Perturbations of a rarefied gas by a resonant optical field

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It was shown that the effects of resonant optical pressure created by a biharmonic field induce undamped coherent perturbations of the Wigner distribution function in a rarefied collisionless gas and these perturbations are manifested in particular by a traveling periodic density wave propagating across the gas. When one of the fields is weak, coherent structures may appear when the parametric resonance condition is satisfied, i.e., when the difference between the frequencies of the two fields is matched in a particular way to the amplitude of the strong field. A spatial grating of the density appears also in echo regimes representing the response of the medium to a sequence of radiation pulses separated on the time scale. The processes analyzed have a long phase memory not limited by radiative relaxation. This is due to parametric phenomena during the action of radiation and after such action the memory of the radiation is retained directly in the distribution function perturbed by recoil effects.

1. INTRODUCTION

The coherent kinetics of a medium in a resonant electromagnetic field is manifested most clearly in such thoroughly investigated phenomena as the decay of free polarization, optical nutation, or photon echo,[1] which are associated with the coherent volume excitation of the macroscopic polarization of an ensemble of particles. It is interesting to consider the possibility of coherent optical perturbation of translational degrees of freedom of a large ensemble of neutral noninteracting particles, which may be manifested by—for example—matched periodic pulsations (in space and time) of the density, directional motion, etc. The methods of resonant radiation pressure[2,3] provide a range of ways for manipulating neutral atoms and, in principle, they can be used to induce coherent structures in a rarefied gas if the transfer of momentum from radiation to the gas is organized in a suitable manner. For example, it is reported in Ref. 4 that coherent beams of particles can be created in separate standing waves by the Kapitza-Dirac resonance effect[5] which produces periodic gratings in the distribution of atoms under spatial echo conditions.

We shall show that coherent perturbations of a rarefied resonant gaseous medium can be induced by parametric processes in two-level atoms excited by a biharmonic electromagnetic field. Such perturbations give rise to temporal echo processes in the amplitudes of the resultant periodic spatial density gratings. The phase-memory time is then not limited by radiative relaxation.

We shall consider a collisionless gas of two-level particles interacting with a quasiresonant biharmonic (bichromatic) field in which the main components are a strong plane wave and a weak wave with a shifted frequency and active as the controlling component. The action on a rarefied gas of one high-power monochromatic traveling wave which saturates a spectral line profile ($dE_0/\hbar \gg ku_0, \gamma_\perp$) is relatively simple after the end of transient processes. In the quasiclassical limit when the photon momentum is considerably less than the thermal momentum in an ensemble of atoms ($\hbar k \ll p_0$) it is found that after times exceeding the spontaneous relaxation time $t \gg \gamma^{-1}$ the action of such a monochromatic field reduces to a directional drift of particles induced by the resonant radiation pressure force $F_0 \approx \hbar k \gamma/2$ complicated by quantum fluctuations that result in slow heating of the gas.[2] We shall show that the situation changes radically when the main field is supplemented by a component shifted on the frequency scale, because parametric processes[6] then create undamped beats of the amplitudes of the polarization and population difference and, consequently, of the directional momentum.[1)] In general, the amplitude of these oscillations is low ($p_1 \ll \hbar k$) and it is manifested in higher orders when the density matrix is expanded in reciprocal powers of the strong field. However, for certain values of the intermode interval in such a two-level system we can expect a parametric Rabi resonance[7,8] (this term was introduced in Ref. 7) in which the amplitudes of the harmonics of elements of the density matrix reach their extremal values. In the problem under consideration a parametric resonance of the amplitude of oscillations of the directional momentum in an ensemble of particles occurs when the frequency difference between the interacting fields is equal to the frequency of nutations in the strong field. An important feature is the dependence of the resultant perturbation of the Wigner distribution function $F = \mathrm{Tr}(\rho)$ on the phases of the interaction fields, so that for $\gamma t \gg 1$ a coherent structure is established in the form of a traveling periodic wave with a slowly varying amplitude. The main consequence of this perturbation $\tilde{f}$ is the existence of a nontrivial first-order moment representing a traveling wave moving across the gas:

$$\mathbf{u} = (\mathbf{p}_1/M) \sin(\delta\omega t - \mathbf{QR} + \Phi), \tag{1}$$

where $p_1 \sim \hbar k$; $\delta\omega$ is the detuning between the field frequencies; M is the atomic mass. Similar oscillations of the spatial density are higher-order effects found on expansion in terms of reciprocal powers of the strong field during the action of radiation. However, in the case when the radiation (field) is removed abruptly, perturbation waves similar to the Van Kampen waves in the plasma echo theory[9–11] are generated:

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$$\tilde{f}=f_1 \exp j\left(\mathbf{Q}_1\mathbf{R}-\frac{\mathbf{Q}_1\mathbf{p}}{M}t\right)+\text{c.c.}, \quad (2)$$

which give rise to periodic spatial density oscillations that are damped out as a result of mixing of the phases. Information on the initial perturbation is nevertheless retained in the distribution and can be retrieved by the application of a second radiation pulse: it is then manifested by echo oscillations of the density due to the interaction of the perturbations induced by the first and second pulses. In this sense the resultant two-pulse echo can be regarded as belonging to an overall system of all known forms of the echo phenomena,[12] which are possible if the dephasing is not irreversible and the nonlinearity reverses the evolution of the oscillation phases.

2. INITIAL EQUATIONS. MATHEMATICAL MODEL

We shall assume that a rarefied gas of two-level atoms with the frequency of a quantum transition ω_{21} is in a resonant bichromatic field of the type

$$\mathbf{E}=\sum_{m=0}^{1}\mathbf{E}_m \exp j(\mathbf{k}_m\mathbf{R}-\omega_m t)+\text{c.c.} \quad (3)$$

In accordance with the comments in the Introduction, the partial amplitudes of the fields obey the inequality

$$|\mathbf{E}_0|\gg|\mathbf{E}_1|. \quad (4)$$

We shall write down the kinetic equations for the density matrix considered in the Wigner approximation $\hat{\rho}(\mathbf{p}, \mathbf{R}, t)$ (see, for example, Ref. 6) using the resonance approximation and assuming a dipole interaction of the atoms in the gas with the field:

$$\frac{\partial Z}{\partial t}+\frac{\mathbf{p}}{M}\frac{\partial Z}{\partial \mathbf{R}}=\frac{1}{\mu}(\hat{A}_0+j\hat{\Delta})Z+(\hat{\Gamma}+\hat{A}_1)Z, \quad (5)$$

where Z is a column vector[2)]:

$$Z=\text{col}(\rho_{21}, \rho_{12}, q, f),$$

$$q=(\hat{\rho}_{22}-\hat{\rho}_{11}), \quad f=\text{Sp}(\hat{\rho}),$$

$$\rho_{21}=\hat{\rho}_{21}\exp(j\omega_0 t), \quad \rho_{12}=\hat{\rho}_{12}\exp(-j\omega_0 t) \quad (6)$$

and the following operator matrices are introduced ($m = 1, 2$):

$$\hat{A}_m=\begin{vmatrix} 0 & D_m \\ D_m' & 0 \end{vmatrix},$$

$$D_m=\begin{vmatrix} -j(\bar{\Omega}_m/2)(\hat{T}_+^{(m)}+\hat{T}_-^{(m)}) & -j(\bar{\Omega}_m/2)(\hat{T}_+^{(m)}-\hat{T}_-^{(m)}) \\ j(\bar{\Omega}_m^*/2)(\hat{T}_+^{(m)}+T_-^{(m)}) & j(\bar{\Omega}_m^*/2)(\hat{T}_+^{(m)}-\hat{T}_-^{(m)}) \end{vmatrix},$$

$$D_m'=\begin{vmatrix} -j\bar{\Omega}_m^*(\hat{T}_+^{(m)}+\hat{T}_-^{(m)}) & j\bar{\Omega}_m(\hat{T}_+^{(m)}+\hat{T}_-^{(m)}) \\ j\bar{\Omega}_m^*(\hat{T}_+^{(m)}-\hat{T}_-^{(m)}) & -j\bar{\Omega}_m(\hat{T}_+^{(m)}-\hat{T}_-^{(m)}) \end{vmatrix}. \quad (7)$$

The square matrices A_m and D_m have the dimensions (4×4) and (2×2), respectively, and they are written down introducing momentum shift operators in accordance with the rule

$$\hat{T}_\pm^{(m)}u(\mathbf{p})=u(\mathbf{p}\pm\hbar\mathbf{k}/2), \quad (8)$$

as well as quantities $\bar{\Omega}_m$ described by the relations

$$\bar{\Omega}_m=\chi_m \exp j(\mathbf{k}_m\mathbf{R}), \quad \chi_m=\mathbf{d}\mathbf{E}_m/\hbar,$$
$$\bar{\Omega}_0=(\bar{\Omega}_0/\mu), \quad \bar{\Omega}_1=\bar{\Omega}_1 \exp(j\delta\omega t), \quad (9)$$

where d is the matrix element of the dipole moment; μ is a formally small parameter ($\mu\ll1$), the meaning of which will be explained later; $\delta\omega$ is the frequency difference between the two fields, which we shall represent by a sum of "rough" and "fine" detuning:

$$\delta\omega=\omega_0-\omega_1=\Delta_0/\mu+\delta. \quad (10)$$

Moreover, Eq. (5) contains a diagonal matrix governed by the frequency shift of the strong field

$$\hat{\Delta}=\text{diag}(-\Delta', \Delta', 0, 0), \quad \Delta'/\mu=-\Delta=\omega_{21}-\omega_0 \quad (11)$$

as well as the spontaneous relaxation matrix

$$\hat{\Gamma}=\begin{vmatrix} \hat{\gamma}_0 & 0 \\ 0 & \hat{\gamma}_1 \end{vmatrix}, \quad \hat{\gamma}_0=\text{diag}(-\gamma_\perp, -\gamma_\perp),$$

$$\hat{\gamma}_1=\begin{vmatrix} -\gamma+\hat{R}_0 & -\gamma+R_0 \\ -R_0 & -\hat{R}_0 \end{vmatrix}, \quad (12)$$

where γ and $\gamma_\perp$ are the longitudinal and transverse relaxation rates: $\hat{R}_0$ is an integral operator defined in terms of the probability of spontaneous emission of photons in the direction of n (Refs. 2 and 3):

$$\hat{R}_0 u(\mathbf{p})={}^1/_2\gamma\left[u(\mathbf{p})-\int u(\mathbf{p}+\hbar k\mathbf{n})\Phi(\mathbf{n})d\mathbf{n}\right], \quad k=\omega_{21}/c. \quad (13)$$

The method of introduction of the dimensionless parameter $\mu\ll1$ corresponds to the inequality (4) and to the following relationships between the principal physical parameters of the problem[3)]:

$$|\bar{\Omega}_0|\sim|\Delta|\sim|\delta\omega|, \quad \gamma, \gamma_\perp, k_0(p_0/M)\ll|\bar{\Omega}_0|. \quad (14)$$

Therefore, in the initial formulation the Rabi frequency of the strong field is comparable with the frequency shift and intermode interval, but is considerably greater than the rates of the processes due to relaxation, controlling field, and motion of atoms. An allowance for all these circumstances makes it possible to consider a parametric resonance.

We shall ignore the reaction of the resonant medium on the fields by assuming that the gas is sufficiently rarefied.

3. ASYMPTOTIC EXPANSION OF THE DENSITY MATRIX

We shall now specify the nature of a pulse of the strong component of the resonant radiation assuming that this radiation is applied instantaneously:

$$\mathbf{E}_0=\mathbf{E}_0(t-\mathbf{n}_0\mathbf{R}/c), \quad \mathbf{n}_0=\mathbf{k}_0/|\mathbf{k}_0|,$$
$$\mathbf{E}_0(t)=0, \quad t<0. \quad (15)$$

We shall also assume that the state Z_0 of the gas is specified at the instant $\mathbf{n}_0\cdot\mathbf{R}/c=t$ of arrival of the strong field pulse at the plane. We shall introduce a delayed time $t_1=t-\mathbf{n}_0\mathbf{R}/c$ and the Fourier transformation in respect of the variable p in order to remove the finite differences in the initial equation (5):

$$\bar{Z}(\boldsymbol{\sigma},\mathbf{R},t)=\hat{F}_{p\to\sigma}(Z)=\int Z\exp[j(\boldsymbol{\sigma}\mathbf{p})]d^3p,$$
$$Z(\mathbf{p},\mathbf{R},t)=\hat{F}^{-1}_{\sigma\to p}(Z)=\int\frac{1}{(2\pi)^3}\bar{Z}\exp[-j(\boldsymbol{\sigma}\mathbf{p})]d^3\sigma. \tag{16}$$

Z satisfies then the following evolution equation:

$$\mu\left(\frac{\partial}{\partial t_1}+\frac{j}{Mc}\mathbf{n}_0\frac{\partial^2}{\partial\boldsymbol{\sigma}\partial t_1}-\frac{j}{M}\operatorname{div}_R\operatorname{grad}_\sigma\right)Z$$
$$=(\bar{A}_0+j\hat{\Delta})Z+\mu(\bar{\Gamma}+A_1)Z,$$
$$\bar{Z}|_{t_1=0}=\bar{Z}_0=\hat{F}_{p\to\sigma}(Z_0), \tag{17}$$

where the matrices $\bar{A}_m$ and $\bar{\Gamma}$ are obtained from the matrices $\hat{A}_m$ and $\hat{\Gamma}$ by transforming to the delay time and replacing the difference operators with the functions

$$(\hat{T}^{(m)}_{+}+T^{(m)}_{-})\to 2\cos(\hbar\mathbf{k}_m\boldsymbol{\sigma}/2),$$
$$(\hat{T}^{(m)}_{+}-\hat{T}^{(m)}_{-})\to -2j\sin(\hbar\mathbf{k}_m\boldsymbol{\sigma}/2), \tag{18}$$
$$\hat{R}_0\to R_0(\boldsymbol{\sigma})={}^1/_2\gamma(1-R_1(\boldsymbol{\sigma})),$$
$$R_1(\boldsymbol{\sigma})=\int\exp[-\hbar k(\mathbf{n}\boldsymbol{\sigma})]\Phi(\mathbf{n})d\mathbf{n}.$$

The small parameter μ occurs in Eqs. (5) and (17) in a singular manner and it is responsible for the fast motion of the system with a characteristic time scale $\sim\mu/|\bar{\Omega}_0|$, which is considerably less than the times of the processes induced by the weak field, relaxation, Doppler frequency shift, and recoil effect. We shall separate the fast and slow components of the density matrix by the method of multiscale asymptotic expansions (see, for example, Ref. 13). With this in mind we shall introduce fast variables describing a singular dependence of the solution on μ and related to the spectrum in the limiting case $\mu=0$ (regularization of the spectrum of the limiting operator[14]):

$$t_2=\frac{j}{\mu}\int_0^{t_1}\Delta_0dt_1,\quad t_{3,4}=\frac{1}{\mu}\int_0^{t_1}\lambda_{3,4}dt_1, \tag{19}$$

where $\lambda_{3,4}$ are nonzero eigenvalues of the matrix $(\bar{A}_0+j\hat{\Delta})=B$, given by

$$\lambda_{3,4}=\pm jG=\pm j(\Delta'^2+4|\bar{\Omega}_0|^2)^{1/2},$$

and the variable t_2 allows for the rapidly oscillating time dependences of the coefficients in Eq. (17). A solution of the more general problem described in terms of the variables t_i

$$\mu\frac{\partial\bar{Z}}{\partial t_1}+\hat{D}\bar{Z}=B\bar{Z}+\mu\hat{B}_1\bar{Z},\quad \bar{Z}|_{t_i=0}=\bar{Z}_0,$$
$$\hat{D}=\sum_{i=3}^{4}\lambda_i(t_1)\frac{\partial}{\partial t_i}+j\Delta_0\frac{\partial}{\partial t_2}, \tag{20}$$
$$\hat{B}_1=(\bar{\Gamma}+\bar{A}_1)-\frac{j}{M}\left(\mathbf{n}_0\frac{\partial^2}{\partial\boldsymbol{\sigma}\partial t_1c}-\operatorname{div}_R\operatorname{grad}_\sigma\right),$$

can be sought in the form of a regular asymptotic expansion

$$\bar{Z}=\bar{Z}^{(0)}+\mu\bar{Z}^{(1)}+\dots. \tag{21}$$

We then obtain a sequence of linear problems

$$(\hat{D}-B)\bar{Z}^{(0)}=0,\quad \bar{Z}^{(0)}|_{t_i=0}=\bar{Z}_0,$$
$$(\hat{D}-B)\bar{Z}^{(1)}=(\hat{B}_1-\partial/\partial t_1)Z^0,\quad \bar{Z}^{(1)}|_{t_i=0}=0,$$
$$\dots\dots\dots\dots\dots\dots \tag{22}$$
$$(\hat{D}-B)\bar{Z}^{(n)}=(\hat{B}_1-\partial/\partial t_1)Z^{(n-1)},\quad \bar{Z}^{(n)}|_{t_i=0}=0.$$

In accordance with the comments made in the Introduction, the greatest interest lies in the case of a parametric resonance when the beat frequency $\delta\omega$ is tuned to a resonance with the nutation frequency in the strong field:[4)]

$$\Delta_0=G. \tag{23}$$

Equation (23) leads to resonance relationships which, introducing arbitrarily a variable $t_0=0$, can be written in the form

$$mt_2(t_1)+t_j(t_1)=t_i(t_1),\quad i\neq j, \tag{24}$$

where the indices j and i assume the values 0, 3, 4, and m (integer) and correspond to the resonance sets $m\,(j,i)$: 1 (4, 0); -1 (3, 0); 1 (0, 3); -1 (0, 4); 2 (4, 3); -2 (3, 4). The Fredholm alternative[15] ensures solubility of each of the problems described by Eq. (22) in the form of linear combinations of exponental functions of fast variables (mt_2+t_j) provided the arguments of the resonance components on the right-hand sides are replaced in accordance with Eq. (24) (this procedure makes it possible to avoid the appearance of diverging terms on return to the initial time variable). The conditions of solubility of the problem in the nth approximation $(n\geqslant 1)$ eliminate the indeterminacy of the solution of the previous $(n-1)$th approximation, found with an accuracy of a solution of the corresponding homogeneous problem that includes four unknown functions of the slow variable t_1.

A general solution of the problem in the zeroth approximation [corresponding to the first equation in the system (22)] is

$$\bar{Z}^{(0)}=\bar{\alpha}_1\varphi_1+\bar{\alpha}_2\varphi_2+\bar{\alpha}_3\varphi_3e^{t_3}+\bar{\alpha}_4\varphi_4e^{t_4}. \tag{25}$$

Here $\varphi_i(\boldsymbol{\sigma},\mathbf{R},t_1)$ are linearly independent eigenvectors of the matrix $B=(\bar{A}_0+j\hat{\Delta})$, the explicit form of which is given in the Appendix; $\alpha_i(\boldsymbol{\sigma},\mathbf{R},t_1)$ are scalar functions describing slowly varying components of the density matrix. They can be determined by considering the first approximation

$$(\hat{D}-B)Z^{(1)}=\sum_{i=0,3,4}Y_je^{t_j}+X_2e^{t_2+t_3}+X_{-2}e^{-t_2+t_4}, \tag{26}$$

where the right-hand side is formed from the column vector $(B_1-\partial/\partial t_1)Z^{(0)}$ allowing for the resonance relationships (24) and $Y_i\exp t_i$ includes a sum of all the components corresponding to all possible sets of the $\pm 1(i,j)$ type. Solubility of the problem (26) is ensured by the orthogonality conditions:

$$\langle b_k,Y_0\rangle=\sum_{i=1}^{4}b_{ki}{}^*Y_{0i}=0,\quad k=1,2,$$
$$\langle b_k,Y_k\rangle=0,\quad k=3,4. \tag{27}$$

Linearly independent eigenvectors b_k, where $k=1$-4, of the conjugate matrix B^+ (see the Appendix) correspond to the eigenvalues $\bar{\lambda}_k$: $\bar{\lambda}_{1,2}=0$, $\bar{\lambda}_3=jG$, $\bar{\lambda}_4=+jG$. Bearing in mind the structure of the operator B_1, we can readily see that

Eq. (27) describes four differental evolution equations for unknown functions α_i (i = 1-4) with the initial conditions governed, in the case when the $\{\varphi_m, b_n\}$ system vectors is selected, by biorthogonal vectors ($\langle n_b, \varphi_m\rangle \propto \delta_{mn}$) in accordance with the relationships that follow from Eq. (25):

$$\bar{\alpha}_k|_{t_1=0}=\langle b_k, \bar{Z}_0\rangle/\langle b_k\varphi_k\rangle=\bar{\alpha}_{k0}. \tag{28}$$

The higher approximations are obtained in the same way. A series found for

$$t_i=\frac{1}{\mu}\int_0^{t_1}\lambda_i dt_1, \quad i=2-4$$

describes an asymptotic expansion of the density matrix in terms of the small parameter:

$$Z=\hat{F}^{-1}_{\sigma\to p}(\bar{Z}^{(0)})+\mu\hat{F}^{-1}_{\sigma\to p}(\bar{Z}^{(1)})+\dots \,. \tag{29}$$

4. COHERENT QUASISTATIONARY PERTURBATIONS OF THE WIGNER DISTRIBUTION OF ATOMS

We shall show that under the parametric resonance conditions in a system of translational degrees of freedom of an ensemble of atoms we can expect coherent perturbations which are not affected by radiative spontaneous relaxation processes. We shall consider only the zeroth approximation in μ and write down the explicit equations corresponding to Eq. (27) and describing the evolution of the slow variables $\bar{\alpha}_j$ in the case when $p_0/Mc \ll \hbar k/p_0$:

$$\left(\frac{\tilde{d}}{dt_1}-\frac{\hbar\mathbf{k}_0\Delta'^2}{2MG^2}\frac{\partial}{\partial\mathbf{R}}\right)\bar{\alpha}_1+\frac{j}{M}\frac{\hbar\mathbf{k}_0\Delta'|\bar{\Omega}_0|}{2G^2}\frac{\partial\bar{\alpha}_2}{\partial\mathbf{R}}$$
$$=-\gamma_1(\boldsymbol{\sigma})\bar{\alpha}_1-\frac{\Delta' j}{4|\bar{\Omega}_0|}\gamma_2(\boldsymbol{\sigma})\bar{\alpha}_2$$
$$+j\left[\cos\beta_1-j\frac{\Delta'}{G}\sin\beta_1\right][G_1{}^*\tilde{\alpha}_3-G_1\tilde{\alpha}_4],$$

$$\left(\frac{\tilde{d}}{dt_1}+\frac{\hbar\mathbf{k}_0\Delta'^2}{2MG^2}\frac{\partial}{\partial\mathbf{R}}\right)\alpha_2-\frac{2j}{M}\frac{\hbar\mathbf{k}_0\Delta'|\bar{\Omega}_0|}{G^2}\frac{\partial\alpha_1}{\partial\mathbf{R}}$$
$$=\frac{4|\bar{\Omega}_0|}{G}j\sin\beta_1(G_1{}^*\tilde{\alpha}_3-G_1\tilde{\alpha}_4)+\gamma[1-R_1(\sigma)\exp(j\hbar\mathbf{k}_0\boldsymbol{\sigma})]$$
$$\times\left[\frac{2j|\bar{\Omega}_0|\Delta'}{G^2}\bar{\alpha}_1-\frac{1}{2}\left(1+\frac{\Delta'^2}{G^2}\right)\bar{\alpha}_2\right],$$

$$\left(\frac{\tilde{d}}{dt_1}+j\delta-\mathbf{Q}_0\frac{\partial}{M\partial\boldsymbol{\sigma}}\right)\tilde{\alpha}_3=-\gamma_+(\boldsymbol{\sigma})\tilde{\alpha}_3+\frac{1}{2}jG_1\bar{\alpha}_1\cos\beta_1$$
$$-\frac{G_1G}{8|\bar{\Omega}_0|}\left(\frac{\Delta'}{G}\cos\beta_1-j\sin\beta\right)\bar{\alpha}_2, \tag{30}$$

$$\left(\frac{\tilde{d}}{dt_1}-j\delta+\mathbf{Q}_0\frac{\partial}{M\partial\boldsymbol{\sigma}}\right)\tilde{\alpha}_4=-\gamma_+(\boldsymbol{\sigma})\tilde{\alpha}_4-\frac{1}{2}jG_1{}^*\bar{\alpha}_1\cos\beta_1$$
$$+\frac{G_1{}^*G}{8|\bar{\Omega}_0|}\left(\frac{\Delta'}{G}\cos\beta_1-j\sin\beta_1\right)\bar{\alpha}_2,$$

where

$$\beta_1=\frac{\hbar\mathbf{Q}\boldsymbol{\sigma}}{2}, \quad \mathbf{Q}=\mathbf{k}_0-\mathbf{k}_1, \quad \frac{\tilde{d}}{dt_1}=\frac{\partial}{\partial t_1}-\frac{j}{M}\operatorname{div}_R\operatorname{grad}_\sigma,$$
$$\mathbf{Q}_1=\frac{\omega_1}{\omega_0}\mathbf{k}_0-\mathbf{k}_1, \quad \tilde{\alpha}_{3,4}=\bar{\alpha}_{3,4}\exp[\mp j(\delta t_1-\mathbf{Q}_1\mathbf{R})],$$

$$G_1=\frac{\chi_1\chi_0{}^*}{|\chi_0|}\left(1-\frac{\Delta'}{G}\right), \quad \mathbf{Q}_0=\mathbf{Q}_1+\mathbf{k}_0\frac{\Delta'}{G},$$
$$\gamma_1(\boldsymbol{\sigma})=\frac{\gamma_\perp}{a_0{}^2}+\frac{R_1(\boldsymbol{\sigma})\gamma\Delta'^2}{G^2}\exp(j\hbar\mathbf{k}_0\boldsymbol{\sigma}), \quad a_0{}^2=\frac{G^2}{4|\bar{\Omega}_0|^2},$$
$$\gamma_2(\boldsymbol{\sigma})=\frac{\gamma_\perp}{a_0{}^2}+\left(1+\frac{\Delta'^2}{G^2}\right)\gamma R_1(\boldsymbol{\sigma})\exp(j\hbar\mathbf{k}_0\boldsymbol{\sigma}),$$
$$\gamma_+(\boldsymbol{\sigma})=\frac{\gamma_\perp}{2}\left(1+\frac{\Delta'^2}{G^2}\right)+\frac{\gamma|\bar{\Omega}_0|^2R_1(\boldsymbol{\sigma})}{G^2}\exp(j\hbar\mathbf{k}_0\boldsymbol{\sigma}).$$

In the usual situation the de Broglie wavelengths of the particles composing a gas are considerably less than the radiation wavelength ($\hbar k \ll p_0$). It is well known that in this case the evolution of the distribution of the particles acted upon by the resonant radiation pressure is described in a wide range of realistic conditions by the Fokker-Planck equation. This reduction of the problem to the Fokker-Planck equation and its subsequent solutions have been considered in several papers[1,2,16–20] and the reduction procedure itself has been analyzed from various points of view (the most detailed analysis can be found in, for example, Refs. 16 and 18). The system of equations for slow motion (30) can also be simplified similarly for $\hbar k \ll p_0$ on the assumption that the spontaneous relaxation time $\tau_s \sim \gamma^{-1}$ and the characteristic time of changes in the dependence differ greatly from the duration of the pulses of the slow components of the density matrix $\tau_p \sim (\hbar k^2/2M)^{-1} = \omega_R^{-1}$. We shall use the inverse Fourier transformation to introduce the functions

$$\alpha_m=\hat{F}^{-1}_{\sigma\to p}\bar{\alpha}_m, \quad m=1,2; \quad \alpha_n=F^{-1}_{\sigma\to p}\tilde{\alpha}_n, \quad n=3,\,4,$$

and we shall employ the differential approximation for the finite differences [in the system (30) this is equivalent to expansion of the right-side in terms of σ (compare with Eq. (18)]. Then α_m and α_n satisfy a system of equations obtained from the system (30) by the substitutions

$$\bar{\alpha}_m\to\alpha_m, \quad \tilde{\alpha}_n\to\alpha_n, \quad \sin(\beta_1)\to j\frac{\hbar\mathbf{Q}}{2}\frac{\partial}{\partial\mathbf{p}}, \quad \cos(\beta_1)\to 1,$$

$$\partial/\partial\boldsymbol{\sigma}\to j\mathbf{p}, \quad \gamma_a(\sigma)\to\tilde{\gamma}_a-\gamma_a{}'\hbar\mathbf{k}_0\partial/\partial\mathbf{p}, \quad \tilde{\gamma}_a=\gamma_a(0),$$
$$\gamma_1{}'=\gamma\frac{\Delta'^2}{G^2}, \quad \gamma_2{}'=\gamma\left[1+\frac{\Delta'^2}{G^2}\right], \quad \gamma_+{}'=\gamma\frac{|\bar{\Omega}_0|^2}{G^2}, \tag{31}$$
$$1-R_1(\sigma)\exp(j\hbar\mathbf{k}_0\boldsymbol{\sigma})\to\hbar\mathbf{k}_0\frac{\partial}{\partial\mathbf{p}}-\frac{(\hbar k)^2}{2}\sum_{i,j}m_{ij}\frac{\partial^2}{\partial p_i\,\partial p_j},$$

$$m_{ij}=\int n_i n_j\Phi(\mathbf{n})\,d\mathbf{n}.$$

Then, eliminating α_1, α_3, and α_4 in the same way as is done in the case of the component of the Bloch vector (see, for example, Refs. 16–18), we obtain the Fokker-Planck equation for $\alpha_2(\mathbf{p}, \mathbf{R}, t_1)$ in agreement with the physical meaning of this variable which is proportional to a sum of the Wigner distributions with shifted momenta in the states 1 and 2:

$$\alpha_2\propto\left[\rho_{22}\left(\mathbf{p}+\frac{\hbar\mathbf{k}_0}{2}\right)+\rho_{11}\left(\mathbf{p}-\frac{\hbar\mathbf{k}_0}{2}\right)\right]+O(\mu).$$

In the case of a weak spatial inhomogeneity

$$\left|\frac{1}{\alpha_2}\frac{\partial\alpha_2}{\partial R}\frac{p}{M}\right|\ll\gamma$$

this equation becomes

$$\frac{\partial\alpha_2}{\partial t_1}+\left(\frac{\mathbf{p}}{M}-\mathbf{v}_R\right)\frac{\partial\alpha_2}{\partial\mathbf{R}}+\frac{\partial}{\partial\mathbf{p}}(\mathbf{F}_0\alpha_2)=\sum_{i,j}\frac{\partial^2}{\partial p_i\,\partial p_j}(D_p{}^{ij}\alpha_2),$$

$$\alpha_2|_{t_1=0}=\hat{F}^{-1}_{\sigma\to p}(\bar{\alpha}_{20})=\alpha_{20}, \tag{32}$$

where the shift coefficient $\mathbf{F}_0$ acts as the force due to the spontaneous resonant radiation pressure, D_p^{ij} are the diffusion coefficients due to fluctuations of the momentum and

$$\mathbf{v}_R=\frac{\hbar\mathbf{k}_0\Delta'^2}{2MG^2}\frac{\gamma}{\tilde{\gamma}_1+|G_1|^2\mathscr{L}},$$

$$\mathscr{L}=\frac{\tilde{\gamma}_+}{\tilde{\gamma}_+{}^2+\tilde{\delta}^2},\qquad \tilde{\delta}=\delta-\mathbf{Q}_0\frac{\mathbf{p}}{M}. \tag{33}$$

It should be noted that in allowing for the influence of transient processes we can correct α_2 by an exponentially rapidly decaying (in a time $\sim\gamma^{-1}$) function $\alpha_2\sim O(\hbar k/p_0)$, so that the initial condition of the system (32) for α_2 is simplified by omitting an unimportant small term $\hbar k/p_0\ll 1$.

If $t_1\gg\gamma^{-1}$ and T_{n_1}, where the latter is the characteristic time of a possible delay of the arrival of the weak-field pulse in the region of the gas under consideration, the variables α_1, α_3, and α_4 follow adiabatically the changes in α_2 (we are now assuming that the amplitude of the controlling weak field behind the leading edge of a pulse is stationary):

$$\alpha_m=M_{m2}^{(0)}\alpha_2,\qquad m=1,3,4,$$

$$M_{12}^{(0)}=-j\frac{\Delta'}{4|\Omega_0|}(1+\gamma\bar{y}),\qquad \bar{y}=(\tilde{\gamma}_1+|G_1|^2\mathscr{L})^{-1},$$

$$M_{42}^{(0)}=-\frac{\Delta'}{8|\Omega_0|}L_1{}^*G_1{}^*\gamma\bar{y}, \tag{34}$$

$$M_{32}^{(0)}=-M_{42}^{(0)*},\qquad L_1=\frac{1}{\tilde{\gamma}+j\tilde{\delta}}.$$

The structure of the dependences of F_0 and D_p^{ij} on the momenta and their magnitudes $F_0\sim\hbar k\gamma$ and $D_p\sim(\hbar k)^2\gamma$ are such that the time evolution of the variable α_2 ($\mathbf{p}$, $\mathbf{R}$, t_1) is characterized by all the principal elements of the evolution of the distribution in the case when a single traveling wave acts on a rarefied gas[19,20]: the drift of the momentum space, monochromatization of the pulses occurring at times $t_1\gtrsim\tau_p\sim\omega_R^{-1}$, and diffusion spreading of the pulses which appear at $t_1\gg\tau_p$.

We shall consider the specific case of unidirectional (along the x axis) waves of the same polarization when we can ignore the difference between the photon momenta $|\hbar\mathbf{k}_1-\hbar\mathbf{k}_0|/\hbar k\ll 1$:

$$D_p{}^{yy}=2D_p{}^{zz}=\frac{(\hbar k)^2}{10}\gamma\left[1-\frac{\Delta'^2\gamma\bar{y}}{G^2}\right],$$

$$D_p{}^{xx}=\frac{(\hbar k)^2}{4}\gamma\left[\frac{7}{5}-\frac{7}{5}\frac{\Delta'^2\gamma\bar{y}}{G^2}-\frac{\gamma\Delta'^2\bar{y}^2\bar{y}_1}{G^2}\right], \tag{35}$$

$$\bar{y}_1=\left[\mathscr{L}^2|G_1|^2\gamma\left(1+\frac{2|\Omega_0|^2-\gamma\bar{y}\Delta'^2}{G^2}\right)\left(1-\frac{\tilde{\delta}^2}{\tilde{\gamma}_+{}^2}\right)\right.$$

$$\left.+\bar{y}\left(\left(\frac{\gamma_\perp}{a_0{}^2}\right)+|G_1|^2\mathscr{L}\right)\left(\frac{\gamma_\perp-\gamma}{a_0{}^2}+|G_1|^2\mathscr{L}\right)\right],$$

$$\mathbf{F}_0=\frac{\hbar\mathbf{k}_0\gamma}{2}\left[1-\frac{\Delta'^2}{G^2}\gamma\bar{y}\right].$$

(The expression for $\mathbf{F}_0$ is simplified by dropping a small correction $\hbar k/p_0$ due to the dependence of the diffusion coefficient on the momentum.)

The most important circumstance is that after a long time $t>\gamma^{-1}$ the dynamics of the translational degrees of freedom is not limited to the slow evolution described by the Fokker-Planck approximation of Eqs. (32) and (34), because the Wigner distribution function has fast coherent components. In fact, using Eqs. (29), (25), (A.1), (34), and (8), we obtain the following expression which describes the distribution during the quasisteady stage of the evolution:

$$f=\bar{f}+\tilde{f}+O(\mu), \tag{36a}$$

$$\bar{f}(\mathbf{p},\mathbf{R},t)=a_0{}^2\left[(\hat{T}_+^{(0)}-\hat{T}_-^{(0)})M_{12}\alpha_2-\frac{j}{a_0{}^2}(\hat{T}_+^{(0)}+\hat{T}_-^{(0)})\alpha_2\right.$$

$$\left.-j\frac{\Delta'^2}{G^2}\hat{T}_-^{(0)}\alpha_2\right], \tag{36b}$$

$$\tilde{f}(\mathbf{p},\mathbf{R},t)=a_0{}^2[(\hat{T}_-^{(0)}-\hat{T}_+^{(0)})(M_{32}\alpha_2)e^{j(\delta\omega t-\mathbf{QR})}+\text{c.c.}], \tag{36c}$$

where

$$M_{12}=\frac{2\Delta'|\Omega_0|}{G^2}M_{12}^{(0)},\qquad M_{i2}=\frac{M_{i2}^{(0)}}{a_0{}^2},\qquad i=3,4.$$

Therefore, after the passage of the leading edges of the pulses of the resonant radiation at times $(t-\mathbf{n}_i\cdot\mathbf{R}/c)\gg\gamma^{-1}$, T_{n_1}, the Wigner distribution of the gas includes a coherent component representing a periodic perturbation wave with a slowly varying amplitude, a wave vector $\mathbf{Q}=\mathbf{k}_0-\mathbf{k}_1$, and a frequency $\delta\omega=\omega_0-\omega_1$. The phase velocity of the wave c_{ph} varies, depending on the geometry of the resonant fields, between $c_{ph}\approx c$ (for unidirectional waves with $\mathbf{n}_0=\mathbf{n}_1$) to $c_{ph}=c\delta\omega/(\omega_0+\omega_1)\ll c$ (opposite waves $\mathbf{n}_1=-\mathbf{n}_0$). The evolution of the wave amplitude is a much slower process than the radiative relaxation processes and it is governed entirely by the dynamics of the variable $\alpha_2(\mathbf{p},\mathbf{R},t_1)$ calculated from the Fokker-Planck Eq. (32). We can readily see that the calculated coherent component $\tilde{f}$ is associated with a first-order moment which is not identically zero ("directional" or hydrodynamic velocity), which in the case of a sufficiently cold gas ($kp_0/M\ll\gamma$) can be written in the following form:

$$\tilde{\mathbf{u}}=\frac{\langle\tilde{f}\mathbf{p}/M\rangle_p}{\langle\bar{f}\rangle_p}=\frac{2\hbar\mathbf{k}_0}{M}\operatorname{Re}jM_{32}\exp(j(\delta\omega t-\mathbf{QR})),$$

$$\langle\ldots\rangle_p\equiv\int(\ldots)d^3p. \tag{37}$$

The zeroth-order moment of $\tilde{f}$ (density perturbation) is identically equal to zero: $\langle\tilde{f}\rangle_p=0$, so that a perturbation of the particle density appears in the first order of the expansion in μ. It can be found without calculating completely the value of $Z^{(1)}$, because linearization of the equation of continuity

$$\partial N/\partial t+\operatorname{div}(N\mathbf{u})=0,\qquad N=\langle f\rangle_p,\qquad \mathbf{u}=\langle\mathbf{p}f\rangle_p/MN,$$

which follows directly from Eq. (5), yields the following expression if we allow for Eqs. (37), (32), and (36), assume that the gas before irradiation is spatially homogeneous and has the density N_0, and ignore the edge and transient effects:

$$N=N_0+\mu\frac{2\hbar\mathbf{k}_0\mathbf{Q}}{MG}N_0\,\mathrm{Re}\,jM_{32}\exp j(\delta\omega t-\mathbf{QR})+O(\mu^2). \tag{38}$$

Consequently, a coherent perturbation of the density is also a harmonic traveling wave with wave characteristics and amplitude dependent on the relative orientations, amplitudes, and phases of the interacting electromagnetic fields.

We note in conclusion that the significance of Eq. (36c) for $\tilde{f}$ goes beyond the Fokker-Planck approximation and its structure remains the same in the case of an ultracold gas, when $p_0\sim\hbar k$ and the quasiclassical condition is no longer obeyed. Simplifications of the type represented by Eq. (31) are no longer possible because the differential approximation is inadmissible in the case of finite differences and the relationship between $\alpha_{3,4}$ and α_2 is strongly nonlocal, i.e., the quantity M_{32} in Eq. (36c) should be regarded as an integral operator

$$M_{32}\alpha_2\to\hat{M}_{32}\alpha_2=\hat{F}^{-1}_{\sigma\to p}(\bar{M}_{32}(\sigma)\,\hat{F}_{p\to\sigma}(\alpha_2)).$$

However, because of atom-momentum fluctuations that result in the heating of the gas, the evolution of this operator in the case when $(\gamma t)^{1/2}>1$ reaches rapidly a stage corresponding to the Fokker-Planck approximation.

5. ECHO EFFECTS

We shall show that a coherent response of a resonant gas to the action of biharmonic optical field pulse separated in time is manifested by spatial periodic pulsations of the gas (echo). These pulsations are associated with the leading term of the asymptotic expansion (29), and in contrast to the oscillations described by Eq. (38), represent a zeroth-order effect in respect of the parameter μ.

We shall assume that two exciting rectangular biharmonic field pulses of the type described by Eq. (3) enter a gas in succession and that

$$E_0(t_1)=\begin{cases}E_{01}, & 0\leqslant t_1<T_1,\\ 0, & T_1\ll t_1<(T_1+T),\quad t_1>(T_1+T+T_2),\\ E_{02}, & (T_1+T)\leqslant t_1<(T_1+T+T_2).\end{cases} \tag{39}$$

The delay between the arrival of the weak and strong field pulses at a given point in the gaseous medium can be ignored in the formulation of the initial conditions for Z [see Eq. (17)] provided the following inequalities are obeyed:

$$|\bar{\Omega}_1|T_d\ll1,\qquad \gamma T_d\ll1, \tag{40}$$

where T_d is the minimum possible delay time that depends on the radiation geometry. We shall also assume that the gas is sufficiently cold and that the diffusion of the pulses in space is not significant during the action of the radiation, i.e.,

$$\hbar k\ll p_0\ll(\gamma/k)M,\qquad \hbar k\gamma^{1/2}T_i\ll p_0. \tag{41}$$

These limitations are not of fundamental nature from the point of view of the physics of the investigated processes, but they do simplify greatly all the calculations. If we assume that before the irradiation with the optical field pulses the gas is unexcited and spatially homogeneous with a distribution $N_0f_0(\mathbf{p})$, ignore the effects of the first and higher orders in μ, use Eqs. (36), (32), (40), (41), and (5), and allow for the smallness of the thermal velocities of the particles compared with the velocity of light ($p_0/Mc\ll1$), we find that in the interval I ($\gamma^{-1}\ll t_1<T_1$), we now have

$$\alpha_2=\alpha_2^{(1)}\approx\frac{jN_0}{(a_0^{(1)})^2}f_0\left(\mathbf{p}-\mathbf{F}_0^{(1)}t_1-\frac{\hbar\mathbf{k}_0}{2}\right), \tag{42}$$

and the function $f^{(1)}(\mathbf{p},\mathbf{R},t_1)$ is defined by Eq. (36). In the interval II $[T_1\leqslant t_1<(T_1+T)]$, ignoring a small correction $\sim(\hbar k/p)^2$ due to the radiative relaxation after the passage of the first radiation pulse, we have

$$f=f^{(2)}(\mathbf{p},\mathbf{R},t_1)=f^{(1)}(\mathbf{p},\mathbf{R}-\mathbf{v}(t_1-T_1),T_1), \tag{43}$$

where $\mathbf{v}=\mathbf{p}/M$. Similarly, in the interval III $[(T_1+T+\gamma^{-1})\ll t_1<(T_1+T+T_2)]$, we find that

$$\alpha_2=\alpha_2^{(3)}(\mathbf{p},\mathbf{R},t_1)=\frac{j}{(a_0^{(3)})^2}f^{(2)}\Big(\mathbf{p}-\mathbf{F}_0^{(3)}[t_1-(T_1-T)]$$

$$-\frac{\hbar\mathbf{k}_0}{2};\quad \mathbf{R}-(\mathbf{v}-\mathbf{v}_R^{(3)})(t_1-[T_1+T]);T_1+T\Big)$$

and in the interval IV $(t_1\geqslant(T_1+T_2+T))$], the corresponding expression is

$$f=f^{(4)}(\mathbf{p},\mathbf{R},t)=f^{(3)}(\mathbf{p},\mathbf{R}-\mathbf{v}[t_1-(T_1+T+T_2)],T_1+T+T_2). \tag{44}$$

Here and henceforth a superscript in parentheses represents the number of the interval in which the function of the physical parameter is calculated.

The force $\mathbf{F}_0$ is given by the following expression in the case of an arbitrary orientation of the vectors $\mathbf{k}_0$ and $\mathbf{k}_1$:

$$\mathbf{F}_0=\frac{\hbar\mathbf{k}_0\gamma}{2}\left[1-\frac{\Delta'^2}{G^2}\gamma\bar{y}\right]+\frac{\hbar\mathbf{Q}\gamma}{2}\frac{\mathscr{L}|G_1|^2\Delta'\bar{y}}{G}. \tag{45}$$

Using Eqs. (42)–(44), we can readily calculate the density $N=\langle f\rangle_p$. In the interval I, we obviously have $N(\mathbf{R},t_1)=N_0$. In the interval II, we find that

$$N=N^{(2)}=N_0[1+4|H(t_1)|\sin(\mathbf{Q}_1^{(1)}\mathbf{R}-\Phi_1)],$$

$$H(t_1)=M_{32}^{(1)}\bar{f}_0\left(\frac{\mathbf{Q}_1^{(1)}}{M}[t_1-T_1]\right)\sin\left(\frac{\hbar\mathbf{k}_0\mathbf{Q}_1^{(1)}}{2M}[t_1-T_1]\right), \tag{46}$$

where

$$\bar{f}_0=\hat{F}_{p\to\sigma}(f_0(\mathbf{p})),$$

$$\Phi_1=b^{(1)}T_1+\left(\frac{Q_1^{(1)}}{M}\left[F_0^{(1)}T_1+\frac{\hbar k_0}{2}\right]\right)(t_1-T_1)+\psi_1+\frac{\pi}{2},$$

$$\psi_1=\mathrm{Arg}\left[M_{32}^{(1)}\bar{f}_0\left(\frac{\mathbf{Q}_1^{(1)}}{M}[t_1-T_1]\right)\right],\qquad b=\frac{G}{\mu}.$$

In the interval III there is a change in the expression for the phase of spatial fluctuations of the density:

$$\Phi_1\to\Phi_2=\Phi_1+\mathbf{Q}_1^{(1)}\mathbf{S}(t_1-[T_1+T]),$$

$$\mathbf{S}=\frac{2\mathbf{F}_0^{(3)}(t_1-[T_1+T])+\hbar\mathbf{k}_0-2M\mathbf{v}_R^{(3)}}{2M}. \tag{47}$$

The amplitude of the density perturbation $|H(t_1)|$ is bell-shaped with $H(T_1)=0$ and $|H(t_1)|\to0$ in the limit $t_1\to\infty$, i.e., the coherent response to the action of a biharmonic radiation pulse is manifested more strongly after a time t^* from the passage of the radiation (Fig. 1). We shall consider the specific case when

$$f_0=\pi^{-3/2}p_0^{-3}\exp[-(\mathbf{p}-\mathbf{p}_1)^2/p_0^2],$$
$$\bar{f}_0=\exp(j\sigma\mathbf{p}_1-p_0^2\sigma^2/4),$$

the maximum of the amplitude is reached at $t^*=T_1+(2^{1/2}M/p_0Q_1^{(1)})$, and the decay at $t_1-T_1>t^*-T_1$ is exponentially rapid:

$$|H(t_1)|<\exp[-(Q_1^{(1)}p_0[t_1-T_1])^2/4].$$

The reason for the delay of the quasiperiodic perturbations of the density is the light-induced spatial modulation of the directional (hydrodynamic) velocity which (immediately after the passage of the radiation) contains, as indicated by Eq. (37), the component

$$\tilde{\mathbf{u}}(R)=\frac{2\hbar\mathbf{k}_0}{M}|M_{32}^{(1)}|\cos(\mathbf{Q}_1^{(1)}\mathbf{R}-\Phi_1(T_1)). \tag{48}$$

Near its nodes $\mathbf{R}_q$ $[\mathbf{u}(\mathbf{R}_q)=0]$ the initial periodic bunching or antibunching of the particles takes place along the $Q_1^{(1)}$ direction with the period $(2/Q_1^{(1)})$ and it disturbs the homogeneity of the gas. It follows from Eq. (43) that the appearance of a spatial grating of the density is directly related to micro-oscillations of the distribution of a freely evolving gas:

$$\bar{f}\propto\left\{\exp\left(-j\frac{\mathbf{Q}_1^{(1)}\mathbf{p}}{M}(t_1-T_1)+j\mathbf{Q}_1^{(1)}\mathbf{R}\right)\right.$$
$$\left.\times(\bar{T}_+^{(0)}-T_-^{(0)})\alpha_2^{(1)}(\mathbf{p},T_1)-\text{c.c.}\right\} \tag{49}$$

(this should be compared with the Van Kampen waves in the theory of a plasma-wave echo in Refs. 9 and 10). If $(t_1-T_1)>(Q_1^{(1)}p_0/M)^{-1}$, then the phases of these oscillations are mixed thoroughly and this destroys the macroscopic response, but dephasing is not irreversible and the application of a second pulse may result in an interaction of perturbations caused by the first and second pulses, which may lead to renewed phase matching.

In the interval IV the density can be represented by

$$N^{(4)}=N_0+N_1+N_2+N_3+N_{ech}. \tag{50}$$

The terms N_1 and N_2 represent the "wake" of the decaying perturbation induced by the first pulse and distorted by the second pulse. The amplitude of the corresponding oscillations are small if

$$(T+T_2)(Q_1^{(1)}p_0/M)\gg1,$$
$$(N_i<N_0\exp[-(t_1-T_1)^2(Q_1^{(1)}p_0/2M)^2]),$$

so that they can be ignored in the interval IV. The term N_3 is the primary response to the action of the second radiation pulse, found allowing for the deformation of the noncoherent component of the distribution caused by the first pulse, and N_{ech} is the interference term associated with reversal of the phase evolution.

We shall introduce the notation

$$\mathbf{q}_R=\mathbf{F}_0^{(1)}T_1+\mathbf{F}_0^{(3)}T_2,$$

$$\theta=\mathbf{Q}_1^{(1)}\left[\mathbf{v}_R^{(3)}T_2+\mathbf{F}_0^{(3)}\frac{T_2T}{M}-\frac{\hbar\mathbf{k}_0}{2M}T_2\right],$$

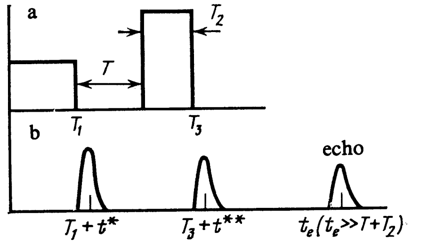

FIG. 1. Echo of resonant optical pressure: a) sequence of excitation pulses; b) relative amplitudes of a light-induced density grating.

$$\eta_\pm(t_1)=\frac{\mathbf{Q}_1^{(3)}}{M}[t_1-T_3]\pm\frac{\mathbf{Q}_1^{(1)}}{M}[t_1-T_1],$$
$$T_3=T_1+T+T_2,\qquad\psi_2=\operatorname{Arg}\left\{f_0\left(\frac{\mathbf{Q}_1^{(3)}}{M}[t_1-T_3]\right)M_{32}^{(3)}\right\},$$
$$\psi_\pm=\operatorname{Arg}\{\bar{f}_0(\eta_\pm(t_1))M_{32}^{(3)}M_{32}^{(1)*}\},\qquad\hbar\mathbf{k}_0\mathbf{Q}_1^{(1)}/2M=\omega_R^{(1)},$$
$$\hbar\mathbf{k}_0\mathbf{Q}_1^{(3)}/2M=\omega_R^{(3)},\qquad(\hbar\mathbf{k}_0/2)\eta_\pm=\rho_\pm. \tag{51}$$

Then, N_3 and N_{ech} are found from the expressions

$$N_3=4\left|M_{32}^{(3)}\xi\bar{f}_0\left(\frac{\mathbf{Q}_1^{(3)}}{M}[t_1-T_3]\right)\sin(\omega_R^{(3)}[t_1-T_3])\right|$$
$$\times\sin(\mathbf{Q}_1^{(3)}\mathbf{R}-\Phi_3), \tag{52}$$

$$\xi=-M_{12}^{(1)}j[\exp(j\omega_R^{(3)}[t_1-T_3])-1]+\frac{1}{2(a_0^{(1)})^2}$$
$$\times[\exp(j\omega_R^{(3)}[t_1-T_3])+1]+\left(\frac{\Delta'^{(1)}}{G^{(1)}}\right)^2\exp(j\omega_R^{(3)}[t_1-T_3]), \tag{53}$$

$$\Phi_3=b^{(3)}T_2+\operatorname{Arg}\xi+\left(\frac{\mathbf{Q}_1^{(3)}\mathbf{q}_R}{M}+\omega_R^{(3)}\right)(t_1-T_3)+\psi_2+\frac{\pi}{2},$$

$$N_{ech}=N_{ech}^{(+)}+N_{ech}^{(-)},$$

$$N_{ech}^{(\pm)}=8|M_{32}^{(3)}M_{32}^{(1)*}|\,|\bar{f}_0(\eta_\pm)\sin\rho_\pm\sin[(\omega_R^{(3)}\pm\omega_R^{(1)})$$
$$\times(t_1-T_3)]|$$
$$\times\cos([\mathbf{Q}_1^{(1)}\pm\mathbf{Q}_1^{(3)}]\mathbf{R}\mp\Phi_\pm), \tag{54}$$
$$\Phi_\pm=b^{(3)}T_2\pm b^{(1)}T_1\mp\theta+(\omega_R^{(3)}\pm\omega_R^{(1)})$$
$$\times(t_1-T_3)+\rho_\pm+\psi_\pm+\frac{\mathbf{q}_R\eta_+}{M}.$$

The oscillation amplitude N_2, representing the primary response to the second radiation pulse, rises from zero to the maximum value in a time $t^{**}\sim2^{1/2}M/p_0Q_1^{(3)}$ and then decays exponentially at times

$$t_1>T_3+t^{**},\qquad|N_2|<N_0\exp(-(t_1-T_3)^2(Q_1^{(3)}p_0/2M)^2).$$

The preferential condition for the appearance of echo oscillations of the density N_{ech} after decay of the primary response (Fig. 1) is, as indicated by Eq. (54), collinearity of the vectors $\mathbf{Q}_1^{(1)}$ and $\mathbf{Q}_1^{(3)}$. The echo growth time t_e is then given by the following relationships (which should be compared with the plasma echo described in Refs. 9–12)

$$(t_e-T_3)\approx\frac{2^{1/2}M}{p_0(Q_1^{(3)}-Q_1^{(1)})}+\frac{Q_1^{(1)}}{(Q_1^{(3)}-Q_1^{(1)})}(T+T_2),$$

$$Q_1^{(3)}>Q_1^{(1)}, \tag{55}$$

where the echo amplitude is of the same relative order as the primary responses if the interval between the radiation pulses and the duration of the second pulse are selected in such a way that

$$(\omega_R^{(3)}-\omega_R^{(1)})(t_e-T_3)\approx{}^1/_2\pi(2m+1).$$

We shall now consider a specific example when $(\hbar k_0/M)=6$ cm/sec, $\gamma=10^{-8}$ sec^{-1}, $\mathbf{n}_0=-\mathbf{n}_1$, $(\omega_1^{(3)}-\omega_1^{(1)})=3\times10^{10}$ Hz, $k=10^5$ cm^{-1}, $p_0/M=100$ cm/sec; we then obtain $(t_1^*-T_1)\approx(t_1^{**}-T_3)\approx10^{-7}$ sec and the period of the spatial grating is $\lambda_p\approx3\times10^{-5}$ cm. The amplitude of the echo oscillations with the spatial period $\lambda_{ech}\approx3$ cm reaches its maximum value in 0.25 sec if $T\approx T_2\approx10^{-6}$ sec. The relative amplitudes of all the responses are maximized by the optimal selection of the parameters of the interacting fields $|G_1|^2\sim\bar{\gamma}_1\bar{\gamma}_1$, $|\Delta'/\bar{\Omega}_0|\sim1.3$ and are of the order of $\hbar k_0/p_0$.

It should be noted that the factors complicating the appearance of an echo are collisions between particles and the boundary effects (collisions with a wall, free expansion of a gas, etc.). The phase memory is then destroyed as a result of interatomic collisions and diffuse reflection from the walls, but is retained in the case of specular reflection from the walls. Moreover, interference between perturbations that appear in different orders in respect of μ is also possible.

APPENDIX

Eigenvectors of the matrix $(\bar{A}_0+j\hat{\Delta})=B$

The matrix B has a simple structure and a double eigenvalue, identically equal to zero, and two complex-conjugate eigenvalues $\lambda_{3,4}=\pm jG$. The linearly independent system of eigenvectors φ_i is

$$\varphi_1=\mathrm{col}(g_0,\ g_0^*,\ -d_0\cos\beta,\ -jd_0\sin\beta),\qquad \lambda_1=0,$$

$$\varphi_2=\mathrm{col}\left\{j\frac{g_0d_0}{4},\quad j\frac{g_0^*d_0}{4},\quad\left(\sin\beta-j\frac{d_0^2}{4}\exp j\beta\right),\right.$$
$$\left.-j\left(\cos\beta+\frac{d_0^2}{4}\exp j\beta\right)\right\},\qquad\lambda_2=0,$$

$$\varphi_3=\mathrm{col}\{-{}^1/_2g_0(s_0-d_0),\ -{}^1/_2g_0^*(s_0+d_0),\quad 2\cos\beta,\ 2j\sin\beta\},$$
$$\lambda_3=+jG, \tag{A.1}$$

$$\varphi_4=\mathrm{col}\{{}^1/_2g_0(s_0+d_0),\ -{}^1/_2g_0^*(s_0-d_0),\ 2\cos\beta,\ 2j\sin\beta\},$$
$$\lambda_4=-jG,$$

where

$$g_0=\frac{\Omega_0}{|\Omega_0|},\qquad d_0=\frac{\Delta'}{|\Omega_0|},\qquad s_0=\frac{G}{|\Omega_0|},$$

$$G=(4|\Omega_0|^2+\Delta'^2)^{1/2},\quad \beta=\hbar\mathbf{k}_0\boldsymbol{\sigma}/2.$$

The linearly independent eigenvectors of the conjugate matrix B^+

$$B^+b_k=\bar{\lambda}_kb_k,\qquad \bar{\lambda}_k=\lambda_k^*,\qquad k=1-4, \tag{A.2}$$

for a biorthogonal system with φ_k are determined uniquely by the normalization conditions adopted in the present study

$$\langle b_k,\varphi_{k'}\rangle=\delta_{kk'}a_{k'}^2,\qquad a_{k'}^2=\begin{cases}a_0^2=(s_0/2)^2, & k'=1,2\\ 4a_0^2, & k'=3,4\end{cases}.$$

[1]We shall formally define the directional momentum as the first-order moment of the Wigner distribution function $f=f(\mathbf{p},\mathbf{R},t)$, which is local in respect of the spatial coordinates.

[2]The symbol "col" is introduced for a column vector written down on a line.

[3]A small parameter appears in a natural manner as we go over to dimensionless variables, provided the conditions (4) and (14) are satisfied.

[4]It should be noted that, generally speaking, the function G may be slow in time on the μ/G scale (this does not affect the asymptotic expression). Then, the conditions for parametric resonance are ensured by varying the frequency of the controlling field. The value $\Delta_0=-G$ also exhibits resonant behavior.

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