\sim\varphi^3$, we obtain

$$\frac{d^2\varphi}{dz^2} + \frac{2\varepsilon_a S_z}{c(\varepsilon_\parallel\varepsilon_\perp)^{1/2}(\varepsilon_\perp-\varepsilon\sin^2\alpha)^{1/2}K_2}\cdot\left[\varphi\varepsilon\sin^2\alpha+\varphi^3\left(\frac{5}{4}\varepsilon\sin^2\alpha-\varepsilon_\perp\right)\right]=0. \quad (18)$$

Next, as is usual, we shall substitute in Eq. (18) the value of $\varphi = \varphi_m \sin(\pi z/L)$ and then, ignoring higher harmonics, we obtain

$$\varphi_m^2 = u_\perp\frac{S_z-S_{z.\text{th}}}{S_{z.\text{th}}},\quad u_\perp = \frac{4}{3}\frac{\varepsilon\sin^2\alpha}{\varepsilon_\perp - {}^5/_4\varepsilon\sin^2\alpha}, \quad (19)$$

where $S_{z.\text{th}}$ is defined by Eq. (13). We can see that if $\varepsilon\sin^2\alpha > 4\varepsilon_\perp/5$, then $u_\perp$ is negative. This corresponds to a hysteresis, as shown in Ref. 13. Equation (19) is then invalid and it is necessary to obtain a more rigorous solution of Eq. (1b).

We shall now consider the planar LIFT-II in a planar cell. We shall assume that a homogeneous magnetic field is applied at right-angles to the unperturbed director and that there is an intensity H of this field which is sufficient to reduce the threshold value of the intensity for the planar LIFT-II below the threshold intensity for the transverse effect. We shall also assume that $H < H_F = (\pi/L)(K_1/\chi_a)^{1/2}$, where $\chi_a > 0$ is the diamagnetic anisotropy of a nematic liquid crystals and H_F is the Fréedericksz transition threshold in a static magnetic field of the geometry under discussion.[14] The presence of such a magnetic field gives rise to a term $\chi_a H^2 \sin\varphi\cos\varphi$ on the left-hand side of Eq. (1a). Substituting $\varphi = 0$ in Eq. (1a) and the expressions for the light field taken from Ref. 8, we obtain

$$\frac{d^2\theta}{dz^2} - k\theta^2\frac{d^2\theta}{dz^2} - k\theta\left(\frac{d\theta}{dz}\right)^2 + \left(\frac{\pi}{L}\right)^2\left[\left(\frac{S_z}{S_{z.\text{th}}}+\eta\right)\varphi - \frac{2}{3}\left(\frac{S_z}{S_{z.\text{th}}}v+\eta\right)\varphi^3\right]=0, \quad (20)$$

where $\eta = \chi_a H^2(K_1\pi^2/L^2)^{-1}$,

$$v = 1+\frac{9}{4}\frac{\varepsilon_a}{\varepsilon_\perp}+\frac{3}{4}\frac{\varepsilon_a\varepsilon}{\varepsilon_\perp}\sin^2\alpha\frac{2\varepsilon\sin^2\alpha-3\varepsilon_\perp}{(\varepsilon_\perp-\varepsilon\sin^2\alpha)(\varepsilon_\perp-2\varepsilon\sin^2\alpha)}.$$

Equation (20) corresponds to the maximum deviation of the director:

$$\theta_m^2 = u_\parallel\frac{S_z-S_{z.\text{th}}}{S_{z.\text{th}}},\quad u_\parallel = \frac{1-\eta}{v+k-(v-1)\eta}, \quad (21)$$

where $k = (K_3 - K_1)/K_1$ and $S_{z.\text{th}} = S_{z.\text{th}}(H=0)(1-\eta)$.

The value of θ_m for the LIFT-II given by Eq. (21) is governed in a very complex manner by the parameters of a nematic liquid crystal and by the angle of incidence α of light. This provides an additional opportunity for inducing a hysteresis of the LIFT-II or for controlling its parameters (if it exists when $\alpha = 0$) by a suitable selection of the value of α. In the case of MBBA when the angle of incidence is $\alpha = 60°$, the intensity of a magnetic field sufficient to equalize the threshold values of the intensities for the planar and transverse LIFT-II cases amounts to $H = 540$ G. The angle α_{hyst} above which a hysteresis is exhibited by the dependence of θ_m on S amounts to $\alpha_{\text{hyst}} = 20.7°$ in a field $H = 700$ G. The threshold intensity for the planar LIFT-II is then $S_{\text{th}} \approx 0.6\times10^2$ W/cm^2 for each of the waves incident on a cell (the threshold intensity for the transverse LIFT-II under the same conditions is $S_{\text{th}} \approx 1.5\times10^2$ W/cm^2). Since $\alpha_{\text{hyst}} < \alpha_{\text{lim}} = 38°$, it follows that in this case a hysteresis occurs throughout the full range of angles of existence of the LIFT-II.

§5. DISCUSSION

We shall first consider the characteristics of the optical manifestations of the LIFT-II. An inhomogeneous orientation of the director alters the phase and polarization of light transmitted by a nematic crystal. By way of example, we shall consider a planar cell in which the planar LIFT-II occurs.

A nonlinear phase advance for small perturbations of the director is given by[8]

$$\delta\Phi=-\frac{\omega}{c}\frac{\varepsilon_a\varepsilon\sin\alpha}{\varepsilon_\perp}\int_0^z\delta\theta(z)\,dz.$$

We have seen already that the quantity $\delta\theta$ grows from thermal fluctuations and can be positive or negative. Consequently, we can expect either self-focusing (if $\delta\theta<0$) or self-defocusing (if $\delta\theta>0$), which varies from one experiment to another. We recall that in the case of normal incidence ($\alpha=0$) the nonlinear phase advance is proportional to $(\delta\theta)^2$ and it is always of the self-focusing nature.

The optical effects are of greater variety in the transverse LIFT-II case. In addition to self-focusing $[\delta\Phi\propto(\delta\theta)^2]$ we can expect also a change in the state of polarization of an initially linearly polarized light beam. The magnitude of the E_y component, which appears because of the departure from the adiabatic conditions, is given by Eqs. (A7) or (A15). In fact, the E_y component of the field appears in the LIFT-II case also in the adiabatic regime [Eqs. (A6) and (A14)], but in this case the component in question vanishes at the exit from the cell. If one of the cell substrates in no way affects the orientation of the director in an (x, y) plane, the n_y component of the director in the LIFT-II case does not vanish for the same substrate. Consequently, the polarization of a wave follows adiabatically the rotation of the director and it leaves the cell in a rotated configuration.

We have considered so far the geometries which are easiest to realize experimentally. For example, a single incident beam and a mirror placed directly behind a cell can produce the situation shown in Fig. 1a or its "homeotropic" analog. One of the characteristics of the configurations in which the angles of incidence of the wave are not equal is the possibility of modulation of the field along the x axis. As pointed out in §1, in some cases such modulation plays no significant role. In a situation of this kind (this is true, for example, of noncoherent beams), a calculation of the threshold intensity of the planar LIFT-II in a homeotropic cell gives

$$\left(\frac{S_{1z}}{\sin\alpha_2}\right)_{\rm th}=\left(\frac{S_{2z}}{\sin\alpha_1}\right)_{\rm th}=\frac{c\varepsilon_\parallel^{1/2}K_3}{\varepsilon_a\varepsilon_\perp^{1/2}}\left(\frac{\pi}{L}\right)^2\times\left[\frac{(\varepsilon_\parallel-\varepsilon\sin^2\alpha_1)^{1/2}}{\sin\alpha_2(\varepsilon_\parallel-2\varepsilon\sin^2\alpha_1)}+\frac{(\varepsilon_\parallel-\varepsilon\sin^2\alpha_2)^{1/2}}{\sin\alpha_1(\varepsilon_\parallel-2\varepsilon\sin^2\alpha_2)}\right].\qquad(22)$$

As expected, $(S_{1z})_{\rm th}\to0$ when $\alpha_2\to0$ and $(S_{2z})_{\rm th}\to0$ when $\alpha_1\to0$.

When two coherent waves are incident symmetrically on a cell from one side, the threshold perturbations of the director are found to be modulated along the x axis: $\theta(x, z)\propto\sin(\pi x/\Lambda)\sin(\pi z/L)$, where Λ is the transverse size of the cell. The threshold intensity then increases somewhat, proportionally to $\sim K_1(\pi/\Lambda)^2$.

We have calculated so far only the threshold values of the z components of the Poynting vector S_z in a medium. In the case of normal incidence of light on a cell, we may assume that S_z is equal to the intensity of the incident light. In the case of oblique incidence we have to allow for the Fresnel reflection at the boundary with a nematic liquid crystal. Therefore, a complete solution of the problem of the threshold intensity requires the use of the relationship between S_z inside the medium and the incident intensity, which is given in the Appendix.

Compensation of the threshold-free effects is also possible in static electric and magnetic fields, but static fields cannot induce a transverse reorientation of the director in the situations discussed above. Such a reorientation is specific to light fields and it is associated with the conditions of validity of the adiabatic approximation when the propagation of light is described.

We shall consider also the following interesting effect. We shall assume that the planar LIFT-II occurs in, for example, a homeotropic cell. Naturally, the application of any third field allows us to reestablish the original unperturbed homogeneous distribution of the director. In particular, this can be achieved by increasing the intensity of one of the waves (the selection of this wave is governed by the direction of orientation of the director in the LIFT-II case). We therefore have a situation in which the total intensity of the waves exceeds the LIFT-II threshold, the compensation condition is disobeyed, but nevertheless there is no reorientation of the director. The excess intensity for the symmetric configuration of the beams (Fig. 1a) is characterized by

$$\delta I=|\theta_m|\frac{\pi^3\varepsilon_\parallel^{1/2}(\varepsilon_\perp-2\varepsilon\sin^2\alpha)}{4L^2\varepsilon\sin\alpha(\varepsilon_\perp-\varepsilon\sin^2\alpha)^{1/2}},\qquad(23)$$

where $|\theta_m|\ll1$ is given by Eq. (21).

APPENDIX. LIGHT FIELD IN A NEMATIC WITH NONPLANAR PERTURBATIONS OF THE DIRECTOR

We shall consider a cell in which the director of a nematic liquid crystal suffers a deformation of the $n_z=0$, $n_x=\cos\varphi(z)$, and $n_y=\sin\varphi(z)$ type. Such a deformation appears in the transverse LIFT-II in a planar cell and, since we are interested only in the near-threshold situation, we can assume that $|\varphi(z)|\ll1$. Let us postulate that an extraordinary monochromatic light wave is incident on a cell in an (x, z) plane. The Maxwell equations for the field in the cell become

$$\frac{\partial^2E_z}{\partial x^2}-\frac{\partial^2E_x}{\partial x\partial z}+k_0^2\varepsilon_\perp E_z=0,\qquad(\mathrm{A1a})$$

$$\frac{\partial^2E_x}{\partial z^2}-\frac{\partial^2E_z}{\partial x\,\partial z}+k_0^2[(\varepsilon_\perp+\varepsilon_a\cos^2\varphi)E_x+\varepsilon_a\sin\varphi\cos\varphi E_y]=0,\qquad(\mathrm{A1b})$$

$$\frac{\partial^2E_y}{\partial x^2}+\frac{\partial^2E_y}{\partial z^2}+k_0^2[\varepsilon_a\sin\varphi\cos\varphi E_x+(\varepsilon_\perp+\varepsilon_a\sin^2\varphi)E_y]=0,\qquad(\mathrm{A1c})$$

where $k_0^2=(\omega/c)^2$.

Perturbations of the medium are homogeneous only along x, i.e., $\varphi = \varphi(z)$. Therefore, $E(x,z) = E(z)\exp(ik_x x)$ and Eq. (A1a) allows us to expression E_z in terms of E_x as follows

$$E_z(z) = \frac{ik_x}{k_0^2\varepsilon_\perp - k_x^2}\frac{\partial E_x(z)}{\partial z}. \tag{A2}$$

Substituting Eq. (A2) into Eq. (A1b), we find that in the zeroth approximation with respect to φ we obtain

$$E_x^{(0)}(z) = A\exp(ik_z z), \quad E_y^{(0)} = 0, \quad k_z = \varepsilon_\parallel^{1/2}\left(k_0^2 - \frac{k_x^2}{\varepsilon_\perp}\right)^{1/2}. \tag{A3}$$

The constant A is related to the z component of the Poynting vector, which is conserved only in a medium with a z-dependent refractive index. The relationship is

$$S_z = \frac{c(\varepsilon_\parallel\varepsilon_\perp)^{1/2}|A|^2}{8\pi}\frac{k_0}{(k_0^2\varepsilon_\perp - k_x^2)^{1/2}} = \frac{c(\varepsilon_\parallel\varepsilon_\perp)^{1/2}|A|^2}{8\pi(\varepsilon_\perp - \varepsilon\sin^2\alpha)^{1/2}}.$$

We can determine E_y in the first nonvanishing approximation by solving the equation

$$\partial^2 E_y^{(1)}(z)/\partial z^2 + (k_0^2\varepsilon_\perp - k_x^2)E_y^{(1)}(z) = -k_0^2\varepsilon_a\varphi(z)E_x^{(0)}(z). \tag{A4}$$

Separating from $E_y^{(1)}$ the rapidly oscillating component $E_y^{(1)} = B(z)\exp(ik_z z)$, we find that the slowly varying amplitude $B(z)$ is described by

$$\frac{\partial B}{\partial z} + i\mu B = i\frac{\varepsilon_a k_0^2}{2k_z}\varphi(z)A,$$
$$\mu = \frac{k_x^2 + k_z^2 - k_0^2\varepsilon_\perp}{2k_z} = \frac{\varepsilon_a}{2(\varepsilon_\parallel\varepsilon_\perp)^{1/2}}(k_0^2\varepsilon_\perp - k_x^2)^{1/2}. \tag{A5}$$

If $\mu \gg \pi/L$, it then follows from Eq. (A5) that

$$B = \frac{\varepsilon_a k_0^2}{2k_z\mu}\varphi(z)A = \frac{k_0^2\varepsilon_\perp}{k_0^2\varepsilon_\perp - k_x^2}\varphi(z)A, \tag{A6}$$

i.e., the polarization of the extraordinary wave follows adiabatically the rotation of the director. This breaks down only near the angle of total internal reflection. We then find from Eq. (A5) that

$$B = i\frac{\varepsilon_a k_0^2}{2k_z}A\int_0^z \varphi(z')\exp\{-i\mu(z-z')\}dz'. \tag{A7}$$

The first nonvanishing correction to $E_x^{(0)}$ is of the second order in φ.

Using Eq. (A3) and, for example, (A6), we obtain

$$E_x^{(2)}(z) = i\frac{\varepsilon_a k_x^2}{2\varepsilon_\perp k_z}E_x^{(0)}(z)\int_0^z \varphi^2(z')dz'. \tag{A8}$$

The correction to $E_y^{(1)}$ is then of the form

$$E_y^{(3)}(z) = E_y^{(1)}\left\{\frac{k_0^2\varepsilon_\perp + k_x^2}{2(k_0^2\varepsilon_\perp - k_x^2)}\varphi^2 + i\frac{\varepsilon_a k_x^2}{2\varepsilon_\perp k_z}\int_0^z \varphi^2(z')dz'\right\}. \tag{A9}$$

In the case of the transverse LIFT-II in a homeotropic cell we are dealing with deformations of the director field of the $n_x = 0$, $n_y = \sin\theta$, and $n_z = \cos\theta$ type. We shall consider the propagation of an extraordinary wave in such a medium and postulate, as before, that the wave is incident in an (x, z) plane. The Maxwell equations are

$$\frac{\partial^2 E_x}{\partial z^2} - \frac{\partial^2 E_z}{\partial x\,\partial z} + k_0^2\varepsilon_\perp E_x = 0, \tag{A10a}$$

$$\frac{\partial^2 E_z}{\partial x^2} - \frac{\partial^2 E_x}{\partial x\,\partial z} + k_0^2[\varepsilon_a\sin\theta\cos\theta E_y + (\varepsilon_\perp + \varepsilon_a\cos^2\theta)E_z] = 0, \tag{A10b}$$

$$\frac{\partial^2 E_y}{\partial x^2} + \frac{\partial^2 E_y}{\partial z^2} + k_0^2[(\varepsilon_\perp + \varepsilon_a\sin^2\theta)E_y + \varepsilon_a\sin\theta\cos\theta E_z] = 0. \tag{A10c}$$

In the zeroth approximation with respect to θ, we can use Eq. (A10b) to find E_z:

$$E_z^{(0)}(z) = \frac{ik_x}{k_0^2\varepsilon_\parallel - k_x^2}\frac{\partial E_x}{\partial z}. \tag{A11}$$

Substitution of the above expression in Eqs. (A10a) and (A10c) gives

$$E_x^{(0)} = Ae^{ik_z z}, \quad E_y^{(0)} = 0, \quad k_z = \varepsilon_\perp^{1/2}\left(k_0^2 - \frac{k_x^2}{\varepsilon_\parallel}\right)^{1/2}, \tag{A12}$$

$$\frac{\partial B}{\partial z} + i\mu B = -i\frac{\varepsilon_a k_0^2 k_x}{2(k_0^2\varepsilon_\parallel - k_x^2)}A\theta, \quad \mu = \frac{\varepsilon_a k_x^2}{2\varepsilon_\parallel k_z}. \tag{A13}$$

The constant A is related to the z component of the Poynting vector by

$$S_z = \frac{c(\varepsilon_\parallel\varepsilon_\perp)^{1/2}|A|^2}{8\pi}\frac{k_0}{(k_0^2\varepsilon_\parallel - k_x^2)^{1/2}} = \frac{c(\varepsilon_\parallel\varepsilon_\perp)^{1/2}|A|^2}{8\pi(\varepsilon_\parallel - \varepsilon\sin^2\alpha)^{1/2}}.$$

Once again if $\mu \gg \pi/L$, Eq. (A13) yields

$$B = -\frac{\varepsilon_a k_0^2 k_x}{2(k_0^2\varepsilon_\parallel - k_x^2)\mu}A\theta = -\frac{(\varepsilon_\parallel\varepsilon_\perp)^{1/2}k_0^2}{k_x(k_0^2\varepsilon_\parallel - k_x^2)^{1/2}}A\theta. \tag{A14}$$

The adiabatic condition is no longer satisfied when the angles are sufficiently small so that $\sin^2\alpha \sim \lambda/\varepsilon\varepsilon_a L$. Then, a solution of Eq. (A13) becomes

$$B = -i\frac{\varepsilon_a k_0^2 k_x}{2(k_0^2\varepsilon_\parallel - k_x^2)}A\oint_0^z \theta(z')\exp\{-i\mu(z-z')\}dz'. \tag{A15}$$

It is clear from Eq. (A15) that the value of E_y decreases on approach of the angle of incidence to the normal and, consequently, the threshold intensity for the transverse LIFT-II rises without limit.

On the other hand, the threshold of the planar LIFT-II in a homeotropic cell is minimal for the normal incidence. Therefore, it is the planar LIFT-II that occurs in a homeotropic cell at low angles. Consequently, we shall not give expressions nonlinear in respect of θ for the fields in this case.

As already pointed out, in the case of oblique incidence of light on a cell the reflection effects become important. The most general formulas describing refraction of light at the boundary between an isotropic medium and a uniaxial crystal are given in Ref. 15. We shall write down here the expressions for the geometry of interest to us when an extraordinary wave is incident on a cell in an (x, z) plane. If a nematic liquid crystal has the planar orientation, then

$$\frac{(S_{\text{int}})_z}{(S_{\text{inc}})_z}=\frac{4(\varepsilon_\parallel\varepsilon_\perp\varepsilon_i)^{1/2}(\varepsilon_\perp-\varepsilon_i\sin^2\alpha_i)^{1/2}\cos\alpha_i}{[\varepsilon_i^{1/2}(\varepsilon_\perp-\varepsilon_i\sin^2\alpha_i)^{1/2}+(\varepsilon_\parallel\varepsilon_\perp)^{1/2}\cos\alpha_i]^2}. \tag{A16}$$

For a homeotropic cell we have

$$\frac{(S_{\text{int}})_z}{(S_{\text{inc}})_z}=\frac{4(\varepsilon_\parallel\varepsilon_\perp\varepsilon_i)^{1/2}(\varepsilon_\parallel-\varepsilon_i\sin^2\alpha_i)^{1/2}\cos\alpha_i}{[\varepsilon_i^{1/2}(\varepsilon_\parallel-\varepsilon_i\sin^2\alpha_i)^{1/2}+(\varepsilon_\parallel\varepsilon_\perp)^{1/2}\cos\alpha_i]^2}. \tag{A17}$$

Equations (A16) and (A17) should be taken into account in calculations of the threshold intensities under specific experimental conditions.

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