Theory of formation of periodic structures on the surfaces of metals and semiconductors by laser radiation

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Some properties of the production of periodic evaporation-damage structures on conductor surfaces by laser radiation are considered theoretically. The nonlinear state of the process, during which an important role is played by the interaction of surface plasma waves in scattering from a periodic profile, is investigated. It is shown that, depending on the incidence angle and on the laser-radiation polarization, the amplitudes of the periodic profile can vary during its formation both monotonically and nonmonotonically with time. The case of a *TE*-polarized wave of laser radiation is considered and the phase trajectories of the periodic profiles are determined for this case.

I. INTRODUCTION

Numerous experiments have shown (see Ref. 1) that high-power laser radiation produces on metal and semiconductor surfaces periodic damage structures with periods of the order of the radiation wavelength. The periods of the structures are determined by the geometry of the experiment and are practically independent of the characteristics of the material. This has led to the conclusion[2] that the structures are due to excitation of surface plasma waves (SPW) whose interference with the incident irradiation wave causes spatial modulation of the energy released from the surface. The periodic structures can develop via various physical mechanisms,[3] one of which is surface evaporation. It was shown in Ref. 4 that a plane metal or semiconductor surface-evaporation front is unstable to the periodic perturbation due to the SPW excitation, and the corresponding growth rates were obtained.

Emel'yanov and Seminogov[5,6] have also confined their investigations to the instability properties of the growth rates and of the ensuing geometric characteristics of the structures. It is known, however, that the pertubations grow at an exponential rate only during the initial, i.e., nonlinear, stage of the unstable process.

We consider here theoretically the succeeding stage of the process, whose character determines the nonlinear phenomena. We shall be interested in that stage during which an important role is played by electrodynamic nonlinearities due to renormalization of the resonance relative to the SPW through scattering by periodic surface distortions. We assume, as in Ref. 4, that the periodic structure are produced by surface evaporation, which we shall describe by using the model developed in Ref. 7.

2. ENERGY RELEASED WHEN LIGHT IS INCIDENT ON A CONDUCTING SURFACE HAVING A PERIODIC PROFILE

Resonant excitation of SPW by light incident on a conductor surface with a periodic profile leads to a considerable increase of the absorption,[8] and in turn to the onset of positive feedback and hence of periodic damage structures. The resonance condition for a plane monochromatic wave corresponds to a continuous set of wave vectors g of the periodic structures. Since the time evolution of the structure formation is initially exponential, and in view of the smallness of the priming fluctuations the typical arguments of the exponentials are large compared with unity, the only significant vectors $\mathbf{g}$ among all that satisfy the resonance condition will be those corresponding to relative maxima of the instability growth rates. We denote by $\{\mathbf{g}_i\}$ the corresponding set of discrete wave vectors. Owing to the fast growth of the exponentials, a substantial role is played in the entire picture of the process only by vicinities of the wave vectors near the values $\mathbf{g}_i$. On this basis, we represent the real structure of the surface as a superposition of a finite number of periodic harmonics. We shall call this description the discrete-mode approximation.

Since the absorbed energy is quadratic in the fields produced in the material, the expression for the energy release will contain, besides the periodic components corresponding to the wave vectors $\mathbf{g}_i$, also harmonics with difference vectors $\mathbf{g}_i - \mathbf{g}_j$, which will generate corresponding harmonics in the surface structure. It will be shown below that the set of wave vectors contains one pair of vectors that differ only in sign: $\mathbf{g} = \mathbf{g}_+$ and $\mathbf{g} = \mathbf{g}_- = -\mathbf{g}_+$. The fact that a surface distortion belongs to one and the same harmonic (degeneracy) makes these vectors exclusive and in need of special treatment. With this taken into account, we specify the equation for the sample surface at an instant of time t in the form

$$z = z_*(\mathbf{r}, t),$$
$$z_*(\mathbf{r}, t) = a(t) + {}^1/_2\Big\{\sum_i{}' b_i(t)e^{i\mathbf{g}_i\mathbf{r}} + \sum_{i>j} b_{ij}(t)e^{i\mathbf{g}_{ij}\mathbf{r}} + \text{c.c.}\Big\},\qquad (2.1)$$
$$\mathbf{g}_{ij} = \mathbf{g}_i - \mathbf{g}_j.$$

Here $z = (\mathbf{n}\cdot\mathbf{r})$ and $(\mathbf{g}_i\cdot\mathbf{n}) = 0$, where $\mathbf{n}$ is an inward unit vector normal to the initially unperturbed (flat) surface. The prime on the sign of summation over i means that the terms with $i = +$ and $i = -$ are taken with half the weight, since they repeat in the complex-conjugate expression: $b_-(t) = b_+^*(t)$.

We assume hereafter that the characteristic absolute amplitudes of the periodic components $b \sim |b_i|$ are small compared with the incident-radiation wavelength, so that we have the inequality

$$bk \ll 1, \quad k=\omega/c, \tag{2.2}$$

in which ω is the frequency of the incident radiation and c is the speed of light.

To solve the electrodynamic problem of the field distribution upon incidence of a plane monochromatic radiation wave on a conducting surface specified by Eq. (2.1), we use the Leontovich boundary condition (see Ref. 9):

$$\left\{\mathbf{E}-\mathbf{m}(\mathbf{mE})-i\frac{\zeta}{k}[\mathbf{m}\times\operatorname{rot}\mathbf{E}]\right\}_{z=z_s(\mathbf{r},t)}=0, \tag{2.3}$$

in which $\mathbf{E}=\mathbf{E}_0+\mathbf{E}_1$ is the sum of electric field intensity vectors of the incident and reflected radiation waves, $\mathbf{m}$ is the normal to the surface (2.1), and $\zeta=\zeta(\omega)$ is the surface impedance. We recall that the condition (2.3) can be used in place of complete system of boundary conditions only if the following inequalities are satisfied:

$$|\zeta|^2 \ll 1, \quad |\zeta|/kR \ll 1, \tag{2.4}$$

where R is the minimum curvature radius of the surface. The first inequality of (2.4) limits the incident-radiation frequency, requiring that it be small compared with the frequency of the bulk plasma oscillations. Since $|\mathbf{g}_i|\sim k$, we get, taking (2.1) into account, the estimate $(kR)^{-1}\sim bk$. The second inequality of (2.4) reduces therefore to the condition $|\zeta|bk \ll 1$, which is certainly satisfied in view of (2.2) and of the first inequality of (2.4). When we expand later in powers of the parameter bk, we shall neglect, in comparison, those powers of ζbk whose inclusion takes us outside the Leontovich approximation.

We write the electric field vector of the incident plane monochromatic light wave in the form

$$\begin{gathered}\mathbf{E}_0(\mathbf{r},t)=[E_{0s}\mathbf{e}_s(\mathbf{k})+E_{0p}\mathbf{e}_p(\mathbf{k})]\,e^{i(\mathbf{kr}-\omega t)},\\ \mathbf{e}_s(\mathbf{k})=[\mathbf{nk}]/|[\mathbf{nk}]|,\quad \mathbf{e}_p(\mathbf{k})=[[\mathbf{nk}]\mathbf{k}]/|[[\mathbf{nk}]\mathbf{k}]|.\end{gathered} \tag{2.5}$$

Here E_{0p} and E_{0s} are the projections of the wave amplitude on the polarization directions in the incident plane and in the plane normal to it (T_E and T_M polarizations), $\mathbf{k}=\mathbf{q}+\mathbf{n}k\cos\theta$ is the wave vector of the incident wave, $(\mathbf{q}\cdot\mathbf{n})=0$, $q\equiv|\mathbf{q}|=k\sin\theta$, and θ is the incidence angle. The field intensity of the wave reflected from the surface (2.1) can be represented in the form

$$\mathbf{E}_1(\mathbf{r},t)=\sum_{\{\mathbf{K}\}}[E_{1s}(\mathbf{K})\mathbf{e}_s(\mathbf{K})+E_{1p}(\mathbf{K})\mathbf{e}_p(\mathbf{K})]\,e^{i(\mathbf{Kr})-\omega t}, \tag{2.6}$$

here $\{\mathbf{K}\}$ is a system of wave vectors defined by the equations

$$\mathbf{K}=\mathbf{Q}-\mathbf{n}kW_{\mathbf{Q}},\quad \mathbf{Q}=\mathbf{q}+\sum_i l_i\mathbf{g}_i,\quad W_{\mathbf{Q}}=(1-Q^2/k^2)^{1/2}, \tag{2.7}$$

and l_i are integers. The polarization vectors $\mathbf{e}_s$ and $\mathbf{e}_p$ in (2.6) are defined in (2.5).

Substitution of (2.5) and (2.6) in the boundary condition (2.3) and expansion of the latter in the set of functions $\exp(i\mathbf{Q}\cdot\mathbf{r})$ reduces the problem to an infinite system of algebraic equations in the amplitudes $E_{1s}(\mathbf{K})$ and $E_{1p}(\mathbf{K})$. In the zeroth order in the small parameter bk the solution of the system is described by the known Fresnel formulas[9] for the reflection of radiation from a flat surface.

In first order in bk, the solution contains terms $\propto bke_p(\mathbf{K})\times\exp[i(\mathbf{K}\cdot\mathbf{r}-\omega t)]/B(\mathbf{Q})$, in which $B(\mathbf{Q})=\zeta+W_{\mathbf{Q}}$, $\mathbf{Q}=\mathbf{q}\pm\mathbf{g}_i$, or $\mathbf{Q}=\mathbf{q}\pm\mathbf{g}_{ij}$. A distinguishing feature of such terms is that they contain denominators $B(\mathbf{Q})$ that can have at $|\mathbf{Q}|\approx k$ an absolute value much smaller than unity (resonance situation). The equation $B(\mathbf{Q})=0$ in $k=\omega/c$ determines the well-known spectrum and damping of the PSW on a flat surface. Naturally, the resonance is preserved in the exact solution, but undergoes a rescaling whose role increases with increasing parameter bk. The rescaling describes the frequency shift and the additional SPW damping, which are due to the periodic profile of the surface. The smallness of the SPW damping (the sharpness of the resonance) makes the rescaling of the resonance, as the bk parameter increases, substantial long before the condition (2.2) is violated. In this situation it is not legitimate to use for the solution of the problem a finite expansion in powers of the parameter bk.

An analysis of the system of equations for the amplitude $E_{1s,p}(\mathbf{K})$, which follows from the condition (2.3), shows that the resonant contributions to the nonresonant harmonics of the reflected field (which correspond to $\mathbf{Q}\neq\mathbf{q}+\mathbf{g}_i$) appear in higher orders of perturbation theory and thus contain, compared with the resonant harmonics ($\mathbf{Q}_r=\mathbf{q}+\mathbf{g}_i$) and additional power of the small parameter bk, whose exponent is equal to the minimum number of steps (as measured by the vectors $\mathbf{g}_i$ and $\mathbf{g}_{ij}$) that separate a given vector $\mathbf{Q}$ from its nearest neighbor $\mathbf{Q}_r$. This allows us to express, accurate to small quantities of order $\sim bk$), the solution (2.6) in the form

$$\begin{aligned}E_1(r,t)=\Big\{\Big[&-\frac{1-\zeta\cos\theta}{1+\zeta\cos\theta}E_{0s}\mathbf{e}_s(\mathbf{K}_0)\\&+\frac{-\zeta+\cos\theta}{\zeta+\cos\theta}E_{0p}\mathbf{e}_p(\mathbf{K}_0)\Big]e^{i\mathbf{K}_0\mathbf{r}'}\\&+(|E_{0s}|^2+|E_{0p}|^2)^{1/2}\sum_i\mathscr{E}_i\mathbf{e}_p(\mathbf{K}_i)e^{i\mathbf{K}_i\mathbf{r}'}\Big\}e^{i(\mathbf{kn}a(t)-\omega t)}.\end{aligned} \tag{2.8}$$

Here

$$\mathbf{r}'=\mathbf{r}-\mathbf{n}a(t),\quad \mathbf{K}_0=\mathbf{q}-\mathbf{n}k\cos\theta,\quad \mathbf{K}_i=\mathbf{q}+\mathbf{g}_i-\mathbf{n}kW_{q+g_i}.$$

The term proportional to $\exp(iK_0r')$ in (2.8) corresponds to a wave reflected from a plane surface. Substitution of (2.8) and (2.5) in (2.3) and projection of the latter on the resonance harmonics (the functions $\mathbf{e}_p(\mathbf{K}_i)\,e^{i\mathbf{K}_i\mathbf{r}}$) leads to the sought system of equations for the resonance amplitudes $\mathscr{E}_i$ in (2.8):

$$\begin{aligned}&B_i\mathscr{E}_i=ib_ikA(\alpha_i)-i\sum_j b_{ij}k\sin^2\frac{\alpha_{ij}}{2}\mathscr{E}_j,\\&B_i\equiv B(\mathbf{q}+\mathbf{g}_i)=\zeta+W_{q+g_i},\\&A(\alpha_i)=(|E_{0s}|^2+|E_{0p}|^2)^{-1/2}\\&\qquad\times[E_{0p}(\cos\alpha_i-\sin\theta)-E_{0s}\cos\theta\sin\alpha_i].\end{aligned} \tag{2.9}$$

Here $\alpha_{ij}=\alpha_i-\alpha_j$, and α_i is the angle between the vectors $\mathbf{q}+\mathbf{g}_i$ and $\mathbf{q}$.

The second term on the right in the system (2.9) for $\mathscr{E}_i$ describes the interaction between the resonant modes, and leads to rescaling, in this case of order bk, of the resonant denominators B_i. A rescaling of higher order of smallness $[\sim(bk)^2]$ appears when there is only one resonant harmonic.[8] This rescaling is due to the interaction of the resonant harmonic and the nonresonant harmonics that are located one step away from it; the latter harmonics must be taken into account in this case.

We proceed now directly to calculate of the energy release $\Phi(\mathbf{r})$ determined by the energy absorbed per unit volume and per unit time and averaged over the temporal period of the radiation. We note that under conditions when the Leontovich approximation can be used the electric and magnetic field-intensity vectors inside the sample reduce to their components tangential to the surface, and differ from their values on the surface by an additional factor $\exp[i(z - z_*(\mathbf{r}))k/\zeta]$. Taking into account in the expression for the energy flux density [Eq. (87.4) of Ref. 9] and the energy-continuity equation, we get therefore

$$\Phi = \frac{c\mu\zeta'}{8\pi k^2}|[\mathbf{n}\,\mathrm{rot}\,\mathbf{E}]|^2_{z=z_*}e^{-\mu(z-z_*)},$$

$$\mu = 2k\,\mathrm{Im}\,\zeta^{-1}, \quad \zeta' = \mathrm{Re}\,\zeta. \tag{2.10}$$

Substituting here Eqs. (2.8) and (2.5) and neglecting small corrections $\sim bk$ and $|\zeta|$, we arrive at the following equations for the energy release:

$$\Phi(\mathbf{r}) = \left\{\Phi_0 + \left[\sum_i{}' \Phi_i e^{i\mathbf{g}_i\mathbf{r}} + \sum_{i>j}\Phi_{ij}e^{i\mathbf{g}_{ij}\mathbf{r}} + \text{K. c.}\right]\right\}e^{-\mu(z-z_*)}; \tag{2.11}$$

$$\Phi_0 = \bar{\Phi}\left[\frac{|E_{0s}|^2\cos^2\theta + |E_{0p}|^2}{|E_{0s}|^2 + |E_{0p}|^2} + \frac{1}{4}\sum_i|\mathscr{E}_i|^2\right]; \tag{2.12}$$

$$\Phi_i = {}^1/_2\bar{\Phi}\bar{A}^*(\alpha_i)\mathscr{E}_i, \quad i \neq +, -; \tag{2.13}$$

$$\Phi_+ = {}^1/_2\bar{\Phi}(\bar{A}^*(\alpha_+)\mathscr{E}_+ + \bar{A}(\alpha_-)\mathscr{E}_-^*), \quad \Phi_- = \Phi_+^*; \tag{2.14}$$

$$\Phi_{ij} = {}^1/_4\bar{\Phi}\cos\alpha_{ij}\mathscr{E}_i\mathscr{E}_j^*; \tag{2.15}$$

$$\bar{\Phi} = \frac{c\mu\zeta'}{2\pi}(|E_{0s}|^2 + |E_{0p}|^2); \tag{2.16}$$

$$\bar{A}(\alpha_i) = \frac{1}{(|E_{0s}|^2 + |E_{0p}|^2)^{1/2}}[E_{0p}\cos\alpha_i - E_{0s}\cos\theta\sin\alpha_i]. \tag{2.17}$$

We note that in view of the isotropy of the SPW dispersion law, the resonance condition for the resonant harmonic $(\mathbf{g} = \mathbf{g}_+, \mathbf{g}_-)$ leads to the requirement $|\mathbf{q} + \mathbf{g}_+| = |\mathbf{q} - \mathbf{g}_+|$, which means that the vectors $\mathbf{q}$ and $\mathbf{g}_+$ are mutually perpendicular. For this reason and as a result of the equalities $|\mathbf{q} + \mathbf{g}_i| \approx k$, and $q = k\sin\theta$ we have

$$\alpha_+ \approx \pi/2 - \theta, \quad \alpha_- \approx -(\pi/2 - \theta). \tag{2.18}$$

3. EVAPORATION WITH PERIODIC FRONT

Assuming that the laser intensity is in the range in which the processes that occur in the gas phase do not influence the evaporation,[11] we use the formulation of Ref. 7 for the problem of the motion of an evaporation front. It reduces to the heat-conduction equation with appropriate boundary conditions

$$c_p\frac{\partial T}{\partial t} = \varkappa\Delta T + \Phi, \tag{3.1}$$

$$\varkappa\left(\mathbf{m}\frac{\partial T}{\partial \mathbf{r}}\right)_* = v(\mathbf{r}, t)\Delta W, \tag{3.2}$$

$$v(\mathbf{r}, t) = c_0\exp\left[-\frac{U}{T_*(\mathbf{r}, t)}\right]. \tag{3.3}$$

Here c_p is the specific heat per unit volume, $\varkappa$ is the thermal conductivity coefficient, ΔW the jump of the enthalpy density on the phase boundary, $v(\mathbf{r}, t)$ the velocity of the evaporation at the surface point $\mathbf{r}$, U the activation energy (of the order of several eV), c_0 a constant of the order of the speed of sound in the condensed phase, and the asterisk indicates that the argument of $\mathbf{r}$ pertains to the sample boundary.

When account is taken of the structure of Eq. (2.11) for the energy release $\Phi(\mathbf{r})$, the temperature function can be written as

$$T(\mathbf{r}, t) = T_0(\xi, t) + \sum_l(T_l(\xi, t)e^{i\mathbf{g}_l\mathbf{r}} + \text{K. c.}), \quad \xi = z - z_*(\mathbf{r}, t). \tag{3.4}$$

The symbol l takes on here and elsewhere the value i or ij $(i>j)$. Substitution of (3.4) in Eqs. (3.1)–(3.3) and expansion of the latter in terms of harmonics $\exp(i\mathbf{g}_l\cdot\mathbf{r})$ leads to a system of differential equations and to boundary conditions in which the unknowns T_0 and T_l are entangled with each other. With account taken of the inequalities $bk, B_i \ll 1$ and upon satisfaction of the condition

$$\frac{U\Delta W_*}{c_p T_*^2}\frac{v}{|g_l|\chi} \ll 1, \quad \chi = \frac{\varkappa}{c_p}, \tag{3.5}$$

the system indicated becomes formally disentangled, with the part pertaining to $T_0(\xi, t)$ reduced to the one-dimensional problem of evaporation[11] with an energy release given by Eq. (2.12), while the equations and boundary conditions for the remaining $T_l(\xi, t)$ take the form

$$c_p\frac{\partial T_l}{\partial t} = \varkappa\left(\frac{\partial^2 T_l}{\partial\xi^2} - g_l^2 T_l\right) + \Phi_l e^{-\mu\xi}, \tag{3.6}$$

$$(\partial T_l/\partial\xi)_{\xi=0} = 0, \tag{3.7}$$

$$\frac{db_l}{dt} = v_0\frac{U}{T_0^2(0)}T_l(0), \quad v_0 = c_0\exp(-U/T_0(0)). \tag{3.8}$$

We shall be interested hereafter in processes that take place in times (reckoned from the start of the laser-pulse action) that are sufficient for the heat to propagate over distances that are larger with the characteristic dimension $|\mathbf{g}_l|^{-1}$ of Eq. (3.6), i.e., $t > (\chi\,\mathbf{g}_l^2)^{-1}$. We assume also that the periodic energy-release components Φ_l, which depend on the time via the quantities $\mathscr{E}_i$, satisfy the condition of slow variation in the time scale $(g_l^2\chi)^{-1}$. In this case we can neglect in (3.6) the time derivatives so that its solution, with boundary condition (3.7) and with allowance for the inequality $|\mathbf{g}_l|/\mu \sim |\zeta|/2 \ll 1$, becomes

$$T_l = \frac{\Phi_l}{\varkappa\mu|g_l|}e^{-|g_l|\xi}. \tag{3.9}$$

Taking into consideration expressions (2.12–2.14) and the system (2.9), we find from the condition (3.8) and with allowance for expression (3.9) and for the ensuing heat-conduction equation, that the required slow rate of change of Φ_l

is achieved under the inequality

$$\left(\frac{v}{|g_i|\chi}\right)^2 \frac{U}{T_0(0)}\frac{1}{\zeta'} \ll 1. \qquad (3.10)$$

Substituting (2.13) and (2.14) in (3.9) and next (3.9) in (3.8) we get the equation of motion for the dimensionless amplitudes $\tilde{b}_l = b_l k$ of the periodic structure

$$\frac{d\tilde{b}_i}{dt} = F\frac{k}{|g_i|}\bar{A}^*(\alpha_i)\mathscr{E}_i, \quad i=+,-; \qquad (3.11)$$

$$\frac{d\tilde{b}_+}{dt} = F\frac{k}{|g_+|}(\bar{A}^*(\alpha_+)\mathscr{E}_+ + \bar{A}(\alpha_-)\mathscr{E}_-^*); \qquad (3.12)$$

$$\frac{d\tilde{b}_{ij}}{dt} = \frac{1}{2}F\frac{k}{|g_{ij}|}\cos\alpha_{ij}\mathscr{E}_i\mathscr{E}_j^*. \qquad (3.13)$$

The quantity F in these equations has the dimension of reciprocal time and is defined as

$$F = v_0\frac{U}{T_0^2(0)}\frac{\Phi}{\varkappa\mu}. \qquad (3.14)$$

4. STEADY-STATE EVAPORATION REGIMES

From the set of differential equations (3.11–3.13) together with the algebraic system (2.9) we determine the time evolution of the periodic evaporation front. It must be borne in mind that the quantity F in (3.11)–(3.13) is a fluctuating function of the amplitudes b_i via the solution of the evaporation problem with a plane front, since the corresponding expression (2.12) for the energy release Φ_0 contains the resonant fields $\mathscr{E}_i$. Even this alone makes it difficult to determine analytically in general form the evolution of the amplitudes of the periodic profile. Nonetheless some characteristics and furthermore important features of the solutions can be fully explained.

We note for this purpose that according to (2.9) and (3.11)–(3.13) the characteristic scale of the dimensionless amplitudes $\tilde{b}_i$ and $\tilde{b}_{ij}$ is the quantity $|B_i| \sim \zeta'$. Therefore, after a sufficiently long time, two different situations are possible for any of the amplitudes $\tilde{b}_l$: the amplitude $\tilde{b}_l$ either remains finite ($|\tilde{b}_l| \lesssim \zeta'$), or becomes large compared with ζ' and continues to increase. We shall distinguish hereafter between the two situations by calling them, respectively, finite and infinite motion. To establish the situation for a particular amplitude $\tilde{b}_l$ we assume that some of the amplitudes correspond to the case of infinite motion. At a sufficiently long time we can then assume the left-hand side of (2.9) to be zero, after which it follows from an examination of (2.9) and (3.11)–(3.13) that $\mathscr{E}_i = \text{const}$. Then $\mathscr{E}_i = 0$ corresponds to finite motion and $\mathscr{E}_i \neq 0$ to infinite motion.

Taking the foregoing into account and combining Eqs. (2.9) and (3.11)–(3.13), we get

$$\left\{\frac{k}{|g_i|}A(\alpha_i)\bar{A}^*(\alpha_i) - \frac{1}{4}\sum_j \cos\alpha_{ij}\left|\sin\frac{\alpha_{ij}}{2}\right| |\mathscr{E}_j|^2\right\}$$

$$\times\mathscr{E}_i = 0, \quad i \neq +,-; \qquad (4.1)$$

$$\frac{k}{|g_+|}A(\alpha_+)(\bar{A}^*(\alpha_+)\mathscr{E}_+ + \bar{A}(\alpha_-)\mathscr{E}_-^*)$$

$$-\frac{1}{4}\sum_j \cos\alpha_{+j}\left|\sin\frac{\alpha_{+j}}{2}\right| |\mathscr{E}_j|^2\mathscr{E}_+ = 0; \qquad (4.2)$$

$$\frac{k}{|g_-|}A(\alpha_-)(\bar{A}^*(\alpha_-)\mathscr{E}_- + \bar{A}(\alpha_+)\mathscr{E}_+^*)$$

$$-\frac{1}{4}\sum_j \cos\alpha_{-j}\left|\sin\frac{\alpha_{-j}}{2}\right| |\mathscr{E}_j|^2\mathscr{E}_- = 0. \qquad (4.3)$$

This system of equation reveals the substantial difference, with respect to formation of periodic structures, between the general case of elliptic polarization of the incident radiation and that of linear polarization. In the former case the product $A(\alpha_i)\bar{A}^*(\alpha_j)$ is, in accordance with the definition of its factors [see (2.9) and (2.17)], a complex quantity and as a result, as shown by analysis of the system (4.1)–(4.3), the evolution of all the harmonics for elliptically polarized light corresponds to finite motion. The situation is different in the case when the radiation is linearly polarized ($A(\alpha_i)\bar{A}^*(\alpha_j)$ is real), which we shall now consider in greater detail.

We begin with finding the possible periodic structures. In accord with the meaning of the discrete-mode approximation, the wave vectors $\mathbf{g}_i$ are determined from the condition that the growth rate be a maximum during the initial stage of the periodic-structure formation. It follows from (2.9), (3.11), and (3.12) that at $|\tilde{b}(\mathbf{g},t)| \ll \zeta'$ the time dependence of the periodic-profile amplitudes is of the form

$$\tilde{b}(\mathbf{g},t) = \tilde{b}(\mathbf{g},0)\exp\left[R(\mathbf{g})A(\alpha_g)\bar{A}^*(\alpha_g)\frac{k}{|\mathbf{g}|}\int_0^t dt' F(t')\right],$$

$$R(\mathbf{g}) = \frac{i}{B(\mathbf{q}+\mathbf{g})} \quad \mathbf{g} \neq \mathbf{g}_\pm; \quad R(\mathbf{g}_\pm) = -2\,\mathrm{Im}\frac{1}{B(\mathbf{q}+\mathbf{g}_\pm)}. \qquad (4.6)$$

Hence, taking into account the expression for $B(\mathbf{q}+\mathbf{g})$ [see (2.9) and (2.7)] we find that the amplitudes $\tilde{b}(\mathbf{g}, t)$ as functions of the vector $\mathbf{g}$ have a maximum exponential growth with time under the following conditions

$$|\mathbf{q}+\mathbf{g}| \approx k, \quad R(\mathbf{g}) = \frac{1+i}{2\zeta'}, \quad \mathbf{g} \neq \mathbf{g}_\pm; \quad R(\mathbf{g}_\pm) = \frac{1}{\zeta'}; \qquad (4.7)$$

$$\frac{\partial}{\partial\alpha}\left[A(\alpha)\bar{A}^*(\alpha)\frac{1}{(1+\sin^2\theta - 2\sin\theta\cos\alpha)^{1/2}}\right] = 0. \qquad (4.8)$$

The condition (4.8) is mandatory only for nondegenerate modes. The degenerate one, in view of its inherent advantage $R(\mathbf{g}_\pm) = 2\,\mathrm{Re}\,R(\mathbf{g} \neq \mathbf{g}_\pm)$, is automatically one of the preferred modes that correspond to a relative maximum of the degree of the exponential growth.

We restrict ourselves now to incident radiation polarized perpendicular to the incidence plane, to which most present-day research into this topic is devoted. After substituting the expressions for $A(\alpha)$ and $\bar{A}^*(\alpha)$ in (4.8) we find that for the TE wave ($E_{0p} = 0$) it has only two solutions

$$\alpha_{1,2}=\pm\left(\frac{\pi}{2}-\Delta\right),$$

$$\Delta=\arcsin\left[\frac{\sin\theta}{1+\sin^2\theta+(1-\sin^2\theta+\sin^4\theta)^{1/2}}\right]. \quad (4.9)$$

We have thus alongside the doubly degenerate mode only four harmonics, corresponding to the amplitudes $\tilde{b}_1, \tilde{b}_2, \tilde{b}_+$, $\tilde{b}_-=\tilde{b}_+^*$ and to the resonant modes $\mathscr{E}_1, \mathscr{E}_2, \mathscr{E}_+, \mathscr{E}_-$. In view of the equality $\overline{A}(\alpha_-)=-\overline{A}(\alpha_+)$ that follows from (2.17), the system (4.1)–(4.3) leads to the relations $\mathscr{E}_-=-\mathscr{E}_+^*$, $|\mathscr{E}_1|^2=|\mathscr{E}_2|^2$, in terms of which our system takes the form

$$[8\cos^3\theta-C|\mathscr{E}_1|^2+\cos\theta\cos 2\theta|\mathscr{E}_+|^2]\mathscr{E}_+=0,$$
$$\left[4\frac{\cos^2\theta\cos^2\Delta}{(1+\sin^2\theta-2\sin\theta\sin\Delta)^{1/2}}-C|\mathscr{E}_+|^2+\cos\Delta\cos 2\Delta|\mathscr{E}_1|^2\right]\mathscr{E}_1=0, \quad (4.10)$$

where

$$C(\theta)=\cos(\theta-\Delta)\sin\frac{\theta-\Delta}{2}-\cos(\theta+\Delta)\cos\frac{\theta+\Delta}{2}.$$

The solution of (4.10) is

$$|\mathscr{E}_1|^2=0,\quad |\mathscr{E}_+|^2=\begin{cases}0, & \theta<\pi/4;\\ 8\cos^2\theta/|\cos 2\theta|, & \theta>\pi/4.\end{cases} \quad (4.11)$$

Only the degenerate mode at an incidence angle $\theta>\pi/4$ is thus infinite.

It must be noted that the growth rates of the degenerate mode and of modes 1 and 2 turn out to be equal if the incidence angle ranges from $\pi/3$ to $\pi/2$. The absolute maximum of the growth rate corresponds here to the degenerate mode at $\theta>\theta_*$, and to modes 1 and 2 to $\theta<\theta_*$. In view of the appreciable difference between the growth rates when the angle θ is not too close to θ_*, the interaction between the degenerate mode, on the one hand, and modes 1 and 2, on the other, is shown by analysis to be negligibly small in the principal range of the variables. Therefore the general system of equations (3.11)–(3.13) and (2.9) breaks up into two subsystems, on for $\tilde{b}_+, \tilde{b}_-, \tilde{b}_{+-}, \mathscr{E}_+, \mathscr{E}_-$ and the other for $\tilde{b}_1, \tilde{b}_2, \tilde{b}_{12}, \mathscr{E}_1, \mathscr{E}_2$.

We start with the first of the system (those equations of (2.9) and (3.11)–(3.13) which contain $\tilde{b}_+, \tilde{b}_-, \tilde{b}_{+-}, \mathscr{E}_+, \mathscr{E}_-$, with $\tilde{b}_{\pm,1}$ and $\tilde{b}_{\pm,2}$ set equal to zero). Analysis do not depend on the time and that those quantities can be represented in the form

$$\tilde{b}_\pm(t)=|\tilde{b}_\pm(t)|e^{\pm i\varphi},\quad \tilde{b}_{+-}(t)=|\tilde{b}_{+-}(t)|\frac{\cos 2\theta}{|\cos 2\theta|}e^{2i\varphi}. \quad (4.12)$$

Here $\varphi=\operatorname{Im}\ln\tilde{b}_+(0)$ is the initial phase of the amplitude of the $\tilde{b}_+(t)$ profile. Taking (4.12) into account, after excluding the fields $\mathscr{E}_+$ and $\mathscr{E}_-$ from the subsystem of equations for $\tilde{b}_+, \tilde{b}_-, \tilde{b}_{+-}, \mathscr{E}_+, \mathscr{E}_-$ we arrive at a system of equations for the absolute values of the amplitudes:

$$\frac{d|\tilde{b}_+|}{dt}=2F\frac{|A(\alpha_+)|^2}{\cos\theta}|\tilde{b}_+|\left(\zeta'-|\tilde{b}_{+-}|\frac{\cos 2\theta}{|\cos 2\theta|}\cos^2\theta\right)\times\left[\zeta'^2+\left(\zeta'-|\tilde{b}_{+-}|\frac{\cos 2\theta}{|\cos 2\theta|}\cos^2\theta\right)^2\right]^{-1},$$
$$\frac{d|\tilde{b}_{+-}|}{dt}=\frac{1}{4}F|A(\alpha_+)|^2\frac{|\cos 2\theta|}{\cos\theta}|\tilde{b}_+|^2\times\left[\zeta'^2+\left(\zeta'-|\tilde{b}_{+-}|\frac{\cos 2\theta}{|\cos 2\theta|}\cos^2\theta\right)^2\right]^{-1}. \quad (4.13)$$

This leads directly to the equation

$$\frac{d}{dt}\left\{\frac{1}{8}|\tilde{b}_+|^2\cos 2\theta\cos^2\theta+\left(\zeta'-|\tilde{b}_{+-}|\frac{\cos 2\theta}{|\cos 2\theta|}\cos^2\theta\right)^2\right\}=0,$$

whose solution, assuming that the inequalities $|\tilde{b}_+(0)|$, $|\tilde{b}_{+-}(0)|\ll\zeta'$ hold, is

$$\frac{1}{8}|\tilde{b}_+|^2\cos 2\theta\cos^2\theta+\left(\zeta'-|\tilde{b}_{+-}|\frac{\cos 2\theta}{|\cos 2\theta|}\cos^2\theta\right)^2=\zeta'^2. \quad (4.14)$$

By establishing the relation between $|\tilde{b}_+|$ and $|\tilde{b}_{+-}|$, this equation specifies the trajectory for the process considered by us, that of formation of periodic structures. At $\theta<\pi/4$ this trajectory is a semi-ellipse in the $(|\tilde{b}_{+-}|, |\tilde{b}_+|)$ plane, with vertical semi-axis $(\zeta'/\cos\theta)(8/\cos 2\theta)^{1/2}$ and a horizontal semi-axis $\zeta'/\cos^2\theta$. At $\theta=\pi/4$ the trajectory is straight vertical line drawn from the origin, and at $\theta>\pi/4$ it is part of a hyperbola whose asymptote has an intercept $\zeta'/\cos^2\theta$ on the abscissa axis and a slope $\cos\theta(8/|\cos 2\theta|)^{1/2}$ (see Fig. 1).

The trajectories for the profile amplitudes $\tilde{b}_1, \tilde{b}_2, \tilde{b}_{12}$ (second subsystem of the equations) are determined similarly. It turns out that $|\tilde{b}_2|=|\tilde{b}_1|$ and the connection between $|\tilde{b}_1|$ and $|\tilde{b}_{12}|$ is

$$^1/_4|\tilde{b}_1|^2\cos 2\Delta\cos\Delta(1+\sin^2\theta-2\sin\theta\sin\Delta)^{1/2}+(\zeta'-|\tilde{b}_{12}|\cos^2\Delta)^2=\zeta'^2. \quad (4.15)$$

Since it can be shown by using (4.9) that $\cos 2\Delta>0$, Eq. (4.15) describes at all incidence angles an elliptic curve, so that the corresponding trajectory takes the form shown in Fig. 1a.

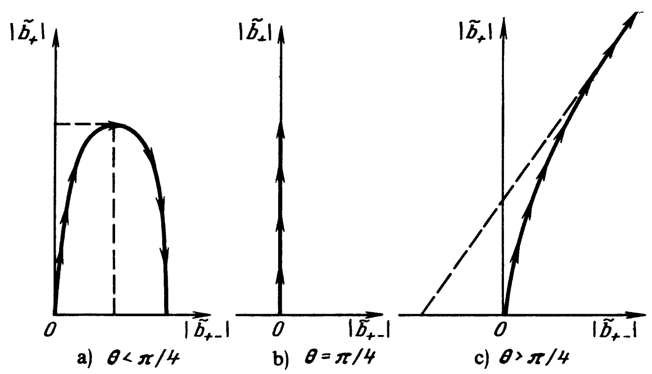

FIG. 1.

5. CONCLUSION

In this section we discuss briefly the meaning of the approximation assumed and touch upon the connection between the results and experiment. One of the main assumptions was to replace the continuous distribution, in terms the wave vectors of the Fourier expansion for the amplitude of the periodic distortion of the surface, by a discrete set of harmonics (the discrete-mode approximation). This replacement is permissible if the exponentially growing part of the amplitude profile (the initial evolution stage), $\tilde{b} \sim \zeta'$, is significantly larger than the initial value $\tilde{b}(0)$. It follows from (4.6)–(4.8) that the size of the two-dimensional region of the wave vectors g, where the surface-perturbations growth is unstable, is of the order of $|\zeta|\zeta' k^2$ for the nondegenerate modes and $|\zeta|^2\zeta'^2 k^2$ for the degenerate one. Therefore the effective value of the initial profile amplitude, corresponding to the indicated g-space region, is of the order of $\tilde{b}(0) \sim b_0 k^2[|\zeta|\zeta' f(\mathbf{g})]^{1/2}$ for the nondegenerate modes and $\tilde{b}(0) \sim b_0 k^2 |\zeta|\zeta' [f(\mathbf{g})]^{1/2}$ for the degenerate one. Here b_0 is the characteristic value of the initial roughness amplitude, $f(\mathbf{g})$ is the distribution function, normalized by the condition $\int d^2\mathbf{g} f(\mathbf{g}) = 1$, of the squared absolute value of the Fourier component of the initial roughness amplitude. Using this estimate for $\tilde{b}(0)$, condition $\tilde{b}(0) \ll \zeta'$ takes the form

$$b_0 k^2 \left[\frac{|\zeta|}{\zeta'} f(\mathbf{g}) \right]^{1/2} \ll 1, \quad \mathbf{g} \neq \mathbf{g}_\pm;$$

$$b_0 k^2 |\zeta| [f(\mathbf{g})]^{1/2} \ll 1, \quad \mathbf{g} = \mathbf{g}_\pm.$$

We see thus that if the characteristic scale of the argument of the function $f(\mathbf{g})$ is $g^* \gg k$, and if it is proportional to $|\mathbf{g}|^\alpha$ at small $|\mathbf{g}|$, the condition for the validity of the discrete-mode approximation takes the form of the inequalities

$$b_0 k (|\mathbf{g}|/g_*)^{1+\alpha/2} (|\zeta|/\zeta')^{1/2} \ll 1, \quad \mathbf{g} \neq \mathbf{g}_\pm;$$

$$b_0 k (|\mathbf{g}|/g_*)^{1+\alpha/2} |\zeta| \ll 1, \quad \mathbf{g} = \mathbf{g}_\pm,$$

which are readily satisfied, for example at the values $b_0 \lesssim 10^{-6}$ cm, $g^* \gtrsim 10^6$ cm^{-1}, $2\pi/k \gtrsim 10^{-4}$ cm, $\alpha > 1$, $\zeta' \gtrsim 10^{-2}$, $|\zeta| \sim 10^{-1}$.

Two other approximations made by us reduce to the inequalities (3.5) and (3.10). It is shown in Ref. 11 that in the problem with a plane evaporation front the parameter $v/\mu\chi \sim (v/|\mathbf{g}|\chi)|\mathrm{Im}\,\zeta|/2$ increases with increasing incident-radiation intensity, and in the range of problem parameters of interest it takes on values in the interval 10^{-3}–10^{-1}. The values of U and $\Delta W_*/c_p$ are in the range 1–3 eV. Assuming $|\mathrm{Im}\,\zeta| \sim 10^{-1}$, $T_* \sim (3\text{–}5)10^{-1}$ eV, and $\zeta' \gtrsim 10^{-2}$ we find then that at not too high incident-radiation intensities the conditions (3.5) and (3.10) are both satisfied.

One of the important features of the results of the preceding section is the conclusion that if a *TE* wave is incident on a conductor at $\theta < \pi/4$ the absolute value of the amplitude $\tilde{b}_+$ depends on $|\tilde{b}_{+-}|$ nonmonotonically. On the other hand, according to the second equation of (4.13), $|\tilde{b}_{+-}|$ is a non-decreasing function of time at all θ. Therefore the dependence of $|\tilde{b}_+|$ on $|\tilde{b}_{+-}|$ obtained by us reflects also qualitatively the dependence of $|\tilde{b}_+|$ on the time. Reference 1 cites experimental data on the diffractive intensity, corresponding to the amplitude b_+, as a function of the number of laser pulses acting on a germanium surface. The experiment yielded a plot with a maximum. It is important that the incidence angle θ in Ref. 1 was 28°, i.e., $\theta < \pi/4$. One can therefore speak of a qualitative agreement between our results and the experimental data of Ref. 1. A more detailed comparison of the theory with experiment will be made possible by data at incidence angles $\theta > \pi/4$, which will confirm experimentally the theoretical conclusion that the $|b_+(t)|$ plot changes its character at $\theta = \pi/4$, and accordingly that the maximum observed on this plot at $\theta < \pi/4$ will vanish. A quantitative comparison will become possible once data are available on the dependence of the diffractive intensity of scattering with wave-vector transfers g_+ and $2g_+$ on the number of laser pulses.

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