Periodic redistribution of the density of parametrically excited spin waves in an antiferromagnet

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An inhomogeneous distribution of the density of parametrically excited magnons was observed experimentally in a sample of antiferromagnetic $CsMnF_3$. This distribution appeared at high pump powers when oscillations of the above-threshold susceptibility of unknown origin were observed. As the absorbed power was increased, the density of spin waves increased in the center of a sample in the course of oscillations. This increase in the density was the result of a nonlinear shift of the magnon spectrum and was similar to self-focusing of light in nonlinear media. A study was made of electromagnetic radiation emitted by parametrically excited spin waves. A periodic redistribution of the density had an additional limiting effect on the amplitude of parametrically excited magnons. The width of the frequency interval of parametrically excited magnons was much less than the width of the magnon resonance curve and it did not increase in the pump power.

1. INTRODUCTION

We shall consider the properties of parametrically excited spin waves (magnons) at relatively high magnon densities. We shall report an attempt to detect a nonlinear self-interaction of parametrically excited magnons, resulting in an instability of a homogeneous distribution of their density in a sample. This inhomogeneity may be expected for the following reasons.

The interaction between spin waves alters their spectrum when they are excited in a sample. This change in the spectrum occurs (see, for example, Refs. 1 and 2) as a result of a four-wave interaction in which two initial magnons with the wave vectors $\mathbf{k}_1$ and $\mathbf{k}_2$ are converted into two secondary magnons with the same vectors. In the final analysis, such conversion is due to the exchange and relativistic interactions. A change in the spectrum of spin waves as a result of the magnon interaction has been studied experimentally in antiferromagnetic $CsMnF_3$ (Ref. 3). Excitation of nonequilibrium magnons reduces the natural frequency of a spin wave with a specific value of $\mathbf{k}$. When such a change in the natural frequency takes place, selffocusing of a weakly diverging wave beam is possible,[4] depending on the amplitude of spin waves, since the phase velocity at the beam edge is less than at the center. On the basis of the quasiparticle formalism we can regard a reduction in the natural frequency of magnons as corresponding to an attraction between magnons. Attraction between quasiparticles present in a relatively high density may disturb the homogeneity of the distribution and cause condensation of particles in a part of the available volume.

In the case of real experiments on parametric excitation of spin waves (see, for example, Refs. 3 and 5), we can expect excitation of about 10^{18} magnons per cm^3. The maximum spectral shift is $\Delta \approx 2.5$ MHz and the magnon frequency is ~ 10 GHz. The wave vector of the excited magnons varies, depending on the magnetic field, from zero to $\sim 10^5$ cm^{-1}. Such magnons are clearly distributed isotropically in the $\mathbf{k}$ space. These experimental results and the magnon spectrum in Sec. 2 are used to estimate the possibility of a departure from homogeneity in the magnon distribution because of the interaction in question. In Secs. 3 and 4 we shall describe the experimental method used to detect a redistribution of the magnon density and the results obtained. In Sec. 5 we shall give the results of some control experiments.

In 1977, Prozorova and Kotyuzhanskiĭ[6] reported that when the threshold for parametric excitation of magnons in $CsMnF_3$ was exceeded, oscillations appeared in a system of parametrically excited magnons and this was manifested by periodic peaks of the absorbed pump power. The nature of these oscillations was not determined. We shall attribute such oscillations to a periodic redistribution of the magnon density in a sample because of the interaction mentioned above.

2. INSTABILITY THRESHOLD OF A HOMOGENEOUS DISTRIBUTION

The investigated compound $CsMnF_3$ is an antiferromagnet with the easy plane anisotropy and a Néel temperature 53.5 K. The spectrum of spin waves of the low-frequency branch of an unperturbed crystal of this antiferromagnet is[7]

$$\omega_{\mathbf{k}} = g\,(H^2 + H_\Delta^2/T + \alpha^2 k^2)^{1/2}, \tag{1}$$

where $g = 2\pi \times 2.8$ GHz/kOe is the magnetomechanical ratio; H is a magnetic field applied in the easy magnetization plane; $H_\Delta^2 = 6.3$ kOe²·K is the hyperfine interaction constant; T is the absolute temperature; $\alpha = 0.95$ kOe·cm is the exchange constant; $\mathbf{k}$ is the wave vector.

In a crystal in which magnons are excited the magnon spectrum changes as follows[2]:

$$\tilde{\omega}_{\mathbf{k}} = \omega_{\mathbf{k}} + 2\sum_{\mathbf{k}'} T_{\mathbf{k}\mathbf{k}'} n_{\mathbf{k}'}, \tag{2}$$

where $T_{\mathbf{k}\mathbf{k}'}$ is the matrix element of the interaction described

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above; n_k are the magnon occupation numbers. We shall denote this shift of the spectrum $2\Sigma_{\mathbf{k}'} T_{\mathbf{kk}'} n_{\mathbf{k}'}$ by Δ.

The condition for self-focusing of a weakly diverging beam can be written as follows (see, for example, Ref. 8):

$$\Delta^* > \frac{1}{2} \frac{\partial^2 \omega_{\mathbf{k}}}{\partial k_\perp^2} \varkappa^2. \tag{3}$$

Here, $k_\perp$ is the component of the wave vector perpendicular to the principal direction of propagation of the beam; $\varkappa = k\theta/2$ is the indeterminacy of the wave vector in the transverse direction describing the beam divergence (θ is the divergence angle). This relationship describes the condition for compensation of the beam divergence by convergence because of the difference between the phase velocities in the middle and at the edge of the beam; Δ^* represents the shift of the spectrum due to the waves forming the beam. In the experimental situation described above and characterized by an isotropic distribution of waves in the **k** space this condition (representing coalescence of narrow beams into which the isotropic distribution can be divided) becomes

$$\alpha^2 k^2/\omega_{\mathbf{k}} < 2T_{\mathbf{kk}}N. \tag{4}$$

Here, N is the total number of parametrically excited magnons. It should be pointed out that the condition (4) agrees, apart from a factor of 2, with the condition for the condensation of those quasiparticles for which the work function for the emission from a drop is $\hbar\Delta$ and the energy measured from the bottom of a band is $\varepsilon_{\mathbf{k}} = \hbar(\omega_{\mathbf{k}} - \omega_0)$. We can see that in the case of such condensation it is necessary to reach a certain level N and that in the case of low values of N only the long-wavelength magnons may condense. Under the conditions attainable experimentally, i.e., for $\Delta \approx 2.5$ MHz, this applies to magnons of frequency differing from the homogeneous precession frequency ω_0 by no more than the width of the frequency interval of magnetostatic modes. In parametric excitation experiments it is usually possible to distinguish reliably magnons with $\mathbf{k} \approx 10^4$ cm^{-1} from the homogeneous precession and magnetostatic modes. In the case of such magnons the condition (4) is satisfied if N is increased by some method by a factor of 20 compared with the experimental values attained in our experiments.

However, the condition (4) is far too stringent: it represents the collapse of all the magnons. It is clear that under certain conditions we can expect some increase in the magnon density also for $N < N_1 = \alpha^2 k^2/2T_{\mathbf{kk}}\omega_{\mathbf{k}}$, where N_1 is deduced from Eq. (4).

We shall consider the passage of a spin wave across a boundary separating excited and unexcited parts of a crystal. Such a boundary may be located, for example, near the edge of a sample where because of the damping introduced by the boundaries the amplitude of spin waves is less than far from the boundaries. The spectral shift Δ has the effect that the wave is refracted and when the glancing angle is sufficiently small, total internal reflection (TIR) is possible. If the glancing angle corresponding to such reflection θ_{TIR} exceeds the angle of divergence of a single spin wave, we can then encounter magnons which on first incidence on the boundary do not escape from the high-density region so that the density in the middle of this region rises. This condition on the total internal reflection angle leads to the inequality

$$q^2 \partial^2 \omega_{\mathbf{k}}/\partial k_\perp^2 < 2T_{\mathbf{kk}}N, \tag{5}$$

or, subject to Eq. (1),

$$q^2 \alpha^2/\omega_{\mathbf{k}} < 2T_{\mathbf{kk}}N. \tag{5a}$$

Here, **q** is the wave vector describing the divergence of a spin wave (q/k is the divergence angle). The minimum value of the divergence angle is due to diffraction by the boundaries of a sample. In our case the sample size is of the order of 1 mm, so that in the case of waves characterized by $k \approx 10^5$ cm^{-1} the condition of Eq. (5) is satisfied when $\Delta/\omega_{\mathbf{k}} \sim 10^{-10}$, i.e., it is satisfied by a large margin (in excess of five orders of magnitude) in respect of N. The condition (5) can be realized if the characteristic size of inhomogeneities responsible for the divergence of a spin wave does not exceed 0.005 mm. Therefore, the condition for the total collapse of magnons is not satisfied for the values of N attainable by parametric excitation, but a partial increase in the magnon density is quite likely. It is possible that the process of a partial increase in the density will change to total collapse. It should also be mentioned that the fraction of magnons participating in this partial increase in the density is determined by the solid angle $\Omega = (\theta_{\mathrm{TIR}})^2 = 2\Delta\omega_{\mathbf{k}}/\alpha^2 k^2$ and it increases as k^{-2} in the limit $k \to 0$, i.e., this partial process should be stronger in the range of small wave vectors.

In the derivation of the conditions (4) and (5) we have ignored the damping of spin waves and the effects of pumping. Therefore, the relationships (4) and (5) are approximate and give only the orders of magnitude. On the one hand, pumping maintains a high density of magnons N and compensates the damping, but on the other hand it tends to maintain this density near its steady-state value (see Refs. 2 and 9). An analysis of the process of selffocusing of a weakly diverging beam of spin waves subject to an allowance for the damping and for the effects of parametric pumping can be found in Ref. 4.

The authors of Ref. 4 used a numerical experiment to show that even in the case of a small excess of the threshold power for parametric excitation and a spatial modulation depth of the order of unity, we can expect collapse of parametrically excited waves. This collapse occurs in the transverse direction and the size of the collapse region is considerably greater than the wavelength. The characteristic instability growth time is of the order of the magnon lifetime and it decreases on increase in the pump amplitude.

3. EXPERIMENTAL METHOD

Excitation of magnons was parametric and the parallel pumping method was used.[2,5,9] The essence of this method was as follows.

A nonlinear coupling between oscillations of the longitudinal and transverse components of the magnetization is usually encountered in magnetic crystals. The effect of this coupling is that a pair of spin waves of frequency $\omega_{\mathbf{k}}$ and with the wave vectors **k** and $-\mathbf{k}$ has a longitudinal magnetic moment which oscillates homogeneously at a frequency $2\omega_{\mathbf{k}}$ (Ref. 10). Therefore, in the case of parallel orientation of the high-frequency and static magnetic fields it is possible to excite standing waves of very short wavelengths at a frequency equal to half the pump frequency ω_p. Parametric excitation, i.e., an exponential increase in the wave amplitude with time until some limitation mechanism is activated occurs when the amplitude of the magnetic pump field h exceeds a certain threshold value h_c. This threshold field h_c is governed by the rate of magnon relaxation and by the experimental conditions.[10] The wave vector of excited magnons is found from the parametric resonance condition

$$\tilde{\omega}_k=\omega_p/2 \qquad (6)$$

and from the expressions (1) and (2) for $\tilde{\omega}_k$. It follows that the excitation of spin waves is possible in the range of magnetic fields $0<H<H_c$, where the field H_c is found from the condition $\omega_0(H_c)=\omega_p/2$ and it corresponds to the parametric excitation of homogeneous precession.

Experimental evidence of parametric excitation is the absorption of microwave power by a sample deduced from a reduction in the power transmitted by a resonator or indicated by a change in the signal reflected from the resonator.

Our measurements were carried out on a sample kept at $T=1.65$ K. It follows from the results of Refs. 5 and 9 that the lifetime τ of magnons of frequency 10 GHz is of the order of 1 μsec and the distance traveled in the case when $k\sim10^5$ cm^{-1} is of the order of 1 mm, i.e., it is of the order of the size of the sample.

The main task in our experiments was to determine the value of the density n of parametrically excited magnons at different points in a sample. The value of n was measured using a probe pump and the nonlinear frequency shift described above; we basically followed Ref. 3, where this effect was investigated. The measurements were carried out as follows. The main pump wave of frequency $\omega_{p1}/2\pi=18$ GHz created parametrically excited magnons of frequency $\tilde{\omega}_{k1}=\omega_{p1}/2$ and of density n_1, which had to be measured. A probe pump wave of frequency $\omega_{p2}=2\pi\times23$ GHz or $2\pi\times36$ GHz, of power slightly higher than the threshold for the excitation of parametric excitons of frequency $\tilde{\omega}_{k2}=\omega_{p2}/2$, was applied continuously. The value of n_1 was determined by switching off the first pump and recording changes which were then exhibited by the probe pump signal. The pump was switched off in a time of 0.1 μsec, i.e., in a time considerably shorter than the lifetime of parametrically excited magnons. When the first pump wave was switched off, the value of n_1 decreased so that the natural frequency of

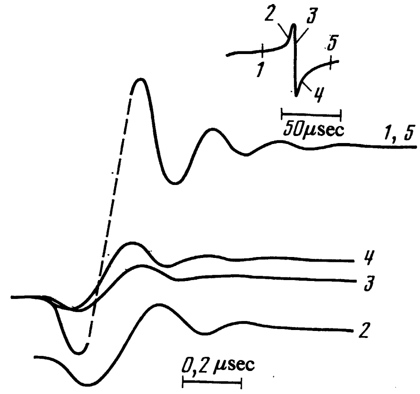

FIG.. 2. Oscillograms of the microwave power transmitted by the resonator in the setup 1. The upper trace represents the main pump signal. The numbers alongside this trace represent the moments of switching off the main pump at which a transient signal of the probe pump (lower trace) was observed in a field of $H=2.01$ kOe. The peak repetition period was 100 μsec. The position of the moment of switching off the main pump relative to the horizontal axis of the figure was the same for all the probe pump oscillograms.

magnons excited up to the moment of switching off was affected. Then, the parametric resonance condition (6) for the second pump wave was no longer observed and an oscillogram of the power transmitted by the resonator containing the sample revealed a transient process dependent on $\Delta_{12}=2T_{12}n_1$ and on the parametrically excited magnon lifetime. These transient processes were investigated in Ref. 3 in order to determine Δ and in Ref. 11 in order to find the spin wave lifetime. The equations describing the change in the absorbed power during a transient process were given in Ref. 3.

If $\Delta_{12}\lesssim(2\pi\tau)^{-1}$, then the transient process is in the form of a gradual increase and then a fall of the absorbed power (see Fig. 1 in Ref. 11). For $\Delta_{12}\gg(2\pi\tau)^{-1}$ this transient process was in the form of damped oscillations of the absorbed power near its zero level (see the text following Fig. 2). The appearance of such oscillations can be easily explained as follows. Under the conditions of a large departure from the parametric resonance condition (6) by an amount considerably greater than $(2\pi\tau)^{-1}$ we can expect parametrically excited magnons, existing at the moment of switching off, to decay when they no longer absorb (on the average) energy from the pump wave. The oscillations of the longitudinal component of the magnetic moment of a sample asso-

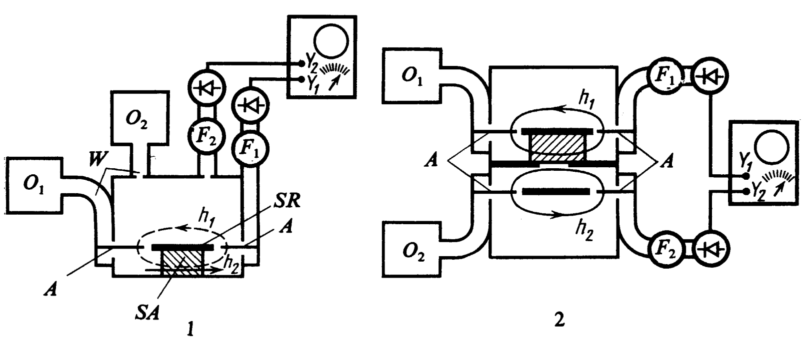

FIG. 1. Schematic diagram of the experimental setup. Two modifications are shown. Here, O_1 and O_2 are microwave oscillators; W are waveguides; A are antennas coupling resonators to waveguides; SR is a strip of a resonator; F_1 and F_2 are microwave filters; SA is a sample. The long sides of the strips are perpendicular to the plane of the figure.

ciated with the old parametrically excited magnons now occur at a new frequency $2\tilde{\omega}_{\mathbf{k}\,2}$. Beats of these oscillations with the field of a probe pump wave of constant frequency are manifested by damped oscillations in the absorbed power oscillogram. Therefore, the frequency of these oscillations is $2\Delta_{12}$.

The amplitude of the waves which exhibit a parametric resonance with the pump after switching off is significant only if a certain time passes from the decay of the old parametrically excited magnons. This was established by observations of the moment at which the absorbed power deviated from zero.

Similar oscillations of the absorbed power were reported also in Refs. 9 and 11 after a rapid change in the pump frequency by an amount exceeding $(2\pi\tau)^{-1}$. The frequency of such oscillations was equal to the change in the pump frequency. This was because in both cases the appearance of a transient process was due to a departure from the parametric resonance condition $\tilde{\omega}_{\mathbf{k}\,2} = \omega_{p2}/2$ [see also Eq. (7) in Ref. 3]. The damping of the group of spin waves alters $\tilde{\omega}_{\mathbf{k}\,2}$ and if the pump frequency is altered, there is also a change in ω_{p2}.

We can thus see that the frequency of oscillations of the power absorbed from the probe pump in the transient process following switching off the main pump was directly proportional to the magnon density. The frequency of these oscillations was approximately $2\Delta_{12} = 4T_{12}n_1$. When the duration of several oscillation periods exceeded the time 2τ, which was true—for example—of the oscillogram in Fig. 3 (curves 2 and 3), then a qualitative measure of n_1 was provided by the reciprocal of the transient growth time. In this case the half-period of oscillations of frequency $2\Delta_{12}$ can be determined by assuming that it is equal to the time for the change in the absorbed power from the maximum to the minimum value.

The above method for determining n_1 was used by us in two modifications.

1. Microwave magnetic fields $\mathbf{h}_1$ and $\mathbf{h}_2$ of both pump waves penetrated the whole sample and were distributed approximately homogeneously inside it. The sample was a disk 2 mm in diameter and 1.2 mm high. We used a two-mode resonator tuned to the frequencies $\omega_{p1}/2\pi = 18$ GHz (strip mode) and $\omega_{p2}/2\pi = 36$ GHz (volume mode); the resonator was the same as in Ref. 3.

2. The field $\mathbf{h}_1$ of the main pump wave ($\omega_{p1}/2\pi = 18$ GHz) was distributed homogeneously in a sample, whereas the field $\mathbf{h}_2$ ($\omega_{p2}/2\pi = 24$ GHz) was concentrated near the center of the end face of the sample in a region about 1 mm in diameter and 0.3 mm high. This configuration of the microwave fields was created by two strip resonators with a common wall of thickness 0.1 mm (Fig. 1). The sample was in the resonator of the main pump wave and the probe pump field $\mathbf{h}_2$ was created in the sample by an aperture 1.5 mm in diameter made in the shared resonator wall. The sample was bonded so that the center of its end face coincided with the center of the aperture in the wall. The field $\mathbf{h}_2$ penetrated slightly into the aperture and interacted effectively only near the end of the sample. The dimensions of the region of the effective action of the field $\mathbf{h}_2$ inside the sample were estimated by comparing the intensities of ESR signals from a sample placed inside the resonator and behind the wall at room temperature.

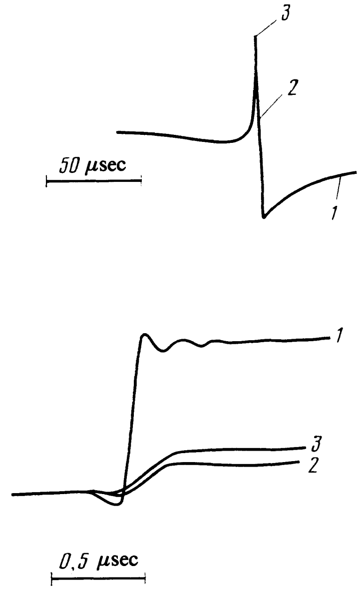

FIG. 3. Oscillograms of the microwave power transmitted by resonators in the setup 2 in a field of $H = 2.01$ kOe. The peak repetition period was 100 μsec.

Therefore, by observing the transient process described above by means of a measuring cell 1 we were able to estimate the density of magnons in the whole sample and we could use a cell 2 to find the same density near the end.

The output channels of the pumps were protected by filters to suppress the signal due to the other pump, so that the signals from each pump could be observed separately. A reduction in the signal transmitted by the resonator compared with the signal in the absence of parametric excitation (for example, in fields $H > H_c$) was a measure of the absorbed power.

The action of the main pump, particularly the high values of its amplitude, was to increase the rate of relaxation of spin waves, i.e., there was some nonlinear damping. When the power of the main pump was 70 times higher than the threshold value, the lifetime of the spin waves excited by the probe pump was halved and the mean free path then did not exceed 0.3 mm.

4. RESULTS OF THE MAIN EXPERIMENT

At low values of the main pump power ($P/P_c < 50$, where P_c is the threshold pump power) throughout the range of magnetic fields capable of parametric excitation we found that both modifications of the experimental setup revealed a transient process with the same frequency of oscillations of the absorbed power or the same growth time. The frequency of these oscillations increased on increase in the main pump power. In the range $P > P^* \sim 100P_{cr}$ we observed, as described in Ref. 6, periodic peaks of the absorbed power. The period of these peaks was considerably longer than all the characteristic times of parametrically excited magnons and it varied within the range 50–1000 μsec when the pump power, temperature, or magnetic field were varied. When the

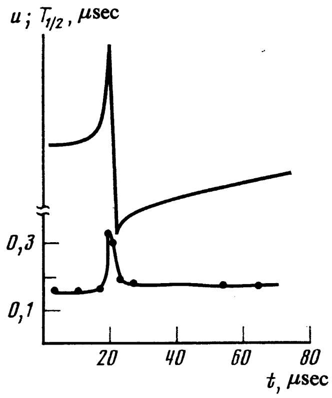

FIG. 4. Half-period $T_{1/2}$ of the beats plotted as a function of the moment of switching off the main pump in the setup 2. The upper curve is an oscillogram of the main pump signal in the vicinity of the absorbed power peak.

pump power was sufficiently high or close to the field H_c, the time interval between the peaks could be up to 10 times higher than the durations of the peaks themselves (5 μsec). The dependences of the period of these peaks on the pump power, magnetic field, and temperature were reported in Ref. 6 and it was pointed out that during a peak there was an increase in the absolute value not only of the imaginary χ'', but also of the real χ' part of the highfrequency susceptibility. A typical oscillogram of the signal of the transmitted power observed in the absorption oscillation regime is given in Fig. 2 (upper trace).

It was found that when the pump power was sufficiently high for the observation of the peaks reported in Ref. 6, the two experimental setups used in the present study revealed different kinds of transient processes if the main pump was switched off during a peak and approximately the same transient processes when the observations were made during the time intervals between the peaks.

Figure 2 (upper part) shows an oscillogram of the main pump power in the vicinity of an absorption peak. The lower part of Fig. 2 shows oscillograms of the probe pump power obtained for different moments of switching off relative to the peak. These oscillograms were recorded using the experimental setup 1. The oscillogram of the main pump showed a tendency for the signal to increase or decrease when the tuning of the resonator was not quite exact; moreover, there were changes in χ' during the peaks. When the moment of switching off the power was shifted, the frequency of beats of the transient process did not change by more than 20%. However, the oscillogram was distorted. When the moments of switching off were those identified by numbers 1 and 5, the oscillogram was in the form of a damped oscillation of approximately the same frequency in both cases, whereas at the moments 2, 3, and 4 the minima of the curves were shifted relative to the midpoint between the maxima. This was evidence of enrichment of the spectral composition of the damped oscillations. For example, the oscillogram denoted by 4 could be represented satisfactorily by a sum of damped oscillations at the fundamental and doubled frequencies. The amplitude of the oscillations at the doubled frequency was then about 20% of the amplitude of the fundamental component. Additional contributions due to lower frequencies predominated in the oscillogram 2.

A reduction in the amplitude of the transient process as a result of a change in the moment of switching off was due to a reduction in the absorbed power of the probe pump because of an increase in the damping of parametrically excited magnons, which occurred during an absorption peak. This increase in the damping by about 20% was clearly due to additional heating during a peak because of an increase in the power absorbed from the main pump.

Figure 3 shows similar oscillograms for the setup 2 in which the probe pump was directed only to the periphery of the sample. The results indicated clearly a reduction in the half-period of the beats by a factor of approximately 2. Figure 4 shows the dependence of the beat half-period on the moment at which the main pump was switched off.

This behavior of the beat frequency indicated that in the region of the most rapid variation of the absorbed power during a peak the density of spin waves at the edge of a sample decreased, whereas the total number of magnons in a sample at least did not decrease. The fact that the total number of magnons did not decrease was deduced, with an accuracy of the order of 20%, from the nature of the transient process observed using the experimental setup 1. Moreover, as described below, experiments were carried out in which electromagnetic radiation of magnons was observed at a frequency $\omega_{p1}/2$. These experiments indicated that the intensity of this radiation increased during the peaks, which was clear evidence of an increase in the total number of magnons in a sample during this time.

A reduction in the magnon density at the periphery of a sample accompanied by a simultaneous increase in the total number implied an increase in the density of magnons somewhere near the middle of the sample or at the other boundary. Unfortunately, the size of the region of a strong increase in the magnon density could not be deduced accurately from our observations. We could only say that it did not exceed 0.5 mm (representing the total size of the sample minus the dimensions of the zone investigated by the probe pump). The frequency and amplitude of the spectral component of the beats in Fig. 2 (line 4) could be used to estimate very approximately that in this central region the magnon density was doubled and the volume of this region was of the order of 0.2 of the total volume. However, it should be remembered that in the presence of a small (in respect of the volume) region with a high density of spin waves, the contribution of this region to the peak amplitude could also be small because of the small volume.

5. OSCILLATIONS OF ABOVE-THRESHOLD SUSCEPTIBILITY[6]

We shall show that the main features of the long-period oscillations of the absorbed power[6] can be accounted for if we consider a nonlinear self-interaction of spin waves which alters their spectrum. In fact, if in a small part of the total volume of a sample the number of spin waves increases in an avalanche-like manner, their relaxation in this region results in a strong evolution of energy and overheating. The rate of relaxation of spin waves rises rapidly on increase in tempera-

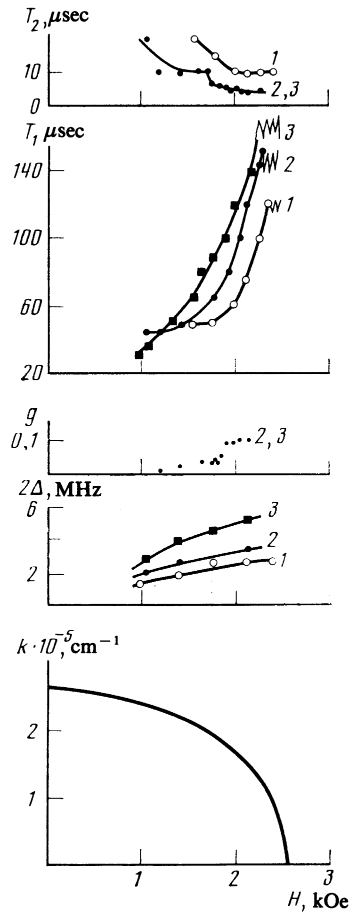

FIG. 5. Dependences of the principal characteristics of oscillations of the above-threshold susceptibility on the magnetic field. Curves 1, 2, and 3 correspond to different values of the power: P_1 exceeding by a factor of 60 the value of P_c in a field $H = 1.4$ kOe; $P_1 = 1.7 P_1$; $P_3 = 2.7 P_1$. The broken lines at the ends of the $T_1(H)$ plots represent the random oscillation regime. The extreme-left points on the $T_1(H)$ plots correspond to the lower limit of the range of magnetic fields in which oscillations appear at a given power P.

ture ($\tau^{-1} = \tau_0^{-1} + \beta T^7$—see Ref. 5) so that a strong overheating results in a rapid depletion of spin waves in the high-density region and the density falls practically to zero. The initial magnon density in such a region is restored only after a time of the order of several relaxation times of the temperature of a sample and then the process is repeated. This cyclic behavior was predicted in Ref. 4 for self-focusing of a weakly diverging standing wave under parametric excitation conditions.

Obviously, the condition for the absence of a steady-state increase in the density of parametrically excited magnons and in temperature is a difference between the characteristic times for the increase in the magnon density which, according to the results reported in the preceding section, is of the order of 2–10 μsec and the thermal relaxation time of the spin system estimated[12] to be of the order of 10 μsec.

An increase in the absorbed power and in the number of parametrically excited magnons during a peak is explained by the continuous action of the pump. The pump causes parametric excitation of new magnons in that part of the crystal from which they have departed to the region of high density. When the microwave power corresponds to the oscillation regime, the excitation of parametric magnons makes 2 μsec from the thermal noise level to the steady-state value.

Figure 5 shows the dependences of the principal characteristics of periodic changes in the absorbed power (peak period T_1, peak duration T_2, relative amplitude of changes in the absorption during peaks g) on the magnetic field, obtained for different pump powers reaching the same sample for which the results are reported in the preceding section. This figure gives also the magnetic field dependences of the wave vector of parametrically excited magnons and of the frequency shift Δ_{12}. We can see that in the case of a relatively small change in Δ_{12} (and, consequently, in the number of parametrically excited magnons) the principal characteristics of the effect undergo significant changes. A reduction in the wave vector (i.e., in the limit $H \to H_c$) enhances the effect: the rate of growth of the instability and the amplitude of the peaks rise steeply for $k < k^*$. An increase in the pump power also increases k^*. This is clearly due to the fact that the fraction of spin waves that undergo total internal reflection and do not escape outside the region of high density increases as k^{-2} (see Sec. 2) and the energy concentration becomes stronger on reduction in k. This explains also the dependences $T_1(H)$ and $T_1(h_1)$: an increase in the number of magnons characterized by an increase in the density increases the evolution of energy in this region and a longer time is needed for the purpose of cooling. If the number of magnons participating in the increase in the density is small and, consequently, the overheating is slight, an increase in the pump power excites parametric magnons also in an overheated sample, which reduces the period of the peaks as h_1 rises in the range of fields $H < 1.05$ kOe.

In a narrow interval of magnetic fields $H^* < H < H_c - 80$ Oe the susceptibility peaks become nonperiodic and random. This interval becomes wider on increase in the pump power. The transition to the random regime of peaks on increase in the field can be explained by a rapid reduction in the group velocity of magnons

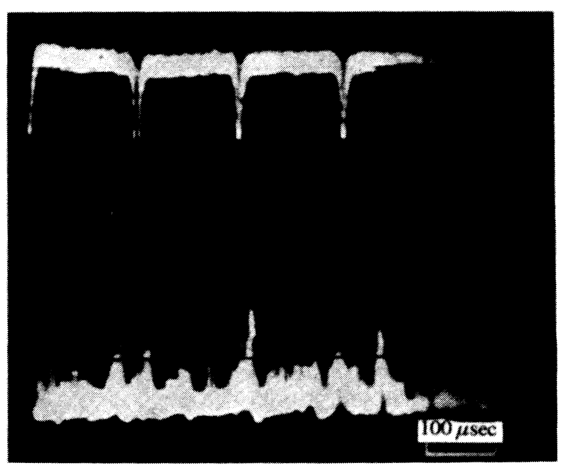

FIG. 6. Oscillogram of the envelope of the radiation of parametrically excited magnons in the regime of oscillations of the above-threshold susceptibility. The upper trace represents the main pump signal.

$v_{gr} = \partial\omega_k/\partial k$, because of which the mean free path decreases and the middle part of the sample ceases to be the preferred region for the increase in the density of spin waves. Consequently, the subsequent increase in the density can occur at different points in a sample which is not related to the rigorous periodicity of the process. The average period of the peaks in the random regime is 30–40% less than in the periodic case. As in Ref. 6, we observed a narrow range of fields free of these peaks. This range was about 80 Oe wide and was located near H_c. The maximum period of the peaks, the field corresponding to the transition to the random regime, and the lower limit of the magnetic fields in which oscillations were observed varied slightly with the shape and size of the sample.

It follows that the hypothesis of an instability of a homogeneous distribution of magnons accounts for the principal features of oscillations of the above-threshold susceptibility of parametrically excited magnons in $CsMnF_3$.

6. ELECTROMAGNETIC RADIATION OF PARAMETRICALLY EXCITED MAGNONS IN $CsMnF_3$

It was shown in Ref. 4 that a periodic collapse of parametrically excited magnons as a result of their selffocusing is an additional mechanism which limits the amplitude of these magnons since the energy of the collapsing waves is lost from the regions of increasing magnon density because of overheating or nonlinear damping. We investigated the behavior of the number of parametrically excited magnons as a function of the pump power by observing electromagnetic radiation emitted by magnons. Moreover, a study of such electromagnetic radiation should make it possible to determine the behavior of the total number of magnons in the regime of oscillations of the above-threshold susceptibility when an inhomogeneous distribution of the density of spin waves in a sample is observed. The behavior of the susceptibility itself cannot give reliable information. A study of the radiation emitted by parametrically excited magnons in $CsMnF_2$ is also of intrinsic interest because it has been observed so far only for substances with a strong magnetization[13,14] under conditions of a strong influence of magnetostatic modes[13] or in the case of a decay spectrum of parametrically excited magnons and, consequently, when the nonlinear damping is strong.

The method for reception of the radiation was similar to that employed in Ref. 14 and it differed only by the frequency of the received signal (9 GHz) and by the fact that oscillations of the magnetic moment of a sample were excited by a coaxial wire loop and not by a waveguide matched to the input channel of a superheterodyne receiver. The polarization of the received radiation was perpendicular to the pump polarization.

The radiation signal was observed in our case, as in Ref. 14, in the form of noise peaks (oscillogram in Fig. 6) with a characteristic duration 10–20 μsec and the same intervals between the peaks. However, in contrast to the radiation of spin waves in $FeBO_3$ (Ref. 14), in our case such radiation was concentrated in a narrow spectral interval of half-width not exceeding 30 kHz throughout the investigated range of pump powers. Weak magnetization of the sample ($4\pi M < 25$ Oe) made it possible to eliminate the influence of magnetostatic modes on the radiation. Therefore, the dependence of the radiation intensity on the magnetic field should be governed primarily by the dependence on the magnon wave vector, i.e.,[14]

$$I \propto 1/k^2, \tag{7}$$

which is due to compensation of the radiation from parts of a standing spin wave where oscillations are in antiphase. This was supported well by our dependence $I(H)$ and the dependence $I(k)$ deduced from the former (Fig. 7). The range of validity of the dependence (7) in terms of the magnetic field can be determined if we exclude the interval of fields near H_c, where a large contribution to the radiation comes from homogeneous precession and magnetostatic modes because

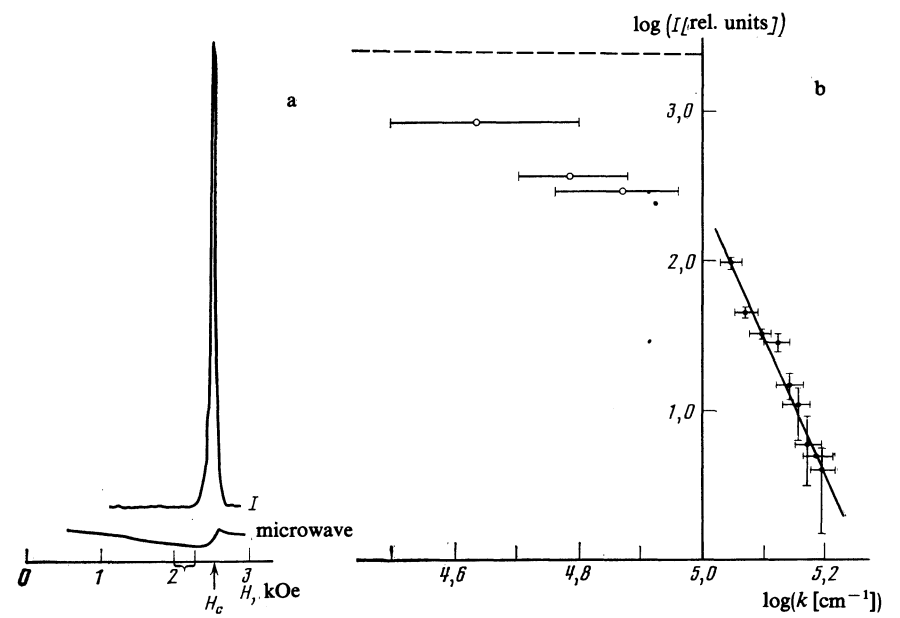

FIG. 7. a) Dependence of the intensity of the radiation of parametrically excited magnons on the magnetic field. The lower curve represents the pump signal transmitted by the resonator. b) Dependence of the intensity of the radiation on the wave vector. The horizontal dashed line represents the maximum intensity of the radiation attained for $k \approx 0$. The continuous straight line corresponds to $I \propto k^{-2}$. The range of magnetic fields where $5.0 < \log k < 5.2$ is denoted by a brace below the abscissa in Fig. 7a.

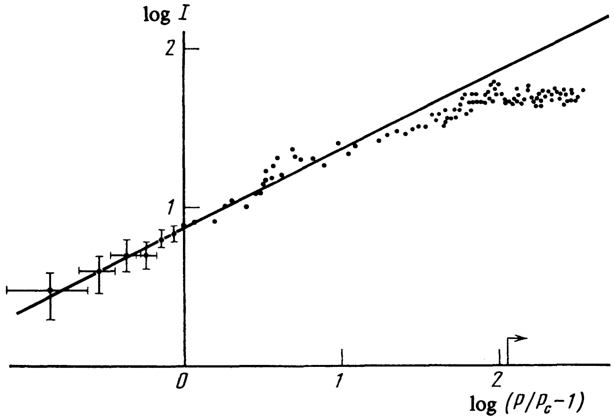

FIG. 8. Dependence of the intensity of radiation of parametrically excited magnons on the pump power in a field $H = 2.31$ kOe. The arrow identifies the power at which oscillations appear. The straight line represents the dependence $I \propto (P/P_c - 1)^{1/2}$ (Ref. 2).

of inhomogeneities of the spin wave spectrum. This interval can be estimated as the width of the range of fields to the right of H_c where the intensity of the radiation exceeds the receiver noise. With a view of determining H_c, we included in Fig. 7 also the dependence of the microwave power transmitted by the resonator on the magnetic field. The values of I obtained outside this interval of fields are identified by black dots in Fig. 7b In these measurements the sensitivity of the receiver was increased by a factor of 12 compared with that used to obtain the dependence shown in Fig. 7a and to record the data identified in Fig. 7b as open circles. The conditions such that for $P/P_c \approx 70$ and $H = 2.30$ kOe the average power radiated by the sample was of the order of 10^{-12} W.

Oscillograms of the radiation envelope recorded by a single sweep or by repeated study of the process showed no significant correlation of the radiation and periodic peaks of the susceptibility. However, an investigation of the envelope with the aid of a phase-sensitive amplifier using as the reference signal the voltage across the detector recording the susceptibility peaks established a correlation between the envelope of the radiation and these peaks. The emission of spin waves during the peaks increased. Modulation of the radiation intensity was approximately the same as the modulation of the absorbed power (about 10%), but changes in the intensity were not as fast as the changes in the susceptibility.

The influence of a redistribution of the density of parametrically excited magnons on the process of limitation parametrically excited magnons on the process of limitation of their number was studied by recording the dependence $I(h_1^2)$. This method for the diagnostics of the above-threshold state has the advantage over the susceptibility[2] or spin wave phase[9] measurements because a quantity proportional to the total number of spin waves is determined. Naturally, a study of the radiation characteristics does not supplant completely the methods described in Refs. 2 and 9. The dependence $I(h_1^2)$ obtained by us is plotted in Fig. 8. When the threshold power for the excitation of parametric magnons is exceeded by a factor up to 20, the intensity of the radiation obeys $I \propto (P/P_c - 1)^{1/2}$, as expected in the case of the phase mechanism of the limitation[2,9] which is realized in the case of a nondecay spectrum of spin waves. In the range $P > 20P_c$ the growth of the number of parametrically excited magnons slows down compared with this dependence and this may be explained by overheating of a sample and nonlinear damping. In the vicinity of the power at which the susceptibility oscillations appear an increase in the number of spin waves ceases and the intensity of the radiation becomes constant. This may be explained by the above additional dissipation of the energy of parametrically excited magnons in the region where this density rises.

7. CONCLUSIONS

In the course of the present investigation we discovered an instability of a homogeneous distribution of parametrically excited magnons. This instability appeared in the form of oscillations of the above-threshold susceptibility.[6] A study of electromagnetic radiation of magnons showed that a periodic distribution of magnons has an additional limiting influence on the amplitude of parametrically excited spin waves.

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[1]F. J. Dyson, Phys. Rev. **102**, 1217 (1956).

[2]V. E. Zakharov, V. S. L'vov, and S. S. Starobinets, Usp. Fiz. Nauk **114**, 609 (1974) [Sov. Phys. Usp. **17**, 896 (1975)].

[3]L. A. Prozorova and A. I. Smirnov, Zh. Eksp. Teor. Fiz. **74**, 1554 (1978) [Sov. Phys. JETP **47**, 812 (1978)].

[4]V. S. L'vov, A. M. Rubenchik, V. V. Sobolev, and V. S. Synakh, Fiz. Tverd. Tela (Leningrad) **15**, 793 (1973) [Sov. Phys. Solid State **15**, 550 (1973)].

[5]B. Ya. Kotyuzhanskiĭ and L. A. Prozorova, Zh. Eksp. Teor. Fiz. **65**, 2470 (1973) [Sov. Phys. JETP **38**, 1233 (1974)].

[6]B. Ya. Kotyuzhanskiĭ and L. A. Prozorova, Pis'ma Zh. Eksp. Teor. Fiz. **25**, 412 (1977) [JETP Lett. **25**, 385 (1977)].

[7]A. S. Borovik-Romanov, Zh. Eksp. Teor. Fiz. **36**, 766 (1959) [Sov. Phys. JETP **9**, 539 (1959)].

[8]S. A. Akhmanov, A. P. Sukhorukov, and R. V. Khokhlov, Usp. Fiz. Nauk **93**, 19 (1967) [Sov. Phys. Usp. **10**, 609 (1968)].

[9]L. A. Prozorova and A. I. Smirnov, Zh. Eksp. Teor. Fiz. **67**, 1952 (1974) [Sov. Phys. JETP **40**, 970 (1975)].

[10]V. I. Ozhogin, Zh. Eksp. Teor. Fiz. **48**, 1307 (1965) [Sov. Phys. JETP **21**, 874 (1965)].

[11]A. I. Smirnov and S. V. Petrov, Zh. Eksp. Teor. Fiz. **80**, 1628 (1981) [Sov. Phys. JETP **53**, 838 (1981)].

[12]A. I. Smirnov, Zh. Eksp. Teor. Fiz. **73**, 2254 (1977) [Sov. Phys. JETP **46**, 1180 (1977)].

[13]I. V. Krutsenko, V. S. L'vov, and G. A. Melkov, Zh. Eksp. Teor. Fiz. **75**, 1114 (1978) [Sov. Phys. JETP **48**, 561 (1978)].

[14]B. Ya. Kotyuzhanskiĭ, L. A. Prozorova, and L. E. Svistov, Zh. Eksp. Teor. Fiz. **868**, 1101 (1984) [Sov. Phys. JETP **59**, 644 (1984)].

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