Stimulated emission and phase transitions

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Attention is called to a far-reaching analogy between a laser and induced generation of a Bose condensate of Cooper pairs in a superconductor. The analogy with the laser is used to postulate nonstationary equations for the order parameter of the superconductor. Examples are cited of other processes in whose dynamics stimulated transitions can play a substantial role.

1. INVERSION CONDITION FOR A SUPERCONDUCTOR

Haken[1] has shown that lasing can be regarded as phase transition in an active medium + electromagnetic field system. This raises the question: is this analogy reciprocal, i.e., can at least some phase transitions can be treated from the standpoint of coherent-state generation via stimulated transitions? This question can be answered in the affirmative, at any rate, with respect to formation of a superconducting Bose Condensate of Cooper pairs.[2]

The present paper is devoted to development of the ideas of Ref. 2. In particular, it traces more consistently the analogy between the equations that describe generation of radiation in lasers, on the one hand, the formation of a coherent Cooper-pair Bose Condensate, on the other.

For stimulated transitions to contribute to the formation of a superconducting Bose condensate it is necessary that the induced "production" of cooperons prevail over their "absorption." Such a state of a system, using laser terminology, can be called "inverted."

It is convenient to formulate the inversion conditions in terms of chemical potentials of quasiparticles. Formation of a Copper pair (cooperon) can be treated as recombination of two quasiparticles ("positive" and "negative") accompanied by phonon emission. For stimulated formation of cooperons to be possible, it is necessary that the stimulated "emission" of the cooperons exceed their stimulated absorption. In the language of quasiparticles, this condition can be written in the form

$$w_k^{\to}|\Psi|^2 n_p^+ n_p^- \prod_\nu (N_\nu+1) > w_k^{\gets}|\Psi|^2(1-n_p^+)(1-n_p^-)\prod_\nu N_\nu. \quad (1)$$

Here n_p^+ and n_p^- are the densities of positive and negative quasiparticles of energy ε_p. Next $w_k^{\to}$ is the probability of copperon production per quasiparticle pair at zero temperature with emission of a set of phonons with energies $\hbar\Omega_{\nu_1}, \hbar\Omega_{\nu_2}..., \hbar\Omega_\nu$, while $w_k^{\gets}$ is the probability of the inverse process; N_ν is the density of phonons of energy $\hbar\Omega_\nu$; Ψ is the order parameter, and $|\Psi|^2$ is a quantity proportional to the density of the Cooper pairs.

We assume the quasiparticle density to have a quasiequlibrium Fermi distribution

$$n_p^\pm = \{\exp[(\varepsilon_p-\mu_\pm)/kT]+1\}^{-1}. \quad (2)$$

This distribution has a limited validity range.[3] It is convenient to use it, however, as a model distribution, to gain a deeper insight in the meaning of relation (1).

Recognizing that $w_k^{\to} = w_k^{\gets}$ and

$$N_\nu = [\exp(\hbar\Omega_\nu/kT)-1]^{-1},$$

we find (1) is equivalent to the inequality

$$\mu_+ + \mu_- > 2\varepsilon_p - \sum_\nu \hbar\Omega_\nu. \quad (3)$$

Since the pairing energy $2\varepsilon_p$ is transferred to the lattice, it follows that $2\varepsilon_p = \Sigma_\nu \hbar\Omega_\nu$, and we arrive at an ultimate inversion condition

$$\mu_+ + \mu_- > 0. \quad (4)$$

We emphasize that condition (4) does not mean inversion in its usual meaning when the particle density at the bottom of the gap exceeds 0.5. Condition (4) is much less stringent, and is governed by the fact that, within the framework of the mechanism considered, the rate of stimulated cooperon production exceeds their destruction rate by a factor $(N_\nu + 1)/N_\nu$, with N_ν small enough at low temperatures. It follows from condition (4) that even an insignificant deviation of the quasiparticle density from the equilibrium $\mu_+ = \mu_- = 0$ corresponds to an inverted distribution. It explains the phase stability of a Bose condensate of copper pairs. Indeed, if a fluctuation breaks some pair of the Bose condensate into two unbound particles, an inversion state satisfying condition (4) sets in immediately. The process of induced cooperon production restores the broken pair to a state coherent with the Bose Condensate.

Coherent processes with probability proportional to $|\Psi|^2$ were taken into account in a number of papers (see, e.g., Ref. 3 and the literature cited there) devoted to the analysis of nonstationary phenomena in superconductors. No notice was taken, however, of the rather profound analogy between induced production of cooperons and the processes that take place in lasers. In our opinion this is not merely a change of terminology. Such an approach has heuristic significance, the gist of which is the following.

Lasers operate in a large variety of dynamic regimes, which have been well investigated theoretically and experimentally. In particular, depending on the parameters, one can observe smooth and pulsating approaches to a stationary regime, a regime with undamped radiation pulsations, and regimes corresponding to the strange attractor.[4–6] Trains of

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picosecond radiation pulses can be generated in a multimode laser.

A consistent development of the theory of analogous processes in a superconductor can be not only of scientific-theoretical but also of applied significance. It is necessary for this purpose to forgo the assumptions made in the treatment of nonequilibrium processes in a superconductor, viz., that they are quasistationary with respect to individual dynamic variables.

The complete system of equation should include a nonstationary equation for the order parameter and nonstationary equations for the densities of the quasiparticles and phonons.

A rigorous derivation of such a set of equations on the basis of the microscopic theory is outside the scope of the present paper. Qualitatively, however, the structure of the system can be visualized by using the analogy with a laser. According to this analogy, the order parameter Ψ of the superconductor corresponds to the electromagnetic field of the laser. The Ginzburg-Landau (GL) equation for the superconductor is equivalent in this case to the equation for a laser in which the polarization of the medium and the number of active particles are replaced by their quasistationary values. Knowing the technique for this substitution, we can attempt to "reinstate" the initial nonstationary equation. This calls for a transition from complete nonstationary system of laser equations to the quasistationary equation for a coherent electromagnetic field.

2. SYSTEM OF LASER EQUATIONS

The theory of the dynamic laser regimes is based in its simplest form on the following system of equations, which was corroborated[1)] in Refs. 7 and 8:

$$\begin{gathered}\frac{\partial^2\mathbf{E}}{\partial t^2}-c^2\nabla^2\mathbf{E}=-2\gamma\frac{\partial\mathbf{E}}{\partial t}-4\pi\frac{\partial^2\mathbf{P}}{\partial t^2},\\ \frac{\partial^2\mathbf{P}}{\partial t^2}+\frac{2}{\tau_2}\frac{\partial\mathbf{P}}{\partial t}+\omega_0{}^2\mathbf{P}=-2\omega_0\frac{|\mu|^2}{\hbar}\Delta N\mathbf{E},\\ \frac{\partial\Delta N}{\partial t}+\frac{1}{\tau_1}\Delta N=I+\frac{2}{\hbar\omega_0}\mathbf{E}\frac{\partial\mathbf{P}}{\partial t}.\end{gathered}\tag{5}$$

The meaning of this set of equations is quite simple. The first is the wave equation for one of the components of the electromagnetic field generated by the laser. In our case this means the electric field $\mathbf{E}(r,t)$, for in most cases the decisive role is played by the electric dipole interaction of the medium with the electromagnetic field. The wave equation contains the field sources in the form of the polarization $\mathbf{P}(r,t)$ of the active medium used in the laser. The same equation takes into account the electromagnetic-radiation losses, described by the term $2\gamma\partial\mathbf{E}/\partial t$. The losses are due primarily to the extraction of the laser radiation from the active volume for further utilization. There are also additional loss sources, due to the imperfections of the laser elements, such as scattering of the radiation by inhomogeneities of the active medium, radiation absorption by the optical elements used in the laser, and others.

The equations for the polarization were obtained in an approximation in which the radiation interacts resonantly with the medium. In this case only two energy levels of the energy spectrum of the medium are taken into account, with the frequency of the transition between them chosen close to the frequency of the electromagnetic field. The polarization relaxation time determines the spectral line width $\Delta\omega_0=2\tau_2{}^{-1}$; ΔN is the density of the population difference between the upper and lower resonant ("working") levels of the medium, frequently called the number of active particles; τ_1 is the time in which equilibrium over the energy levels of the medium is established; μ is the dipole-moment matrix element and corresponds to a transition between resonant energy levels. The quantity I describes the so-called pump—the supply of energy from an external source to produce resonant-energy-level inverted population needed for the laser operation.

Since the equations for the polarization and for the field are close to the equations of harmonic oscillators, the field and the polarization can be represented in the form

$$\mathbf{E}(r,t)=\mathbf{A}(r,t)e^{-i\omega t},\qquad \mathbf{P}(r,t)=\mathbf{B}(r,t)e^{-i\omega t},\tag{6}$$

with the frequency ω close to the resonant frequency ω_0 of the transition, so that $|\omega-\omega_0|\ll\omega_0$, and $\mathbf{A}$ and $\mathbf{B}$ slow function of the time compared with $\exp(-i\omega t)$, i.e., $\partial\mathbf{A}/\partial t\ll\omega\mathbf{A}$ and $\partial\mathbf{B}/\partial t\ll\omega\mathbf{B}$.

With allowance for (5), the equations for $\mathbf{A}(r,t)$ and $\mathbf{B}(r,t)$ take the form[9]

$$\begin{gathered}\frac{\partial\mathbf{A}}{\partial t}+\gamma\mathbf{A}-\frac{i}{2\omega}(\omega^2+c^2\nabla^2)\mathbf{A}=i2\pi\omega\mathbf{B},\\ \frac{\partial\mathbf{B}}{\partial t}+\left[\frac{1}{\tau_2}+i(\omega_0-\omega)\right]\mathbf{B}=-i\frac{|\mu|^2}{\hbar}\Delta N\mathbf{A},\\ \frac{\partial\Delta N}{\partial t}+\frac{1}{\tau_1}\Delta N=I+\frac{i}{2\hbar}(\mathbf{A}\mathbf{B}^*-\mathbf{A}^*\mathbf{B}).\end{gathered}\tag{7}$$

Monochromatic lasing is achieved by exciting one definite oscillation mode in the cavity. In this case $c^2\nabla^2A=-\omega_j^2A$, where ω_j is the natural frequency of this mode. If furthermore $\omega_j=\omega_0$, the lasing frequency ω coincides with the resonant frequency ω_0 of the spectral line of the medium, and

$$\begin{gathered}\frac{\partial\mathbf{A}}{\partial t}+\gamma\mathbf{A}=i2\pi\omega\mathbf{B},\\ \frac{\partial\mathbf{B}}{\partial t}+\frac{1}{\tau_2}\mathbf{B}=-i\frac{|\mu|^2}{\hbar}\Delta N\mathbf{A},\\ \frac{\partial\Delta N}{\partial t}+\frac{1}{\tau_1}\Delta N=I+\frac{i}{2\hbar}(\mathbf{A}\mathbf{B}^*-\mathbf{A}^*\mathbf{B}).\end{gathered}\tag{8}$$

Further simplifications of the system (8) are achieved by specifying in detail the type of the laser. For solid-state lasers based on ruby, neodymium glass, and similar media, we have $\tau_2\ll\tau_1$, $\tau_2\ll\gamma^{-1}$. In this case $\partial B/\partial t\ll B/\tau_2$, the equation for the polarization becomes algebraic, and its solution permits the polarization to be eliminated from Eqs. (8):

$$\frac{\partial\mathbf{F}}{\partial t}+\left(\gamma-\frac{1}{2}\sigma\Delta N\right)\mathbf{F},\tag{9a}$$

$$\frac{\partial \Delta N}{\partial t}+\frac{1}{\tau_1}\Delta N=I-2\sigma\Delta N|\mathbf{F}|^2,$$

where, to make the equations compact, we introduce the notation

$$\mathbf{F}=\left(\frac{1}{8\pi\hbar\omega}\right)^{1/2}\mathbf{A},\quad \sigma=4\pi\frac{|\mu|^2}{\hbar}\omega\tau_2.$$

The values of τ_1 and τ_2 are close in order of magnitude for lasers of a definite type, so that the relations $\tau_1, \tau_2 \ll \gamma^{-1}$ hold. In this case we can neglect not only $\partial\mathbf{B}/\partial t$ compared with $\tau_2^{-1}\mathbf{B}$, but also $\partial\Delta N/\partial t$ compared with $\tau_1^{-1}\Delta N$. The system (8) reduces then to the single equation

$$\frac{\partial \mathbf{F}}{\partial t}+\left(\gamma-\frac{1}{2}\sigma\frac{I\tau_1}{1+2\sigma\tau_1|\mathbf{F}|^2}\right)\mathbf{F}=0. \qquad (10)$$

At relatively low laser power we have $\sigma\tau_1|\mathbf{F}|^2\ll 1$, so that

$$(1+2\sigma\tau_1|\mathbf{F}|^2)^{-1}\approx 1-2\sigma\tau_1|\mathbf{F}|^2.$$

Equation (10) takes then the form

$$\frac{\partial \mathbf{F}}{\partial t}+\left[\gamma-\frac{1}{2}\sigma I\tau_1(1-2\sigma\tau_1|\mathbf{F}|^2)\right]\mathbf{F}=0. \qquad (11)$$

The stationary amplitude of the generated radiation is determined by Eq. (11) under the condition $\partial\mathbf{F}/\partial t=0$:

$$\left(\gamma-\frac{1}{2}\sigma I\tau_1\right)\mathbf{F}+\sigma^2\tau_1^2 I|\mathbf{F}|^2\mathbf{F}=0. \qquad (12)$$

3. NONSTATIONARY EQUATIONS FOR THE ORDER PARAMETER

Notation aside, Eq. (12) coincides with the GL equation for the spatially homogeneous order parameter of a superconductor.[11]

This circumstance was used in Ref. 1 to treat the onset of lasing as a phase transition. The inverse analogy is used here to obtain a nonstationary set of equations that describe the superconducting state.

Just as Eq. (12) is a stationary approximation of the more general Eq. (10), we shall consider the GL equation[2)]

$$\left[\alpha-\beta|\Psi|^2+D\left(i\nabla+\frac{2e}{\hbar c}\mathbf{A}\right)^2\right]\Psi=0 \qquad (13)$$

as a stationary approximation of the following equation:

$$\left(\frac{\partial}{\partial t}+i\delta\right)\Psi+\left[\alpha-\beta|\Psi|^2+D\left(i\nabla+\frac{2e}{\hbar c}\mathbf{A}\right)^2\right]\Psi=0. \qquad (14)$$

The term $i\delta\Psi$ was introduced in (14) for the sake of generality, since replacement of Ψ by $\Psi\exp\{-i\int\delta dt\}$ does not alter the form of (13). Equation (14) coincides with the nonstationary GL equation derived in Ref. 12 on the basis of microscopic theory for a zero-gap superconductor. It was found in the same reference that $\delta=2e\hbar^{-1}\varphi$, where φ is the electrochemical potential (see also Ref. 13).

The laser equation (10) is the consequence of the quasistationary approximation for the number of active particles (the assumption $\partial\Delta N/\partial t\ll\tau_1^{-1}\Delta N$). In the case of a superconductor, the analogs of active particles are quasiparticles corresponding to elementary excitations, and it can be assumed that Eq. (14) results from a quasistationary approximation for the quasiparticle density. The nonstationary equations can be "reconstituted," by analogy with laser theory, in the following manner.

The term in (9b) with $\sigma\Delta N|F|^2$ is none other than the rate of the induced transition, and $\sigma\Delta N$ is the same rate per unit density of the generated photons. This term enters in Eq. (9a) for the laser field in the form $\frac{1}{2}\sigma\Delta NF$. The analogous quantity for a superconductor is given by the difference, summed over the quasiparticle momenta and over all possible sets of emitted photons, between the terms in the right- and left-hand sides of (1). We denote this quantity by $W|\Psi|^2$. Then

$$W=\sum_p W_p=\sum_{p,k} w_k n_p^+ n_p^- \prod_\nu (N_\nu+1)$$
$$\times\left[1-\frac{1-n_p^+}{n_p^+}\frac{1-n_p^-}{n_p^-}\prod_\nu\frac{N_\nu}{N_\nu+1}\right], \qquad (15)$$

and the equation for the order parameter takes the form

$$\frac{\partial\Psi}{\partial t}+i\delta\Psi+\left(\frac{1}{\tau_\Psi}-\frac{1}{2}W\right)\Psi+D\left(i\nabla+\frac{2e}{\hbar c}\mathbf{A}\right)^2\Psi=0, \qquad (16)$$

where τ_Ψ is the time of relaxation of the order parameter by the impurities.

Since W depends on the number of quasiparticles, Eq. (16) is not a closed one. We need also equations that describe the variation of the quasiparticle density $n_p^\pm$ in space and in time. The quasiparticle dynamics can be described by using the kinetic equation

$$\frac{\partial n_p^\pm}{\partial t}+\frac{\partial n_p^\pm}{\partial r}\frac{\partial\varepsilon_p}{\partial p}-\frac{\partial n_p^\pm}{\partial p}\frac{\partial\varepsilon_p}{\partial r}$$
$$=\frac{2}{\tau_{\Psi p}}|\Psi|^2-S_p^R+S_p^+-S_p^-+S_p^e+S_p^{im}+Q_p, \qquad (17)$$

where $\tau_{\Psi p}$ is the partial relaxation time that describes the breakup of the pairs at the impurities into quasiparticles with momentum p; $\tau_\Psi^{-1}=\Sigma_p\tau_{\Psi_p}^{-1}$; Q_p is the source of the nonequilibrium density of the quasiparticles; $S_p^R, S_p^+, S_p^-, S_p^e$, and S_p^{im} are the collision integrals obtained in Ref. 13 and given in the review of Elesin and Kopaev.[3] Their explicit forms are unwieldy and will not we written out here. We note only that S_p^+ and S_p^- describe the energy relaxation of quasiparticles by phonons: S_p^+ describes the arrival of particles with momentum p, and S_p^- their departure with the same momentum; S_p^e is the electron-electron collision integral; S_p^{im} is an integral that describes the energy relaxation of the quasiparticles by the impurities; S_p^R is the recombination collision integral. The quantity $W_p|\Psi|^2$ is in fact the "coherent part of the recombination integral S_p^R and is proportional to Δ^2 [see Ref. 3, Eq. (24)].

We have neglected in (16) the incoherent (spontaneous) recombination of the quasiparticles. Spontaneous recombination creates a Cooper pair in a state of arbitrary phase, and is a source of noise for the order parameter, just as spontaneous emission in a laser is a source of noise that causes the finite spectral width of the laser emission.[14]

If the phonons have no equilibrium distribution function, the system (16) and (17) must be supplemented with the kinetic equation derived in Ref. 15 for the phonons (see also Eq. (29) of Ref. 3).

Equations (16) and (17) describes also the electron pairing in the case when the interaction between them is repulsive. This phenomenon should become manifest in high quasiparticle density ($n_p > 0.5$) in a certain energy interval located above the Fermi level (see Ref. 3).

Assuming for the quasiparticle density a quasiequilibrium Fermi distribution with potentials $\mu_\pm$ that depend on the coordinates and time, and assuming also an equilibrium phonon distribution and a small deviation of the quasiparticle distribution from the equilibrium value ($\mu_\pm/kT \ll 1$), we can write down a relatively simple closed system of equations. In this case

$$1-\frac{1-n_p^+}{n_p^+}\frac{1-n_p^-}{n_p^-}\prod_\nu \frac{N_\nu}{N_\nu+1}\approx\frac{\mu_++\mu_-}{kT}, \quad (18)$$

and W takes the simple form

$$W=G\frac{\mu_++\mu_-}{kT}\equiv G\xi, \quad (19)$$

where

$$G=\sum_{p,k} w_k n_{p0}^+ n_{p0}^- \prod_\nu (N_\nu+1),$$

while n_{p0}^+ and n_{p0}^- are the equilibrium Fermi distributions with $\mu_\pm = 0$.

As a result we obtain from (16) and (17)

$$\frac{\partial\Psi}{\partial t}+i\delta\Psi+\left(\frac{1}{\tau_\Psi}-\frac{1}{2}G\xi\right)\Psi+D\left(i\nabla+\frac{2e}{\hbar c}\mathbf{A}\right)^2\Psi=0, \quad (20a)$$

$$\frac{\partial\xi}{\partial t}-D_\xi\nabla^2\xi=\frac{4}{ab}\left(\frac{1}{\tau_\Psi}-\frac{1}{2}G\xi\right)|\Psi|^2+\Phi(|\Psi|)\frac{\partial|\Psi|^2}{\partial t}, \quad (20b)$$

where

$$a=\frac{1}{2}\frac{p_F^2kT}{h^3v_F},\quad b=\frac{1}{n_0}\frac{\Delta_0^2}{(kT)^2},$$

$$\Phi(x)=\int_0^\infty\frac{\exp(x^2+u^2)^{1/2}}{[\exp(x^2+u^2)^{1/2}+1]^2}\frac{du}{(x^2+u^2)^{1/2}}, \quad (21)$$

v_F and p_F are the electron velocity and momentum on the Fermi surface; $2\Delta_0$ is the gap width, and $\frac{1}{2}n_0$ is the density of the Cooper pairs at absolute zero. D_ξ is the quasiparticle diffusion coefficient.

Equation (20b) is derived by substitution in (17) the distribution function (2) and summing the result over all momenta. It must be recognized here that the energy ε_p is independent of the energy gap $\Delta(r,t)$:

$$\varepsilon_p=[v_F^2(p-p_F)^2+\Delta^2(r,t)]^{1/2}.$$

We have retained in (20b) terms of the first order in Δ/kT, and introduced the notation $|\Psi|^2=\Delta^2/(kT)^2$.

To determine G we consider a homogeneous superconductor in the absence of a magnetic field and of currents. In this case the quasiparticle density is conserved:

$$\sum_p(n_p^++n_p^-)+(2/b)|\Psi|^2=n_0. \quad (22)$$

In the absence of impurities ($\tau_\Psi^{-1}=0$) the equilibrium state of the superconductor corresponds to a zero chemical potential $\mu_\pm$ of the quasiparticles.

Expanding n_p^+ and n_p^-, expressed in the form (2), in powers of $\mu_\pm$ we obtain

$$a\xi+(2/b)|\Psi|^2=n_0-n_T, \quad (23)$$

where n_T is the equilibrium number of unpaired quasiparticles at the temperature T. Since, however, $n_0-n_T=n_0(1-T/T_c)$, where T_c is the critical superconducting-transition temperature of the impurity-free metal, we get

$$\xi=\frac{n_0}{a}\left(1-\frac{T}{T_c}\right)-\frac{2}{ab}|\Psi|^2. \quad (24)$$

It follows from (20a) and (24) that

$$\frac{\partial\Psi}{\partial t}+i\delta\Psi-\frac{1}{a}G\left[\frac{n_0}{2}\left(1-\frac{T}{T_c}\right)-\frac{1}{b}|\Psi|^2\right]\Psi=0. \quad (25)$$

Comparing the coefficients in (25) with the GL-equation coefficients calculated form the microscopic theory,[11] we obtain

$$G=ga/\hbar n_0, \quad (26)$$

where $g=N_FV$, with N_F the state density in phase space near the Fermi level and V the electron-phonon interaction constant.

Nonstationary equations for the quasiparticles and a stationary equation for the order parameter (the energy gap) have been used in a number of investigations of nonequilibrium processes in superconductors.[3] The argument customarily used to justify this approach is that the time required to establish an equilibrium state in a system of Cooper pairs is substantially shorter than the time to establish equilibrium in a quasiparticle system. Experience with lasers shows that these arguments are debatable. In a solid-state laser, for example, the time to establish equilibrium in the active medium (which is the analog of the quasiparticles) is much longer than the electromagnetic-field transient time in the cavity. The use of a stationary equation for the field, however, is categorically inadmissible. A stationary field equation in the laser-equation system (9) leads inevitably to a stationary equation for the number of active particles.

The quasistationary approximation of the order parameter does not lead in the case of superconductors to a formal contradiction, since the equation for the order parameter contains the magnetic-diffusion term $[i\nabla+(2e\mathbf{A}/\hbar c)]^2\Psi$.

The use of the quasistationary equation for Ψ, however, can lead to loss of solutions that correspond to modulation regimes. For example, relatively shallow modulation of the pump intensity I of the cavity losses γ (in a range from several percent to a fraction of a percent, depending on the laser parameters) can lead to deep modulation of the radiation generated by solid-state lasers.[16] The lasing consists in this case of periodically repeated radiation pulses whose maximum values can substantially exceed the radiation corresponding to the stationary lasing regime. The frequency of the external modulation of the laser parameters should be chosen to be of the same order as the so-called relaxation frequency $\Omega_0 \sim (\gamma\tau_1)^{1/2}$ (Ref. 16). Starting from the analogy of the dynamic equations of the superconductor and the laser, it can be assumed that a modulation regime can be achieved also in a superconductor if the material and the magnitude and spatial distribution of the flowing current are suitably chosen. Weak modulation can be produced in a superconductor by applying an alternating magnetic field modulated by microwave electromagnetic radiation, and by excitation of sound. One cannot exclude beforehand the possible onset, under certain conditions, of a self-modulation regime. Finally, a great decrease of a magnetic field whose initial value is close to critical, or even exceeds it, can produce an order-parameter pulse of large amplitude, analogous to the "giant" pulse of a laser. Instead of using a magnetic field, one can abruptly turn off another agent that destroys the superconducting state.

Of course, $|\Psi|^2$ cannot exceed in any of the processes the maximum value determined by the quantity $\Delta n_T = n_0 - n_T$ [see (23)]. If, however, the stationary value of $|\Psi|^2$ is noticeably smaller than Δn_T under certain conditions, the maximum of the order parameter will exceed noticeably its stationary value. Recognizing that the current pulse density $\mathbf{j} \sim \mathbf{A}|\Psi|^2$, supercritical currents can flow through the sample at the instant when an order-parameter pulse is produced.

4. CONCLUSION

In view all the foregoing, a more detailed review of the role of induced transitions in a great variety of processes is called for.

It is necessary first of all to refine the widely used statement that a Bose condensate is "attracting." This statement stems from the fact that the probability of the transition of a system into a state characterized by the presence of a certain number of Bose particles is proportional to this number. If the transition takes place, however, from a state with a certain number of Bose particles, the transition probability is also proportional to the number of these bosons. Thus, a Bose condensate can equally well attract to itself or destroy itself. The only question is which of these processes predominates. For the "creation" process to predominate, special conditions must be satisfied. In a laser, such a condition is the need for inverted population that exceeds a threshold value; for the quasiparticles of a superconductor, this is condition (4), etc. In analogy with lasers, such conditions in other system can be called inversion conditions.

We present examples of some processes in whose nonstationary dynamics a substantial role can be played by induced transitions.

1. First, processes that form the ferroelectric state. The analogy between the onset of radiation in a laser and the formation of the ferroelectric state seems even more natural than the analogy between a laser and the superconducting state.[3)]

2. According to contemporary opinions concerning the nature of elementary particles, a physical vacuum contains Bose condensate of singular Higgs particles, and these divide the universal interactions into strong, weak, and electromagnetic.[18,19] These condensates were produced by phase transitions that occurred in the universe during the initial stage of its expansion after the big bang.[19] One cannot exclude the possibility that inverted states were produced at the instants of the phase transitions in the expanding universe, and that induced transitions played an essential role in the formation of both the Higgs bosons and of their coherent Bose condensates.

The complicated dynamics of nonequilibrium processes, which appears when induced transitions are taken into account, may have played and still plays a substantial role in the evolution of the universe. Rapid expansion of matter can be accompanied by "quenching" of states, and this contributes to the appearance of an active medium that is equivalent to inverted population.[20] As it expanded, the universe could therefore pass through a state of inversion relative to the induced generation of various bosons. It is not excluded that inverted states can occur also in other cosmologic processes, e.g., supernova explosions.

3. Interest in self-organization—synergetics—has greatly increased of late.[21] It seems to us that a deeper insight into the nonstationary dynamics of self-organization processes can be gained on the basis of the concepts of inverted population and induced transitions. It is precisely induced transitions which can cause the complicated dynamics of the process; in the presence of inversion they cause a rapid development of the process and ensure the succession of the generations.

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[1)]The derivation of Eqs. (5) can be found in a large number of books (see, e.g., Refs. 9 and 10).
[2)]Here and elsewhere $\mathbf{A}$ is the vector potential of the electromagnetic field.
[3)]This circumstance was called to my attention by Yu. V. Kopaev during the discussions of the present results (see also Ref. 17).

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