Nonlinear anomalous skin effect in metals

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An investigation was made of the nonlinear impedance of metals under the conditions of the anomalous skin effect in the absence of an external static magnetic field. It was established experimentally that the impedance Z is a nonmonotonic function of the amplitude of the magnetic field of an electromagnetic wave. The position of a maximum of the dependence of the impedance $Z(\mathscr{H})$ on this magnetic field $\mathscr{H}$ depended strongly on the wave frequency f and on the mean free path of electrons l. A theoretical analysis and the experimental results demonstrated that the ratio $\mathrm{Re}\{Z(\mathscr{H})\}/\mathrm{Re}\{Z(0)\}$ is a universal function of the magnetodynamic nonlinearity parameter $b_0 \propto \mathscr{H}^{-1/2} f^{-1/6} l^{-1}$.

The anomalous skin effect occurs when the depth of the skin layer δ becomes less than the mean free path l of the conduction electrons. In the purest metals at low temperatures this condition is satisfied beginning from frequencies of tens of hertz. We shall be interested in the nonlinear skin effect when the magnetic field of an electromagnetic wave incident on a metal affects significantly the dynamics of the conduction electrons. A nonlinearity of this type was first observed by Gantmakher[1] and by Cochran and Shiffman[2] in measurements of the impedance of bismuth and gallium in a weak static magnetic field H parallel to the surface. The subsequent numerous experimental investigations were concerned with a variety of nonlinear effects in metals (for a review see, for example, Ref. 3). Electric current states, spontaneous oscillations of the magnetic moment of a sample, etc. were discovered and investigated theoretically. In spite of the considerable interest in the nonlinear effects, the behavior of an important electromagnetic characteristic of metals, which is its surface impedance $Z(\mathscr{H})$, has not yet been investigated sufficiently thoroughly in the $H = 0$ case (here, $\mathscr{H}$ is the amplitude of an alternating magnetic field on the surface of a metal).

The behavior of $Z(\mathscr{H})$ under the conditions of the almost normal skin effect was considered in Ref. 4. Boĭko *et al.*[5] Studied experimentally the dependence of the impedance of tungsten on $\mathscr{H}$ in the anomalous skin effect case and they found that an increase in the field amplitude should increase significantly the surface resistance. An increase in the frequency weakens the nonlinear rise of the impedance on increase in the field. Lyubimov *et al.*[6] developed a theory of the nonlinear skin effect and obtained an asymptotic expression for $Z(\mathscr{H})$ in the cases of weak and strong nonlinearity. According to the theoretical conclusions, an increase in $\mathscr{H}$ should convert the rise of the impedance to a fall.

The present describes an investigation of the nonlinear impedance of metals in the absence of a static magnetic field. The measurements were carried out on silver and tungsten samples in an alternating field of amplitudes up to 400 Oe at frequencies in the range 0.1–100 kHz. It was found that the surface impedance depended monotonically on the field $\mathscr{H}$. The experimental results were compared with the predictions of a theory in which an interpolation expression was obtained for $Z(\mathscr{H})$.

§ 1. EXPERIMENTAL METHOD

We investigated the surface impedance of metal plates under conditions such that a depth of penetration δ was much less than the plate thickness d. In the nonlinear case the impedance defined as the ratio of the first harmonics of the electric E_f and magnetic $H_f = \mathscr{H}/2$ fields on the surface of a crystal, in accordance with Ref. 7, was a convenient electrodynamic characteristic:

$$Z(\mathscr{H}) \equiv R(\mathscr{H}) - iX(\mathscr{H}) = \frac{4\pi}{c}\frac{E_f}{H_f}, \qquad (1)$$

where R and X are the surface resistance and surface reactance, respectively. A block diagram of the apparatus used in the impedance measurements is shown in Fig. 1. An alternating magnetic field was created by a solenoid L_0 wound on a fabric-based laminate cylinder using a copper wire 0.03 mm in diameter. The solenoid was supplied with a sinusoidal current ($\cos \omega t$, $\omega = 2\pi f$), produced by an oscillator 1. The low ($<0.2\%$) content of the harmonics was ensured by a strong negative feedback in a power amplifier 2. At frequencies $f < 1$ kHz a negative feedback in respect of the current was used in the amplifier; at frequencies $f > 1$ kHz a capacitor was connected in series with the solenoid and its capacitance was selected to ensure resonance; in this case a voltage feedback loop was used. The solenoid current and the associated field $\mathscr{H}$ were deduced from the voltage drop across an active resistor r_0 included in the current circuit of the solenoid.

A sample S with a tightly wound single-layer coil L_1 was fixed securely in a cylindrical Textolite holder. The alternating field in the coil excited an emf which could be represented conveniently by a sum of several terms:

$$\mathscr{E} = (E^R - iE^X)\lambda n - i\mathscr{E}_\Phi + \mathscr{E}_\Pi{}^R + i\mathscr{E}_\Pi{}^X. \qquad (2)$$

The real terms on the right-hand side of Eq. (2) represent the components varying in phase with the field $\mathscr{H}$, whereas the imaginary components are phase-shifted by $\pi/2$. The first term describes a signal associated with the sample (E^R and

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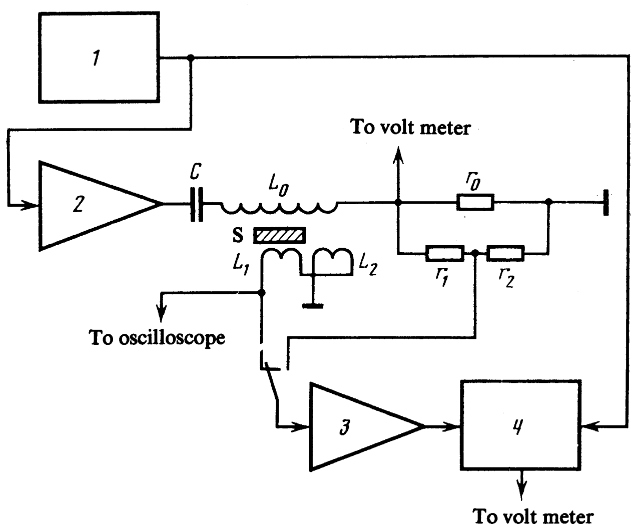

FIG. 1. Block diagram of the apparatus.

E^X are the components of the electric field on the surface of metal, λ is the perimeter of the cross section of the sample in a plane perpendicular to the alternating magnetic field, and n is the number of turns in the measuring coil). The term $\mathscr{E}_\Phi$ is due to an alternating magnetic flux penetrating into the space between the sample and the coil and also due to the strays induced in the measuring circuit; $\mathscr{E}^R_\Pi$ and $\mathscr{E}^X_\Pi$ are the components associated with the currents induced in the metal parts of the apparatus.

In the surface resistance measurements the active component of $\mathscr{E}$ associated with the sample was found by a null method. A compensating coil L_2 was wound on the same holder as the measuring coil. A longitudinal shift of the holder in the solenoid made it possible to suppress practically completely the imaginary part of the signal $\mathscr{E}$. When a glass cryostat without a metal coating was employed and metal parts were removed from the low-temperature part of the apparatus, the unavoidable stray signal was mainly due to eddy currents in the coil wires. We were also able to determine experimentally changes in the reactance $X(\mathscr{H})$.

The resistance $R(\mathscr{H})$ or the changes in $X(\mathscr{H})$ were deduced from the emf $\mathscr{E}$ of Eq. (2) employing a selective Unipan-237 amplifier 3, which selected the first harmonic. The amplified signal was applied to a phase-sensitive detector 4, which enabled us to select the real or imaginary part of $\mathscr{E}$ containing information on the surface resistance and reactance, respectively. The phase in the detector was found and the absolute values of the measured signals were calibrated with the aid of an auxiliary voltage from an active divider (r_1, r_2) connected in parallel with the resistor r_0. In the measurements of $R(\mathscr{H})$ the influence of the imaginary part of $\mathscr{E}$ was eliminated by reducing it by four orders of magnitude; in the reactance measurements the active part of $\mathscr{E}$ was suppressed by about two orders of magnitude. The errors in the determination of the surface resistance did not exceed 2% in any specific case. It was possible to determine experimentally the dependence of the impedance on a static magnetic field H parallel to the alternating field. The field H was created by an additional solenoid. A system of Helmholtz coils used to compensate the terrestrial magnetic field was placed outside the cryostat.

Measurements were carried out on tungsten samples with the resistivity ratio $\rho_{300\mathrm{K}}/\rho_{4.2\mathrm{K}}$ amounting to 35 000 (sample W1), 50 000 (W2), and 80 000 (W3), as well as of silver (Ag1) with the resistivity ratio $\rho_{300\mathrm{K}}/\rho_{4.2\mathrm{K}} = 16\,000$. The sample W1 was a parallelepiped of $5\times5\times2$ mm dimensions; the normals to its faces coincided with the C_4 crystal axes; W_2 was a slant cylinder of height 1.3 mm and diameter 7 mm, with the normal to the bases parallel to the C_4 axis; W3 was a parallelepiped of $7\times4\times0.9$ mm dimensions and its normal was parallel to the [110] axis; Ag1 was a plate of 0.8 mm thickness, its maximum length and width were approximately 12 and 6 mm, and the normal to the plate was parallel to [110]. Samples were treated by methods described in Ref. 8 (in the case of tungsten) and in Ref. 9 (in the case of silver). The mounting was such that liquid helium had free access to the whole surface of a sample not occupied by the coil. In these measurements we monitored the temperature of the helium bath. The maximum power dissipated in the samples at the frequency of $f = 100$ kHz in a field of $\mathscr{H} = 400$ Oe did not exceed 0.2 W/cm.[2]

§ 2. RESULTS OF MEASUREMENTS

The proper operation of the apparatus was checked by measuring the surface resistance R_0 of tungsten in the linear regime. It was established that in the frequency range $f > 10$ Hz the surface resistance of the investigated samples varied with the frequency in accordance with the law $R_0 \propto f^{2/3}$, typical of the anomalous skin effect. The value of R_0 for tungsten sample W1, the perimeter of which could be determined with high accuracy, amounted to 1.6×10^{-19} Gaussian units at $f = 1.09$ kHz. This value of R_0 and the expression for the impedance of an isotropic metal were used to calculate the ratio $l/\sigma_0 = 1.2\times10^{-11}\ \Omega\cdot\mathrm{cm}^{-2}$, which was in satisfactory agreement with the available data of a tungsten (σ_0 is the static conductivity).

When the amplitude of the magnetic field was sufficiently large, current states could appear in a sample and then the impedance was no longer a single-valued function of $\mathscr{H}$. The results reported below were obtained in the absence of current states when the function $\mathscr{E}(t)$ had no even harmonics.

1. Figure 2 shows the measured $R(\mathscr{H})$ of a silver plate at different frequencies. The ordinate gives the value of $R(\mathscr{H})$ reduced to R_0, which was defined as the limit of $R(\mathscr{H})$ when $\mathscr{H}\to0$. The abscissa is the product $\mathscr{H}f^{1/3}$ (we shall explain later in § 3 why this product was selected). The experimental curves obtained at different frequencies are qualitatively similar and they demonstrate that the dependence $R(\mathscr{H})$ is nonmonotonic. The observed maximum of $R(\mathscr{H})$ decreases in amplitude and shifts to the right along the abscissa when the frequency is increased.

The behavior of the nonlinear surface resistance of tungsten samples W1 and W2 is generally similar to the corresponding behavior of silver. The main difference is that the amplitude of the maximum is 2–3 times greater for tungsten.

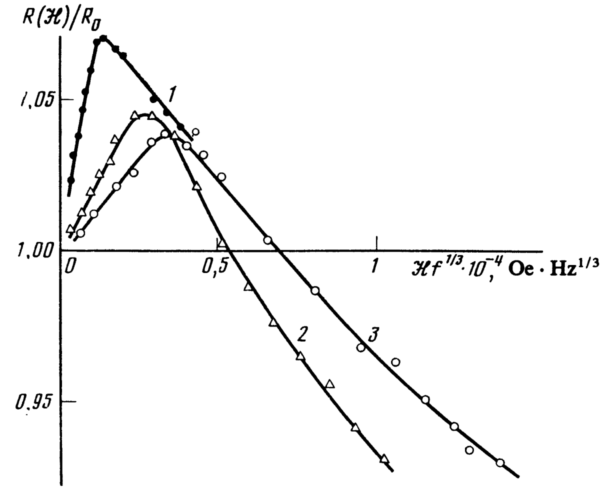

FIG. 2. Dependence $R(\mathscr{H})/R_0$ for silver. Curve 1 corresponds to $f = 1.75$ kHz, curve 2 to 23 kHz, and curve 3 to 98 kHz. $T = 4.2$K.

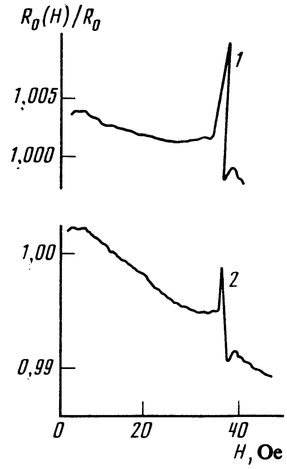

FIG. 4. Records of the *rf* size effect in tungsten (sample W3) obtained for H‖[110] and $\mathscr{H} = 0.3$ Oe at $T = 4.2$K for two values of the frequency: $f = 20$ kHz (curve 1) and 98 kHz (curve 2).

A change in the direction of the alternating magnetic field in the plane of the sample has practically no influence on the dependence $R(\mathscr{H})$. For example, in the case of sample W2 the difference between the values of $R(\mathscr{H})$ for $\vec{\mathscr{H}} \| C_4$ and $\measuredangle(\vec{\mathscr{H}}, C_4) = 30°$ does not exceed 2–3%.

2. We can see that the frequency dependences of the amplitude and position of the maximum of the nonlinear surface resistance $R(\mathscr{H})$ are different for samples W1 and W2. We shall assume that this is due to an inhomogeneity of the conductivity σ_0 near the surfaces of the plates. This assumption was checked by a study of lines representing the rf size effect in a parallel magnetic field of different frequencies applies to samples W1, W3, and Ag1. In the case of sample W1 at $T = 1.5$ K and $f = 20$ kHz the $R_0(H)$ curve shows a clear line due to the rf size effect (identified by an arrow in Fig. 3). [For the sake of convenience, the ordinate gives the value of $R_0(H)$ reduced to the linear value of the surface resistance in $H = 0$.] At the frequency of 100 kHz the line due to the rf size effect is practically indistinguishable against the background of a smooth variation of $R_0(H)$. This means that the effective mean free path of electrons decreases considerably on approach to the surface. The same conclusion follows from the results for sample Ag1. In the case of sample W3, subjected to mechanical polishing and then to a deeper etching (to a depth of approximately 60 μ on each side), the amplitude of the line due to the rf size effect decreases much more slowly on increase in the frequency, as illustrated by the $R_0(H)/R_0$ curves in Fig. 4 obtained at two frequencies. A fivefold increase in the frequency reduces the amplitude of the lines by a factor of about 1.5. This is in reasonable agreement with the theory of Ref. 10 which predicts that the relative amplitude should be proportional to $f^{-1/6}$. It follows from our measurements that the conductivity inhomogeneity of sample W3 is considerably less than that of samples W1 and Ag1.

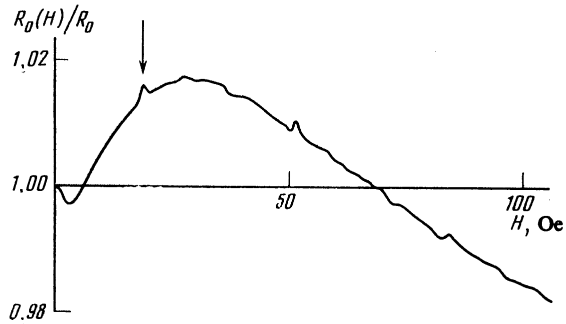

FIG. 3. Dependence of the linear surface resistance of tungsten (sample W1) on the static field (H)‖[100]); $\mathscr{H} = 0.3$ Oe, $f = 20$ kHz, $T = 1.5$K.

3. The results of measurements of the nonlinear surface resistance of sample W3 are presented in Fig. 5. It follows from them that, on the selected scale, the position of the maximum $R(\mathscr{H})$ is the same for all the curves obtained at different frequencies. Moreover, if an allowance is made for the error in the calibration, the curves obtained at frequencies 20 and 98 kHz are practically indistinguishable.

A nonlinearity of the impedance was greatly enhanced by cooling. Lowering of T from 4.2 to 1.6 K reduced the field $\mathscr{H}$ in which $R(\mathscr{H})$ reached its maximum by a factor of 1.4 and 2.2 for samples W1 and W3, respectively. The behavior of $R(\mathscr{H})$ in the case of sample W3 at two temperatures is shown in Fig. 6.

We shall conclude this section with a comment on the nonlinear reactance. In view of the experimental error in the determination of the reactance mentioned above we shall not give the graphs illustrating the behavior of $X(\mathscr{H})$. We shall simply note that the dependence $X(\mathscr{H})$ is similar to $R_0(H)$ shown in Fig. 3. In the case of sample W1 at 22 kHz and $T = 4.2$K a minimum of the function $X(\mathscr{H})$ is observed at $\mathscr{H} \approx 30$ Oe and a maximum at $\mathscr{H} \approx 200$ Oe.

§ 3. DISCUSSION OF RESULTS

1. The nonlinear dependence of the surface impedance on the amplitude of an alternating magnetic field is related to

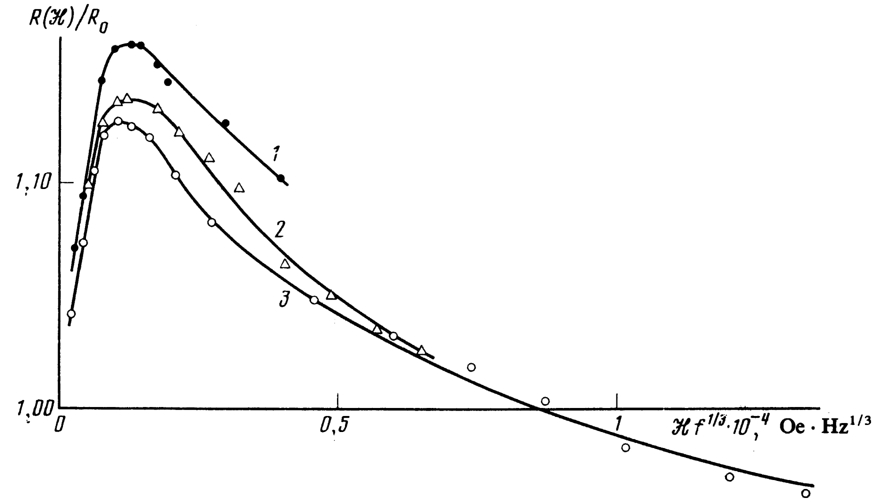

FIG. 5. Nonlinear surface resistance of tungsten (sample W3) at different frequencies. The alternating magnetic field is directed along the [110] axis. Frequency (kHz): 1) 1.8; 2) 20; 3) 98.

its influence on the electron paths and, therefore, to the conductivity of the investigated metal. The nonlinearity mechanism is known as magnetodynamic. According to Ref. 6, the degree of nonlinearity is governed by the ratio of the path L traveled by an "effective" electron in the skin layer and its mean free path:

$$b=L/l=(8r\delta)^{1/2}/l, \qquad r=cp_F/e\mathscr{H}, \tag{3}$$

where r is the characteristic radius of an electron path, p_F is the Fermi momentum, and e is the electron charge. When the amplitude of the alternating field is low ($b\gg 1$), the paths of the "effective" electrons in the skin layer are almost straight lines and the nonlinearity is weak. The strong nonlinearity corresponds to the inequality $b\ll 1$.

We shall show how it is possible to obtain an analytic dependence $Z(\mathscr{H})$ in a wide range of amplitudes of the incident wave for any value of the nonlinearity parameter b.

2. We shall use the well-known formula for the surface impedance:

$$Z(\mathscr{H})=-\frac{8i\omega}{c^2}\int_0^\infty \frac{dk}{k^2-(4\pi i\omega/c^2)\sigma(k)}. \tag{4}$$

Equation (4) is exact when the Fourier components of the current density $j(k)$ and of the electric field $E(k)$ are linked by the local relationship $j(k)=\sigma(k)E(k)$. Moreover, the validity of Eq. (4) requires that the conductivity $\sigma(k)$ should be independent of time. These requirements are not satisfied in the nonlinear case under discussion. However, if we replace the actual conductivity operator with the effective conductivity $\sigma_{\rm eff}(k)$ which gives the correct (in respect of the order of magnitude) value of the current density $j(k)$, we then find that Eq. (4) describes qualitatively the dependence of the impedance on the amplitude of the incident wave.

Using the asymptotically accurate results,[6] we shall write down the effective conductivity of a metal in the form

$$\sigma(k)=\frac{3\pi}{4}\frac{\sigma_0}{kl}F(b), \tag{5}$$

where $\sigma_0=ne^2/m\nu$; m and n are the electron mass and the electron density, respectively; ν is the collision frequency. The factor $(kl)^{-1}$ in Eq. (5) describes the spatial dispersion under the conditions of the anomalous skin effect and the function $F(b)$ governs the dependence of the conductivity on the magnetodynamic nonlinearity parameter. Since the main contribution to the integral (4) is made by $k\propto\delta^{-1}$, the value of b depends on the wave number k:

$$b=(8r/kl^2)^{1/2}\equiv b_0(k\delta_0)^{-1/2}, \qquad \delta_0=(c^2l/3\pi^2\omega\sigma_0)^{1/3}. \tag{6}$$

We have introduced here a real nonlinearity parameter b_0 related to the depth of the skin layer δ_0 in the linear theory. Substituting Eqs. (5) and (6) in the expression for the impedance (4), we obtain

$$Z(\mathscr{H})=-\frac{8i\omega\delta_0}{c^2}\int_0^\infty \frac{q\,dq}{q^3-iF(b_0/q^{1/2})}. \tag{7}$$

Some conclusions relating to the dependence of the impedance on the amplitude $\mathscr{H}$, frequency ω, and mean free path l can be drawn already at this stage without knowing

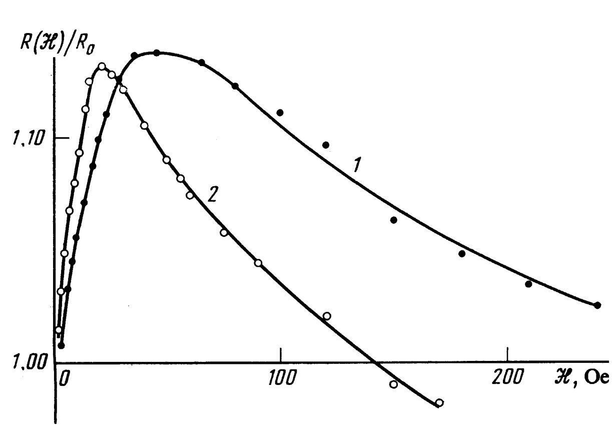

FIG. 6. Influence of temperature on the nonlinear surface resistance $R(\mathscr{H})$ of tungsten (sample W3) obtained at $f=20$ kHz. Temperature (K): 1) 4.2; 2) 1.6.

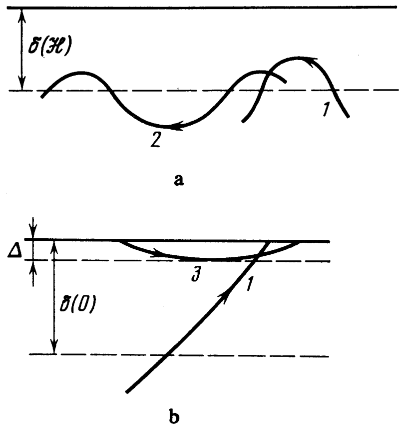

FIG. 7. Paths of "effective" electrons in a magnetic field of a wave in the skin layer: a) strong nonlinearity; b) weak nonlinearity; 1) untrapped electrons; 2) trapped electrons; 3) localized electrons.

the actual form of $F(b)$. It follows from Eq. (7) that the impedance $z = Z(\mathscr{H})/|Z_0|$ normalized to $|Z_0|$ is a universal function of the parameter $b_0 \propto \mathscr{H}^{-1/2} f^{-1/6} l^{-1}$

$$z=\frac{3^{1/2}}{2\pi}\int_0^\infty \frac{q\,dq}{F(b_0/q^{1/2})+iq^3}. \tag{8}$$

Variation of the wave frequency and of the mean free path simply alter the scale of $\mathscr{H}$ in the dependence $z(\mathscr{H})$.

3. We shall now find the function $f(b)$ because its nature governs the actual dependence $z(\mathscr{H})$. It is necessary to write down the expressions for the conductivities of various electron groups. We shall consider the specific case of purely diffuse reflection.

Untrapped electrons. We shall use this term for the electrons that do not have a turning point as they move into the metal (Fig. 7). The conductivity due to these electrons can be described qualitatively by the expression

$$\sigma=N_{\text{eff}}e^2\tau/m. \tag{9}$$

Here, N_{eff} is the number of untrapped electrons and τ is the duration of their interaction with the electromagnetic field. In the weak nonlinearity case we have $N_{\text{eff}} \sim N\delta/l$ because only those electrons are "effective" which move at an angle of δ/l to the surface. The time τ for these electrons is ν^{-1}. Therefore, if $b \gg 1$, the untrapped-electron conductivity is described by the usual anomalous skin formula $\sigma_{\text{untr}} = \sigma_0\delta/l$. In the strong nonlinearity case the number of "effective" electrons increases and becomes equal to $N_{\text{eff}} \sim N(\delta/r)^{1/2} \sim N(\delta/l)b^{-1}$, but the time spent by these electrons decreases by a similar amount: $\tau \sim (r\delta)^{1-2}/v_F \sim b\nu^{-1}$. Therefore, the untrapped-electron conductivity is independent of the degree of nonlinearity and we shall write it in the form

$$\sigma_{\text{untr}}=(3\pi\sigma_0/8kl),\quad F_{\text{untr}}={}^1/_2. \tag{10}$$

Trapped electrons. Such electrons appear because the spatial distribution of the magnetic field in the metal is of variable sign. The electrons trapped by Lorentz force spent their mean free time in the skin layer and in the strong nonlinearity case they made the greatest contribution to the conductivity because their relative number δ/L is $b^{-1} \gg 1$ times greater than the number of the "effective" electrons δ/l. In the weak nonlinearity case the conductivities due to the trapped and transit electrons are of the same orders of magnitude. An estimate of the trapped-electron conductivity for an arbitrary value of b, obtained from Eq. (5), gives

$$\sigma_{\text{trap}}=(3\pi\sigma_0/8kl)\coth b,\quad F_{\text{trap}}=(\coth b)/2. \tag{11}$$

Here, $\coth b = \coth \nu T_0$ allows for multiple returns of the trapped electrons to the skin layer [T_0 is the half-period of the motion of these electrons and $T_0 = (8r\delta)^{1/2}/v_F = b\nu^{-1}$].

Localized electrons. If $b \gg 1$, then the nonlinear correction to the impedance is mainly due to the electrons concentrated in a narrow surface layer of the metal of thickness $\Delta = \delta b^{-2} \ll \delta$ (see Ref. 7b). The path traveled by these electrons in the layer Δ is $(8r\Delta)^{1/2} \sim l$ and they do not leave this layer throughout the mean free time. The relative number of localized electrons $N_{\text{eff}}/N \sim (\Delta/r)^{1/2} \sim (\delta/l)b^{-2}$ is $b^2 \gg 1$ times less than the number of the untrapped particles. We can estimate the contribution of the localized electrons to the conductivity if we allow also for the fact that the associated current is of surface nature and concentrated in a layer of thickness $\Delta \sim \delta b^{-2}$. Consequently, if $b \gg 1$, we find that

$$\sigma_{\text{loc}} \sim \sigma_0 b^{-4}/kl,\quad F_{\text{loc}}=Ab^{-4}. \tag{12}$$

An increase in $\mathscr{H}$ increases the contribution of the localized electrons to the conductivity and at $b \sim 1$ this contribution becomes comparable with that of the transit particles. A further increase in $\mathscr{H}$ causes the value of σ_{loc} to decrease, because in the strong nonlinearity case there are no localized electrons.

We shall write down the following interpolation formula for the resultant conductivity, which is valid for arbitrary values of the parameter b:

$$\sigma(k)=\frac{3\pi}{4}\frac{\sigma_0}{kl}F(b),\qquad F(b)=\frac{1+\operatorname{cth} b}{2}+\frac{A\exp(-B/b^4)}{b^4}. \tag{13}$$

The complex number A can be found from the requirement that the impedance in the $b \gg 1$ case is described by the results of Ref. 6:

$$A=(3^{5/2}/2\pi)\exp(-5\pi i/6). \tag{14}$$

The real parameter B, which is of the order of unity, is used to match the results of the calculations of $z(\mathscr{H})$ in the experimental data. Equation (13), together with Eqs. (14) and (8), describes the required dependence $z(\mathscr{H})$. In the limiting cases ($b \gg 1$ and $b \ll 1$) this dependence is identical with the results obtained in Ref. 6.

Figure 8 shows the results of a calculation of $R(\mathscr{H})/R_0 = 2\text{Re}\, z(\mathscr{H})$ for the value $B = 2$. The abscissa gives $b_0^{-2} \propto \mathscr{H} f^{1/3} l^2$. It should be noted that a change in the parameter b has a strong influence on the amplitude of the maximum dependence $R(\mathscr{H})$; the position of the maximum on the abscissa ($b_0 \sim 1$) then changes only slightly. For convenience of comparison with the theory, the experimental data in Figs. 2

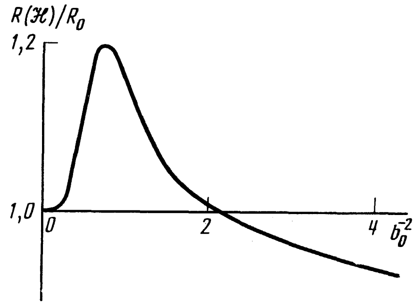

FIG. 8. Calculated dependence of $R(\mathscr{H})/R_0$ on b_0^{-2}.

and 5 are plotted as a function of the parameter $\mathscr{H}f^{1/3} \propto b_0^{-2}$. The absence of accurate data on the mean free path prevented us from using the same coordinates as in Fig. 8.

The calculated curves are in quantitative agreement with the experimental dependences of Figs. 5 and 6. Moreover, the results of measurements of the surface resistance carried out using different values of $\mathscr{H}$, f, and T allow us to draw the conclusion confirming the main prediction of the theory that the dependence of $R(\mathscr{H})/R_0$ on the nonlinearity parameter b_0 is universal. In fact, the curves in Fig. 5 are practically coincident if we allow for the experimental error. Curves 1 and 2 in Fig. 6 can also be made to fit with satisfactory accuracy if the scale at the abscissa for curve 1 is reduced by a factor of 2.2. According to the theory, this reduction factor should be equal to the square of the ratio of the mean free paths. Unfortunately, we are unable to check this quantitatively because we do not know the exact values of l. For the same reason we cannot determine the numerical value of b_0 for given $\mathscr{H}$ and f. We shall obtain a rough estimate of b_0 using the fact that, according to the theory, the maximum of the function $R(\mathscr{H})$ is reached at $b_0 \sim 1$. A quantitative comparison of the experiment and theory is, in principle, possible in the weak ($b_0 \gg 1$) and strong ($b_0 \ll 1$) nonlinearity cases when the asymptotic formulas of Ref. 6 are valid. Under our experimental conditions these inequalities are satisfied in the fields $\mathscr{H} < 2$Oe and $\mathscr{H} > 2$kOe, respectively. In weak fields the resolution of our apparatus was insufficient to determine the dependence $R(\mathscr{H})$. In strong fields the overheating of the samples limited the range of possible investigations. In intermediate fields the experimental values of $R(\mathscr{H})/R_0$ behaved in the same way as the calculated curve, but in this range the theory gave only qualitative results.

In the light of this investigation we can see why the results of measurements of $R(\mathscr{H})/R_0$ for silver (Fig. 2) are not described by a universal function of the parameter $\mathscr{H}f^{1/3}$. According to the results given in §2.2, an increase in the frequency reduces the effective mean free path l_{eff} of the electrons in the skin layer so that the parameter b_0 is greater than for $l = \text{const}$. Consequently, the maximum of $R(\mathscr{H})$ which occurs when $b_0 \sim 1$ is reached in higher fields $\mathscr{H}$. A reduction in l on approach to the surface weakens the role of the localized electrons and, consequently, the role of the associated nonlinear effects, particularly the amplitude of the maximum of $R(\mathscr{H})$. We may assume that the weakening of the nonlinear effects in tungsten on increase in the frequency[5] is of the same origin.

It is of interest to compare the function $R(\mathscr{H})$ with the dependence of the surface resistance on the external static magnetic field H parallel to the surface of a metal. Figure 3 shows this dependence for sample W1. A wide smooth maximum of $R_0(H)$ occurring at $8r\delta_0/l^2 \sim 1$ is known as the background signal. It was already observed in the first cyclotron resonance experiments (see, for example, Refs. 11–13). The theory of this background signal can be found in Ref. 14.

The maxima of the $R(\mathscr{H})$ and $R_0(H)$ curves are of the same origin. They are due to a change in the nature of the interaction of electrons with a wave, which occurs in a magnetic field in which the length of the arc traveled by the "effective" electrons in the skin layer becomes comparable with the mean free path. However, an intrinsic inhomogeneous magnetic field of the wave acts as the external field in the case of the dependence $R(\mathscr{H})$. This is of fundamental importance and the curves $R(\mathscr{H})$ and $R_0(H)$ differ considerably. In the weak nonlinearity case the value of $R(\mathscr{H})$ increases with the field, so that a low static field H makes a negative correction to the impedance. The functions $R(\mathscr{H})$ and $R_0(H)$ differ also in strong fields, because the impedance $Z(\mathscr{H})$ corresponding to high values of $\mathscr{H}$ is specific to a group of trapped electrons in the nonlinear regime, whereas in a static field the reduction in $Z_0(H)$ is due to the return of the electrons to the skin layer along closed paths.

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