Nonlinear resonance and stochasticity in a system of surface electrons

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We present the conditions for the onset of an isolated nonlinear resonance and the criterion for the overlap of such resonances in an electron system over a helium surface in a clamping field. The analysis is carried out in the classical and quantum approximations. We discuss the feasibility of experimental study of the dynamics of an isolated quantum resonance and of quantum stochasticity.

1. INTRODUCTION

The dynamic properties of quantum systems under conditions when the classical approach ($\hbar = 0$) leads to stochasticity have recently attracted much attention.[1–8] According to Refs. 3–8 allowance for quantum effects can give rise to substantial anomalies in the manifestation of stochasticity even when the initial population of the system is quasiclassical. For example, as demonstrated for very simple models of nonlinear quantum systems excited by a periodic sequence of δ function pulses, the quantum correlation functions attenuate much more slowly than in the classical limit, which in the end, reduces the rate of quantum diffusion.[4–8] It must be noted, however, that under certain conditions quantum dynamics can be brought closer to the classical stochastic approach for systems evolving over finite times.[3–8] It is therefore of interest to study under real experimental conditions the dynamics of nonlinear quantum systems that are stochastic at $\hbar = 0$.

The feasibility of a stochastic mechanism of collisionless dissociation of polyatomic molecules that interact with coherent IR laser emission is discussed in Refs. 9 and 10. Stochastic ionization of a hydrogen-atom electron excited beforehand by a microwave field is analyzed analytically and numerically in Refs. 11–14. It was proposed in Ref. 15 that a suitable object for the study of stochasticity in quantum systems is a system of electrons located over a liquid-helium surface and interacting with a periodic microwave field polarized perpendicular to the surface. The system proposed in Ref. 15 is convenient primarily because it is effectively one-dimensional, since the electron motion can then be resolved into free motion in a plane parallel to the helium surface and motion perpendicular to this surface; the latter is described by the Hamiltonian[16–18]

$$H=P_x^2/2m+V(x)+e\mathscr{E}x\cos\omega t, \tag{1.1}$$

$$V(x)=V_0, \quad x<0, \tag{1.2}$$

$$V(x)=-Ze^2/(x+\xi), \quad x>0.$$

Here x is the distance of the electron from the helium surface, m and e are the electron mass and charge, Ze is the imaginary charge induced by the helium surface; $\mathscr{E}$ is the amplitude of the external field polarized along the x axis, V_0 is the potential barrier to electron penetration into the helium ($V_0 = 1$ eV), $\xi = 10^{-10}$ cm, and ω is the frequency of the external field. Assuming an energy < 1 eV, we shall approximate hereafter the potential (1.2) by the function[17,18]

$$V(x)=\infty, \quad x<0,$$

$$V(x)=-Ze^2/x, \quad x>0. \tag{1.3}$$

Note that in this case the system considered is directly related to the problem of stochastic ionization of a hydrogen atom in an electromagnetic-wave field.

The condition for the onset of stochasticity in a system with the Hamiltonian (1.1), (1.3) was investigated in Ref. 15 in the classical approximation ($\hbar = 0$). The stochasticity threshold was estimated from the overlap of the nonlinear resonances. It was shown in Ref. 15 that random motion is produced in the system when external-field amplitudes exceed some threshold $\mathscr{E} > \mathscr{E}_{cr}(\omega)$ and leads to a diffusive growth of the energy right up to the ionization energy.

The concept of isolated nonlinear resonances (first used in the quasiclassical approximation[9]) can be introduced also in the quantum case, and the overlap of these resonances can be considered.

Numerical investigations of the quantum-mechanical system (1.1), (1.3) with an initially high level population ($n \approx 50$) have demonstrated the essential role of quantum effects in the electron-excitation mechanism.[13] According to Ref. 13, the level-population distribution function is strongly resonant, and the diffusion is substantially reduced compared with the classical case. Similar effects were observed also in numerical experiments[14] with a time-dependent perturbation operator in the form of a periodic train of δ-function pulses.

These differences between the dynamics of quantum and classical systems are due both to the considerable differences in the dynamics of the individual nonlinear resonances and to differences in the way they interact, relative to the classical case. It follows from Refs. 19–21 that how closely the dynamics of interacting nonlinear quantum resonances approximate stochastic motion of the corresponding classical system is determined not only by their overlap and population conditions at high level numbers, but also by the requirement that each resonance capture a large number ($\delta n \gg 1$) of levels. It will be shown below that the last condition is difficult to meet in an actual experiment in the case of the system (1.1), (1.3).

We present below a classical and quantum analysis of

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the dynamics of the system (1.1), (1.3) with allowance for an additional clamping field $\mathscr{E}_0$. The parameters that describe the dynamics of an isolated resonance are presented, and the case of resonance overlap is considered. We show that variation of the strength of the clamping field permits the system dynamics to be studied at different values of δn. The results are discussed in connection with the feasibility of observing an isolate quantum nonlinear resonance and of studying the stochasticity in a system of surface electrons under real experimental conditions.

2. CLASSICAL LIMIT OF NONLINEAR RESONANCE IN A CLAMPING FIELD

We consider in this section in the classical approximation an isolated nonlinear resonance for the system (1.1), with allowance for the additional clamping field. To facilitate the subsequent comparison with the quantum case, we transform in (1.1) to new dimensionless variables

$$q=\alpha x, \quad p=P_x/\hbar\alpha. \tag{2.1}$$

The quantity α has the meaning of the reciprocal Bohr radius: $\alpha = mZe^2/\hbar \approx 0.14\cdot 10^6$ cm^{-1}. In the variables q and p, the Hamiltonian for a surface electron in a clamping field takes the form

$$\begin{aligned} H&=H_0+H_{int}(t),\\ H_0&=R(p^2/2-1/q+\varepsilon_0 q)=RE,\\ H_{int}&=R\varepsilon q\cos\nu\tau, \end{aligned} \tag{2.2}$$

where

$$\begin{aligned} &R=\hbar^2\alpha^2/m\approx 1.3\cdot 10^{-3}\ \text{eV},\\ &\varepsilon_0=e\mathscr{E}_0/\alpha R, \quad \varepsilon=e\mathscr{E}/\alpha R,\\ &\nu=\omega\hbar/R, \quad \tau=Rt/\hbar. \end{aligned} \tag{2.3}$$

R has the dimension of energy and the meaning of the Rydberg constant, while E is the dimensionless unperturbed energy.

We introduce the dimensionless action J for the Hamiltonian H_0:

$$J=\frac{1}{2\pi}\oint p\,dq=J(E). \tag{2.4}$$

From (2.2) and (2.4) we obtain

$$J(E)=\frac{2\sqrt{2}}{3\pi\varepsilon_0^{1/4}}\left(a+\frac{1}{a}\right)^{1/2}\left[\left(a-\frac{1}{a}\right)E(k)+\frac{1}{a}K(k)\right], \tag{2.5}$$

where $K(k)$ and $E(k)$ are complete elliptic integrals of the first and second kind, with the modulus k of the integrals and the quantity a given by

$$k=\left(\frac{a^2}{a^2+1}\right)^{1/2}, \quad a=\frac{E}{2\varepsilon_0^{1/2}}+\left[\left(\frac{E}{2\varepsilon_0^{1/2}}\right)^2+1\right]^{1/2}. \tag{2.6}$$

The parameter a is connected with the turning point q_0 by the relation $q_0 = a\varepsilon_0^{1/2}$. In terms of the action (J) and angle (θ) variables the dimensionless Hamiltonian (2.2) takes the form

$$\mathscr{H}=\frac{H}{R}=E(J)+\varepsilon\cos\nu\tau\sum_{l=-\infty}^{\infty}q_l(J)e^{il\theta}. \tag{2.7}$$

The explicit $E(J)$ dependence in (2.7) is determined from (2.5), and the dimensionless coordinate q of the electron is assumed to be expressed in terms of J and θ:

$$q=q(J,\theta)=\sum_{l=-\infty}^{\infty}q_l(J)e^{il\theta}.$$

The Fourier harmonic $q_l(J)$ in (2.7) is given by

$$\begin{aligned} q_l(J)&=-\frac{2}{l\varepsilon_0^{1/2}}\int_0^q \sin\left[\frac{\pi l\Phi(y)}{\Phi(a)}\right]dy,\\ \Phi(y)&=\int_0^y\frac{y'}{(a-y')(y'+1/a)}dy' \quad (l\neq 0). \end{aligned} \tag{2.8}$$

An external alternating field produces nonlinear resonances in the vicinities of the points J_l defined by the equation

$$\omega_l\equiv(dE(J)/dJ)_{J=J_l}=\nu/l. \tag{2.9}$$

Each nonlinear resonance is characterized by the following parameters: the action width $\delta J_l = 2(J-J_l)_{\max}$ and the phase-oscillation frequency Ω_l. The effective Hamiltonian corresponding to the resonance numbered l is of the form[22]

$$\begin{aligned} &\mathscr{H}_l=l\gamma_l(J-J_l)^2/2+l\varepsilon q_l(J_l)\cos\psi_l,\\ &\gamma_l=(d^2E(J)/dJ^2)_{J=J_l}, \quad \psi_l=l\theta-\nu\tau. \end{aligned} \tag{2.10}$$

From (2.10) we have

$$\delta J_l=4[\varepsilon q_l(J_l)/\gamma_l]^{1/2}, \tag{2.11}$$

$$\Omega_l=l[\varepsilon q_l(J_l)\gamma_l]^{1/2}. \tag{2.12}$$

The isolated-resonance approximation is justified if the width δJ_l of an individual resonance is less than the spacing between the resonances

$$\Delta J_l=J_{l-1}-J_l\approx dJ_l/dl. \tag{2.13}$$

The system (2.7) in the absence of a clamping field ($\varepsilon_0 = 0$) was considered in Ref. 15. In that case, as follows from (2.5), $E(J)$ takes the form $E(J) = -1/2J^2$. From (2.9) and (2.1) we have then $J_l = J_1 l^{1/3}$, $\Delta J_l \approx J_1/3l^{2/3}$, where $J_1 = \nu^{1/3}$ is the value of the action for the fundamental resonance. At $\varepsilon_0 = 0$ the maximum width at which the resonance can be regarded as isolated is of the order of

$$\delta J_l \lesssim \Delta J_l \approx J_1/3l^{2/3}. \tag{2.14}$$

Note that for our chosen dimensionless variables the quasiclassical quantization rule corresponds to choosing integer values $J = n$. In the quantum-mechanical treatment of the system (2.2), the maximum number of levels δn_l included in the isolated resonance numbered l is specified in the quasiclassical region by Eq. (2.14). Substitution of the numerical values shows that when the system is excited by an alternating field with $\omega = 10^{-2}$–10^2 GHz the fundamental resonance corresponds to values n_1 from 20 to 1. Consequently the number of levels included in the resonance in this case is at most 7, i.e., $\delta n_l \leqslant 7$. As shown in Refs. 22 and 23, the interaction of quantum nonlinear resonances when a small number of levels are included ($\delta n \lesssim 20$) is essentially quantum-mechanical, and the residual correlation level is quite

high in this case. We note in this connection that essentially quantum effects were observed in a computer experiment[13] near a quasiclassical initial population with $n_1 = 50$. Further increase of n in the case of surface-electron system (in contrast to the hydrogen atom) is physically untenable, since the ionization energies become $\lesssim 3\cdot 10^{-7}$ eV, much lower than the thermal-fluctuation energy.

The situation changes substantially in the presence of an external clamping field. In this case Eq. (2.5) can have two asymptotic forms, depending on the value of a. The first ($a \ll 1$) corresponds to a weak clamping field $\mathscr{E}_0$ and $E < 0$:

$$E = -\frac{1}{2J^2} + \frac{29}{6}\varepsilon_0 J^2. \tag{2.15}$$

In the other limiting case ($a \gg 1$) we get from (2.5)

$$E = AJ^{4/3}\left[1 - \frac{\varepsilon_0 \ln 4\varepsilon_0^{-1/4} AJ^{5/3}}{A^2 J^{4/3}}\right], \quad A = \left(\frac{3\pi\varepsilon_0}{2\sqrt{2}}\right)^{2/3}. \tag{2.16}$$

The boundary between the asymptotic form (2.15) and (2.16) corresponds to the value $a = 1$ ($E = 0$), which yields for J

$$J^* = 4K(1/\sqrt{2})/(3\pi\varepsilon_0^{1/4})^{-1} \approx 0.8\varepsilon_0^{-1/4}. \tag{2.17}$$

Typical clamping fields $\mathscr{E}_0$ used in experiments with surface electrons lie in the range $(1\text{–}2)\cdot 10^3$ V/cm, equivalent to the dimensionless $\varepsilon_0 = 0.5\cdot 10^{-3}\mathscr{E}_0$ ($\mathscr{E}_0$ is measured in V/cm). Substituting the numerical value of ε_0 in (2.17) we obtain $J^* \leqslant 4$ i.e., at $\mathscr{E}_0 > 1$ V/cm the boundary between the asymptotics (2.15) and (2.16) lies in the vicinity of the first four levels. The quasiclassical-approximation condition ($n \gg 1$) necessitates the use of the asymptotic form (2.16) for an external clamping field $\mathscr{E}_0 > 1$ V/cm.

In this case the expressions for the Fourier harmonic $q_l(J)$, for the nonlinearity γ_l, and for the resonant values of the actions J_l (2.9) take the form

$$q_l(J) \approx -\frac{2AJ^{3/2}}{\pi^2\varepsilon_0 l^2} \quad (l \neq 0), \tag{2.18}$$

$$\gamma_l \approx -\frac{2}{9}AJ_l^{-4/3},$$

$$J_l \approx J_1 l^3, \quad J_1 \approx \left(\frac{2A}{3\nu}\right)^3 = \frac{\pi^2\varepsilon_0^2}{3\nu^3}. \tag{2.19}$$

Substituting (2.18) and (2.19) in (2.11) and (2.12) we obtain the characteristic parameters of the l th nonlinear resonance for $a \gg 1$:

$$\delta J_l = \frac{12}{\pi} l^2 J_1 \left(\frac{\varepsilon}{\varepsilon_0}\right)^{1/2}, \tag{2.20}$$

$$\Omega_l = \frac{1}{l}\left(\frac{2}{\pi}\right)^{1/2}\left(\frac{\varepsilon}{\varepsilon_0}\right)^{1/2}. \tag{2.21}$$

Since the distance between resonances is $\Delta J_l = 3J_1 l^2$, according to (2.13) and (2.19), the isolated-resonance approximation is valid subject to satisfaction of the inequality

$$K = \frac{\delta J_l}{\Delta J_l} = \frac{4}{\pi}\left(\frac{\varepsilon}{\varepsilon_0}\right)^{1/2} \ll 1. \tag{2.22}$$

Consequently, if the alternating field is much weaker than the clamping field, the resonances are isolated. Recall that $J_1 \propto \varepsilon_0^2$ according to (2.19). Thus, if the ratio $\varepsilon/\varepsilon_0$ and ν are fixed, δJ_l can be smoothly varied in a wide range by varying the clamping field. It becomes possible then to study the nonlinear resonance under conditions when it is localized, when the frequency Ω_l of the phase-oscillation frequency is fixed, and when different values of the number δJ_l of the levels were made resonant.

By way of example, we present numerical estimates for the fundamental resonance ($l = 1$) at an alternating-field frequency $\omega/2\pi = 1$ GHz($\nu = 3\cdot 10^{-3}$). At $\mathscr{E}_0 = 1$ V/cm ($\varepsilon_0 = 0.56\cdot 10^{-3}$) we have from (2.9) $J_1 \approx 40$. In this case $RE(J_1) = 2.3\cdot 10^{-4}$ eV. Raising the clamping field strength to 8 V/cm increases J_1 by a factor of 64, i.e., $J_1 \approx 2560$. Let the oscillating field strength be weaker by a factor 10^3 than that of the clamping field ($\varepsilon/\varepsilon_0 = 10^{-3}$); we find then for the latter case from (2.20) that the number of levels trapped by the resonance is $\delta n_1 \approx 300$.

It follows from the foregoing estimates that the number of trapped levels can be made large enough to permit analysis of quantum nonlinear resonance under conditions close to the classical approximation. In the next section we present a quantum-mechanical analysis of the system (2.2). An abbreviated Schrödinger equation is obtained for the isolated quantum nonlinear resonance, the parameters of the resonance are obtained, and the resonance is compared with the classical limit described by the Hamiltonian (2.10).

3. QUANTUM-MECHANICAL DESCRIPTION

We shall find it convenient to use the quantum-mechanical "action angle" representation introduced in Ref. 8. According to this reference, the Schrödinger equation for the system (2.2) takes in the "action angle" representation the form

$$i\frac{\partial\Phi(\theta,\tau)}{\partial\tau} = \mathscr{H}(\hat{n},\theta,\tau)\Phi(\theta,\tau), \quad \Phi(\theta+2\pi,\tau) = \Phi(\theta,\tau), \tag{3.1}$$

where the dimensionless Hamiltonian is [cf. (2.7)]

$$\mathscr{H}(\hat{n},\theta,\tau) = E(\hat{n}) + \varepsilon\cos\nu\tau\sum_{l=0}^{\infty}[q_l(n)e^{-il\theta} + \text{c.c}],$$

$$\hat{n} = -i\frac{\partial}{\partial\theta}. \tag{3.2}$$

The wave function $\Phi(\theta,\tau)$ is connected with the wave function $\psi(q,\tau)$ of the initial q-representation by the relation[8]

$$\psi(q,\tau) = \sum_{m=0}^{\infty}\left[\frac{1}{2\pi}\int_0^{2\pi}\Phi(\theta,\tau)e^{-im\theta}d\theta\right]\varphi_m(q), \tag{3.3}$$

and the operator functions $E(n)$ and $q_l(n)$ in (3.2) are obtained by substituting $n \to \hat{n}$ in the following complex-valued functions:

$$E(n) = \int_0^{\infty} dq\varphi_n^*(q)\left[\frac{\hat{p}^2}{2} - \frac{1}{q} + \varepsilon_0 q\right]\varphi_n(q),$$

$$q_l(n) = \int_0^{\infty} dq\,\varphi_{n-l}^*(q)q\varphi_n(q), \quad n \geqslant l, \tag{3.4}$$

$$q_l(n) = 0, \quad 0 \leqslant n < l.$$

In (3.3) and (3.4) $\varphi_n(q)$ is the wave eigenfunction of the unperturbed Hamiltonian

$$\hat{\mathcal{H}}_0=\hat{p}^2/2-1/q+\varepsilon_0 q. \tag{3.5}$$

We consider the electron-energy region with large value of a in (2.6): $a\gg 1$. In this case the system is in the quasiclassical region and the matrix elements (3.4) can be estimated in the quasiclassical approximation. As shown in Ref. 8, the functional dependence of the complex-valued functions in (3.4) on the parameter n coincides in this case with the classical expressions (2.16) and (2.18). Thus, in the energy region considered ($a\gg 1$) the Hamiltonian (3.2) takes the form

$$\mathcal{H}(\hat{n},\theta,\tau)=A\hat{n}^{2/3}-\frac{4A\varepsilon}{\pi\varepsilon_0}\cos\nu\tau\sum_{l=1}^{\infty}\frac{1}{l^2}(\hat{n}^{2/3}e^{-il\theta}+\text{c.c}), \tag{3.6}$$

where A is defined in (2.16).

We consider now an isolated quantum nonlinear resonance. Let the system with Hamiltonian (3.2) be populated at the initial instant of time in the vicinity of a level n_l determined from the condition of the l th resonance:

$$E(n_l+l)-E(n_l)\approx l(dE(n)/dn)_{n=n_l}=\nu. \tag{3.7}$$

We represent the wave function in (3.1) in the form

$$\Phi(\theta,\tau)=\exp[-iE(n_l)\tau+in_l\theta]\chi(\theta,\tau). \tag{3.8}$$

Substituting (3.8) in (3.1) we obtain the equation

$$i\partial\chi(\theta,\tau)/\partial\tau=[\mathcal{H}(\hat{n}+n_l,\theta,\tau)-E(n_l)]\chi(\theta,\tau), \tag{3.9}$$

where the Hamiltonian $\hat{\mathcal{H}}$ is defined by (3.2). We expand this Hamiltonian about the point n_l, confining ourselves in the perturbation to the first term of the expansion, and expand the unperturbed Hamiltonian up to the terms in n^2. This approximation is tantamount to the classical condition of moderate nonlinearity[22] and means in the quantum case that the number δn_l of levels actually participating in the dynamics is much less than the number of levels n_l, i.e., $\delta n_l/n_l\ll 1$ (Refs. 8 and 30). Using (3.2), we obtain from (3.9) in this approximation an equation for the function $\chi(\theta,\tau)$ in the form

$$i\frac{\partial\chi(\theta,\tau)}{\partial\tau}=\omega_l\hat{n}+\frac{1}{2}\gamma_l\hat{n}^2+\varepsilon\cos\nu\tau\sum_{m=1}^{\infty}[q_m(n_l)e^{-im\theta}+\text{c.c}], \tag{3.10}$$

where by virtue of the foregoing discussion ω_l, γ_l, and $q_m(n)$ coincide with the classical expressions (2.9) and (2.10). We replace the variable θ in (3.10) by a new one, $\psi_l=l\theta-\nu\tau$. Discarding the nonresonant terms we obtain ultimately from (3.10)

$$i\frac{\partial\chi(\psi_l,\tau)}{\partial\tau}=\left[-\frac{\gamma_l l^2}{2}\frac{\partial^2}{\partial\psi_l{}^2}+\varepsilon q_l(n_l)\cos\psi_l\right]\chi(\psi_l,\tau),$$
$$\chi(\psi_l+2\pi,\tau)=\chi(\psi_l,\tau). \tag{3.11}$$

Equation (3.11) describes the isolated quantum nonlinear resonance whose dynamics was investigated analytically and numerically in Refs. 19–21. The solution of (3.11) can be represented as an expression in periodic Mathieu functions:

$$\chi(\psi_l,\tau)=\sum_{m=0}^{\infty}\sum_{c,s}a_m^{(c,s)}\exp[-i\lambda_m^{(c,s)}]\chi_m^{(c,s)}(\psi_l), \tag{3.12}$$

where the coefficients $a_m^{(c,s)}$ are determined from the initial conditions, and the superscripts c and s denote the even and odd Mathieu functions:

$$\chi_m^{(c)}(\psi)=ce_{2m}(\psi/2,\mu),$$
$$\chi_m^{(s)}(\psi)=se_{2m}(\psi/2,\mu),$$
$$\lambda_m^{(c,s)}=\tfrac{1}{2}\gamma_l l^2\alpha_{2m}^{(c,s)}(\mu),\quad \mu=\varepsilon q_l(n_l)/2\gamma_l l^2. \tag{3.13}$$

In (3.13), cs and se are periodic Mathieu function and $\alpha(\mu)$ is the spectrum of the Mathieu equation.[23]

It is easy to estimate the number of levels δn_l that participates in the dynamics of the l th resonance. Since it follows from (3.7) that an external resonant field causes transitions to l levels, we have $\delta n_l\sim l\delta m_l$, where δm_l is the characteristic number of levels in the potential well $\varepsilon q_l(n_l)\cos\psi_l$ in (3.11), i.e.,

$$\delta m_l\approx\frac{1}{l}\left[\frac{\varepsilon q_l(n_l)}{\gamma_l}\right]^{1/2}. \tag{3.14}$$

Thus, we obtain $\delta n_l\sim\delta J_l$, where δJ_l determines the width of the classical resonance in terms of the dimensionless action J [see (2.11)]. A more accurate estimate of the number of trapped levels δn_l confirms the functional relation (3.14):

$$\delta n_l=c\delta J_l. \tag{3.15}$$

In (3.15), c is of order unity and depends little on J_l (e.g., in the region $\delta n_l\approx 50$ we have $c\approx 1.4$ (Refs. 20, 23).

We estimate now the frequencies of the phase oscillations in the vicinity of the l th resonance. To this end we represent the asymptotic form of the $\lambda_m^{(c,s)}$ spectrum at $m\ll\mu^{1/2}$ in the form

$$\lambda_m^{(c)}\approx-\varepsilon q_l(n_l)+\Omega_l(2m+\tfrac{1}{2}),\quad m=0,1,\ldots,$$
$$\lambda_m^{(s)}\approx-\varepsilon q_l(n_l)+\Omega_l(2m-\tfrac{1}{2}),\quad m=1,2,\ldots, \tag{3.16}$$
$$\Omega_l=l[\varepsilon q_l(n_l)\gamma_l]^{1/2}.$$

The quantity Ω_l is the frequency of the small oscillations of the system near the bottom of a potential well, and coincides with the frequency of the phase oscillations (2.12) of a classical system. Thus it follows from (3.12) and (3.16) that arbitrary quantum-mechanical mean values will oscillate in the vicinity of the l th resonance with frequencies $\sim m\Omega_l$.

4. CONCLUSION

It follows from the above estimates of the nonlinear-resonance parameters that application of an additional clamping field permits a study of the dynamics of a resonance in a quantum system close enough to the classical limit. By varying the strength of the clamping field one can trap a large number of levels (say, $\delta n\sim 10^3$) into the quantum resonance. This capability is in our opinion important for experimental attempts to study stochasticity in the quantum case. Indeed, since classical stochasticity is directly related to overlap of nonlinear resonances, it is reasonable in the quantum approach to bring the isolated quantum nonlinear resonance as close as possible to its classical limit. This system of surface electrons in a clamping field, which is experimentally accessible, makes it indeed possible to realize an

isolated resonance under the condition $n_l \gg 1$.

Quantum stochasticity can be studied in such a system by letting the resonances overlap. This can be accomplished by using additional external alternating-field sources that permit the production of mutually overlapping resonance systems. In addition, resonance overlap can be achieved also by varying the parameter K [Eq. (2.22)] all the way to $K \sim 1$. We note, however, that in this case we must put $\mathscr{E} \sim \mathscr{E}_0$, and analysis of the system stochastization conditions on the basis of the resonance-overlap conditions may turn out to be invalid. Further research, including computer simulations, is necessary to ascertain the character of the system motion in this case.

Quantum stochasticity should manifest itself in the system investigated by a diffusive change of the electron energy, and close enough to the classical limit ($\delta n_l \gg 1$) one can expect quantum dynamics and the law of diffusion in a classical system to be close.

It should also be noted that the system considered permits a study of the transition from pure quantum dynamics of resonance interaction (small δn_l, $K \sim 1$) to the classical limit.

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