Nonlinear resistance of thin metal plates

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Nonlinear effects in the resistance of thin metal plates to dc currents are studied experimentally. The bending of the electron trajectories in the magnetic field of the current causes the resistance R to depend on the current I; $R(I)$ has a characteristic minimum at $I = I_{\min}$ which is independent of temperature and depends only slightly on the geometry of the plate. The dependence $R(I)$ oscillates with large amplitude when I exceeds a value which depends on the temperature.

A new area of solid-state physics has developed recently which deals with nonlinearity in metals. Most of the work along these lines has been concerned with rf and microwave frequencies (see, e.g., the bibliography in Ref. 1). Examples of nonlinear effects include current states, the dependence of the impedance on the amplitude of incident radiowaves, etc. Many of these effects are caused by electrons which are trapped by a magnetic field alternative in direction and follow curved trajectories. This type of behavior is referred to as magnetodynamic nonlinearity.

Kaner *et al.*[2,3] demonstrated theoretically that pronounced magnetodynamic nonlinearity occurs in thin conductors that carry a dc current and diffusely scatter the carriers at a boundary. The electrons trapped by the magnetic field of the current contribute greatly to the conductivity, and this is responsible for the nonlinearity. According to Refs. 2 and 3, if

$$d \ll (rd)^{1/2} \ll l \tag{1}$$

the conductivity of a plate is determined primarily by electrons which move along the electric field a distance $\sim l$ and wind around the plane of vanishing magnetic field (here r is the characteristic radius of curvature of the electron trajectory in the magnetic field of the current, $2d$ is the thickness of the plate, and l is the electron mean free path). The relative number of these "effective" electrons is proportional to $(d/r)^{1/2}$ and greatly exceeds the fraction $\approx d/l$ of the electrons that contribute effectively to the conductivity in metals when $I = 0$. The concentration of effective carriers increases with I, so that R drops. The nonlinearity is pronounced for currents exceeding the characteristic value

$$I_0 = 2Ddc^2 p_F \ln(2d/l)/el^2, \tag{2}$$

where c is the speed of light, p_F is the Fermi momentum, $2D$ is the width of the plate, and e is the electron charge. The voltage-current (I-V) characteristic for a plate with $D \gg l \gg d$ is of the form

$$U = R_0 (I_0 I)^{1/2} \tag{3}$$

for $I > I_0$, where R_0 is the linear value of the resistance. The influence of an external magnetic field on the nonlinear resistance of metal plates was also studied theoretically in Ref. 3.

In spite of the interest in magnetodynamic nonlinearity for static currents, little experimental work has been done in this area. We are aware of only one paper,[4] in which $R(I)$ was studied for fine single-crystal gallium wires. It was suggested there that the observed decrease in R was due to distortion of the electron trajectories by the magnetic field of the current.

The present work is concerned with an experimental study of the nonlinear resistance in thin cadmium and tungsten plates. A strong dependence was observed experimentally—as I increased, the differential resistance of the highest-quality plates dropped to just 50% of the linear value, after which a further increase in I caused R to rise and the smooth dependence $R(I)$ began to oscillate with large amplitude. We concluded from measurements at various temperatures and external magnetic fields that the observed smooth dependence $R(I)$ was due to the contribution to the conductivity from the spiraling electrons. The nonlinearity occurred at relatively low currents for which plate heating was negligible.

1. SAMPLES AND MEASUREMENT TECHNIQUE

We measured the electrical resistances of the rectangular plates by the four-contact method. The plates were cut from crystal bars on an electrical arc machine. The ratio $\rho_{300}/\rho_{4.2}$ of the resistances at 300 and 4.2 K was equal to 50,000 for cadmium and 80,000 for tungsten. The cut cadmium plates were then chemically polished and supplied with indium-solder current and voltage contacts. The tungsten plates were mechanically polished and then etched in a mixture of concentrated nitric, fluoric, and orthophosphoric acids. In order to fabricate reliable current contacts, we electroplated a 10-μm-thick layer of tin on the ends of the plates and then soldered copper wires to the tin. The voltage leads (0.06-mm-diameter platinum wire) were welded to the plates by discharging a capacitor. The current and voltage contacts were separated by a distance of 2–3 mm, and the voltage contacts were spaced at least 4 mm apart.

The cadmium plate (Cd1) measured $12 \times 0.25 \times 0.12$ mm^3 and its large face was normal to the [0001] crystallographic axis. The dimensions of the tungsten plates were $9 \times 0.4 \times 0.14$ mm^3 (W1), $9 \times 0.27 \times 0.09$ mm^3 (W2), $13 \times 2 \times 0.03$ mm^3 (W3), and $12 \times 0.7 \times 0.12$ mm^3 (W4). All the tungsten plates had the same orientation, with the large face along the (100) plane. In order to make the exposition clearer, we take the **x**, **y**, **z** coordinate axes parallel to

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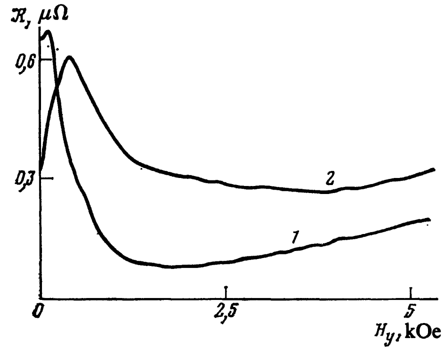

FIG. 1. Differential resistance of tungsten plate W1 in a longitudinal magnetic field, $T = 4.2$ K. Curves 1 and 2 correspond to $\mathscr{R}(I=0) \approx R_0$ and $\mathscr{R}(I = 7.5\ \text{A})$, respectively.

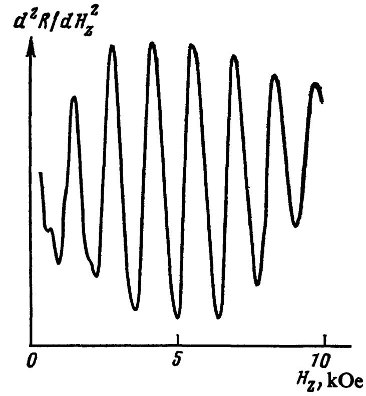

FIG. 2. Sondheimer oscillations in tungsten plate W1; $H \parallel [100]$, $T = 4.2$ K.

the edges and the coordinate origin at the center of the plate; the large face is parallel to the xy plane, and the current flows along the y axis, which is parallel to the binary axis of the crystal.

In the experiment we recorded the voltage-current (I-V) characteristics of the plates and the current-dependence of the resistance. The I-V characteristics were recorded by a Hewlett-Packhard model 3390 integrating potentiometer. A dc current potentiometer measured the resistance. However, this method did not permit us to determine the limit $R_0 = R(I \to 0)$ sufficiently accurately. Considerably more accurate results were obtained by the modulation technique, which enabled us to measure the differential resistance $\mathscr{R} = dU/dI$ and record continuous traces $\mathscr{R}(I)$ by harmonically modulating the current at low frequency and using a selective amplifier and phase detector to select the signal (proportional to dU/dI) at the modulation frequency. The phase in the detector was clamped to the signal from an active resistor, which was connected in series with the plate in the current circuit. The modulation frequency was low enough so that the skin effect was negligible. Most of the measurements were made at 10 Hz [the dependence (I) remained the same to within the measurement error when the frequency was doubled]. The amplitude of the modulation current was 0.2–0.3 A.

An electromagnet or superconducting solenoid generated the external magnetic field. We were able to record signal proportional to dR/dH and d^2R/dH^2 by modulating the magnetic field.

The ratio $\rho_{300}/\rho_{4.2}$ was 4000–10,000 for our plates, which was much less than in the starting bars. Resistance measurements in a longitudinal magnetic field H_y revealed that the thinness of the plates rather than plate deformation was primarily responsible for these small values—the application of a field H_y increased $\rho_{300}/\rho_{4.2}$ almost to the value for the initial bar. Thus for sample Wl ($\rho_{300}/\rho_{4.2} = 8000$), the application of the longitudinal field decreased R_0 almost tenfold (cf. curve 1 in Fig. 1). We estimated l from the known values of ρl and found that $l = 2$ and 3 mm at $T = 1.5$ K for Cd and W, respectively. We note that because $l \gtrsim 2D$ for our plates, our results are not quantitatively comparable with those in Refs. 2 and 3.

We measured the plate thicknesses more accurately by measuring the period of the Sondheimer oscillations in a magnetic field H_z normal to the large face of the plate. Figure 2 shows a trace of d^2R/dH_z^2 for sample W1. The oscillations are due to octahedral holes, for which the displacement along the magnetic field is greatest during the cyclotron period. The value of $(\partial S/\partial p_z)_{\text{extr}}$ for these holes is equal to 3.1 Å^{-1} (here S is the area of the section of the Fermi surface cut by a plane perpendicular to H, and p_z is the z-component of the electron momentum). The value $2d = 0.14$ mm found for W1 agrees closely with optical measurements. Figure 2 shows that the oscillations are nearly harmonic; this is presumably because relatively few of the electrons in our samples were specularly reflected by the metal surface. The pronounced dimensional effect (decrease in $\rho_{300}/\rho_{4.2}$) discussed above also indicates that the reflection was primarily diffuse.

We close this section by noting that since $R(I)$ and $\mathscr{R}(I)$ were measured for $I < 50$ A, less than 0.1 W/cm^2 of power was evolved in the samples.

2. EXPERIMENTAL RESULTS AND DISCUSSION

1. We first discuss the measurements of the nonlinear resistance in the absence of an external magnetic field. Figure 3 shows the I-V characteristic of the cadmium plate (sample Cd1). The linear dependence $U = R_0 I$ starts to

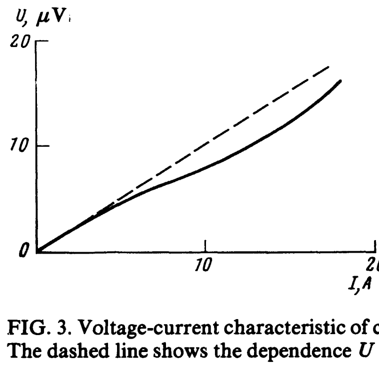

FIG. 3. Voltage-current characteristic of cadmium plate Cd1, $T = 1.5$ K. The dashed line shows the dependence $U = R_0 I$.

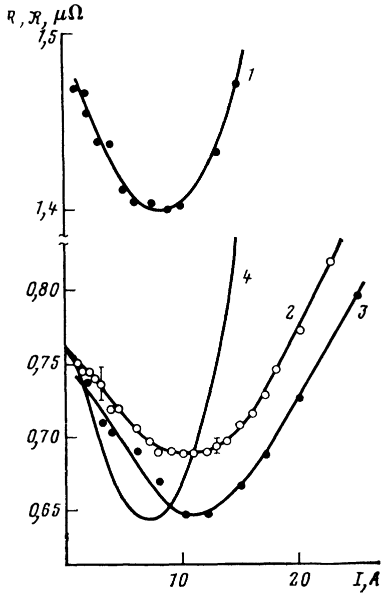

FIG. 4. Resistance (curves 1–3) and differential resistance (curve 4) of tungsten plates as a function of the current. 1) W3 plate, $T = 1.5$ K; 2, 3) $W4$ plate, $T = 4.2$ and 1.5 K; 4) W4 plate, $T = 4.2$ K.

break down for $I \gtrsim 2$ A and the resistance decreases with I. For large currents $I \gtrsim 15$ A the increase in $U(I)$ is faster than linear.

Curves 1–3 and curve 4 in Fig. 4 show $R(I)$ and $\mathscr{R}(I)$, respectively, for tungsten plates (samples W3 and W4). The resistance depends nonmonotonically on the current—it decreases for small I to a minimum at $I = I_{\min}$ and then increases. Cooling from 4.2 to 1.5 K makes the minimum deeper (cf. curves 2 and 3) but has little effect on $I_{\min}$. We note that the curves $R(I)$ and $\mathscr{R}(I)$ (curves 2 and 4) for the same sample are qualitatively similar; however, the minimum in curve 4 is considerably deeper than for curve 2 and occurs at lower I. Curves 2 and 4 cross at $I = I_{\min}$, because the condition for $R(I)$ to have a minimum is that $R(I_{\min}) = (dU/dI)|_{I=I_{\min}} \equiv \mathscr{R}(I_{\min})$.

The qualitative behavior of the nonlinear resistance for $I < I_{\min}$ is as predicted by the theory in Refs. 2 and 3. For $T = 1.5$ K the values of I_0 for Cd1 and W3, W4 estimated by Eq. (2) are equal to 1, 1.8, and 1.2 A, respectively. The experimental curve $R(I)$ for sample Cd1 drops appreciably for $I \gtrsim 2$ A, in good agreement with theoretical estimates. For tungsten, nonlinearity is observed for I well below the calculated values I_0.

We have already noted that the experimental results agree with the theoretical predictions, except that R increases after reaching a minimum at $I = I_{\min}$. The observed increase in R might be due to a breakdown of the conditions under which the theory is applicable, One of these conditions is that the inequalities (1) should hold. The left-hand inequality is certainly violated for currents $I \sim I_d$, where

$$I_d \sim \frac{c^2 p_F}{\pi e} \frac{D}{d}. \tag{4}$$

The currents I_d calculated from (4) for samples Cd1, W3, and W4 are equal to 20, 500, and 50 A, respectively, and are much larger than the experimentally measured values $I_{\min}$. We also note that according to (4), I_d is proportional to the ratio D/d and should therefore depend on the dimensions of the plates. However, the experimental values $I_{\min}$ were similar for the different samples. Another limitation of the theory results from the assumption in Ref. 2 that the plates were unbounded, whereas the dimensions were necessarily finite in the experiment. In this case the field component $\mathscr{H}_z$ should become important in addition to the component $\mathscr{H}_x$ considered above. We can estimate its magnitude by the formula

$$\mathscr{H}_z(D) \approx k \frac{I}{cD}, \tag{5}$$

where the constant k depends on D/d [$k = 1.7$ for $D = d$ and increases with D/d to the limiting value $1 + \ln(2D/d)$]. The influence of $\mathscr{H}_z$ is especially pronounced when the characteristic radius of curvature of the electron trajectory in the xy plane becomes less than the halfwidth D. This occurs for currents[1)]

$$I_D \sim \frac{1}{k} \frac{c^2 p_F}{e}. \tag{6}$$

If I exceeds I_D, the effective electrons are confined to a layer of width

$$2x_0 \sim 2cp_F/(e\mathscr{H}_z(x_0)) < 2D,$$

where $\mathscr{H}_z(x_0)$ is the z-component of the magnetic field at the boundary of the layer and is given by $\mathscr{H}_z(x_0) = \mathscr{H}_z(D)x_0/D$ to lowest order. Since the width of the layer decreases as I increases, we expect R to increase for $I > I_D$. Clearly, $R(I)$ should have a minimum for $I \sim I_D$, where the current I_D depends only on p_F and k. The minimum should occur at lower currents for plates with large D/d, as is confirmed experimentally. For sample W4 with $D/d \sim 6$, $I_{\min}$ was roughly 1.4 times as large as for sample W3, for which $D/d \sim 70$ (cf. curves 1 and 2 in Fig. 4). The current I_D found from Eq. (6) agrees in order of magnitude with $I_{\min}$ (for $k = \pi$, $I_D \sim 6$–8 A).

The theoretical and experimental results can thus be compared for currents $I_0 < I < I_D$. Equations (2) and (3) imply that cooling should accelerate the decrease in R for these currents, and this was acually observed experimentally. As predicted by (6), $I_{\min}$ was independent of temperature.

For large enough currents $I > I_d$ the effective electrons lie within the region

$$|x| \leqslant x_0 = \frac{cp_F}{e\mathscr{H}_z(x_0)}, \qquad |z| \leqslant z_0 = \frac{cp_F}{e\mathscr{H}_x(z_0)}, \tag{7}$$

and their contribution to the conductivity drops faster than for $I_D < I < I_d$. In this case the bulk magnetoresistance and the static skin effect in the magnetic field of the current also become important.[5] We therefore expect $\mathscr{R}(I)$ to increase more rapidly. The same factors could also be responsible for the increase in $d\mathscr{R}/dI$ for $I > 40$ A (Fig. 5); the latter bound is similar to the value $I_d \sim 50$ A estimated by Eq. (4).

2. We now discuss how an external magnetic field along

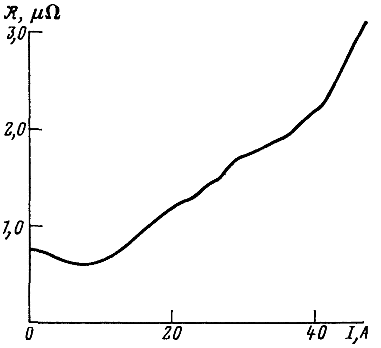

FIG. 5. Differential resistance of plate W4 for a wide range of currents, $T = 2.2$ K.

the x or y axis influences the nonlinear resistance.

Figure 6 shows the differential resistance of tungsten (sample W1) as a function of the current for transverse (H_x) and longitudinal (H_y) magnetic fields. Curves 2 and 3 illustrate how the field H_x influences the behavior $\mathcal{R}(I)$. The chief difference from the case $H = 0$ (curve 1) is the peak at $I = I_{\max}$, which is preceded by a slight increase in $\mathcal{R}$. The behavior of $\mathcal{R}(I)$ for $I > I_{\max}$ is similar to that for $H_x = 0$; however, $\mathcal{R}$ does not decrease by as much, and the minimum occurs at higher currents (curve 2). The current $I_{\max}$ is proportional to H_x, and the magnetic field induced by $I_{\max}$ on the surface of the plate is comparable to the external magnetic field. The longitudinal field H_y (curve 4) also attenuates the nonlinear effect, but it leaves $I_{\max}$ unchanged.

The influence of an external magnetic field on the resistance of plates satisfying (1) was studied in Ref. 3. We first consider how a transverse field H_x alters the dependence $R(I)$. For small currents ($\mathcal{H}_x < H_x$) such that the magnetic field does not vanish anywhere inside the sample, the resistance increases with I:

$$R(I)\propto \ln\frac{cp_F}{ed(H_x+\pi I/cD)}. \qquad (8)$$

The magnetic field in the plate starts to change sign for currents greater than

$$I_{\mathcal{H}=H}=cDH_x/\pi, \qquad (9)$$

which corresponds to the equality $\mathcal{H}_x = H_x$. As I increases, the plane of zero magnetic field moves deeper into the plate, and R drops because more of the electrons contribute effectively to the conduction. Since the plane of zero magnetic field lies closer to the surface for $H_x > 0$ than for $H_x = 0$, the transverse field should decrease the drop in $R(I)$, and $R(I)$ should have a maximum for transitional currents $I \approx I_{\mathcal{H}=H} \propto H_x$. Theory[3] and experiment thus agree for $I_0 < I < I_{\min}$.

For $I > I_D$ the efffective electrons lie within the region

$$|x|\leqslant x_0=\frac{cp_F}{e\mathcal{H}_z(x_0)}, \quad d\left[\frac{2H_x}{\mathcal{H}_x(d)}-1\right]\leqslant z<d, \qquad (10)$$

which becomes narrower and thicker as I increases. The relative number of effective electrons thus peaks at a current

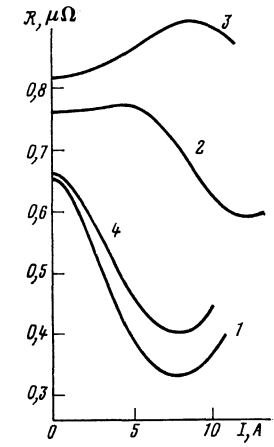

FIG. 6. Influence of an external magnetic field H on the differential resistance of tungsten plate W1 at $T = 4.2$ K. 1) $H = 0$; 2, 3) $H = H_x = 60$ Oe and 120 Oe, respectively; 4) $H = H_y = 100$ Oe.

$I = I^*_{\min} > I_D$, and $I^*_{\min}$ increases with H_x. The resistance should have a corresponding minimum near $I \sim I^*_{\min}$. These arguments account for the observed shift in the minimum of $\mathcal{R}(I)$ toward larger I as H_x increases.

The effects of an external field H_y for an unbounded plate can be readily understood by considering how an effective electron moves in the crossed field H_y and $\mathcal{H}_x$. The result

$$z=\frac{dv_xH_y}{v_F\mathcal{H}_x(d)}+A\sin\left[\left(\frac{ev_F\mathcal{H}_x(d)}{mcd}\right)^{1/2}t+\varphi_0\right],$$
$$A<A_0\equiv d[1-H_yv_x/v_F\mathcal{H}_x(d)], \qquad (11)$$

for the time-dependence of the electron z-coordinate follows by solving the equations of motion for a uniform current distribution over the plate cross section for plates satisfying (1). Here v_x is the x-component of the electron velocity, v_F is the Fermi velocity, and the constants of integration A, φ_0 are determined by the initial conditions. Equation (11) shows that if $v_x \neq 0$ then its plane of equilibrium moves a distance $dH_yv_x/v_F\mathcal{H}_x(d)$ from the center along the z axis. This shift of course decreases the relative number of effective electrons as compared with the case $H_y = 0$.

The above discussion makes it clear why the differential resistance of plates in a magnetic field H_y differs for the linear ($I = 0$) and nonlinear ($I = 7.5$ A) cases (Fig. 1). When I is low enough so that the magnetic field $\mathcal{H}_x$ can be neglected (curve 1), the resistance drops almost tenfold as H_y increases because there is less surface scattering of electrons. In the nonlinear case (curve 2), R for $H_y = 0$ is roughly just 50% of R_0; according to (11), turning on the field H_y will displace the plane of equilibrium of each effective electron for which $v_x \neq 0$, and R will increase. If H_y is $\gg cp_F/ed$, the decreased surface scattering dominates and R drops as in the linear case. A further increase in H_y decreases the difference between curves 1 and 2 caused by the different orientation of the total magnetic field relative to the current.

3. The existing theory can thus account for the experimental dependences of R on the current and external mag-

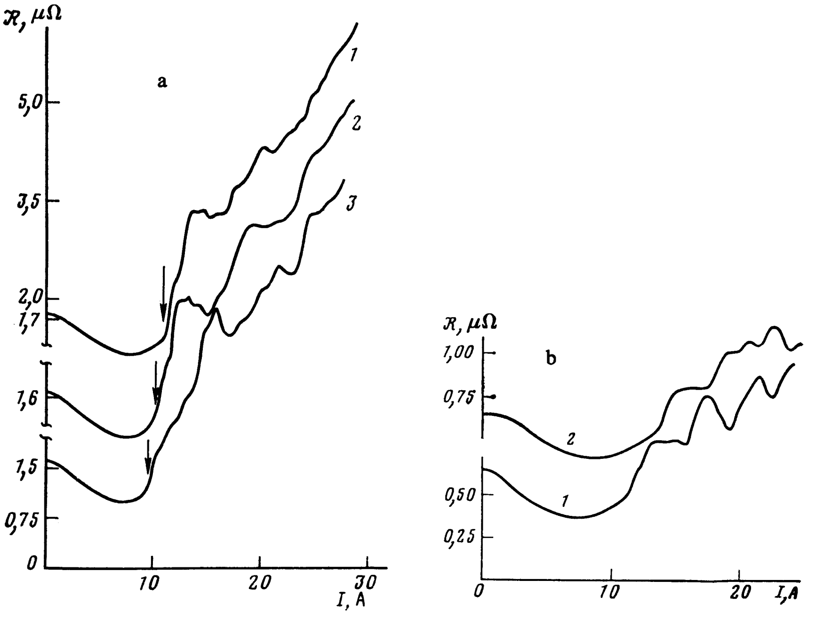

FIG. 7. Oscillations of the differential resistance as a function of the current. a) Plate W2, $H = 0$. 1) $T = 4.2$ K; 2) $T = 3.4$ K; 3) $T = 1.5$ K; b) Plate W1, $T = 4.2$ K; 1) $H = 0$; 2) $H = H_x = 20$ Oe.

netic field. However, some features of the $\mathscr{R}(I)$ dependence of the differential resistance lack a satisfactory explanation.

For example, strong oscillations in $\mathscr{R}(I)$ were observed for several of the plates for $I > I_{\min}$. The oscillations usually start before the abrupt increase in $\mathscr{R}$. The arrows in Fig. 7a show the currents at which the jump occurs. The oscillation amplitude differed appreciably–for sample W4 the oscillations about the smooth dependence $\mathscr{R}(I)$ could barely be distinguished (Fig. 5), while for W1 the oscillation amplitude was comparable to R_0 (cf. curve 1 in Fig. 7b). The oscillations lacked a well-defined period and were anharmonic.

As T drops the current jump (threshold) preceding the oscillations shifts toward lower I, and the nature of the oscillations changes considerably (Fig. 7a). Even a weak transverse field H_x suffices to shift the threshold jump toward higher I; the oscillations then decrease in amplitude and the overall pattern changes (Fig. 7b, curve 2).

Small oscillations of the resistance as a function of current were previously observed in fine gallium wires at 1.2 K in Ref. 4, where an attempt was made to relate them to the Sondheimer effect. We are not convinced that Sondheimer oscillations can occur in the nonuniform magnetic field induced by the current-carrying wire. Even if we grant their existence and estimate $(\partial S/\partial p_z)_{\text{extr}}$ from the distance between adjacent extrema, the resulting value is an order of magnitude lower than the known values $(\partial S/\partial p_z)_{\text{extr}}$ for the tungsten Fermi surface. We add that we did not detect any oscillations with such a short period in our measurements under linear conditions ($I = 0$) for magnetic fields H_x or H_z up to 500 Oe.

Nor can the oscillations in $\mathscr{R}(I)$ be accounted for by the quantization of the electron trajectories considered in Ref. 2. The period and amplitude of our oscillations are several orders of magnitude larger than the estimates given there. Further experimental and theoretical work is needed to establish the nature of these oscillations.

We thank É. A. Kaner, N. M. Makarov, and V. A. Yampol'skiĭ for communicating their results[2,3] prior to publication.

[1)]Even for the case of an isotropic Fermi surface, no rigorous solution is available for the nonlinear resistance of bounded plates; the formulas and estimates given below are therefore only qualitative.

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