Novel types of sound oscillations in a selectively excited gas

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Sound (low frequency) waves in a gas are considered in the case of excitation that selects molecules with respect to velocity as a result of the Doppler effect. By expanding the solutions of the kinetic equation into a series in the eigenfunctions of the linearized collision operator, it is shown that the following events occur in a selectively excited gas: 1) the velocity of ordinary (longitudinal) sound depends on the direction of the sound with respect to the laser beam; 2) two new types of waves exist which, for small angles between the sound wave vector and the direction of the laser beam are a) thermal sound, in which temperature and gas density oscillations occur, and b) transverse sound, in which oscillations of the macroscopic gas velocity occur in a direction perpendicular to the wave vector of the sound. A gasdynamic theory of these processes is developed which yields the same results as the kinetic approach. The anisotropic corrections to the velocity of ordinary sound and the velocities and properties of the new types of waves depend on the frequency and intensity of the laser radiation and the properties of the excited gas. They may be as great as several millimeters per second. An experiment is proposed for observation of the anisotropic character of the correction to the velocity of ordinary sound.

1. The beginning of a systematic investigation of kinetic processes in a gas in the case of velocity-selective excitation was put forward in Refs. 1 and 2. Such excitation takes place when, on account of the Doppler effect, only atoms (or molecules) having certain projections of the velocity along the direction of a laser beam interact efficiently with the laser radiation.

According to Ref. 3, the kinetic equation for a selectively excited gas can be written in the form

$$\frac{\partial f}{\partial t}+\mathbf{v}\frac{\partial f}{\partial \mathbf{r}}=\iint d^3v_1\, d^2\Omega\sigma(w,\cos\vartheta)$$
$$\times w[f'f_1'(1+\xi(\mathbf{v}'))(1+\xi(\mathbf{v}_1'))$$
$$-ff_1(1+\xi(\mathbf{v}))(1+\xi(\mathbf{v}_1))], \qquad (1)$$

under certain conditions. Here $f(t, \mathbf{v}, \mathbf{r})$ is the distribution function, summed over the internal states of the atoms, $\omega = |\mathbf{v}-\mathbf{v}_1|$; $\sigma(\omega, \cos\theta)$ is the differential scattering cross section of atoms in the ground state; $f'\equiv f(\mathbf{v}', \mathbf{r})$, $f_1\equiv f(\mathbf{v}_1, \mathbf{r})$; the velocities $\mathbf{v}'$ and $\mathbf{v}_1'$ are expressed in terms of the velocities $\mathbf{v}$ and $\mathbf{v}_1$, and also the scattering angle is given on the basis of the usual conservation laws for elastic collisions.

The factors $[1+\xi(\mathbf{v})]$ describe the selective interaction and distinguish Eq. (1) from the ordinary Boltzmann equation. For a two-level model of an atom in the case of weak excitation, we have the functions

$$\xi(\mathbf{v})=\frac{\sigma_1-\sigma_0}{\sigma_0}n_1(\mathbf{v}),$$

where σ_1 is the elastic scattering cross section of the excited atom by an unexcited atom, σ_0 is the elastic scattering cross section of two unexcited atoms, and n_1 is the population of the upper level at the point $\mathbf{v}$ of velocity space. When the characteristic time of the radiation process is much smaller than the characteristic time for gaskinetic processes,

$$n_1(\mathbf{v})=n_{1m}\Gamma^2\left[\Gamma^2+\left(\frac{\omega_l}{c}\mathbf{vl}-\Omega\right)^2\right]^{-1},$$

where Γ is the radiation halfwidth of the absorption resonance line, c is the velocity of light, $\mathbf{l}$ is a unit vector along the laser beam, $\Omega=\omega_l-\omega_{01}-$ is the difference between the frequency of the laser radiation ω and the frequency of the resonance transition ω_{01}, and n_{1m} is the maximum population of the upper level. We emphasize that the dependence of $\xi(\mathbf{v})$ on $\mathbf{v}$ reduces to a dependence on $\mathbf{v}\cdot\mathbf{l}$ and we introduce the notation $\xi(\mathbf{v})=\xi(\mathbf{vl})\equiv\xi(v_\parallel)$ where $\mathbf{vl}=v_\parallel$.

The stationary distribution function of the gas, which is described by Eq. (1), and which has the macroscopic velocity $\mathbf{V}$, number density n_0 and the mean kinetic energy density $(3/2)n_0T_0$, has the form[3]:

$$f(\mathbf{v}\,|\,n_0, T_0, \mathbf{V})=n_0\left(\frac{m}{2\pi T}\right)^{3/2}\frac{1+\mu_0(T_0,\mathbf{V})}{1+\xi(\mathbf{vl})}$$
$$\times\exp\left\{-\frac{m}{2T}(\mathbf{v}-\mathbf{V}-\mu_1(T_0,\mathbf{V})v_T\mathbf{l})^2\right\}, \qquad (2)$$

where

$$T=T_0\left\{1-\frac{1}{3}[\mu_0(T_0,\mathbf{V})-\mu_2(T_0,\mathbf{V})]\right\},$$

$$\mu_k(T_0,\mathbf{V})=\frac{1}{(2\pi)^{1/2}}\left(\frac{m}{T_0}\right)^{(k+1)/2}$$
$$\times\int_{-\infty}^{\infty}\exp\left(-\frac{mv_\parallel^2}{2T_0}\right)v_\parallel^k\xi(v_\parallel+\mathbf{Vl})\,dv_\parallel.$$

We assume $|\xi(v_\parallel)|\ll 1$, $|\mu_k|\ll 1$. For the calculations that follow, we shall use a dimensionless velocity for the atoms, $\mathbf{u}=\mathbf{v}/v_T$, where $v_T=(T/m)^{1/2}$, and we shall normalize the distribution function accordingly. The coefficients μ_k, written down without their arguments, will be assumed in what follows to be calculated at the value $\mathbf{V}=0$. Further, we shall

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need μ_k with $k = 0 - 5$. We note that the theory developed below is valid for any relation between the Doppler and homogeneous broadening of the absorption lines. In the particular case in which the homogeneous broadening is much smaller than the Doppler,

$$\mu_k(T_0, \mathbf{V}) = (v_r - \mathbf{V}\mathbf{l})^k (m/T_0)^{k/2} \mu_0(T_0, V),$$

where the resonance velocity of the absorption is $v_T = \Omega_c/\omega_l$.

2. We now investigate the sound oscillations in a gas described by Equation (1) with the stationary distribution function (2). Applying standard techniques (Refs. 4 and 5), we linearize Eq. (1) and seek a distribution function in the form

$$f = f(\mathbf{u}|n_0, T_0, 0)\,[1 + g(\mathbf{u}, \mathbf{r}, t)], \quad |g| \ll 1.$$

For the function g, which contains only terms of first order in μ_k and ξ, we get from (1)

$$\frac{\partial g}{\partial t} + v_T \mathbf{u}\frac{\partial g}{\partial \mathbf{r}} = [1 + \mu_0 + \xi(u_{\parallel})]\hat{\mathscr{L}}_0(g) + \delta\hat{\mathscr{L}}(g), \qquad (3)$$

where the operators $\hat{\mathscr{L}}_0$ and $\delta\hat{\mathscr{L}}_0$ have the form

$$\hat{\mathscr{L}}_0(g) = v_T n_0 (2\pi)^{-3/2} \iint d^3u_1\, d^2\Omega\, w\sigma(w, \cos\theta) \times e^{-u_1^2/2}[g' + g_1' - g - g_1],$$

$$\delta\hat{\mathscr{L}}(g) = v_T n_0 (2\pi)^{-3/2} \iint d^3u_1\, d^2\Omega\, w\sigma(w, \cos\theta) \times e^{-u_1^2/2}\mu_1(\mathbf{u}_1\mathbf{l})\,[g' + g_1' - g - g_1].$$

In the derivation of (3), it is important to note that the factors containing $(1 + \xi)$ vanish under the integral sign.

We seek a solution of (3) in the form $g(\mathbf{u}, \mathbf{r}, t) = \exp(-i\omega t + i\mathbf{kr})\varphi(\mathbf{u})$, where ω is the sound frequency and $\mathbf{k}$ is the wave vector. For the function $\varphi(u)$ we have the equation

$$-i\omega\varphi(\mathbf{u}) = \hat{O}(\varphi(\mathbf{u})) \equiv [(1 + \mu_0 + \xi(u_{\parallel}))\hat{\mathscr{L}}_0 + \delta\hat{\mathscr{L}} - iv_T\mathbf{ku}]\varphi(\mathbf{u}), \qquad (4)$$

i.e., $\varphi(\mathbf{u})$ and $-i\omega$ are the eigenfunction and the eigenvalue of the operator $\hat{O}$ which appears on the right side of (4). We represent $\hat{O}$ in the form

$$\hat{O} = \hat{\mathscr{L}}_0 + \hat{S} + \hat{V}, \quad \hat{S} = -iv_T\mathbf{ku}, \quad \hat{V} = (\mu_0 + \xi(u_{\parallel}))\hat{\mathscr{L}}_0 + \delta\hat{\mathscr{L}}.$$

The operator $\hat{V}$ governs the selective excitation of the gas. The operator $\hat{S}$ has the same form as in the ordinary kinetic theory of sound. The operator $\hat{\mathscr{L}}_0$ is also the ordinary linearized Boltzmann collision operator. We define a Hilbert space through the scalar product

$$\langle f|g\rangle = \int d^3u\, e^{-u^2/2} f^*(\mathbf{u})\, g(\mathbf{u}),$$

containing the weight $e^{-u^{3/2}}$, which is identical with the exponent found in the kernel of the operator $\hat{\mathscr{L}}_0$. Then the operator $\hat{\mathscr{L}}_0$ will be Hermitian in this space, which makes it possible to apply standard stationary perturbation theory (see, for example, Ref. 6). Here we need to take it into account that the operator $\hat{S}$ is anti-Hermitian, while the operator $\hat{V}$ is non-Hermitian. The operator $\hat{S}$ must be regarded as a small increment to the operator $\hat{\mathscr{L}}_0$, since oscillations with the wave vector $k \ll 1/\lambda$, where λ is the free path length, are studied. The operator $\hat{V}$ is also to be regarded as a small correction to $\hat{\mathscr{L}}_0$ (which is always valid when the intensity of the exciting radiation is low), while we shall be interested in effects which are first order in $\hat{V}$.

Let φ_k be the eigenfunctions and λ_k^0 the corresponding eigenvalues of the operator $\hat{\mathscr{L}}_0$. Because $\hat{\mathscr{L}}_0$ is Hermitian, it generates a complete orthogonal set $\{\varphi_k\}$. As is well known,[4,5] the operator $\hat{\mathscr{L}}_0$ has the fivefold degenerate eigenvalue $\lambda_\alpha{}^0 = 0$ $(\alpha = 1 - 5)$; all the other eigenvalues are real and negative and have (for the case of stiff potentials) a value of the order of $1/\tau$ or greater, where τ is the time of free flight.[7] The excited eigenvalues of the operator $\hat{O}$ corresponding to them also have negative real parts of the order of $1/\tau$ and therefore the solutions corresponding to them are damped out within times of the order of τ.

3. We are interested in sound with frequencies $\omega \ll 1/\tau$. Therefore, we shall consider only the eigenfunctions or modes corresponding to the eigenvalues of the operator $\hat{O}$, into which the fivefold degenerate zeroth eigenvalue of the operator $\hat{\mathscr{L}}_0$ transforms under the perturbation. We choose a set of coordinates such that the sound wave vector $\mathbf{k}$ has the coordinates $(k, 0, 0)$, while the unit vector $\mathbf{l}$ in the radiation propagation direction vector has the coordinates $(\cos\beta, \sin\beta, 0)$; it is clear that β is the angle between the sound wave vector and the direction of the radiation.

As the first five unperturbed eigenfunctions of the operator we choose the following orthonormal set:

$$\varphi_{1,2} = \frac{1}{2^{1/2}}\left(\pm u_x + \frac{u^2}{15^{1/2}}\right),$$

$$\varphi_{3,5} = u_y, u_z, \quad \varphi_4 = \left(\frac{2}{5}\right)^{1/2}\left(\frac{5}{2} - \frac{u^2}{2}\right). \qquad (5)$$

The functions $\varphi_1 - \varphi_2$ correspond to five gasdynamic modes in a dissipationless monatomic gas. The functions $\varphi_{1,2}$ describe sound propagation along the x axis in the positive (φ_1) and negative (φ_2) directions. The functions $\varphi_3 - \varphi_5$ describe the stationary distribution of the y- and z-components of the velocity of the gas (φ_3) and (φ_5) and its temperature (φ_4) along the z axis. To find the correct eigenfunctions of the zeroth approximation and the eigenvalues to first order in the perturbing operator $\hat{S} + \hat{V}$, we should solve the secular equation

$$\det[K_{\alpha\beta} - \lambda^{(1)}\delta_{\alpha\beta}] = 0, \quad K_{\alpha\beta} \equiv \langle\varphi_\alpha|S + V|\varphi_\beta\rangle.$$

It is easy to see that the matrix $K_{\alpha\beta}$ has only two nonzero elements: K_{11} and K_{22}. Therefore φ_1 and φ_2 are thecorrect functions of the zeroth approximation in $\hat{S} + \hat{V}$; we use the notation $\varphi_1 = \varphi_{s1}$, $\varphi_2 = \varphi_{s2}$. The corresponding eigenvalues are

$$\lambda_{s1,\,s2} = K_{11},\ K_{22} = \mp(^5/_3)^{1/2} ikv_T. \qquad (6)$$

We note that in this approximation, the degeneracy $\varphi_3 - \varphi_5$ is not removed and the specifics of selective excitation are not affected.

According to perturbation theory, the correction from the second-order approximation in $\hat{S} + \hat{V}$ to the eigenvalue for the acoustic modes is equal to

$$\lambda_{Sa}^{(2)}=K'_{\alpha\alpha}\quad(\alpha=1-2),$$
$$K_{\alpha\beta}'=-\sum_{j>5}\frac{1}{\lambda_j^0}\langle\varphi_\alpha|\hat S+\hat V|\varphi_j\rangle\langle\varphi_j|\hat S+\hat V|\varphi_\beta\rangle. \qquad (7)$$

Although φ_j with $j=5$ enters into the determination of $K'_{\alpha\beta}$, the explicit form of φ_j is not needed for further calculations.

In second order, the degeneracy of $\varphi_3-\varphi_5$ is removed and the correct eigenfunctions are the superpositions $c_3\varphi_3+c_4\varphi_4+c_5\varphi_5$. The corresponding eigenvalue in the second approximation is the root of the secular equation

$$\det(K_{\alpha\beta}'-\lambda\delta_{\alpha\beta})=0\quad(\alpha,\ \beta=3-5). \qquad (8)$$

The coefficients c_α of the superposition ($\alpha=3$–5) are the solutions of the set

$$\sum_{\beta=3}^{5}K_{\alpha\beta}'c_\beta=\lambda c_\alpha\quad(\alpha=3-5). \qquad (9)$$

Taking account of the explicit form of the operators $\hat V$ and $\hat S$, we have

$$K_{\alpha\beta}'=v_T^2k^2\sum_{j>5}\frac{1}{\lambda_j^0}\langle\varphi_\alpha|u_x|\varphi_j\rangle\langle\varphi_j|u_x|\varphi_\beta\rangle$$
$$+iv_Tk\sum_{j>5}\frac{1}{\lambda_j^0}\langle\varphi_\alpha|\delta\hat{\mathscr L}|\varphi_j\rangle\langle\varphi_j|u_x|\varphi_\beta\rangle$$
$$+iv_Tk\sum_{j>5}\langle\varphi_\alpha|\xi(u_\parallel)|\varphi_j\rangle\langle\varphi_j|u_x|\varphi_\beta\rangle. \qquad (10)$$

The first term in (10), which is connected with dissipative processes, can be neglected under the condition that $|kv_T\tau|\ll|\mu_0|$ or $|k\lambda|\ll|\mu_0|$. The second and third terms in (10) describe the effect of the excitation of the gas on the dispersion relations for the first five modes.

To calculate the second term in (10), we note that, to first order in μ,

$$\delta\hat{\mathscr L}(\varphi_j)=\mu_1\mathbf{l}\left[\hat{\mathscr L}_0\left(\frac{\partial\varphi_j}{\partial\mathbf u}\right)-\lambda_j^0\frac{\partial\varphi_j}{\partial\mathbf u}\right]. \qquad (11)$$

Therefore,

$$iv_Tk\sum_{j>5}\frac{1}{\lambda_j^0}\langle\varphi_\alpha|\delta\hat{\mathscr L}|\varphi_j\rangle\langle\varphi_j|u_x|\varphi_\beta\rangle$$
$$=-iv_Tk\mu_1\mathbf{l}\sum_{j>5}\langle\varphi_\alpha|\frac{\partial}{\partial\mathbf u}|\varphi_j\rangle\langle\varphi_j|u_x|\varphi_\beta\rangle$$
$$=iv_Tk\mu_1\mathbf{l}\left\{\langle\varphi_\alpha|\frac{\partial}{\partial\mathbf u}u_x|\varphi_\beta\rangle-\sum_{\gamma=1}^{5}\langle\varphi_\alpha|\frac{\partial}{\partial\mathbf u}|\varphi_\gamma\rangle\langle\varphi_\gamma|u_x|\varphi_\beta\rangle\right\}$$
$$=-iv_Tk\mu_1\cos\beta\delta_{\alpha\beta}-iv_Tk\mu_1\mathbf{l}\left\{\langle\varphi_\alpha|u_x\frac{\partial}{\partial\mathbf u}|\varphi_\beta\rangle\right.$$
$$\left.-\langle\varphi_\beta|u_x|\varphi_\beta\rangle\langle\varphi_\alpha|\frac{\partial}{\partial\mathbf u}|\varphi_\beta\rangle\right\}$$
$$=-iv_Tk\mu_1\cos\beta\delta_{\alpha\beta}-iv_Tk\mu_1\mathbf{l}\langle\varphi_\alpha|\frac{\partial}{\partial\mathbf u}|\varphi_\beta\rangle$$
$$\times\{\langle\varphi_\alpha|u_x|\varphi_\alpha\rangle-\langle\varphi_\beta|u_x|\varphi_\beta\rangle\}=-iv_Tk\mu_1\cos\beta\delta_{\alpha\beta} \qquad (12)$$

(for $(\alpha,\beta)=(1,1),(2,2)$ and for $\alpha,\beta=3$–5). The origin of the matrix elements (12) is associated with the shift of the argument of the exponential of the stationary distribution function (2). For the third term in (10), we have

$$iv_Tk\sum_{j>5}\langle\varphi_\alpha|\xi(u_\parallel)|\varphi_j\rangle\langle\varphi_j|u_x|\varphi_\beta\rangle=iv_Tk\left\{\langle\varphi_\alpha|\xi(u_\parallel)u_x|\varphi_\beta\rangle\right.$$
$$\left.-\sum_{\gamma=1}^{5}\langle\varphi_\alpha|\xi(u_\parallel)|\varphi_\gamma\rangle\langle\varphi_\gamma|u_x|\varphi_\beta\rangle\right\}=uv_Tk\langle\varphi_\alpha|\xi(u_\parallel)u_x|\varphi_\beta\rangle$$
$$+\langle\varphi_\alpha|\xi(u_\parallel)|\varphi_\beta\rangle\langle\varphi_\beta|-iv_Tku_x|\varphi_\beta\rangle=iv_Tk\langle\varphi_\alpha|\xi(u_\parallel)|\varphi_\beta\rangle$$
$$+\lambda_\beta^{(1)}\langle\varphi_\alpha|\xi(u_\parallel)|\varphi_\beta\rangle. \qquad (13)$$

The calculation of the matrix elements according to Eq. (13) is conveniently carried out in the coordinate frame in which $\mathbf{k}=(k\cos\beta,k\sin\beta,0)$ and $\mathbf{1}=(1,0,0)$.

By calculating the matrix elements $K'_{\alpha\beta}$ from Eqs. (10–13), for the longitudinal acoustic modes from (6), (7), we obtain the dispersion relations (the upper sign corresponds to the $S1$ mode the lower to the $S2$ mode)

$$\omega_{S1,S2}=kv_T\left\{\pm\left(\frac{5}{3}\right)^{1/2}\left[1+\frac{1}{30}(-\mu_0-2\mu_2+\mu_4)\right]\right.$$
$$+\frac{1}{30}(-3\mu_1+6\mu_3-\mu_5)\cos\beta$$
$$\left.\pm\frac{1}{10}\left(\frac{5}{3}\right)^{1/2}(3\mu_0+3\mu_2-2\mu_4)\cos^2\beta+\frac{1}{2}(3\mu_1-\mu_3)\cos^3\beta\right\}. \qquad (14)$$

To find the coefficients of the second-order superpositions, we obtain the system

$$\begin{bmatrix}0&0&0\\0&(3\mu_1-\mu_3)\cos\beta\sin^2\beta&\frac{1}{\sqrt{10}}(\mu_0-4\mu_2+\mu_4)\cos\beta\sin\beta\\0&\frac{1}{\sqrt{10}}(\mu_0-4\mu_2+\mu_4)\cos\beta\sin\beta&\frac{1}{10}(-3\mu_1+6\mu_3-\mu_5)\cos\beta\end{bmatrix}\begin{bmatrix}c_3\\c_4\\c_5\end{bmatrix}=\frac{\lambda}{-ikv_T}\begin{bmatrix}c_3\\c_4\\c_5\end{bmatrix}. \qquad (15)$$

The values λ are found from the secular equation corresponding to (15). We use for its roots the designations λ_z, λ_{tr}, λ_{th}, the meaning of which is made clear in the discussion of the results. Solving the secular equation, we find

$$\lambda_z=0,\quad \lambda_{th,tr}=-ikv_T\frac{\cos\beta}{2}\Big\{(3\mu_1-\mu_3)\sin^2\beta+\frac{1}{10}(-3\mu_1+6\mu_3-\mu_5)\pm\Big[\Big((3\mu_1-\mu_3)\sin^2\beta-\frac{1}{10}(-3\mu_1+6\mu_3-\mu_5)\Big)^2+\frac{2}{5}(\mu_0-4\mu_2+\mu_4)^2\sin^2\beta\Big]^{1/2}\Big\}. \quad (16)$$

For λ_z the eigenfunction φ_z is equal to $\varphi_z=\varphi_5$. For λ_{th} and λ_{tr}, the eigenfunctions, which we denote by φ_{th} and φ_{tr}, are superpositions of φ_3 and φ_4 in the general case.

In the special case of small angles β,

$$\lambda_{th}\approx-\frac{ikv_T}{10}(-3\mu_1+6\mu_3-\mu_5),\quad \varphi_{th}\approx\varphi_4,$$
$$\lambda_{tr}\approx-\frac{ikv_T}{2}\Big[6\mu_1-2\mu_3-2\frac{(\mu_0-4\mu_2+\mu_4)^2}{-3\mu_1+6\mu_3-\mu_5}\Big]\sin^2\beta,\quad \varphi_{tr}\approx\varphi_3. \quad (17)$$

The dispersion relations corresponding to the *th*, *tr*, and *z* modes have the form

$$-i\omega_{th,\,tr,\,z}(\mathbf{k})=\lambda_{th,\,tr,\,z}(\mathbf{k}). \quad (18)$$

4. We now discuss the results obtained. We recall that the sound is propagated parallel to the x axis; the direction of the laser beam lies in the (x, y) plane; β is the angle between this direction and the x axis. We consider the $S1$ and $S2$ modes, which correspond to ordinary sound in the unperturbed gas; $S1$ corresponds to sound propagation in the positive x direction, $S2$, to the negative. In our case, the sound velocity $s_\alpha=|\delta\omega_{s2}/\delta k|$ depends on the angle β while, $\omega_{s1}(\beta=0)=\omega_{s2}(\beta=\pi)$ as it should. Let $\beta=0$, i.e., the laser radiation propagates in the positive x direction. The sound velocity s_1 in the direction of the laser beam is different from the sound velocity s_2 propagating in the opposite direction. The velocity difference is equal to

$$\Delta s=s_1-s_2={}^1/_{15}v_T(42\mu_1-9\mu_3-\mu_5). \quad (19)$$

The excitation of the gas in the considered approximation has no effect on the dispersion relation for the mode with $\alpha=5$ (mode z), which corresponds to macroscopic motion of the gas along the z axis.

The oscillations in the two remaining, weakly attenuated modes *th* and *tr* in the excited gas (in the general case) are superpositions of the oscillations of modes φ_3 and φ_4 of the unperturbed gas, i.e., oscillations of the temperature and the transverse velocity v_y. However, in contrast with the unperturbed gas, the modes φ_{th} and φ_{tr} in the excited gas are not stationary distributions, but traveling transverse-thermal waves with velocity $s_{th,\,tr}=i\lambda_{th,\,tr}/k$. We note that at $\beta\ll1$, it follows from (17) that the mode *th* is a thermal traveling wave, in which oscillations of the temperature and density, respectively, of the gas take place; the mode *tr* is a transverse traveling wave, in which oscillations of the y component of the macroscopic velocity of the gas principally occur.

Thus, the presence of velocity-selective excitation in a gas causes the velocity of ordinary (i.e., longitudinal) sound to depend on the direction of its propagation and gives rise to the appearance of two new types of traveling waves (modes *th* and *tr*), which can be called, respectively, the first and second branches of transverse-thermal sound. Both the anisotropic increments to the velocity of ordinary sound and the velocity of the new waves are of order $\varkappa v_T(\sigma_1-\sigma_0)/\sigma_0$, where $\varkappa$ is the fraction of the excited atoms in the gas. For the gas we can use, e.g., neon, on which experiments on velocity-selective excitation have been carried out.[8] Taking the realistic values $(\sigma_1-\sigma_0)/\sigma_0\approx10^{-1}$ and $\varkappa\sim10^{-4}$ as estimates, we obtain a velocity on the order of $10^{-5}\,v_T$, i.e., several millimeters per second. The condition that the damping of the new types of waves be weak is given by the relation $|\tau k v_T|\ll\varkappa(\sigma_1-\sigma_0)/\sigma_0$, which ensures the smallness of the first term in (10). Taking as an estimate $\tau\sim10^{-9}$ s, which is valid for a gas at room temperature and pressure on the order of 100 Torr, we obtain $k\ll0.3$ cm^{-1}. This means that observation of a wave with wavelength of order 10 cm and more is possible.

Corrections to the velocity of the longitudinal wave can be observed, for example, in the following experiment. Let a standing wave be generated in a cavity, with a frequency ω that is the same as one of the eigenfrequencies of the resonator. The gas is excited by laser radiation, directed parallel to the standing wave. Under the action of the radiation, the length of a sound wave of fixed frequency ω begins to depend on the direction of sound propagation; the resonance conditions for the sound in the cavity worsen and the amplitude of the standing wave decreases. It is significant that this decrease cannot be removed by changing the frequency ω, since under the action of the radiation, the effective Q of the cavity decreases. Thus, we can quickly [within the time needed to establish the stationary distribution function (2)] switch the effective Q of the cavity.

APPENDIX

We have used the kinetic approach above. We now obtain and investigate the equations of gasdynamics for a selectively excited gas. Applying the operators

$$m\int d^3v,\quad m\int v d^3v,\quad m\int\frac{v^2}{2}d^3v$$

to Eq. (1), and using only the fact that the collision integral on the right side of (1) conserves the number of particles, momentum and energy, we obtain the equations of gasdynamics in the form of the conservation laws

$$\frac{\partial\rho}{\partial t}+\frac{\partial}{\partial x_k}(\rho V_k)=0, \quad (A1)$$

$$\frac{\partial}{\partial t}(\rho V_i)+\frac{\partial}{\partial x_k}(\rho V_iV_k+\pi_{ik})=0, \quad (A2)$$

$$\frac{\partial}{\partial t}\Big(\rho\frac{V^2}{2}+\frac{1}{2}\mathrm{Sp}\,\pi\Big)+\frac{\partial}{\partial x_k}\Big[V_k\Big(\frac{\rho V^2}{2}+\frac{1}{2}\mathrm{Sp}\,\pi\Big)+\pi_{kl}V_l+q_k\Big]=0, \quad (A3)$$

where $\rho=nm$ is the density of the gas, m is the mass of the atom; the gasdynamic velocity $\mathbf{V}$, the pressure tensor π_{ik} and the additional (uncorrected) energy flux $\mathbf{q}$ that arise in the derivation of the set (A1)–(A3) are determined by the formulas

$$V_i=\langle v_i/n\rangle,$$
$$\pi_{ik}=\langle m(v_i-V_i)(v_k-V_k)\rangle, \tag{A4}$$
$$q_i=\left\langle \frac{m}{2}(v_i-V_i)(v_l-V_l)^2\right\rangle,$$
$$\langle(\ldots)\rangle\equiv\int d^3v f(\mathbf{v})(\ldots). \tag{A5}$$

The set (A1)–(A3) consists of five equations and permits formulation of the problem with initial conditions for $\mathbf{V}$, ρ and some other scalar quantity, for which we choose the pressure p, equal, by definition, to $(1/3)\mathrm{Tr}\pi$. We shall consider gasdynamics as a theory of the field having the five components ρ, $\mathbf{V}$, p. For this purpose, we close the system (A1)–(A3), i.e., we express π_{ik} and q_i in terms of ρ, $\mathbf{V}$, p. The ordinary equations of gasdynamics (without account of dissipative processes) are obtained if we postulate the Pascal's law and the absence of additional energy flux:

$$\pi_{ik}=p\delta_{ik},\quad q_i=0. \tag{A6}$$

We note that (A6) is obtained if we take $f(\mathbf{v},\mathbf{V})=f_M(\mathbf{v}-\mathbf{V})$, where f_M is the equilibrium Maxwell function. In our case, however, f is not a displaced Maxwellian, since there is still another vector in the problem—the wave vector of the radiation, $\mathbf{k}_l$. The function f depends on $\mathbf{k}_l$ by virtue of the Doppler effect. Hence $f(\mathbf{v},\mathbf{V},\mathbf{k}_l)=f(\mathbf{v},\mathbf{V},c|\mathbf{k}_l|-\mathbf{k}_l\mathbf{v})$, where $c|\mathbf{k}_l|-\mathbf{k}_l\mathbf{v}$ is the radiation frequency in the system of an atom having a velocity $\mathbf{v}$ (we assume $v\ll c$). We shall denote by a prime those quantities measured by an observer having the velocity w relative to the laboratory system. It is necessary that

$$f'(\mathbf{v}',\mathbf{V}',c|\mathbf{k}_l'|-\mathbf{k}_l'\mathbf{v}')=f(\mathbf{v},\mathbf{V},c|\mathbf{k}_l|-k_l\mathbf{v})$$

in the case

$$\mathbf{v}'=\mathbf{v}-\mathbf{w},\quad V'=\mathbf{V}-\mathbf{w},\quad c|\mathbf{k}_l'|-\mathbf{k}_l'\mathbf{v}'=c|\mathbf{k}_l|-\mathbf{k}_l\mathbf{v}.$$

The function

$$f(\mathbf{v},\mathbf{V},\mathbf{k}_l)=f(\mathbf{v}-\mathbf{V},\quad c|\mathbf{k}_l|-\mathbf{k}_l\mathbf{V},\quad \mathbf{k}_l/|\mathbf{k}_l|) \tag{A7}$$

satisfies this requirement. Substituting (A7) in (A4), and (A5), we find that π_{ik} and q_i can be represented in the form

$$\pi_{ik}=p\delta_{ik}+a(l_il_k-{}^1/_3\delta_{ik}),\quad q_i=ql_i, \tag{A8}$$

where $l_i=k_i/k$. We have introduced two new functions, a and q, which depend on ρ, p and $V_\parallel=\mathbf{V}\cdot\mathbf{l}$, the projection of the velocity $\mathbf{V}$ in the l direction:

$$a=a(\rho,p,V_\parallel),\quad q=q(\rho,p,V_\parallel).$$

We emphasize that there is no justification for neglecting the dependence of a and q on any of the variables. Aside from the dependence explicitly indicated, a and q the functions naturally depend on the intensity of the scattering radiation, and vanish when there is no radiation. Thus, as has already been pointed out in Ref. 3, an additional energy flux (the function q) and anisotropic pressure tensor (the function a) arise in a selectively excited gas.[1)]

Taking account of (A8), the set (A1)–(A3) become closed; here a and q must be regarded as known functions of their arguments.

We emphasize that in the derivation of (A1)–(A3), (A8), we have not used all the specifics of Eq. (1), but have used only the conservation laws and the symmetry of the problem. If the equilibrium distribution function f has the form (2), which takes into account the specifics of Eq. (1) in full measure, then a and q are expressed in terms of their parameters as follows:

$$a=p\left[\mu_0\left(\frac{mp}{\rho},V_\parallel\right)-\mu_2\left(\frac{mp}{\rho},V_\parallel\right)\right],$$
$$q=\frac{p^{3/2}}{2\rho}\left[3\mu_1\left(\frac{mp}{\rho},V_\parallel\right)-\mu_3\left(\frac{mp}{\rho},V_\parallel\right)\right] \tag{A9}$$

Using the set (A1)–(A3), (A8), we can investigate the various modes of sound oscillations within the framework of gasdynamics. Taking account of (A9), the set (A1)–(A3), (A8), after linearization near the values ρ_0, p_0, $\mathbf{V}_0=0$, we get the same results (14), (16) for sound as in the kinetic approach. Here the velocity of ordinary sound s turns out to be equal to $s=(5p_0/3\rho_0)^{1/2}$ $(\mu_k\to 0)$, as it should.

The possible types of sound oscillations in a selectively excited gas have been investigated in the work of Ref. 10 on the basis of the various gasdynamic relations written there. Assumptions are made in implicit form in Ref. 10 which, in our notation, take the form

$$a=0,\quad q(\rho,p,V_\parallel)=q(\rho,p). \tag{A10}$$

Making the additional assumptions (A10), we can obtain all the results of Ref. 10 from the set (A1)–(A3) for a selectively excited gas that is homogeneous in composition, including the presence of a new type of oscillation—thermal sound, whose velocity is found to be proportional to $\cos\beta$.

However, the assumptions (A10) contradict the kinetic approach and the systematic gasdynamic theory based on it which, in addition to other waves reveal the presence of two new types of oscillations—thermal sound and transverse sound, the velocity of which is a complicated function of β [see (16), (18)]. In spite of this, the work of Ref. 10 has value as the first indication that the sound oscillations in a selectively excited gas do not reduce to ordinary sound.

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[1)]The dependence of a and p on the intensity of the radiation can be used for efficient generation of sound.[9]

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