Anomalous magnetoresistance of superconducting metallic glasses of the metal-metal type

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The magnetoresistance of metallic glasses based on transition metals Zr and Hf is studied at low temperatures. In all the systems studied the magnetoresistance is positive and depends on the magnetic field strength in an anomalous manner. The anomalous behavior of the magnetoresistance can be described satisfactorily by the theory of weak localization and electron-electron interaction in 3D disordered systems, in which the scattering of electrons by superconducting fluctuations is taken into account. The absolute values and temperature dependences of the electron-electron coupling constant and the inelastic scattering times of conduction electrons in the metallic glasses studied are determined.

1. INTRODUCTION

Interest in the study of the kinetic phenomena in disordered systems with a metallic conductivity has increased in recent years because of the development of a new theory[1] which explains several experimental results concerning the dependence of the magnetokinetic coefficients on temperature and magnetic field and, in particular, the existence of negative magnetoresistance which had no satisfactory explanation for a long time. The new theory predicts two effects: a "weak" localization of conduction electrons and an enhancement of the electron-electron interaction in the presence of an impurity scattering, which account for the corrections to the classical conductivity. These corrections depend anomalously on the temperature, magnetic field, and several other parameters.

It is evident from the classical theory that normal magnetoresistance is negligible in amorphous systems in which the mean free path of electrons is comparable to the atomic separation. Many experimental data show, however, that an appreciable magnetoresistance is present in a large group of disordered systems: in thin nonsuperconducting metallic films (see, for example, Refs. 2 and 3 and the bibliography cited there), in three-dimensional superconducting[4,5] and nonsuperconducting amorphous systems,[6,7] and in strongly doped semiconductors.[8] In these studies it was shown experimentally that the dependence of the magnetoresistance on the magnetic field is anomalous; i.e., there is a marked deviation from the H^2 law, which is characteristic of crystal systems in weak, classical magnetic fields ($\omega_c\tau \ll 1$, where ω_c is the cyclotron frequency, and τ is the momentum relaxation time of an electron).

Information obtained from the study of anomalous magnetoresistance in disordered systems can be used to test the basic conclusions of the theory of weak localization of electrons and electron-electron interaction, to determine the absolute values and temperature dependence of the electron-electron-interaction parameter $g(T)$, and to obtain important information about the energy and spin relaxation of the conduction electrons in these systems. Of particular interest are the results of the theory of anomalous magnetoresistance in disordered superconducting systems. These results establish a link between the correction to the conductivity resulting from electron-electron interaction, which is determined at temperatures much higher than the transition temperature to the superconducting state, T_c, and the superconducting transition temperature observed experimentally.

Among the published experimental studies of the anomalous magnetoresistance, virtually no attention has been given to quantum effects in 3D superconducting metallic glasses of the metal-metal type. It is accordingly of definite interest to study 3D superconducting amorphous systems in order to determine the magnitude of the anomalous magnetoresistance, to identify the dominant quantum mechanisms and to check the theoretical relationship between T_c and the amonalous magnetoresistance parameters.

In the present experiments we have studied the magnetoresistance of the following 3D superconducting amorphous systems based on transition metals: $Zr_{75}Rh_{25}$, $Zr_{70}Be_{30}$, $Zr_{60}Be_{40}$, $Zr_{54}Cu_{46}$, and $Hf_{58}Be_{42}$. These studies were carried out at 4.2 K in magnetic fields up to 200 kG. In two Zr_xBe_{1-x} systems ($x = 0.7$ and 0.6) we have studied the temperature dependence of the magnetoresistance in the temperature range 4.2–10 K and in fields up to 60 kG. We have chosen these systems because all of them are 3D metallic glasses with superconducting properties and because detailed thermodynamic and x-ray structural data[9–11] are available for these systems (with the exception of $Hf_{58}Be_{42}$), making it possible to use them for quantitative estimates.

2. BASIC RESULTS OF THE THEORY OF ANOMALOUS MAGNETORESISTANCE FOR THE THREE-DIMENSIONAL CASE

In general, the total magnetic conductivity can be represented as follows:

$$\Delta\sigma(H) = \Delta\sigma^{cl}(H) + \Delta\sigma^{qu}(H), \tag{1}$$

where the first term is a classical contribution to the magnetic conductivity, and the second term, which is of a quantum origin, is described by the anomalous magnetoresistance the-

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ory. The condition under which this theory is applicable is given by the inequality

$$k_F l > 1, \tag{2}$$

where l is the mean free path of an electron, and k_F is its wave vector. A remarkable feature of $\Delta\sigma^{qu}(H)$ is that the effect of the magnetic field is appreciable even in classically weak fields in which

$$\omega_c \tau \ll 1,$$

and the contribution $\Delta\sigma^{cl}(H)$ is negligible. Within the scope of the anomalous magnetoresistance theory, the quantity $\Delta\sigma^{qu}(H)$ can be represented as a sum of various quantum corrections[1]:

$$\Delta\sigma^{qu}(H) = c_3 \Delta\sigma^{L}(H) - c_3^{int} \Delta\sigma^{MT}(H) + c_3^{int} \Delta\sigma^{int}(H). \tag{3}$$

The first quantum correction to the magnetic conductivity in (3) stems from the localization of the noninteracting electrons $\Delta\sigma^{L}(H)$, the second quantum correction to the magnetic conductivity in (3) stems from the scattering of electrons by superconducting fluctuations (the Maki-Thompson correction) $\Delta\sigma^{MT}(H)$, and the third correction stems from the allowance for the interaction between the electrons $\Delta\sigma^{int}(H)$. The coefficients c_3 and c_3^{int} are given in Ref. 1. In the case of a 3D sample, the thickness d and the diffusion length L_φ, which is the scale dimension of the localization, satisfy the condition $d > L_\varphi = (D\tau_\varphi)^{1/2}$, where D is the diffusion coefficient of electrons, and τ_φ is the relaxation time of the wave-function phase associated with the inelastic collisions. In the 3D case, the corrections to the magnetic conductivity are given by

$$\Delta\sigma^{L}(H) = \frac{e^2}{2\pi^2\hbar} f_3\left(\frac{4DeH}{\hbar c}\tau_\varphi\right)\left(\frac{eH}{\hbar c}\right)^{1/2}, \tag{4a}$$

$$f_3(x) = \begin{cases} 0.605, & x \gg 1 \\ x^{1/2}/48, & x \ll 1 \end{cases}; \tag{4b}$$

$$\Delta\sigma^{MT}(H) = -\beta(T)\Delta\sigma^{L}(H), \tag{5a}$$

$$\beta(T) = \begin{cases} (\pi^2/4)\,|g(T)|, & -g(T) \gg 1 \\ (\pi^2/6)\,g^2(T), & |g(T)| \ll 1 \end{cases}, \tag{5b}$$

$$g(T) = -\ln(T/T_c)^{-1}; \tag{5c}$$

$$\Delta\sigma^{int}(H) = -g(T)\frac{e^2}{2\pi^2\hbar}\left(\frac{eH}{\hbar c}\right)^{1/2} \varphi_3\left(\frac{2DeH}{\pi cT}\right), \tag{6a}$$

$$\varphi_3(x_1) = \begin{cases} 1{,}90, & x_1 \gg 1 \\ \approx 0{,}33 x_1^{1/2}, & x_1 \ll 1 \end{cases}, \tag{6b}$$

where $g(T)$ is the electron-electron coupling constant.

In Eqs. (5a)–(5c) we give the asymptotic relations between $\beta(T)$ and $g(T)$, and their functional dependence in the intermediate region is tabulated in Ref. 12. From the condition $x = x_1 = 1$ we can determine two length scales of the magnetic fields: $H_\varphi = \hbar c/4eD\tau_\varphi$ and $H_{int} = \pi cT/2eD$ which appear in the anomalous magnetoresistance theory and which separate the weak magnetic fields from the strong magnetic fields, in which the magnetic conductivity varies as the second power and as the square-root, respectively, of the magnetic field. In strong magnetic fields the absolute value of the magnetic conductivity for the noninteracting electrons, $\Delta\sigma^{L}(H)$, does not depend, according to relations (4a) and (4b), on the parameters of the material but instead depends on H in a universal manner. In the case of electron-electron interactions, the contributions of $\Delta\sigma^{MT}$ and $\Delta\sigma^{int}$ depend solely on $g(T)$ in strong magnetic fields. The sign of $\Delta\sigma^{int}(H)$ is determined, according to (6a) and (6b), by the sign of the constant $g(T)$ if the total momentum is small: In the case of attraction $[g(T) < 0]$, the magnetic conductivity is positive (the anomalous magnetoresistance is less than zero) and in the case of repulsion $[g(t) > 0]$, the magnetic conductivity is negative (the anomalous magnetoresistance is greater than zero). The anomalous magnetoresistance caused by the scattering of electrons by superconducting fluctuations is always positive, irrespective of the sign of $g(T)$.

In the 3D case, allowance for the spin-orbit interaction in the scattering of conduction electrons by impurities changes only the sign of the quantum correction $\Delta\sigma^{L}(H)$ which continues to depend on the magnetic field anomalously. The magnitude of the spin-orbit interaction is taken into account analytically in Eq. (3) by changing the coefficients c_3 and c_3^{int}, which have the following values in the 3D case[1]: $c_3 = c_3^{int} = 1$ for a weak spin-orbit interaction ($\tau_{so} \gg \tau_\varphi$) of the electrons with the impurities and $c_3 = -1/2$, $c_3^{int} = 1$ for a strong spin-orbit interaction ($\tau_{so} \ll \tau_\varphi$).

3. EXPERIMENTAL PROCEDURE

1. Preparation and properties of the samples

The alloy $Zr_{75}Rh_{25}$, $Zr_{70}Be_{30}$, $Zr_{60}Be_{40}$, $Zr_{54}Cu_{46}$, and $Hf_{58}Be_{42}$ were prepared from electrolytically pure Zr (99.99%) and Hf (99.95%) and from pure Be (99.88%), pure Rh (99.95%), and cathode Cu(extremely pure grade 11-4). The samples were melted in an induction furnace in the suspended state at a low helium pressure and then solidified from the liquid state by dosing at the outer surface of a copper disk which rotated at the velocity of 10^6 deg/s. The amorphous samples prepared in this manner were ribbons 1 and 2 mm wide and 0.03 mm in thickness.

The structure of the samples was studied by means of x-ray and electron diffraction.[9–11] The x-ray and electron diffraction patterns of the samples which were solidified from

TABLE I.

Sample	2θ, deg	Δ(2θ), deg	d_{eff}, Å	k_p, Å^{-1}	d, g/cm^3
$Zr_{75}Rh_{25}$	37	5	2.98	2.59	7.52
$Zr_{70}Be_{30}$	38.5	6.5	2.81	2.69	5.71
$Zr_{60}Be_{40}$	36.25	6.2	3.03	2.54	5.54
$Zr_{54}Cu_{46}$	37.25	6.2	2.95	2.60	7.30
$Hf_{58}Be_{42}$	37.25	7	2.95	2.60	10.2

the liquid state are typical of the amorphous system and show an absence of long-range order. Table I gives the experimental results of an x-ray structural analysis obtained from the x-ray-diffraction curve: the position 2θ and the width $\Delta(2\theta)$ at half-maximum of the first peak; the position of the first peak in the structure factor, $k_p = (4\pi/\lambda)\sin\theta$, which is an estimate of the Fermi momentum k_F in Ref. 13, and the distance between the nearest neighbors,[14] $d_{\text{eff}} \approx (3/2)^{1/2}\lambda/2\sin\theta$ ($\lambda = 1.54$ Å for Cu K_α emission). This table also lists the densities (d) of the systems under study which were measured by suspending the samples in air and in CCl_4.

The kinetics of crystallization of the metallic glasses in question was studied by the conventional resistance and calorimetric method using a differential scanning calorimeter (Perkin Elmer DSC-2).[15] These studies have shown that with increasing temperature all amorphous systems undergo a transition first to the metastable state and then to the stable crystalline state at higher temperatures.[1)] The stable crystalline state has a different chemical (composite) order and a multiphase structure, as is evident from the x-ray structural data and data in the literature.[9,10]

Although the metastable crystalline state has a similar short-range order, it generally includes some of a second phase, which in our case is a residue of the initial amorphous state. The structural features of the crystalline state which were indicated above rule out a proper comparison of the magnetokinetic properties with the initial amorphous state.

2. Experiment

The measurements of the magnetoresistance at 4.2 K were carried out directly in liquid helium in the field of a superconducting solenoid with an induction up to 8 T and in a composite KS-250 system in fields[16] up to 20 T. The temperature dependence of the magnetoresistance was studied in an intermediate-temperature cryostat, whose lower part was immersed into the superconducting solenoid. A thermally stabilized metal assembly containing the samples and thermometer was mounted inside the vacuum chamber in which the pressure of the heat-transfer helium was in the range 5–10 torr. A thermal regulation system, in which a TSU-1 resistance thermometer virtually insensitive to the magnetic field was used as a sensor, held the temperature of the assembly constant within 0.05 K.

A direct-current, four-contact method was used to measure the resistance of the test samples. The samples, cut from the same piece of amorphous tape, were studied in longitudinal and transverse magnetic fields with respect to the measuring current. The electrical resistance of the samples was measured using 40-mm-long tape samples when the sample was positioned longitudinally in a magnetic field and 10-mm-long tape samples when the sample was placed at right angles to the field. The sensitivity of the measuring instruments to the change in resistance was 1 $\mu\Omega$. The geometry factor (s/l) of the sample introduced approximately 10% uncertainty in the absolute value of the resistivity ρ.

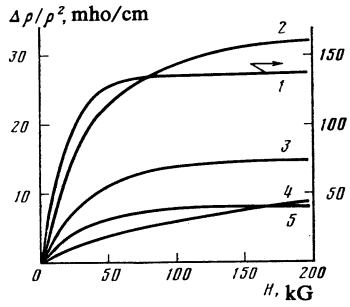

FIG. 1. Dependence of $\Delta\rho/\rho^2$ on H in fields up to 200 kG for the amorphous systems $Zr_{75}Rh_{25}$ (1), $Zr_{70}Be_{30}$ (2), $Zr_{60}Be_{40}$ (3), $Zr_{54}Cu_{46}$ (4), and $Hf_{58}Be_{42}$ (5) at 4.2 K.

4. EXPERIMENTAL RESULTS AND DISCUSSION

We begin the discussion of the experimental results with the analysis of the data on magnetoresistance at $T = 4.2$ K over the entire range of magnetic fields. Figure 1 shows the field dependence of the magnetic conductivity

$$\Delta\sigma(H) = \sigma(0) - \sigma(H) = [\rho(H) - \rho(0)]/\rho(H)\rho(0) \approx \Delta\rho(H)/\rho^2(0)$$

for $Zr_{75}Rh_{25}$, $Zr_{70}Be_{30}$, $Zr_{60}Be_{40}$, $Zr_{54}Cu_{46}$, and $Hf_{58}Be_{42}$.

We will first consider the general features in the behavior of the magnetoresistance of the superconducting metallic glasses studied. It has been found that the observable magnetoresistance is positive and that it is independent of the orientation of the magnetic field with respect to the measuring current. It is evident from Fig. 1 and Table II that for the systems indicated above the value of the magnetoresistance correlates with the temperature of the transition to the superconducting state T_c. Typical of all the systems studied is also the fact that the curve $\Delta\rho(H)/\rho^2(0)$ exhibits, in addition to the quadratic dependence in small magnetic fields, a clearly defined region with $H^{1/2}$. A final characteristic property of the amorphous systems studied, which we will discuss below, is that the magnetoresistance decreases sharply with increasing temperature.

We begin the analysis of the experimental data on the effect of the magnetic field on the resistance by estimating the classical contribution in expression (1). To estimate the

TABLE II. Parameters of the systems studied in the weak spin-orbit interaction approximation ($T = 4.2$ K).

Sample	k_F, Å^{-1}	ρ, $\mu\Omega$ ·	$k_F l$	$\beta(T)$	$\frac{1}{2}-g(T)$	T^*_c, K	T'_c, K	T_c, K	τ_φ, 10^{-11} s	τ_{so}, 10^{-11} s	L_φ, нм	D^{expt}, cm^2/s	D^{calc}, cm^2/s	H_{int}, kG	$N(\varepsilon_F)$, eV^{-1}·at^{-1}	$\Theta_D(0)$, K
$Zr_{75}Rh_{25}$	1,29	230	4,2	25,6	10,4	3,8	–	4,23	7,7	40	56	0,417	0,73	140	0,69	190
$Zr_{70}Be_{30}$	1,35	300	3,0	5,8	2,5	2,8	3,3	3,50	4,9	50	47	0,46	0,55	125	0,89	234
$Zr_{60}Be_{40}$	1,27	307	3,1	4,9	2,0	2,6	2,9	2,70	3,3	8	43	0,55	0,56	100	0,76	289
$Zr_{54}Cu_{46}$	1,30	175	5,3	2,1	1,6	2,2	–	1,22	1,46	0,1	41	–	1,16	50	0,55	221
$Hf_{58}Be_{42}$	1,30	317	3,9	1,78	1,4	2,0	–	1,32	1,88	0,01	39	–	0,81	70	–	187*

Key: 1-*The estimate is based on the data on high-temperature specific heat.

normal magnetoresistance, we used Kohler's rule

$$\Delta\rho_{cl}/\rho \sim (nec)^{-2}(H/\rho)^2$$

(n is the electron density) and the experimental data given in Tables I and II. Estimate of the value of $\Delta\rho_{cl}(H)$ showed that it is four orders of magnitude lower than the magnetoresistance observed experimentally, because $\omega_c\tau$ is small in the amorphous systems studied. It is evident from Table II that condition (2) holds for all samples; the parameter $k_F l$ can be expressed in terms of the diffusion coefficient D or the conductivity σ as follows:

$$k_F l = 3Dm^*\hbar = {}^3/_2 k_F\sigma/G_0,$$

where

$$G_0 = e^2/2\pi^2\hbar \approx 1.23\times 10^{-5}\Omega^{-1}.$$

Here m^* is the effective mass of electrons.

1. Dependence of the resistance on the magnetic field

The measurement results of the magnetoresistance of superconducting amorphous systems at $T = 4.2$ K are shown in Figs. 2 and 3. Figure 2 shows the dependence of the magnetic conductivity on H^2 for $Zr_{75}Rh_{25}$, $Zr_{70}Be_{30}$, $Zr_{60}Be_{40}$, $Zr_{54}Cu_{46}$, and $Hf_{58}Be_{42}$ in fields up to 4.5 kG. As we can see in Fig. 3, the value becomes proportional to $H^{1/2}$ with increasing magnetic field. A comparison of the experimental dependence of magnetic conductivity on H with theoretical relations (4)–(6) shows that the experimental functional dependence $\Delta\sigma(H)$ is characteristic of the behavior of the disordered 3D systems in a magnetic field. In determining whether the equations indicated above are applicable in our specific case, we must quantitatively determine that the 3D condition ($d > L_\varphi$) holds for the systems under study.

Under the assumption that the spin-orbit interaction is weak, we have determined, in accordance with relations (4) and (5), the values of the parameters β and τ_φ (see Table II) from the slopes of the curves of $\Delta\rho/\rho^2$ vs H^2 (Fig. 2) and $\Delta\rho/\rho^2$ vs $H^{1/2}$ (Fig. 3). For all systems the contribution of $\Delta\sigma^{int}(H)$ to $\Delta\sigma^{qu}(H)$ [see Eqs. (3) and (6)] is small, since the $H^{1/2}$ law applies quite accurately at fields 3–50 kG, despite the fact that $\Delta\sigma^{int} \propto H^2$ ($H < H_{int}$). In this respect, the system $Hf_{58}Be_{42}$ differs from the other systems in that the region of square-root dependence of $\Delta\rho/\rho^2$ in it extends to the limiting fields (200 kG) in our experiment, and since $H > H_{int}$ in this case, the contribution of $\Delta\sigma^{int}(H)$ to $\Delta\sigma^{qu}(H)$ cannot be disregarded.

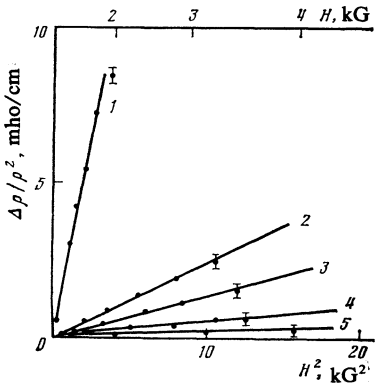

FIG. 2. Dependence of $\Delta\rho/\rho^2$ on H^2 in low fields for the amorphous systems $Zr_{75}Rh_{25}$ (1), $Zr_{70}Be_{30}$ (2), $Zr_{60}Be_{40}$ (3), $Zr_{54}Cu_{46}$ (4), and $Hf_{58}Be_{42}$ (5) at 4.2 K.

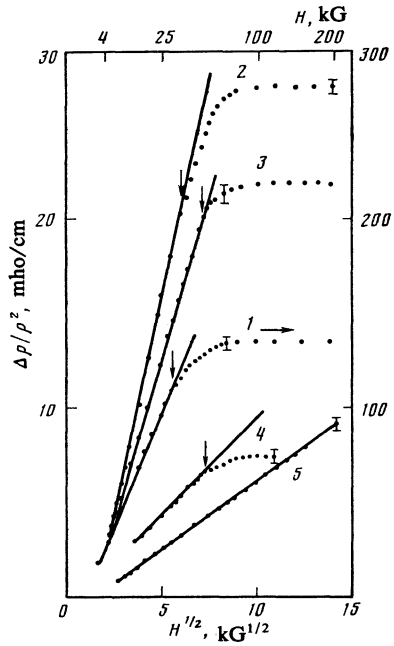

FIG. 3. Dependence of $\Delta\rho/\rho^2$ on $H^{1/2}$ in fields 4–200 kG for the amorphous systems $Zr_{75}Rh_{25}$ (1), $Zr_{70}Be_{30}$ (2), $Zr_{60}Be_{40}$ (3), $Zr_{54}Cu_{46}$ (4), and $Hf_{58}Be_{42}$ (5) at 4.2 K.

Estimate of the maximum contribution of $\Delta\sigma^{int}$ to the field H^* ($H^* < H_{int}$), in which there is a divergence from the $H^{1/2}$ law (see the arrows in Fig. 3), shows that, in the worst case, it is no greater than 5% for $Zr_{75}Rh_{25}$ and $Zr_{70}Be_{30}$ systems with the highest T_c, whereas the contribution is much smaller for the rest of the samples.

Table II also gives the effective electron-electron coupling constants $g(T)$ which were determined by Larkin[12] from the calculated values of the parameter β. The experimental values of the diffusion coefficient D (Ref. 17) and the values found for τ_φ allowed us to quantitatively determine whether the 3D conditions are satisfied and whether the relation $\Delta\sigma^{int} \ll \Delta\sigma^{L} + \Delta\sigma^{MT}$ is valid. For two systems, $Zr_{54}Cu_{46}$ and $Hf_{58}Be_{42}$, we have estimated the diffusion coefficients from our data on the electron state density at the Fermi surface, $N(\varepsilon_F)$, and also from σ, k_F, and l. The calculated values of D^{calc} for all the systems studied are given in Table II. The values of L_φ listed in this table are three to four orders of magnitude smaller than the thickness of the samples tested; i.e., the 3D case is reached safely in all systems.

In determining the parameter $g(T)$ we must ask how accurately the anomalous magnetoresistance (AMR) data can describe the experimental values of T_c quantitatively in terms of relations (5a)–(5c). A comparison of the superconducting transition temperatures T_c^*, calculated from the electron-electron coupling constants $g(T)$ in the weak spin-orbit-interaction approximation, with the values of T_c mea-

sured from the specific heat and by the inductive method (see Table II) shows that the agreement is satisfactory on the whole. In the case of the $Zr_{54}Cu_{46}$ and $Hf_{58}Be_{42}$ systems, the discrepancy between the calculated and experimental values of T_c is, however, large enough (60–80%) to assume that the weak spin-orbit-interaction approximation is inapplicable.

Better agreement between T_c^* and T_c for $Zr_{54}Cu_{46}$ can be obtained from and analysis of the experimental data in the strong spin-orbit-interaction approximation (see Table III). In the case of $Hf_{58}Be_{42}$ we see that despite the change in the calculated value of T_c^*, the quantitative agreement between T_c^* and T_c does not improve. This circumstance and the fact that $\Delta\rho/\rho^2 \propto H^{1/2}$ up to 200 kG show that the contribution of the electron-electron interaction, $\Delta\sigma^{int}(H)$, to the magnetic conductivity must be taken into account. According to relations (3)–(6), all contributions to $\Delta\sigma^{qu}(H)$ have a square-root dependence on H at fields $H > H_{int}$; the term $c_{3,i}^{nt}$ $\Delta\sigma^{int}(H)$ amounts to 40–50% of the observed effect. It is evident from Table III that the agreement between the calculated and experimental values of T_c improves markedly if the contribution of the electron-electron interaction to $\Delta\sigma^{qu}(H)$ is taken into account.

The problem concerning the spin-orbit interaction and its accurate calculation can in principle be resolved through a mathematical description of the experimental dependence of $\Delta\sigma(H)$ on the basis of an expression for the localization contribution which takes the spin-orbit interaction into account:

$$\Delta\sigma^{L}(H) = \frac{e^2}{4\pi^2\hbar}\left(\frac{eH}{\hbar c}\right)^{1/2} \times\left[\frac{3}{2}f_3\left(\frac{4DeH}{\hbar c}\tau_\varphi^*\right)-\frac{1}{2}f_3\left(\frac{4DeH}{\hbar c}\tau_\varphi\right)\right]. \quad (7)$$

The relaxation time of the phase of the electron wave function, τ_φ, and the time which is modified with allowance for the spin-orbit interaction, τ_φ^*, can be determined from the relations[18]

$$\tau_\varphi^{-1}=\tau_\varepsilon^{-1}+2\tau_s^{-1}, \quad (\tau_\varphi^*)^{-1}=\tau_\varepsilon^{-1}+{}^2/_3\tau_s^{-1}+{}^4/_3\tau_{so}^{-1}, \quad (8a)$$

where τ_ε is the inelastic scattering time of electrons, and τ_s is the elastic scattering time of electrons, involving a paramagnetic-impurity-induced spin flip. Since the inequality $\tau_s \gg \tau_\varepsilon$, τ_{so} holds for superconducting systems, we can use the approximation relation

$$(\tau_\varphi^*)^{-1}\approx\tau_\varphi^{-1}+2\tau_{so}^{-1}. \quad (8b)$$

In the case case of weak ($\tau_{so} \gg \tau_\varphi$) and strong ($\tau_{so} \ll \tau_\varphi$, $\hbar c/4DeH$) spin-orbit interactions, the coefficients of the asymptotic values of H are of the form $\beta(T) - 1$ and $\beta(T) + 1/2$, respectively, for the first two contributions [see Eq. (3)].

For the systems under consideration, the value of τ_{so} was estimated on the basis of Eqs. (3), (7), and (8) from several smoothly fitted experimental points. These estimates show that for the systems $Zr_{75}Rh_{25}$ and Zr_xBe_{1-x} ($x = 0.6$ and 0.7) the magnetic fields, in which relations (3), (7), and (8) hold in the approximation of the weak spin-orbit interaction ($H > H_{so} = \hbar c/4De\tau_{so}$), include virtually the whole range of fields within which the H^2 and $H^{1/2}$ laws hold, i.e., $H_{so} < H_\varphi < H_{int}$. In the case of $Zr_{54}Cu_{46}$ and $Hf_{58}Be_{42}$, the estimates of τ_{so} have confirmed the conclusion, drawn on the basis of a comparison of the calculated and experimental values of T_c, that the spin-orbit interaction is strong in these systems.

In a study of the anomalous magnetoresistance in a metallic glass[19] $Zr_{57}Cu_{43}$, which is similar in composition to the system $Zr_{54}Cu_{46}$, it was also concluded that the spin-orbit interaction is strong. The values of τ_φ and τ_{so} obtained in Ref. 19 are in good quantitative agreement with our data.

Analysis of the experimental data within the scope of the anomalous magnetoresistance theory[1] shows that the magnetokinetic effect observed in 3D superconducting amorphous systems gives us an adequate theoretical understanding and that relations (3)–(8) can be used for quantitative calculations. The superconducting-transition temperature of $Hf_{58}Be_{42}$ calculated from the data on magnetoresistance was subsequently confirmed to within acceptable accuracy through a direct measurement of T_c by an inductive method.

Analysis of the experimental data within the context of the theory we used gives a natural explanation of the observed correlations between T_c, the tilt angles of the $\Delta\rho/\rho^2$ vs H^2 curves, $H^{1/2}$, and the magnetic fields H^* (in Fig. 3 they are indicated by an arrow pointing downward ↓) at which a divergence from the $H^{1/2}$ law begins: higher T_c correspond to higher values of β and lower H^*. The first correlation is a consequence of relations (4a) and (4b) which establish a relationship among the quantities T_c, $\beta(T)$, and $g(T)$. In the second case, the deviation of $\Delta\rho/\rho^2$ from the $H^{1/2}$ law stems from the fact that electron-electron interaction begins to play a more dominant roles as H is raised. In superconducting amorphous systems with higher T_c [and hence larger $g(T)$] the electron-electron interaction is a factor at lower H^*. Analytically, this effect is seen in the suppression of the temperature dependence $g(T)$ by the magnetic field and in

TABLE III. Experimental parameters of the systems studied in the approximation of a strong spin-orbit interaction ($T = 4.2$ K).

system	β		$-g(T)$	T^*_c, K	τ_φ, 10^{-11} s	τ_{so}, 10^{-13} s	L_φ, нм
$Zr_{54}Cu_{46}$	0,64		0,73	1,1	1,9	6	47
$Hf_{58}Be_{42}$	$\Delta\sigma^{int}=0$	0,28	0,45	0,5	1,83	0,1–1	37
	$\Delta\sigma^{int}\neq 0$	0,8	0,84	1,3	1,3	0,1–1	31

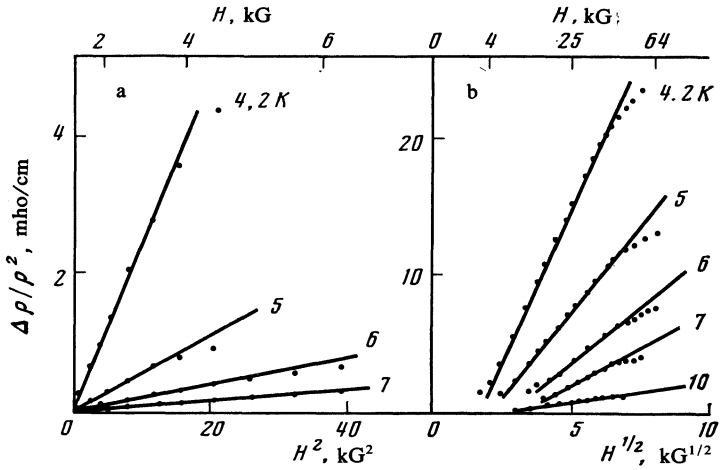

FIG. 4. Magnetoresistance of the amorphous system $Zr_{70}Be_{30}$ in the temperature range 4.2–10 K, plotted as $\Delta\rho/\rho^2$ vs H^2 (a) and $\Delta\rho/\rho^2$ vs $H^{1/2}$ (b).

the appearance of a field dependence $g(H)$. In general, when there is an attraction between electrons, the electron-electron coupling constant can be described by the expression[1]

$$g^{-1}(T,H)=\begin{cases}1/\bar{\lambda}+\ln(\gamma\Theta_D/\pi T)=-\ln(T/T_c), & H\ll cT/2eD\\ 1/\bar{\lambda}+\ln(c\Theta_D/DeH), & H\gg cT/2eD\end{cases}$$

where $\bar{\lambda}$ is a dimensionless bare coupling constant, $\ln\gamma\approx 0.577$, and Θ_D is the Debye temperature. In the case of the system $Hf_{58}Be_{42}$, there is no divergence from the $H^{1/2}$ law in the fields we used since the contribution $\Delta_\sigma^{\text{int}}(H)$ like $[1/2+\beta(T)]\Delta\sigma^L(H)$, has, as we noted above, a square-root dependence. In this case, however, the square-root dependence has the opposite sign.

We wish to note in conclusion that $\Delta\sigma^{\text{int}}(H)$ can also be used in other systems to improve the agreement between the calculated and experimental values of T_c.

2. Temperature dependence of the magnetoresistance

The temperature dependence of the parameter $\beta(T)$, which is related to the electron-electron coupling constant $g(T)$, and the temperature of the scale time for the loss of phase coherence of the electron wave function can be determined by analyzing the curves for $\Delta\rho/\rho^2$ vs H taken at various temperatures in magnetic fields $H<H_{\text{int}}$. The basic mechanism for the loss of phase coherence of the conduction-electron wave function in amorphous superconductors can be identified from $\tau_\varphi(T)$.

In an effort to determine this mechanism, we have studied the dependence of $\Delta\rho/\rho^2$ on H in the temperature range $T_c<T<10$ K in magnetic fields up to 75 kG for amorphous systems Zr_xBe_{1-x} ($x=0.6$, 0.7). It can be seen from the experimental data (Fig. 4) that as the temperature is raised, the coefficients fall off sharply for H^2 and $H^{1/2}$ and at $T=10$ K the magnetoresistance is nearly equal to the experimental error. Figure 5 is a plot of the coefficient β^{-1} as a function of $\ln T$. In the case of each ZrBe system, the curve is linear within the experimental error margin. The value of T_c', averaged over several temperature points, can be determined by extrapolating this curve to the point at which it crosses the $\ln T$ axis (see Table II).

Figure 6 is a logarithmic plot of the time for the loss of phase coherence of the electron wave function, as a function of temperature, for the same systems. Within the error margins indicated in the figure, the experimental points lie on the same line, demonstrating that $\tau_\varphi\propto T^{-2}$. The fact that τ_φ depends on temperature shows that the time it takes the phase of an electron wave function to relax is the same as the time it takes an electron to scatter inelastically, τ_ε [see Eq. (8a)], since a finite value of τ_s, if it does exist, must be independent of T. As for the inelastic scattering of electrons

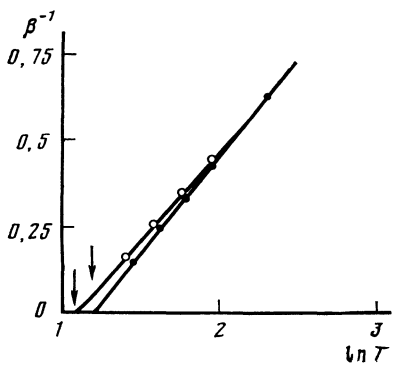

FIG. 5. Temperature dependences of the parameters β for the systems $Zr_{70}Be_{30}$ (●) and $Zr_{60}Be_{40}$ (○). The arrows show the calculated values of T_c', averaged over several temperature points.

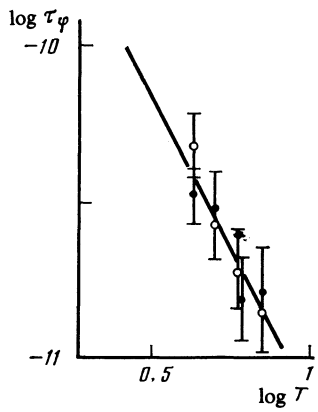

FIG. 6. Temperature dependences of τ_φ for the amorphous systems $Zr_{70}Be_{30}$ (○) and $Zr_{60}Be_{40}$ (●).

this can be done by photons (τ_ε^{ph}), electrons (τ_ε^{e}), and oscillating impurity ions (τ_ε^{imp}).

In the 3D case [$d \gg L_\varphi, L_T$, where $L_T = (D/T)^{1/2}$], the following temperature dependences of the electron-electron collision frequencies have been derived from an analysis of the electron-electron interactions, in which the scattering by impurities was taken into account.[20] In the case of a large energy transfer, with $\hbar\omega \sim kT$ and $q \sim (T/D)^{1/2}$, we have

$$(\tau_\varepsilon^{e})^{-1} \sim T^{3/2}/\mu^2\tau^{1/2}$$

and in the same situation, with $q \sim q_F$, we have

$$(\tau_\varepsilon^{e})^{-1} \sim T^2/\mu$$

(μ is the chemical potential). In each case, estimates of $(\tau_\varepsilon^{e})^{-1}$ show that its contribution to the magnetoresistance is extremely small in comparison with the effects observed experimentally.

The scattering of conduction electrons by oscillating impurity ions should lead[21] to an energy relaxation with the temperature dependence (T^{-2}) observed experimentally:

$$(\tau_\varepsilon^{imp})^{-1} \propto T^2/\mu\tau\Theta_D. \qquad (9)$$

Estimates of the absolute values of τ_ε^{imp} based on Eq. (9) are, however, three orders of magnitude larger than the values found for τ_φ. We cannot conclude, therefore, that this mechanism is responsible for the relaxation of the phase of the conduction-electron wave function. The temperature dependence observed, $\tau_\varphi \propto T^{-2}$, may also be attributable to the inelastic electron-ion interaction. The thermal vibrations in this case interact with the electron which diffuses in the impurity field. This interaction gives rise to the following expression for the "electron-phonon" interaction time[22]:

$$(\tau_\varepsilon^{ph})^{-1} = \frac{2\pi^2 a}{k_F l}\frac{k\Theta_D}{\hbar}\left(\frac{T}{\Theta_D}\right)^2$$

(a is the lattice constant). This relation is valid for an "impure" case, in which the wavelength of thermal vibrations is $\lambda_T \gg l$. An estimate of the energy-relaxation rate based on this relation, and under the assumption that the diffusion coefficient is independent of T over the temperature interval studied, yields plausible values of τ_ε^{ph} which are in satisfactory agreement with τ_φ:

$$\tau_\varepsilon^{ph}(Zr_{70}Be_{30}) \approx 1.4 \cdot 10^{-11}\text{s}, \quad \tau_\varepsilon^{ph}(Zr_{60}Be_{40}) \approx 2 \times 10^{-11}\text{s}.$$

Accordingly, the agreement of τ_φ with the estimates of τ_ε^{ph} based on the temperature dependence observed experimentally suggests that inelastic scattering of electrons by ionic oscillations is the basic mechanism for the relaxation of the phase of the conduction-electron wave function.

5. CONCLUSION

A study of low-temperature magnetokinetic properties of metallic glasses based on Zr and Hf has shown that the magnetoresistance observed experimentally in the systems under study is positive and that it depends on the magnetic field in an anomalous manner, showing regions of quadratic and square-root dependence on H. The magnetoresistance does not depend on the orientation of the magnetic field with respect to the measuring current that flows through the sample. In all systems studied we see a correlation between T_c and the tilt factors of the quadratic and square-root laws in the plot of $\Delta\rho/\rho$ vs H. A deviation from the $H^{1/2}$ law also correlates with T_c.

In the case of Zr_xBe_{1-x} systems ($x = 0.6$ and 0.7), the magnetoresistance depends on the composition. The maximum magnetoresistance is seen in the $Zr_{70}Be_{30}$ system with a higher T_c. Studies of the magnetoresistance of these systems over the temperature interval 4–10 K have shown that the temperature dependence of the coefficients of H^2 and $H^{1/2}$ is appreciable.

The basic features of the low-temperature behavior of the magnetoresistance observed experimentally are described well by the present theory of "weak" localization of electrons and electron-electron interaction in 3D disordered systems. The anomalous magnetoresistance of metallic glasses exhibits both effects which can be distinguished in a magnetic field. The contribution which is linked with electron-electron interaction and which stems from the scattering of electrons by superconducting fluctuations dominate, since $|g(T)| \gtrsim 1$ at $T \gtrsim T_c$.

Using the relations of the anomalous magnetoresistance theory,[1] we found from the experimental data the parameters $\beta(T)$, which determine the effective interaction $g(T)$ between the electrons for all systems. The temperature dependences of $\beta(T)$ were determined for the Zr_xBe_{1-x} systems. The temperatures at which these systems go superconducting were estimated numerically from these data. The calculated values of T_c are in satisfactory agreement with the experimental values of T_c. Since the parameter $g(T)$ for the $Hf_{58}Be_{42}$ system was determined in strong fields $H > H_{int} = \cos k\pi/2eD$, in which the temperature dependence of $g(T)$ is suppressed by the magnetic field, we had to use the contribution of $\Delta\sigma^{int}(H)$ resulting from the interaction between the electrons. The correlation between the deviation of the magnetoresistance from the square-root dependence and T_c is attributable to the enhancement of the interaction of the electrons in superconducting systems with higher T_c and hence larger $g(T)$. This effect is seen in the increase of the contribution of $\Delta\sigma^{int}(H) \propto g(T)$ to $\Delta\sigma^{qu}(H)$.

A comparison of the calculated and experimental values of T_c has shown that in contrast with the $Zr_{75}Rh_{25}$ and Zr_xBe_{1-x} systems, in which a weak spin-orbit interaction is observed,[2)] a strong spin-orbit interaction occurs in the metallic glasses $Zr_{54}Cu_{46}$ and $Hf_{58}Be_{42}$, consistent with the estimates of the scale time τ_{so}.

We see from the data on the temperature dependence of the magnetoresistance of the Zr_xBe_{1-x} system at $T > T_c$ that the relaxation time τ_φ of the phase of the electron wave function depends on the temperature as T^{-2}. Here τ_φ is comparable to the time of the electron-"phonon" interaction calculated in the "impure"-limit approximation. This fact suggests that the basic mechanism for the phase relaxation of conduction electrons in this temperature region is the inelastic scattering of these electrons by ion waves.

In summary, the low-temperature behavior of the magnetoresistance of 3D superconducting metallic glasses of the metal-metal type at $T \gtrsim T_c$ is described well by the present theory of anomalous magnetoresistance in disordered systems, and experimental study of these effects makes it possible to determine the temperature dependence and typical absolute relaxation times of electrons and the electron-electron coupling constants.

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[1)]As a result of annealing, the amorphous system $Zr_{54}Cu_{46}$ immediately undergoes a transition to the stable crystalline phase.

[2)]We should point out that within the scope of our analysis, the spin-orbit interaction is found to be weak in the amorphous systems $Zr_{75}Rh_{25}$ and Zr_xBe_{1-x}. This circumstance, in our view, requires a further study.

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