Endpoint of the rotational band of a nonspherical nucleus and a variable moment of inertia

V. G. Nosov and A. M. Kamchatnov
(Submitted 6 July 1984)
Zh. Eksp. Teor. Fiz. **88**, 1505–1513 (May 1985)

We develop the theory of a continuous phase transition from a rotational band to a Maclaurin spheroid, in which the mechanical moment of inertia of a precessing nucleus serves as the ordering parameter. A general expression for this moment in terms of the rotational band level energy allows us to self-consistently minimize the energy, making it single-valued. The complete solution determines the type of singularities of the band characteristics at its endpoint $J = J_0$. An estimate of the Maclaurin spheroid fluctuations is expressed via the corresponding classical moment of inertia. In the band $J < J_0$ the levels do not fluctuate. It is possible to estimate the spheroid quadrupole radiation resulting from quantum fluctuations in its shape. The intensity of this radiation is small compared to the band radiation, but much larger than that of the single-particle scale.

1. INTRODUCTION. THE ROLE OF CENTRIFUGAL FORCE

In principle, a thorough study of the rotational properties of a cold quantum fluid would include a consideration of the sequence of levels

$$E_J = \min, \tag{1}$$

minimizing the total energy of the entire system for different values of its conserved angular momentum **J**. Even as applied to the nucleus only (a Fermi liquid) the properties of this sequence[1)] are well known to reveal considerable variety. It is by no means obvious that an arbitrary component corresponds to a real rotation in the strict sense (a rotation band). For quantum reasons, a purely mechanical rotation is impossible for a highly symmetric configuration. Even then, however, the levels (1) may exhibit a "pseudorotational" behavior under certain circumstances.

The singular points of the lines defined by (1) are of especial theoretical interest. The simplest cases—discontinuities in the energy curve—seem to have been rather widely discussed.[2)] We have already treated the experimentally most accessible of these, $J_c \approx k_j R$ (where k_j is the limiting momentum and R is the radius of the nucleus) as a singularity of the rotation band (Refs. 2–4). Good agreement with experiment was obtained, and the physical meaning of the singularity $J = J_C$ is closely related to the spin alignment of the nucleons, leading, apparently, to a complete reordering of the angular momentum coupling scheme of the rotating nucleus. It is noteworthy that even at the endpoint $J = J_0$ of the rotation band the derivative of the energy with respect to the spin also decreases in jumps; more will be said about this in the next section.

The aim of this paper is to analyze the unavoidable discontinuity in the band that arises from the increasing influence of the centrifugal force on the shape of a highly nonspherical nucleus, and to shed light on the properties of that $J > J_0$ part of the minimizing sequence (1) of the nuclear levels that lies in the band. For the purposes of preliminary orientation the small drop model[5,6] is best for the study of centrifugal effects. Its behavior for moderate values of the angular momentum has been well studied.[7–10]

The model is extremely primitive and, strictly speaking, pseudorotational (the ground state $J = 0$ is spherical). Since for small perturbations $\alpha \ll 1$ it is known that the principal role belongs to the quadrupole component $\alpha_2 \equiv \alpha$, in the usual expansion of the shape of the nucleus in spherical harmonics

$$R(\cos\theta) = R_0\{1+\alpha_0+\alpha_2 P_2(\cos\theta)\} \tag{2}$$

we immediately limit ourselves to the second Legendre polynomial. For simplicity, we also neglect the Coulomb energy U_C. It can always be taken into account by the substitution

$$U_S \to (1-x)\,U_S,$$

where $x = U_C/2U_S$ is the well known divisibility parameter. This leaves only the surface energy U_S, and for the momentum **J** the equilibrium shape is defined by

$$U_S + (\hbar^2/2I_\parallel)\,J^2 = \min. \tag{3}$$

Here $I_\parallel$ is the classical moment of inertia about an axis of symmetry parallel to the vector **J**.

In our present approximation the dependence of U_S and $I_\parallel$ on the deformation is known, and our problem can be solved by equating to zero the derivative with respect to α of the left side of (3). Besides the rotational velocity

$$\hbar\Omega = (\hbar^2/I_0)\,J(1+\alpha) \tag{4}$$

we introduce the equilibrium value of the deformation:

$$\alpha = -{}^5/_8\,(\hbar^2/I_0)\,J^2/U_S. \tag{5}$$

Here $I_0 = 2/5MR_0^2$ is the rigid body inertial moment of the spherical configuration of the nucleus, and the minus sign of the deformation indicates that the nucleus forms an oblate Maclaurin spheroid; see for instance Ref. 11.

 0038-5646/85/050899-06$04.00

How does this model correspond with reality? Because of the quantum structure of the ground state of a Fermi liquid a nucleus of strongly nonspherical shape undergoes a spontaneous deformation

$$\bar{\alpha}\sim 1/k_f R. \tag{6}$$

Because of its origin this is in no way coupled to the momentum vector **J** of the total system, and precesses freely about it. The deformation (5), which is due to centrifugal forces, is strictly fixed with respect to **J** and displays sharp growth as a function of the spin J. Clearly the competition ends in its favor. On the next level of the rotation band the precession deformation will be finally suppressed by centrifugal forces. From $|\alpha|\sim\bar{\alpha}$ we can estimate the order of magnitude of its spin:

$$J_0\sim[U_S/(\hbar^2/I_0)(k_f R)]^{1/2}. \tag{7}$$

Although purely formal estimates are sometimes quite inexact in these problems, to study the dependence on the dimensions of the nucleus (or on the number of particles in it), we introduce the characteristic energy

$$\bar{\varepsilon}=p_f v_f/2\sim 50\ \text{MeV}. \tag{8}$$

Here p_f is the momentum of the bounding quasinucleon, in the usual units and v_f is its velocity. Here we have

$$U_S\sim\bar{\varepsilon}(k_f R)^2,\quad \hbar^2/I_0\sim\bar{\varepsilon}/(k_f R)^5,\quad J_0\sim(k_f R)^3\sim A. \tag{9}$$

Thus for $J>J_0$ (a Maclaurin spheroid) the precessional deformation is absent and there is no purely mechanical motion (precession).

2. THE ENDPOINT OF THE ROTATION BAND

We now focus our attention on the region $J<J_0$, where the levels (1) are embedded in the rotation band. For angular velocity Ω and moment of inertia I we have (Refs. 2, 4)

$$\hbar\Omega=\frac{dE}{dJ},\quad \frac{\hbar^2}{I}=\frac{d(\hbar\Omega)}{dJ}=\frac{d^2E}{dJ^2}. \tag{10}$$

Here $E(J)=E_J$. The mechanical properties of this region are determined by the presence in the system of the unit vector $\mathbf{n}(\vartheta,\varphi)$ which specifies the direction of the precessional component of the deformation. This characteristic spontaneous symmetry breakdown in the given case is equivalent in its implications to purely mechanical rotation.

Landau[13] emphasized the role of the moment of inertia I. He related the loss of rotational capability—the transition to an unbroken symmetry—to the vanishing of I. This represents a deformation-dependent scalar ($\mathbf{\Omega}\|\mathbf{J}$ in free space) in which the latter enter via their invariant combination. These considerations supports the convenient and natural choice of an ordering parameter that characterizes the breakdown of symmetry for $J<J_0$. Near the endpoint the actual values of I are much smaller than I_0 and must be energetically advantageous. We expand the general expression for the energy into a power series and retain the low order terms:

$$E(J,I)=E_r(J)-A(J)I+{}^1/_2D(J)I^2. \tag{11}$$

The function $E_r(J)$ represents the energy of the disordered (symmetric) state, as a spheroid that actually exists for $J>J_0$. The indices b and r represent the respective regions $J<J_0$ and $J>J_0$; the supplementary index 0 characterizes the value assumed for $J=J_0\mp 0$. In their customary forms we have for the coefficient

$$D(J)\approx D(J_0)\equiv D>0,$$

and $\partial E/\partial I=0$ yields

$$I=\frac{A(J)}{D},\quad E_r-E_b=\frac{1}{2}\frac{A^2(J)}{D}=\frac{1}{2}DI^2. \tag{12}$$

Moreover, even the preferred value of I must be related to the energy of the mechanical relations (10). Introducing the convenient notation $\mathscr{E}=E_r-E_b$, we can simplify the second derivative of the energy variable in the limit that interests us so that we have

$$d^2\mathscr{E}/di^2\approx-\hbar^2/I. \tag{13}$$

Here we assume that

$$\frac{d(\hbar\Omega_r)}{dJ}\sim\frac{\hbar\Omega_r}{J}=\frac{\hbar^2}{I_\parallel}\sim\frac{\hbar^2}{I_0}\ll\frac{\hbar^2}{I}.$$

By eliminating the mechanical moment of inertia I from the relations (12) and (13) we arrive at

$$\mathscr{E}^{1/2}\frac{d^2\mathscr{E}}{di^2}=-\beta,\quad \beta=\frac{\hbar^2}{\sqrt{2}}D^{1/2}>0. \tag{14}$$

This equation determines all the essential characteristics of the phase transition to a Maclaurin spheroid.

We easily transform equation (14) to the implicit form:

$$\beta^{1/2}i={}^2/_3(C-y)^{3/2}-2C(C-y)^{1/2}+{}^4/_3C^{3/2}. \tag{15}$$

Here $y=\mathscr{E}^{1/2}$ and $C>0$ is a constant of integration; the other arbitrary constant is chosen so that i and $\mathscr{E}$ vanish simultaneously. Near the transition point $y\ll C$. We retain the linear contribution from y on the right, and expand (15) through the third order terms:

$$\beta^{1/2}i\approx{}^1/_2C^{-1/2}\mathscr{E}+{}^1/_6C^{-3/2}\mathscr{E}^{3/2}. \tag{16}$$

Inverting this relation with the required precision, we find for $\mathscr{E}$ and its derivatives

$$\begin{aligned}\mathscr{E}&=2(\beta C)^{1/2}i-\frac{2^{3/2}}{3}\frac{\beta^{3/4}}{C^{1/4}}i^{3/2},\\ \frac{d\mathscr{E}}{di}&=\hbar\Omega_b-\hbar\Omega_r=2(\beta C)^{1/2}-\sqrt{2}\frac{\beta^{3/4}}{C^{1/4}}i^{1/2},\\ \frac{d^2\mathscr{E}}{di^2}&=-\frac{\hbar^2}{I}=-\frac{1}{\sqrt{2}}\frac{\beta^{3/4}}{C^{1/4}}\frac{1}{i^{1/2}}.\end{aligned} \tag{17}$$

As $J\to J_0-0$ the angular velocity has a singularity, but remains finite. It is clear from the second equation that at the

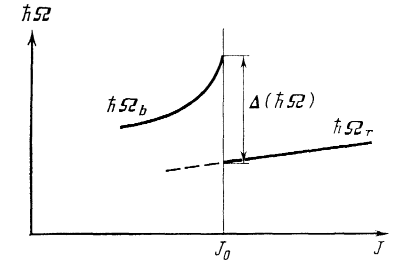

FIG. 1

endpoint of the rotation band the angular velocity jumps to some finite value

$$\Delta(\hbar\Omega) = \hbar\Omega_{b0} - \hbar\Omega_{r0} > 0 \tag{18}$$

(see Fig. 1). The curves of phase energy intersect at an angle, as shown in Fig. 2. The mechanical moment of inertia vanishes like $(J_0 - J)^{1/2}$:

$$\frac{\hbar^2}{I} = \frac{\hbar^2}{I_0}\left(\frac{J_0 - J_1}{J_0 - J}\right)^{1/2}. \tag{19}$$

There is no direct and precise physical meaning for the parameter J_1. Nevertheless, the quantity $J_0 - J_1$ determines the order of magnitude of the distance to the endpoint of the rotation band, at which the moment of inertia begins to fall away from our approximation of it. We may write

$$\left(\frac{J_0 - J}{J_0 - J_1}\right)^{1/2} \ll 1 \tag{20}$$

as a criterion for the applicability of our theory.

We estimate the parameters more precisely by proposing that even before going over to a Maclaurin spheroid the band has begun to approach the rigid body asymptote $I \approx I_0$. Near the transition point the difference in phase velocity is given by equation (17). (The coefficients are conveniently expressed via $\Delta(\hbar\Omega)$ and $J_0 - J_1$.) In the opposite limiting case: $J_0 - J > J_0 - J_1$, this difference is estimated by Eq. (4) if we write $-\alpha \sim \bar{\alpha} \sim 1/k_j R$. After making all the substitutions we have

$$\Delta(\hbar\Omega) \sim \frac{\hbar^2}{I_0}\frac{J_0}{k_f R} \ll \hbar\Omega_0, \qquad J_0 - J_1 \lesssim J_0/k_f R \ll J_0. \tag{21}$$

Given the moment of inertia (19) we easily find by a simple integration all the rotational characteristics of the b-phase:

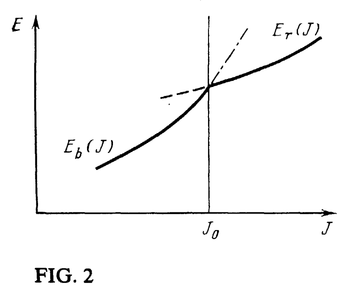

FIG. 2

$$\hbar\Omega = \hbar\Omega_0 - 2\frac{\hbar^2}{I_0}\left[(J_0 - J_1)(J_0 - J)\right]^{1/2},$$
$$E = E_0 - \hbar\Omega_0 (J_0 - J) + \frac{4}{3}\frac{\hbar^2}{I_0}(J_0 - J_1)^{1/2}(J_0 - J)^{3/2} \tag{22}$$

(since from now on we shall discuss only the rotation band, we shall omit the index b). The quantity $\Omega = \Omega_0$ is the limiting attainable velocity of a purely mechanical rotating nucleus. In accordance with the formulae (7)–(9),

$$\hbar\Omega_0 \sim \frac{\hbar^2}{I_0} J_0 \sim \left(\frac{U_s \hbar^2 / I_0}{k_f R}\right)^{1/2} \sim \frac{\bar{\varepsilon}}{(k_f R)^2}. \tag{23}$$

It is worthwhile to compare this with the velocities of the nuclear quasi-particles:

$$\omega_f \sim \frac{v_f}{R}, \qquad \hbar\omega_f \sim \frac{\hbar^2 k_f}{m_n R} \sim \frac{\bar{\varepsilon}}{k_f R}, \qquad \Omega_0 \ll \omega_f. \tag{24}$$

The limiting velocity of the rotational precession of a nonspherical nucleus is still small compared to that of the nucleons in it.

3. FLUCTUATIONS IN THE LEVELS OF A NUCLEAR ROTATION

There have been practically no studies of the possible fluctuations in the levels arising from the rotation of atomic nuclei. The principal difficulty is that the widely accepted conflicting concepts of the "collective" and the "single particle" properties of the nuclear system (or of its corresponding number of degrees of freedom) do little to help, and do not illuminate the problem. We have worked primarily from the uncertainty principle.

We begin with the uncertainty equation

$$\Delta E \Delta x \sim \hbar \dot{x} \tag{25}$$

(see, for example, Ref. 14). We begin by setting $x = \varphi$, $\dot{\varphi} = \Omega$ as the estimates of the required energy uncertainties. In the kinematic sense the azimuthal coordinates satisfy $\Delta\varphi \lesssim 1$ and

$$\Delta E \sim \hbar\Omega. \tag{26}$$

As is customary, in the quasi-classical case the quantum uncertainties of the most essential quantities are estimated at their minimum.[4)]

We write $x = \Omega$ and distinguish two cases:

a) $J < J_0$ (the rotation band). Then the equation $\dot{\Omega} = 0$, arising from the dynamics of free rotation, correctly specifies the position of the body. In the band, where the temperature is strictly zero [see Eq. (1)], the motion is purely mechanical, namely a regular precession of the vector **n**, with respect to the azimuth φ, the state of which is completely determined.[3,4] For maximum polarization $M = J$ along the z-axis the role of the Hamiltonian $H(M)$ is played by the actual distribution of the levels $E(J)$ within the band. The variability (10) of the moment of inertia is not in conflict with this conclusion, and there are no fluctuations.

b) $J > J_0$ (Maclaurin spheroid). For obvious reasons we cannot here have completely closed dynamics of a purely rotational character. For convenience in making the following estimates we introduce the fluctuating torque dM/dt, i.e., the angular momentum, with the usual dimensions of action, transmitted per unit time from degrees of freedom to its rotation. Then the angular velocity is estimated from the equation of motion. We have

$$I_{\|}\dot{\Omega}\sim dM/dt\sim\hbar\Omega,$$

since the elementary quantum of momentum exchange is h and in the system of discrete levels the characteristic time is defined by the frequency of transitions among the levels.

Thus

$$\Delta E\Delta\Omega\sim(\hbar/I_{\|})\hbar\Omega,$$

but $\Delta E\sim\hbar\Omega$ (see above). Returning to the interpretation of the angular velocity as determining the separation $2\hbar\Omega$ between levels we finally obtain the relation

$$\delta E\sim\delta(\hbar\Omega)\sim\hbar^2/I_{\|}\ll\hbar\Omega, \tag{27}$$

where δE is the desired fluctuation (of purely quantum origin) among the actual energy levels of the spheroid. The fact is that a precise value of the energy of the levels cannot be prescribed: even in principle it cannot be derived from purely rotational considerations. The approximate and not wholly consistent dynamics of the pseudorotation of a Maclaurin spheroid allows us to predict such a quantity only to within an approximate rotational energy quantum $\hbar^2/I_{\|}$. In particular, if we average the angular velocities we obtain the simple formlula

$$\bar{\Omega}=M/I_{\|}. \tag{28}$$

Here $M=\hbar J$ is the classical momentum of the motion of the total system.

The formulae (27) and (28) provide a quite specific representation of the nature of the transition to the classical limit in this delicate question. In quantum theory, strictly speaking, a body cannot mechanically rotate about an axis of total axial symmetry. However, in the classical limit such a rotation is possible. If we neglect Planck's constant, the pseudorotation we are contemplating here is not dynamically different from a true rotation with similar properties.

4. INTENSITY OF THE $E2$ TRANSITIONS

Within the ensemble of levels (1) [see also the remarks in footnote 1)] we find almost exclusively $E2$-radiation. The most typical part of the probability of the process is given by the squared modulus of the matrix element of the quadrupole moment tensor. We denote by $\overline{Q}^2$ the transition intensity.

Let us look first at the region $J<J_0$. Here the nonspherical nucleus radiates because it precesses (rotational radiation). At a distance from the endpoint we have

$$\overline{\overline{Q}}{}^2\sim Q_0^2,\quad J_0-J\gg J_0-J_1, \tag{29}$$

where $Q_0=6/5\ ZR_0^2\ \bar{\alpha}$ is the quadrupole moment of the shape of the spontaneously deformed nucleus. In the more difficult case $J_0-J\ll J_0-J_1$ we guide ourselves by a close analogy between the quantities I and $\overline{Q}^2$. The $E2$-intensity is also invariant: the agreement is especially striking if we consider the hypothetical situation in which the shape of the nucleus approaches the spherical. Then the scalar quantities of interest vanish like α^2.

We assume

$$\overline{\overline{Q}}{}^2\sim Q_0^2[(J_0-J)/(J_0-J_1)]^{1/2},\quad J_0-J\ll J_0-J_1 \tag{30}$$

in view of (19). Then we find

$$\alpha_p\propto(J_0-J)^{1/4} \tag{31}$$

for the behavior of the precessional component of α_p of the deformation at its vanishing point. For a rough preliminary estimate of the situation at the endpoint itself we set $J_0-J\sim1$. The formulae (21) and (9) yield

$$\overline{\overline{Q}}{}^2\sim Q_0^2(J_0-J_1)^{-1/2}\gtrsim Q_0^2/k_fR.$$

For this method of estimating, the rotational radiation turns out to be highly significant even in the last energy interval in the band.

We now consider the region $J>J_0$. Since in quantum theory a body cannot rotate about a true symmetry axis, and since there is no change in the quadrupole moment in the classical scheme, we have reason to doubt whether a significant quadrupole radiation intensity exists. It turns out, however, that in our problem concerning the properties of Maclaurin spheroids we cannot obtain sufficient accuracy if we consider only the mean values of the characteristics.

The growth of the spin strongly affects the shape of the nucleus, inducing centrifugal flattening in it. However, this strong interaction has another aspect: the curve (1) is discrete rather than continuous. Together with the centrifugally generated mean deformation α, the spin sequence (1) introduces its own frequency $\omega=2\Omega$, imposing its quadrupole deformation on the shape. Then for a finite rigidity C, quantum fluctuations in the deformation necessarily occur.

The frequency spectrum of the fluctuations $\delta\alpha$ is prescribed by the standard selection rules for quadrupole radiation, but under the observational conditions a detailed geometrical specification is not important for order-of-magnitude estimates. In the narrow band where radiative transitions occur, we may interpret the quasiclassical essential uncertainty $\Delta E\sim\hbar\Omega$ as the order of magnitude of the uncertainty in the energy of the fluctuating deformations:

$$C(\delta\alpha)^2\sim\hbar\Omega. \tag{32}$$

In the present case we have $C\sim U_S$, $\hbar\Omega\sim\hbar\Omega_0$ [formula (23)]. Thus

$$(\delta\alpha)^2\sim\left(\frac{\hbar^2/I_0}{U_Sk_fR}\right)^{1/2}\sim\frac{1}{(k_fR)^4}. \tag{33}$$

Since $|\alpha| \sim \alpha \sim 1/k_j R$ the relation

$$(\delta\alpha/|\alpha|)^2 \sim (\delta\alpha/\bar{\alpha})^2 \sim 1/(k_j R)^2 \quad (34)$$

allows us not only to estimate the relative magnitude of the fluctuations but also to compare the intensity of the radiation in both phases $\gtrless J_0$.

After the disruption of the band the radiation falls off because the precession stops. However, the remaining intensity, the same scale as the vibrational, still far exceeds the characteristic single particle value $\bar{\bar{Q}}_1^2 \sim R^4$. In the chain of strong inequalities

$$\bar{\bar{Q}}_b^2 \gg \bar{\bar{Q}}_r^2 \gg \bar{\bar{Q}}_1^2, \quad (35)$$

each corresponds, roughly speaking, to a ratio of the order of $(k_j R)^2$. As of now there is no obvious reason why any levels of a Maclaurin spheroid should exhibit isomeric properties.

5. DISCUSSION

The problem of the free rotation of a fluid quantum system is neither straightforward nor easy. We have attempted to give a cursory account of the situation with respect to nuclear rotation in the broad general physics sense.

Intuitively, we would expect that random deviations from formulae of rotational type should depend on the total number of particles and should decrease as the number increases. The following quantum effect is less trivial: in a macroscopic Fermi system (spontaneously deformed, $\alpha \sim 1/k_j R$) of finite dimension the sequence (1) of minimizing levels is stratified into two regions $J \lessgtr J_0$. Paradoxically, in the lower region $J < J_0$ the energy levels do not fluctuate, and the exchange of the inertial moment $\hbar^2(d^2E/dJ^2)^{-1}$ and the singularities are problems of a somewhat different kind. As for the upper region $J > J_0$, we have

$$\delta(\hbar\Omega) \sim \frac{\hbar^2}{I_\parallel} \approx \frac{\hbar^2}{I_0},$$
$$\hbar\Omega = \frac{\hbar^2}{I_\parallel} J \sim \frac{\hbar^2}{I_0} J_0 \sim \frac{\hbar^2}{I_0} A, \quad \delta(\hbar\Omega)/\hbar\Omega \sim 1/A \quad (36)$$

in agreement with general physical considerations.

The experimental situation is less favorable, though some hopeful signs have appeared. In a recent paper[15] the sequence (1) for high spin states of $^{84}{}_{40}Zr_{44}$ was investigated up through the level $J_F = 34$, and a Maclaurin spheroid was observed for the first time. This nucleus is close to being magic with $Z, N = 50$, so that the smallness of the deformation possibly helped to display the effect in a relatively accessible region. A case of this kind, however, is not favorable for an investigation of the phase transition from a rotation band to a Maclaurin spheroid. As usually happens in a nucleus of doubtful type, the path of the mechanical moment of inertia here is unpredictable and narrow; the region (20) does not exist because $J_0 - J_1 < 1$. However, qualitatively in favor of the basic idea is the fact that the amount of inertia I reached its minimum at the endpoint $J = 20$ of the band rather than at the beginning.

In the region $J > 22$ of the spheroid the proportionality of the angular velocity and rotational moment was observed to within a small fluctuation. The mean square fluctuation $\delta(\hbar\Omega) \approx 9.3$ keV evidently exceeds the experimental error. The theoretical estimate (27) yields $\delta(\hbar\Omega) \sim \hbar^2/I_\parallel \approx 40$ keV. For $r_0 = 1.1$ fm (the choice of the radius of the nucleus was made by "backbending" according to the method described in Ref. 3) this corresponds to a mean deformation of the spheroid $\alpha \approx -0.3$. The estimate (33) of the fluctuations in the deformation yields $\delta\alpha \approx 0.07$.

Among the clearly nonspherical nuclei special interest attaches to $^{248}{}_{96}Cm_{152}$, $J_F = 30$ (Ref. 16). In experimental observations on the lower portion $J < J_C$ of the basic rotation band the variable moment of inertia proved to be monotonically increasing. There was an exception, however: it went through a maximum at $J \approx 23$ and then decreased. Among the nuclei at this nucleid that were investigated, $Z^2/A = 37.2$ is the largest, and because of the Coulomb forces the onset of the transition to a Maclaurin spheroid may have occurred unusually early [see the Introduction and the text following equation (2)].

If our hypothesis is valid, this is close to the endpoint of the band; the predicted value is $J_0 \approx 33$. The difference $J_0 - J_1 \approx 2.4$ is not large. However, the theory may turn out to be applicable. In the present somewhat exotic instance, the falling off of the inertial moment began at a value almost twice as large as that for a rigid body. The predicted moment of inertia is comparable to the rigid body value only for $J = 30$.

The two examples we have cited are representative. Primary interest attaches to either the relatively light nucleids typified by $^{78}{}_{36}Kr_{42}$. $J_C = 9.2$, $J_F = 16$ or to the heavy actinides, in which the resistance to centrifugal flattening is overcome by Coulomb energy. We suggest that basic experimental study of the breakdown of the rotation band should begin at these two ends of the periodic table.

We thank M. Ya. Amus', G. A. Pik-Pichak, and Ya. A. Smolensky for advice and criticism.

[1)]The change in a two-valued spatial quantum number or in the signature $(-1)^J$ correlates poorly with the thermodynamic considerations that we are limiting ourselves to. The necessary correspondence can be established if we fix on a suitable quantum number; we will discuss this later. In such a sequence the spins of the levels follow one another with an interval $\Delta J = 2$.

[2)]Judging by the literature, it was Ya. B. Zel'dovich who first called attention to the possibility of a break in the energy curve with respect to spin. In Ref. 1 he employed a different set of assumptions on the structure of the nucleus, taking it as a Bose-fluid.

[3)]For the ideal Fermi-gas this was shown in Ref. 12; see also Ref. 4. The essentially equivalent form $\bar{\alpha} \sim A^{-1/3}$ of this estimate has been widely used in the literature.

[4)]Although the coordinate φ does not exist for $J > J_0$, it is clear from the physical nature of the results that the smallest possible energy uncertainty in similar circumstances (quasiclassical) has the same order of magnitude for the flattened spheroid.

[1]Ya. B. Zel'dovich, Pis'ma Zh. Eks. Teor. Fiz. **4**, 78 (1966) [JETP Lett. **4**, 53 (1966)].
[2]V. G. Nosov and A. M. Kamchatnov, Zh. Eks. Teor. Fiz. **73**, 785 (1977)

[Sov. Phys. JETP **46**, 411 (1979)]; **76**, 1056 (1979) [Sov. Phys. JETP **49**, 765 (1979)].
[3]V. G. Nosov and A. M. Kamchatnov, Zh. Eks. Teor. Fiz. **80**, 433 (1981) [Sov. Phys. JETP **53**, 852 (1981)].
[4]V. G. Nosov Makroscopicheskie kvantovye effeckty v atomonykh yadrakh. M.: Atomizdat, 1980. (Macroscopic quantum effects in atomic nuclei).
[5]N. Bohr and J. Wheeler, Phys. Rev. **56**, 426 (1939) (Russian tr. N. Bor. Izbrannye nauchnye trudy. M.: Nauka, 1971, t. 2).
[6]Ya. Frenkel', Zh. Eks. Teor. Fiz. **9**, 641 (1939).
[7]G. A. Pik-Pichak, Zh. Eks. Teor. Fiz. **34**, 341 (1958) [Sov. Phys. JETP **7**, 238 (1958)].
[8]G. A. Pik-Pichak, Zh. Eks. Teor. Fiz. **42**, 1294 (1962); [Sov. Phys. JETP **15**, 897 (1962)]; **43**, 1701 (1962) [Sov. Phys. JETP **16**, 1201 (1963)].
[9]G. A. Pik-Pichak, Yad. Fiz. **31**, 98 (1980) [Sov. J. Nucl. Phys. **31**, 52 (1980)].
[10]O. Bor and B. Mottel'son, Struktura atomnogo yadra, Mir, Moscow (1977) t. 2. [Nuclear Structure, Vol. 2, Benjamin, New York (1969)].
[11]S. Chandrasekhar, Ellipsoidal'nye figury ravnovesiya. M.: Mir, 1973. (Ellipsoidal figures of equilibrium, Yale Univ. Press, New Haven).
[12]V. G. Nosov, Zh. Eks. Teor. Fiz. **31**, 335 (1957) [Sov. Phys. JETP **4**, 263 (1957)].
[13]L. Landau and Ya. Smorodinskii, Lektsii po teorii atomnogo yadra. M.: Gostekhizdat, 1955. (Lectures on Nuclear theory, Consultants Bureau, New York).
[14]L. D. Landau and E. M. Lifshitz, Kvantovaya mekhanika, Nauka, Moscow, 1974. [Quantum mechanics: Non-Relativistic Theory, 3rd ed., Pergamon, Oxford (1977)].
[15]H. G. Price, C. J. Lister, B. J. Varley *et al.* Phys. Rev. Lett. **51**, 1842 (1983).
[16]R. B. Piercey, J. H. Hamilton, A. V. Ramayya *et al.* Phys. Rev. Lett. **46**, 415 (1981).

Translated by A. Brown