Bions and coherent states for a Heisenberg spin-$\frac{1}{2}$ chain

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A bion (a soliton with intrinsic precession) localized near the end of an anisotropic chain of $s=\frac{1}{2}$ spins is investigated quantum-mechanically. The coherent state $|\Phi(t)\rangle$ in which the evolution of the mean spin components S_m^α and the mean energy of the system are described by the solution to the Landau-Lifshitz equations is found. The state $|\Phi(t)\rangle$ is a nonspreading superposition of spin complexes.

1. INTRODUCTION

To investigate the dynamics of ferromagnets we often use the phenomenological equations of motion of the magnetic moment (the Landau-Lifshitz equations)[1–3]:

$$\frac{\partial \mathbf{M}}{\partial t}=\left[\frac{\delta W}{\delta \mathbf{M}},\mathbf{M}\right],\quad |\mathbf{M}(\mathbf{r},t)|=M_0=\text{const}. \tag{1}$$

Here W is the energy functional for the system in question.

In quantum theory the equations of motion for spin operators $\hat{S}_m^\alpha$ in the Heisenberg representation have the form ($\hbar=1$)

$$\frac{\partial \hat{S}_m{}^\alpha}{\partial t}=i[\hat{H},\hat{S}_m{}^\alpha],\quad \alpha=x,y,z. \tag{2}$$

By formally replacing the operators $\hat{\mathbf{S}}_m$ in the Hamiltonian H and in the equations (2) by vectors $\mathbf{S}_m$ of fixed length s, we can obtain in the continuum limit the functional W and the equations (1) (in this limit the discrete spin distribution is replaced by the vector field $\mathbf{M}(\mathbf{r},t)$.[3] It is more consistent to regard the equations (1) as obtained through appropriate averaging of the operator equations (2). In Ref. 2 arguments are presented concerning the derivation of the Landau-Lifshitz equations by averaging (2) over a density matrix. These arguments do not exclude the possibility of deriving the relations (1) by means of another type of averaging, specifically, by averaging (2) over pure quantum states. In the case of the linearized equations this possibility is, in essence, demonstrated and used in Refs. 4 and 5. The appropriate state over which the equations (2) are averaged is[1)] the Glauber coherent state.[5] In this state the evolution of the mean S_m^α is described by a classical spin wave. Let us also note that the linearized Landau-Lifshitz equations for a uniaxial ferromagnet with exchange anisotropy are exact for any value of the spin s.

Numerous investigations of the nonlinear properties of ferromagnets have been carried out in recent years within the framework of the phenomenological theory.[3,6,7–9] Comparison with the exact quantum-mechanical results shows, in particular, that the solutions to the Landau-Lifshitz equations describe a number of nontrivial properties of the Heisenberg spin chains.[3] It is impressive that the quasiclassical soliton energies are equal to the energies of the spin complexes (the magnon bound states), a result which was first proved for the isotropic chain,[7] and later generalized to a number of anisotropic spin-$\frac{1}{2}$ models.[10] Also equal are the z-spin-component densities in the soliton and complex.[8,9] Nearly equal are the contributions of the two-magnon bound states and the solitons to the dynamical form factor of a one-dimensional spin-1 system.[11]

It may be conjectured that these coincidences are a consequence of some deeper connection between the phenomenological equations (1) and the microscopic theory (2) in the nonlinear region. The establishment of such a connection would, besides justifying the relations (1), resolve the question of the existence of solitons in quantum systems. Such a connection can, as is clear from the foregoing, result from the states over which (2) is averaged to obtain the Landau-Lifshitz equations (1).

Since a general derivation of the relations (1) has so far not been published, we are justified in attempting to solve the narrower problem of demonstrating the existence of and determining the states $|\Phi\rangle$ in which the temporal evolution of the mean S_m^α and the mean energy of a specific system are described by a particular characteristic solution of the nonlinear Landau-Lifshitz equations.

In Ref. 9 we found that linear combination $|\Phi_0\rangle$ of the heavy spin complexes $|\Psi_n\rangle$,

$$|\Phi_0\rangle=a_0\sum_{k=-\infty}^{\infty}\exp\left(-\frac{k^2\sigma}{2}\right)|\Psi_{N_0+k}\rangle,\quad N_0\to\infty, \tag{3}$$

which is the full quantum analog of the classical domain wall in a chain with the Hamiltonian

$$H=-\sum_{m=0}^{\infty}\left[\frac{1}{\operatorname{ch}\sigma}(S_m{}^xS_{m+1}^x+S_m{}^yS_{m+1}^y)+S_m{}^zS_{m+1}^z\right],$$
$$\sigma>0,\quad J=1,\quad s={}^1/{}_2. \tag{4}$$

(Here ch is the hyperbolic cosine.) The mean values $\langle\Phi_0|S_m^\alpha|\Phi_0\rangle$ and the energy coincide with the values that follow from the static solution to Eq. (1) (more precisely, their discrete analog). We then have the identity

$$\sum_{\alpha=x,y,z}(\langle\Phi_0|S_m{}^\alpha|\Phi_0\rangle)^2=\frac{1}{4}, \tag{5}$$

which, in our opinion, corresponds to the assumption made in the phenomenological approach (1) that the vector $\mathbf{M}(\mathbf{r})$ has a constant length.

The state (3) can also be represented[12] in the form of a direct product of one-site coherent spin states:

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$$|\Phi_0\rangle = \left(\prod_{m=0}^{\infty} \exp[-i\theta_m \hat{S}_m{}^y]\right)|0\rangle, \tag{6}$$

where θ_m is a solution to the nonlinear difference equation $\partial W/\partial\theta_{m=0}$ A peculiarity of the coherent state $|\Phi_0\rangle$ is that it is stationary. The existence of a stationary state of the unbounded chain (4) in the form (6) for arbitrary spin was first demonstrated by Pokrovskiĭ and Khokhlachev.[13] The stationarity of $|\Phi_0\rangle$ corresponds to the phenomenological theory of the excitations of the system (4).

Also well known are the time-dependent soliton solutions to the equations (1).[3] The temporal evolution of the solitons has a number of characteristics that also attract especially the attention of researchers. The simplest solutions to the Landau-Lifshitz equations of this type describe a bion (a soliton with intrinsic precession).

In the present paper we investigate quantum-mechanically a bion localized near the end of an anisotropic chain (4) of spins of magnitude $s=\frac{1}{2}$. We find the coherent state $|\Phi(t)\rangle$, the system's mean energy and the evolution of the mean spin components in which are described by the corresponding solution to the Landau-Lifshitz equations. The state $|\Phi(t)\rangle$ is a nonspreading packet of spin complexes.

2. THE STATE $|\Phi\rangle$ AS A LINEAR COMBINATION OF SPIN COMPLEXES

The solution to the equations (1) that describes a nonlinear excitation localized near the end of the chain (4) has the following form $(s=\frac{1}{2})$[14]:

$$S_m{}^x = {}^1/_2 \sin\theta_m \cos\omega t,\quad S_m{}^y = -{}^1/_2 \sin\theta_m \sin\omega t,\quad S_m{}^z = {}^1/_2 \cos\theta_m,$$

$$\operatorname{tg}\frac{\theta_m}{2} = \frac{\operatorname{sh} a}{\operatorname{ch}(\sigma m \operatorname{th} a)},\quad W = \frac{1}{2}\sigma \operatorname{th} a,\quad \omega = \frac{\sigma^2}{2\operatorname{ch}^2 a}, \tag{7}$$
$$a \equiv \sigma N \geqslant 1,\ \sigma \ll 1.$$

Here ω is the frequency of the intrinsic precession of the bion; N is the conserved value of S^z; and a is a parameter characterizing the form of the bion (or the degree of nonlinearity). Three regions can, depending on the value of a, be distinguished: the region $a \ll 1$ which is the region of weak nonlinearity, where the equation (1) reduce to the nonlinear Schrödinger equation; $a \sim 1$, the intermediate region; and $a \gg 1$, the region of strong nonlinearity. The limit $a \to \infty$ corresponds to a domain wall.[9,14] In the present paper we shall not discuss the case of weak nonlinearity. Furthermore, the analysis is performed in the continuum approximation, which presupposes a weak interaction anisotropy, i.e., that $\sigma \ll 1$.[9,10,14]

Let us choose the initial state $|\Phi\rangle$ in the quantum picture in the form (6), where the angles θ_m are taken from the solution (7). The mean values $\langle\Phi|\hat{S}_m^\alpha|\Phi\rangle$ then coincide with the values of the S_m^α from (7) at zero time $t=0$. The mean energy does not depend on the time, and will likewise be equal to the energy W. We are interested in the evolution of the quantum-mechanical averages:

$$\langle\Phi(t)|\hat{S}_m{}^\alpha|\Phi(t)\rangle,\quad |\Phi(t)\rangle = e^{-iHt}|\Phi\rangle.$$

Let us first of all establish the connection between the state $|\Phi\rangle$ and the stationary states of the system (4). In the case $\sigma \ll 1$ and $a \gtrsim 1$ under consideration here we can show that $|\Phi\rangle$ can be represented as a linear combination of the spin complexes of the chain (4). To do this, let us first find the expansion of $|\Phi\rangle$ with θ_m set equal to 2 arctg $(A/\operatorname{ch} Bm)$ in terms of a set of orthonormalized vectors of the following type:

$$|\chi_n(B)\rangle = Z_n{}^{-1/2}(B)$$
$$\times \sum_{\{m_i\}} (\operatorname{ch} Bm_1 \ldots \operatorname{ch} Bm_n)^{-1} S_{m_1}^- S_{m_2}^- \ldots S_{m_n}^- |0\rangle, \tag{8}$$

where the sum over the $\{m_i\}$ denotes summation over $m_1, m_2, \ldots, m_n$ with the condition $0 \leqslant m_1 < m_2 < \ldots < m_n$. Notice that the reciprocal of the square of the normalization constant

$$Z_n(B) = \sum_{\{m_i\}} (\operatorname{ch} Bm_1 \operatorname{ch} Bm_2 \ldots \operatorname{ch} Bm_n)^{-2}$$

formally coincides with the partition function of an ideal fermion gas with the single-particle spectrum $E_m = \ln \operatorname{ch}^2 Bm$ at $T=1$.

For the coefficient $C_n = \langle\Phi|\chi_n\rangle$ we can derive the expression

$$C_n = A^n Z_n^{1/2}(B) Z^{-1/2}(A^2, B), \tag{9}$$

where

$$Z(A^2,B) = \prod_{m=0}^{\infty} \cos^{-2}\frac{\theta_m}{2} = \prod_{m=0}^{\infty}\left(1 + \frac{A^2}{\operatorname{ch}^2 Bm}\right)$$

is none other than the partition function of the same gas within the framework of the grand canonical ensemble (A^2 is the activity). It is well known that the partition function Z_n and Z are connected by the relation

$$Z(A^2,B) = \sum_{n=0}^{\infty} A^{2n} Z_n(B).$$

From this it follows that

$$\sum_{n=0}^{\infty} C_n{}^2 = 1,$$

i.e., the state $|\Phi\rangle$ can be expanded only in terms of the set of $|\chi_n\rangle$ vectors (evidently, the $|\chi_n\rangle$ do not be themselves form a complete set). Thus, we have the exact equality

$$|\Phi\rangle = \sum_{n=0}^{\infty} C_n|\chi_n\rangle. \tag{10}$$

The state $|\chi_n\rangle$ is localized near the end of the chain. In the general case this state with n flipped spins is not a stationary state for the system (4). It follows from (8) that, for $n \to \infty$,

$$|\chi_n(B)\rangle = \mathcal{A}_n \sum_{\{m_i\}} \exp[-B(m_1+m_2+\ldots+m_n)]$$
$$\times S_{m_1}^- S_{m_2}^- \ldots S_{m_n}^- |0\rangle.$$

As shown in Ref. 9, the vector $|\Psi_n\rangle$ of the spin complex localized near the end of the chain (4) can also be written in this form (in the same limiting case $n\to\infty$ and with $B=\sigma$). Consequently, for $n\to\infty$ and $B=\sigma$, we have $|\chi_n\rangle=|\Psi_n\rangle$, and $|\Phi\rangle$ is the linear combination (3) of spin complexes. It turns out that, even when n is finite in the chain (4) with a weak anisotropy, the state $|\chi_n\rangle$ with $B=\sigma\,\mathrm{th}\,a$ is close to the state $|\Psi_n\rangle$. To prove the latter assertion, it is sufficient to compute the mean energy[2] $\bar{E}_n=\langle\chi_n|H|\chi_n\rangle$. From the relations (9) and (10) we obtain

$$W(A^2,B,\sigma)\equiv\langle\Phi|H|\Phi\rangle=\sum_{n=0}^{\infty}\frac{A^{2n}Z_n(B)\bar{E}_n(B,\sigma)}{Z(A^2,B)}. \tag{11}$$

Treating the functions $W(A^2)$, $Z(A^2)$, and A^{2n} as functions of the complex variable λ $(A^2\to\lambda)$, and using the Cauchy theorem, we can write

$$Z_n(B)=\frac{1}{2\pi i}\oint_{L_1}\frac{Z(\lambda,B)}{\lambda^{n+1}}d\lambda,$$
$$\bar{E}_n(B)=\frac{1}{2\pi i Z_n(B)}\oint_{L_2}\frac{Z(\lambda,B)W(\lambda,B,\sigma)}{\lambda^{n+1}}d\lambda, \tag{12}$$

where the L_i are closed contours around the point $\lambda=0$ in the complex plane. The first of the equalities (12) is the well-known definition of the partition function Z_n in terms of the partition function of a gas in the grand ensemble.

For small σ values we can compute the integrals in (12) by the method of steepest descent, choosing the contours L_i in the form of circles $\lambda=r_0\exp i\varphi$. The equation for the saddle point r_0 has the form

$$\sigma n=\frac{y_0\,\mathrm{th}\,y_0}{\mathrm{th}\,a}+\frac{\sigma}{2}\mathrm{th}^2\,y_0+O(\sigma^3),\quad r_0\equiv\mathrm{sh}^2\,y_0. \tag{13}$$

In such a calculation the expressions for $Z_n(B)$ and $\bar{E}_n$ are obtained in the form of expansions in powers of σ. As will be seen below, for our purposes it is sufficient to find $\bar{E}_n$ up to and including the terms of order σ^2. The function $W(\lambda,B,\sigma)$ is obtained from the Hamiltonian H by replacing the operators $\hat{\mathbf{S}}_m$ by classical vectors of length $\frac{1}{2}$. With the aid of the Euler-Maclaurin summation formula we can write W to the required accuracy in the form

$$W(\lambda,B,\sigma)=\frac{B\lambda}{2}\int_0^{\infty}d\xi\frac{\mathrm{ch}^2\,\xi}{(\lambda+\mathrm{ch}^2\,\xi)^2}\left(\mathrm{th}^2\,\xi+\frac{\sigma^2}{B^2}\right)+O(\sigma^3).$$

Carrying out the indicated computation, we arrive at the following expression for the mean energy:

$$\bar{E}_n={}^1\!/_2\sigma\,\mathrm{th}\,\sigma n+{}^1\!/_2B^2[f(2a)+(n-N)^2\sigma f_1(a)]+O(\sigma^3),$$
$$f(a)=\frac{a\,\mathrm{sh}\,a-\mathrm{sh}\,a}{(a+\mathrm{sh}\,a)\,\mathrm{sh}^2\,a},\quad f_1(a)=\frac{\tau(1-\tau+2\tau^2\,\mathrm{sh}^2\,a)}{2a(1+\tau)^2\,\mathrm{sh}^2\,a}, \tag{14}$$

where $\tau\equiv 2a/\mathrm{sh}\,2a$.

With the aid of similar computations we can find the coefficients C_n as well. Retaining only the n-dependent factors, we can, as a result, write

$$C_n\sim\left(\frac{\mathrm{sh}\,a}{\mathrm{sh}\,y_0}\right)^n\left[\left(1+\frac{2y_0}{\mathrm{sh}\,2y_0}\right)\mathrm{th}^2\,y_0\right]^{-1/4}\exp\frac{y_0^2}{2B}.$$

If we are not interested in the weak dependence on n of the factor in the square brackets and in the insignificant shift in the index of the exponential function, then from the solution to Eq. (13) we obtain the following simple expression for the $C_\varkappa$ $(k\equiv n-N)$:

$$C_k\sim\exp\left\{-\frac{\sigma k^2}{2}f_2(a)\right\},\quad f_2(a)=\frac{(1+2\tau)\,\mathrm{cth}\,a}{(1+\tau)^2}. \tag{15}$$

It can be seen from the latter result that the width of the distribution $C_\varkappa$ is of order $\sigma^{-1/2}$. The function $f_2(a)$ is monotonic: $f_2(\infty)=1$ and $f_2(1)\approx 1.15$. Thus, in the region $a\gtrsim 1$ the width of the packet (10) decreases slowly with decreasing a.

Using (14) and (15), we can assess how close the state $|\chi_n\rangle$ is to the spin-complex state $|\Psi_n\rangle$.

The complex $|\Psi_n\rangle$, which is localized near the end of the chain (4), is the ground state in the substate of the n-particle states of the Hamiltonian H. The exact solution for the wave function $|\Psi_n\rangle$ and the energy ε_n is found in Ref. 15. For small σ and $n\sigma\gtrsim 1$, the expression for ε_n assumes the form

$$\varepsilon_n={}^1\!/_2\sigma\,\mathrm{th}\,\sigma n+O(\sigma^3). \tag{16}$$

The gap in the spectrum of the n-particle states of the system (4) is equal to[3]

$$\Delta_n=\varepsilon_{n-1}+\varepsilon_1-\varepsilon_n={}^1\!/_2\sigma^2\,\mathrm{th}^2\,\sigma n+O(\sigma^3).$$

It can be seen that, for $n\sigma\gtrsim 1$, the gap Δ_n is of order σ^2. For $n-N\lesssim\sigma^{-1/2}$, we have[4] $\Delta_n=B^2/2+O(\sigma^3)$ recall that $B=\sigma\mathrm{th}\,\sigma N$).

Let us write the general form of the expansion of $|\chi_n\rangle$ in terms of the stationary states of the Hamiltonian (4):

$$|\chi_n\rangle=a_{n,0}|\Psi_n\rangle+\sum_\nu a_{n,\nu}|\Psi_{n,\nu}\rangle,$$

where the $|\Psi_{n,\nu}\rangle$ are the excited states in the n-particle sector. The mean energy $\bar{E}_n$ is then equal to

$$\bar{E}_n=\frac{1}{2}\sigma\,\mathrm{th}\,\sigma n+\sum_\nu|a_{n,\nu}|^2(\varepsilon_{n,\nu}-\varepsilon_n). \tag{17}$$

Taking into account the fact that

$$\varepsilon_{n,\nu}-\varepsilon_n\geqslant\Delta_n=B^2/2+O(\sigma^3),$$

we find from a comparison of (14) and (17) in the case when $n-N\lesssim\sigma^{-1/2}$ that

$$\sum_\nu|a_{n,\nu}|^2\leqslant f(2a)+(n-N)^2\sigma f_1(a).$$

In the cases when the right-hand side of this inequality is small, $|\chi_n\rangle$ is close to the state $|\Psi_n\rangle$. We can obtain an estimate for the overall contribution of the excited states $|\Psi_{n,\nu}\rangle$ in the expansion of $|\Phi\rangle$ at once. Writing

$$|\Phi\rangle=\sum_n C_n a_{n,0}|\Psi_n\rangle+\sum_{n,\nu}C_n a_{n,\nu}|\Psi_{n,\nu}\rangle$$

and carrying out a simple calculation, we obtain

$$\sum_{n,\nu}C_n^2|a_{n,\nu}|^2\leqslant v(a)\equiv f(2a)+\frac{f_1(a)}{2f_2(a)}.$$

The functions f, f_1, and f_2 are defined in (14) and (15).

In the cases when the condition

$$v(a)\equiv f(2a)+f_1(a)/2f_2(a)\ll 1, \qquad (18)$$

is fulfilled, we have the equality

$$|\Phi\rangle=\sum_{n=0}^{\infty}C_n|\Psi_n\rangle. \qquad (19)$$

For $a\gg 1$ the value of $v(a)$ is exponentially small ($v\sim ae^{-4a}$); in the limit $a\to\infty$ we have $v(a)=0$, and the equality (19) is exact. An investigation of the function $v(a)$ shows that the condition (18) is fulfilled for virtually all $a\gtrsim 1$. The function $v(a)$ is monotonic: $V_{\max}=v(0)=\frac{7}{18}$, with $v(1)\approx 0.100$ and $v(2)\approx 0.005$. From this we can conclude that, in the entire $a\gtrsim 1$, the state $|\Phi\rangle$ can be explained only in terms of the spin complexes. Or, more precisely, we should say that the corrections to the expansion (19) are small both to the extent that σ is small and to the extent that the numerical function $v(a)$ is small.

3. COMPUTATION OF THE AVERAGES $\langle\Phi(t)|\hat{S}_m|\Phi(t)\rangle$

Using the results (15) and (19), we easily find the state

$$|\Phi(t)\rangle=e^{-iHt}|\Phi\rangle.$$

Indeed, for $k\lesssim\sigma^{-1/2}$ and $\sigma\ll 1$, the energy of the spin complex consisting of $N+k$ magnons can, as follows from (16), be written in the form

$$\varepsilon_{N+k}\approx\varepsilon_N+k\omega, \qquad (20)$$

where ω is equal to the precession frequency of the classical vectors $\mathbf{S}_m$ [see (7)]. From this it follows that

$$|\Phi(t)\rangle=e^{-i\varepsilon_N t}\sum_{k=-\infty}^{\infty}C_k e^{-ik\omega t}|\Psi_{N+k}\rangle. \qquad (21)$$

It is not difficult to find yet another form of the state $|\Phi(t)\rangle$:

$$|\Phi(t)\rangle=\exp[-i(\varepsilon_N-N\omega)t]\times\left\{\prod_{m=0}^{\infty}\exp[-i\theta_m(\hat{S}_m^x\sin\omega t+\hat{S}_m^y\cos\omega t)]\right\}. \qquad (22)$$

It is easy to verify that the mean values of the operators $\hat{S}_m^\alpha$ in this state coincide with the values of S_m^α that follows from the bion solution (7) to the Landau-Lifshitz equations:

$$\langle\Phi(t)|\hat{S}_m^\alpha|\Phi(t)\rangle=S^\alpha_{m,\text{bion}}.$$

4. PROPERTIES OF THE STATE $|\Phi(t)\rangle$ AND CONCLUSION

Let us enumerate the main properties of $|\Phi(t)\rangle$. This is a nonstationary state for the system (4), that preserves its functional form (22) in time. It is a direct product of one-spin (or Bloch) coherent states, whose properties have been studied in detail by Perelomov and Radcliff.[16] It follows from these properties and the form of (22) that $|\Phi(t)\rangle$ at all times minimizes the products of the uncertainties and factorizes the correlation functions:

$$\overline{\hat{S}_{m_1}^{\alpha_1}\hat{S}_{m_2}^{\alpha_2}}=\overline{\hat{S}_{m_1}^{\alpha_1}}\,\overline{\hat{S}_{m_2}^{\alpha_2}},\qquad m_1\neq m_2.$$

The averaging of the operator equations (2) with the aid of $|\Phi\rangle$ leads to the phenomenological Landau-Lifshitz equations. Consequently, we have constructed for the system (4) a coherent state that establishes a connection between the quantum theory and the solution (7) to the equations (1). The state $|\Phi(t)\rangle$ furnishes a complete description of a bion localized near the end of the quantum spin chain (4).

In the $s=\frac{1}{2}$ case under consideration here the $a\to\infty$ limit exists in which $v(a)=0$, and the Landau-Lifshitz equations are exact in the strict sense of the word. It should be emphasized that this limit is not the classical $\hbar\to 0$, $s\to\infty$ limit discussed in Ref. 17. The possibility, demonstrated above, of describing the evolution of the quantum-mechanical averages S_m^α in a chain of $s=\frac{1}{2}$ spins by phenomenological equations in practically the entire $a\gtrsim 1$ region is, in our opinion, interesting. It may be that this possibility is due to the total integrability[5)] of the system (4) in both the quantum[18] and classical[19] cases. In any case the question of the existence of nonspreading packets with the properties of the state $|\Phi(t)\rangle$ (this is a question of the existence of coherent states) is important for all quantum systems. This question has been completely resolved only for Hamiltonians that are quadratic in the Bose or Fermi operators. The search for such packets in more complicated situations has been conducted even in the last few years almost exclusively in systems with several degrees of freedom.[20]

It is to be expected that in the model (4) with arbitrary s the function $v(a)$ in (18) will contain an additional factor of the type $(2s)^{-1}$ that will make the value of $v(a)$ in (18) go to zero in the limit as $s\to\infty$. To carry out an analysis of the $s\rangle\frac{1}{2}$ case by the method proposed here, we must have the explicit solution to the spin-complex problem, a solution which, for the chain (4), it known only in the $s=\frac{1}{2}$ case.[15] For other s values the exactly soluble generalized ferromagnet model[21] may turn out to be useful for this purpose.

As can be seen from (15) and (19), the state $|\Phi(t)\rangle$ is a linear combination of the spin complexes[6)] $|\Psi_n\rangle$. The connection between the complexes and the soliton solutions to the equations (1) has been pointed out repeatedly in the literature.[3,7–11,14] The main results confirming this connection are presented in the Introduction. Here we must explain how these results can be reconciled with the fact that $|\Phi(t)\rangle$ is a superposition of the complexes, and not simply a single complex. Since the spectrum of coupled states (20) is equally spaced and the $C_\varkappa$ distribution (15) is symmetric, the packet's mean energy $\overline{E}(\overline{N})$ ($\overline{N}$ is the mean magnon number) is equal to the energy ϵ_n ($n\leftrightarrow\overline{N}$, $\epsilon\leftrightarrow\overline{E}$) of a complex. The equality of the S_m^z densities in $|\Phi\rangle$ and $|\Psi_n\rangle$ is more easily understood within the framework of the statistical mechanics problem which was used for comparison throughout this analysis. Equal occupation numbers for the canonical and grand canonical gas ensembles corresponds to equal S_m^z averages in the two states.

The other important property (5), however, is peculiar to $|\Phi(t)\rangle$. The condition (5) shows that the mean values of S_m^α at any site m determine a vector of fixed length $\frac{1}{2}$, and, as we note in Refs. 9 and 12, it cannot, in principle, be fulfilled for the individual complexes $|\Psi_n\rangle$. We can expect that in other cases the coherent states will possess both the characteristics of the individual complexes and specific properties such as property (5).

Our results are inapplicable in the region of weak nonlinearity. The characteristics of the spectrum of the complexes and those of the gap Δ in the region $a \ll 1$ require another type of analysis. It is possible that $|\Phi(t)\rangle$ in the form (22) will describe a bion in this region as well, but, besides the complexes $|\Psi_n\rangle$ a continuous spectrum may equally well be present in the expansion (19).

Summarizing, we can say that we have studied quantum-mechanically with the aid of the solution (7) to the Landau-Lifshitz equations and the exact spectrum (20) of the spin complexes a bion localized near the end of the chain (4). We hope that our method is applicable also to the more complicated moving-soliton and soliton-soliton-collision problems in unbounded systems.

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1) In the spin-wave approximation the Hamiltonian is quadratic in the boson operators. It is for these operators that the equations of motion must be written.

2) For finite n it is difficult to prove the assertion by directly comparing the vectors $|\chi_n\rangle$ and $|\Psi_n\rangle$.

3) In computing Δ_n, we need only take into account the fact that the first excited states describe the scattering of a magnon with energy $\epsilon_1 = \frac{1}{2}\sigma^2 + O(\sigma^3)$ by the $(n-1)$th partial spin complex.

4) The maximum $n - N$ values for which the analysis is being performed are determined by the C_x distribution [see (15)].

5) Let us nonetheless point out that, in Refs. 18 and 19, total integrability is proved for the unbounded systems (4) (or systems with periodic boundary conditions). Here we are investigating a semifinite chain.

6) For comparison, let us recall that the Glauber coherent state, which establishes a connection between the microscopic theory and the classical spin wave, is a linear combination of the spin-wave states of the Hamiltonian.[5]

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