Quantum corrections to the electric conductivity of an inhomogeneous medium

F. T. Vas'ko

Semiconductor Institute, Ukrainian Academy of Sciences
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The quantum corrections are calculated for the electric conductivity and for the Hall coefficient of noninteracting electrons in a medium with small long-wave fluctuations of the density of pointlike scatterers or electrons. In the former case the logarithmic increment to the conductivity of two-dimensional electrons is changed when the phase relaxation length of the wave function is of the order of the correlation length. If the Fermi energy fluctuates, the Hall coefficient acquires a temperature-dependent increment.

Besides the short-range scatterers that determine in the metallic region the quantum corrections to the conductivity, a disordered conductor contains also long-wave fluctuations (over distances exceeding the electron mean free path) of various parameters of the medium. The role of such fluctuations (a metal-insulator transition is possible if their amplitude is large) under conditions of intense short-wave scattering has become the subject of recent studies (see Ref. 2 for a discussion and references). We write out here the quantum corrections to the current, and the equations for the cooperon (we consider the case when the contribution of the Coulomb corrections is small) in the presence of density fluctuations of pointlike scatterers or of the Fermi energy. For small fluctuations we find next the averaged quantum correction to the conductivity of two-dimensional electrons, as well as the Hall coefficient. These quantities are qualitatively different from those of a macroscopically homogeneous medium. The differences are that the coefficient of the logarithmic increment to the two-dimensional conductivity depends on the amplitude fluctuations (this coefficient changes radically in the case of pointlike-scatterer density fluctuations, when the phase relaxation length l_φ is of the order of the correlation length l_c), while the Hall coefficient acquires a temperature-dependent correction due to fluctuations of the Fermi energy.

1. The influence of long-wave fluctuations on the quantum correction to the conductivity can be treated (even in an approximation linear in the electric field) by the Keldysh diagram technique, which is expedient for the quasiclassical transition to the limit. We take into account the pointlike-scatterer density by identifying the impurity dashed line with a correlator

$$w(\mathbf{x}_1, \mathbf{x}_2) = w\left(\frac{\mathbf{x}_1+\mathbf{x}_2}{2}\right)\delta(\mathbf{x}_1-\mathbf{x}_2), \quad (1)$$

in which $w(\mathbf{r})$ varies smoothly over distances on the order of the mean free path l_F for electrons with Fermi energy ε_F. Allowance for the inhomogeneity does not alter the analogous[3,4] derivation of an equation for the cooperon, in which only a variable diffusion coefficient $D_r \propto w(\mathbf{r})$ appears (τ_φ denotes below the phase-relaxation time):

$$(-\nabla D_r \nabla + i\Omega + 1/\tau_\varphi) C(\mathbf{r}, \mathbf{r}') = \delta(\mathbf{r}-\mathbf{r}'). \quad (2)$$

The quantum increment to the current (we consider the contribution to a response at a frequency Ω) differs from the standard expression only in that account is taken of the coordinate dependence of the field $\mathbf{E}_r$:

$$\Delta\mathbf{j}_r = -(2e^2/\pi)\mathbf{E}_r D_r C(\mathbf{r}, \mathbf{r}). \quad (3)$$

Consider also another model of a smooth inhomogeneity: assuming a δ-like correlator (1), we add to the equation a random electric field, so that the Fermi distribution $\theta(\mu_r - \varepsilon)$ contains the electrochemical potential μ. Now $\Delta\mathbf{j}_r$ differs from (3) by the presence of an additional averaging over the energy (the contribution that determines the Hall constant and is linear in the magnetic field has also been written out):

$$\Delta\mathbf{j}_r = -\frac{2e^2}{\pi}\int d\varepsilon\,\delta(\mu_r-\varepsilon)\left\{\mathbf{E}_r + 2\tau_\varepsilon\frac{e}{mc}[\mathbf{E}_r\times\mathbf{H}]\right\} D_\varepsilon C_\varepsilon(\mathbf{r}, \mathbf{r}), \quad (4)$$

and the equation analogous to (2) for the cooperon $C_\varepsilon(\mathbf{r}, \mathbf{r}')$ contains $-D_\varepsilon\nabla^2$, where D_ε is the diffusion coefficient of an electron having an energy ε, while τ_ε is the corresponding momentum-relaxation time.

The classical current $\mathbf{j}_r$ is defined by the usual equation

$$\mathbf{j}_r = \sigma_r\left(\mathbf{E}_r + \tau_r\frac{e}{mc}[\mathbf{E}_r\times\mathbf{H}]\right) \quad (5)$$

with a conductivity σ_r that fluctuates (because of the variation of the scatterer or electron densities) and with a momentum relaxation time τ_r. Equations (3)–(5) are averaged in analogy with Ref. 5 in the small-fluctuation approximation, while $\mathbf{E}_r$ and the total current $\mathbf{j}_r + \Delta\mathbf{j}_r$ satisfy at $\Omega\tau_F < 1$ the electrostatic equations

$$\operatorname{rot}\mathbf{E}_r = 0, \quad \operatorname{div}(\mathbf{j}_r + \Delta\mathbf{j}_r) = 0. \quad (6)$$

The averaging results are given above for the case of two-dimensional diffusion, when small fluctuations alter the universal coefficient of the logarithm. Separation of the contribution of small fluctuations for the one- or three-dimensional case is a more complicated matter, since they enter together with a coefficient that contains material parameters.

2. We seek the solution of (2) in the momentum representation, separating in the cooperon the averaged and fluctuating parts $\langle C(\mathbf{k},\mathbf{k}')\rangle$ and $\delta C(\mathbf{k},\mathbf{k}')$, respectively, for which we obtain, to within ξ^2, the equations

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$$\left(D_F k^2+i\Omega+\frac{1}{\tau_\varphi}\right)\langle C(\mathbf{k},\mathbf{k}')\rangle$$
$$+D_F\int\frac{d\mathbf{k}_1}{(2\pi)^2}(\mathbf{k}\mathbf{k}_1)\langle\xi_{\mathbf{k}-\mathbf{k}_1}\delta C(\mathbf{k}_1,\mathbf{k}')\rangle=(2\pi)^2\delta(\mathbf{k}-\mathbf{k}'),$$
$$\left(D_F k^2+i\Omega+\frac{1}{\tau_\varphi}\right)\delta C(\mathbf{k},\mathbf{k}')$$
$$+D_F\int\frac{d\mathbf{k}_1}{(2\pi)^2}(\mathbf{k}\mathbf{k}_1)\xi_{\mathbf{k}-\mathbf{k}_1}\langle C(\mathbf{k}_1,\mathbf{k}')\rangle=0. \qquad (7)$$

The coordinate dependence of the diffusion coefficient (as well as of the conductivity) is introduced here by the relation $D_r = D_F(1+\xi_r)$. Since $\langle\xi\delta C\rangle$ does not contain a logarithmic contribution that diverges at low momenta, the cooperon fluctuations determine only the rescaling of the diffusion coefficient in the expression

$$\langle C(\mathbf{r},\mathbf{r})\rangle=\int\frac{d\mathbf{k}}{(2\pi)^2}\left[D(k)k^2+i\Omega+\frac{1}{\tau_\varphi}\right]^{-1},$$
$$D(k)=D_F\begin{cases}1-\overline{\xi}^2/2, & k\ll k_c\\ 1, & k\gg k_c,\end{cases} \qquad (8)$$

where $\overline{\xi}^2$ is the mean square fluctuation of D_F and $k_c=2/l_c$. The fluctuation field

$$\delta\mathbf{E}_r=-\int\frac{d\mathbf{k}}{(2\pi)^2}e^{i\mathbf{k}\mathbf{r}}\xi_r\frac{\mathbf{k}(\mathbf{k}\mathbf{E})}{k^2} \qquad (9)$$

determined in the usual manner from (6) does not contain the quantum correction $\Delta\sigma$ to the conductivity, and the latter is expressed in terms of (8) and of the average classical current [that stems from $D_r\mathbf{E}_r$ in (3)].

The integral in (8) is cut off at small momenta for $k_m=\max[(\Omega/D_F)^{1/2},(D_F\tau_\varphi)^{-1/2}]$ in the case of long-wave fluctuations ($k_m>k_c$). For short-wave fluctuations, $k_m(1-\xi^2/2)^{-1/2}<k_c$, the logarithmic divergence is cut off at $k_m(1-\xi^2/2)^{-1/2}$. The result is

$$\Delta\sigma=-\frac{e^2}{2\pi^2\hbar}\begin{cases}(1-\overline{\xi}^2/2)\ln(k_m l_F)^{-1}, & k_m>k_c\\ \ln(k_m\tilde{l}_F)^{-1}, & k_m<k_c(1-\overline{\xi}^2/2)^{1/2}\equiv\tilde{k}_c\end{cases}, \qquad (10)$$

so that the slope of the logarithmic dependence of $\Delta\sigma$ changes over the interval $\tilde{k}_c<k_m<k_c$ and is somewhat lowered, since

$$\tilde{l}_F^{-1}=(1-\overline{\xi}^2/2)(l_F k_c)^{\overline{\xi}^2/2}l_F^{-1}<l_F^{-1}.$$

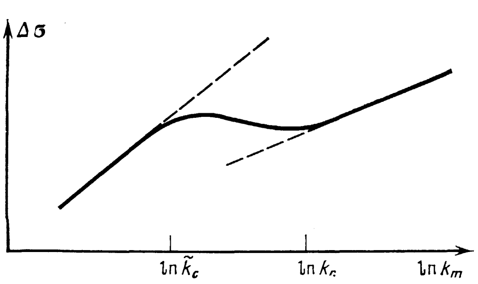

FIG. 1. Change of logarithmic dependence of σ at $k_c>k_m>\tilde{k}_c\equiv k_c(1-\overline{\xi}^2/2)^{1/2}$. The temperature dependence of $\Delta\sigma$ is obtained from this by using $k_m\propto\tau_\varphi^{-1/2}\propto T^{p/2}$.

The temperature or frequency dependences determined by (10) (see Fig. 1) are physically understandable: the short-wave fluctuations can be regarded as an additional scattering mechanism that does not alter the coefficient of the logarithm, whereas in the long-wave case the additional coefficient $a-\overline{\xi}^2/2$ is due to averaging of the fluctuating classical current that enters in (3).

Since the short-wave ($k_m\ll k_c$) fluctuations can be regarded as a supplementary scattering mechanism, the quantum corrections to the Hall constant drop out in this case.[6] Adding a magnetic field to (3), in analogy with (4) and (5) (the quantum correction $\delta\mathbf{E}_r$ remains in this case), we can verify that this conclusion holds for the entire range of variation of k_m, i.e., no temperature dependence of the Hall constant sets in.

3. In the case of electron-density fluctuations, the cooperon in (4) is defined by the expression

$$D_\varepsilon C_\varepsilon(\mathbf{r},\mathbf{r})=\frac{1}{2\pi}\ln[k_m(\varepsilon)l_F]^{-1}, \qquad (11)$$

and the energy dependences of D_ε, τ_ε, and of the cutoff parameter $k_m(\varepsilon)$ are different for two-dimensional electrons (case I) and for two-dimensional diffusion in a film with a three-dimensional energy spectrum (case II). The coordinate dependence of μ_r [and of σ_r and τ_r in (5)] are also different in cases I and II. Assuming the electron density to fluctuate like $n(1+\xi_r)$, we get

$$D_\varepsilon=D_F\varepsilon/\varepsilon_F,\quad k_m(\varepsilon)=k_m(\varepsilon_F/\varepsilon)^{1/2},\quad \mu_r=\varepsilon_F(1+\xi_r)\ \ (\mathrm{I}),$$
$$D_\varepsilon=D_F(\varepsilon/\varepsilon_F)^{3/2},\quad k_m(\varepsilon)=k_m(\varepsilon_F/\varepsilon)^{3/2}, \qquad (12)$$
$$\mu_r=\varepsilon_F(1+{}^2/_3\xi_r)\ \ (\mathrm{II}).$$

Solving (6) in the approximation linear in the fluctuations, we can use (11) at $\varepsilon=\varepsilon_F$, so that the quantum correction δ that enters in δE_r is determined by the relation (σ_B is the conductivity for $\xi_r=0$)

$$\sigma_B\delta=\frac{e^2}{2\pi^2}\ln(k_m l_F)^{-1}, \qquad (13)$$

and the fluctuating field differs from (9) in that $\mathbf{E}$ is replaced by

$$(1+\delta)\left(\mathbf{E}+\frac{e\tau_F}{mc}[\mathbf{E}\times\mathbf{H}]\right) \qquad (\mathrm{I}),$$
$$(1+\delta)\left(\frac{4}{3}\mathbf{E}+\frac{5}{3}\frac{e\tau_F}{mc}[\mathbf{E}\times\mathbf{H}]\right)-\delta\frac{e\tau_F}{mc}[\mathbf{E}\times\mathbf{H}] \qquad (\mathrm{II}). \qquad (14)$$

$\langle\mathbf{j}_r\rangle$ is determined directly from (5) (9), and (14). By averaging the current $\Delta\mathbf{j}_r$ determined by (4) and (11), it is also easy to calculate the contribution proportional to $\ln k_m$, and the small changes of l_F^{-1} and δ due to the contribution $\ln\varepsilon$ can be neglected. The result is a conductivity quantum correction

$$\Delta\sigma\approx-\frac{e^2}{2\pi^2\hbar}\ln(k_m l_F)^{-1}\begin{cases}1+{}^1/_2\overline{\xi}^2 & (\mathrm{I})\\ 1+{}^8/_9\overline{\xi}^2 & (\mathrm{II})\end{cases}, \qquad (15)$$

that does not depend on the correlation length l_c. The Hall constant also acquires a temperature-dependent increment (proportional to $\bar{\xi}^2\delta$)

$$R=\frac{1}{enc}\begin{cases}1-\bar{\xi}^2\delta & \text{(I)}\\ 1-{}^4/_9\bar{\xi}^2-2\bar{\xi}^2\delta & \text{(II)}\end{cases}. \qquad (16)$$

We emphasize that for the calculation of (15) and (16) (as well as for the Hall effect to be independent of temperature) it is important to take into account the logarithmic temperature increments due to the fluctuating electric field $\delta\mathbf{E}_r$.

4. The models considered demonstrate the qualitative features of the temperature dependence (the frequency dispersion was apparently not measured) of the conductivity of weakly localized electrons in a macroscopically inhomogeneous medium. Continuous inhomogeneities alter the observed degree of the temperature dependence of the phase-relaxation time ($\tau_\varphi \propto T^{-p}$), so that an increment proportional to $\bar{\xi}^2$ appears in the coefficient $p/2$ preceding the logarithm. Measurements (see Ref. 7 for the sources) usually yield for p values that differ from a rational number. However, the homogeneities of the samples is not specially monitored (although the technologies employed do not exclude the possibility of long-wave fluctuations of the parameters), so that is not clear whether the obtained values of p are the results the mechanism considered above or, for example, are due to the contribution of several phonon-scattering channels to k_m. The observed temperature increments to the Hall constant are attributed to the contribution of electron-electron quantum additions.[6]

The contribution the long-wave fluctuations can be identified by means of the temperature dependences, provided that $l_\varphi \sim l_c$ in the investigated temperature interval, so that a kink appears on the logarithmic temperature dependence (see Fig. 1). To observe the contribution of the density fluctuations, the correlation described by (15) and (16) should appear between the change of the coefficient of the logarithm in $\Delta\sigma$ and the Hall constant. A direct method of investigating these effects would be measurements of the temperature dependences in samples with controllable (or variable in the course of the treatment) values of the long-wave fluctuations.

Adding the magnetic field to the equation for the cooperon, we can describe in similar fashion the influence of long-wave fluctuations on the magnetoresistance. Other generalizations of the calculation presented are also possible: account can be taken of the simultaneous influence of the scatterer-density and electron density fluctuations (this density does not lead to new properties of $\Delta\sigma$ and R), or consideration of the case of large fluctuations, which calls for a more elaborate classical treatment (see Ref. 8). Similar effects result from smooth fluctuations of the film thickness and from the Coulomb contribution to the quantum corrections to the conductivity.

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