Phase conjugation in nonstationary stimulated Brillouin scattering of focused beams

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The observation conditions, existence limits, and accuracy of phase conjugation (PC) in nonstationary stimulated Brillouin scattering (SBS) are investigated theoretically and experimentally. The experimental results for a wavelength $\lambda = 1.05\,\mu$m corroborate well the theoretical conclusions for light beams with small spatial inhomogeneities. The accuracy of phase conjugation is determined and the reflection coefficients of SBS mirrors are determined for quasihomogeneous saturated beams at a relative pulse duration $t_{\rm pul}/T_2 = 0.15$ to 1 (T_2 is the hypersound relaxation time). It is shown that the maximum duration of a pulse reflected by an SBS mirror is limited by competing processes such as breakdown and self-focusing. The accuracy with which smooth aberrations such as astigmatism are corrected is measured.

INTRODUCTION

Phase conjugation (PC) in stimulated Brillouin scattering (SBS) has been the subject of many papers (see, e.g., Refs. 1 and 2). They all deal, however, with stationary scattering conditions. In practice it is frequently necessary to produce PC in SBS when the pulse duration $t_{\rm pul}$ is comparable with or shorter than the hypersound relaxation time T_2, i.e., under nonstationary conditions. This takes place frequently, for example, in near-infrared apparatus for the investigation of the interaction of radiation with matter.

The feasibility of attaining a near-unity coefficient ($R \approx 0.8$) of pump reflection into a Stokes wave for SBS in compressed gases at $t_{\rm pul} \sim T_2$ was demonstrated experimentally in Ref. 4, but the spatial structure of the reflected radiation was not investigated there. A value $R \approx 0.9$ was obtained for Gaussian beams (in SF_6); the accuracy with which the Stokes wave was conjugated to the pump was found to be practically ideal ($\chi = 0.98$).[5] In SBS of short pulses of nearly uniform intensity distribution in nonlinear liquid, the PC deteriorated as the pump power was increased and $t_{\rm pul}$ was shortened.[6] The feasibility of achieving PC in beams with elaborate structure was noted in experiments,[7,8] but no quantitative investigations were made.

We develop here for nonstationary SBS of spatially inhomogeneous light beams a theory that determines the PC existence limits for both light guides and focused beams. We measured the reflection coefficients and the accuracy of the PC for nonstationary SBS in various media and in a wide range of the radiation parameters. The main conclusions of the theory agree well with the experimental results. We show, in particular, that the PC effect is shut off when the scale of the transverse inhomogeneities of the pump is decreased, and moreover does so earlier the shorter the laser pulse.

1. THEORY OF PC IN NONSTATIONARY SBS

The reason why PC is produced in SBS is[1] preferred enhancement of a structure in the Stokes wave, so that its complex field amplitude E_s is the conjugate of the pump-field amplitude E_p, E_s, E_p^*. The preferred enhancement makes the growth rate M of the Stokes wave reversed towards the pump double the growth rate $\overline{M}$ of the scattered-radiation components that are not correlated with the Stokes wave in the case of SBS in the light guide[9,10] and approximately 1.65 times larger when the radiation is focused into the nonlinear medium.[11–13] For SBS of focused beams, the Stokes wave tends to become localized near the pump-beam axis, where its gain is a maximum. The envelope of the Stokes beam reversed towards the pump is therefore considerably narrower than the exciting-radiation beam envelope, so that additional diffraction losses are produced. It is the last circumstance which raises the main difficulties when it comes to calculate the growth rate of the reversed Stokes wave.

To analyze PC in nonstationary SBS we elaborate in this paper on the approach proposed in an investigation[12] of this effect under stationary scattering conditions. In this approach, in the material polarization responsible for the excitation and amplification of the Stokes wave we separate that component which corresponds to the amplification of the most rapidly growing structure. This part of the polarization is expressed as an expansion in a definite system of orthogonal transverse modes having different envelopes but equal small-scale distributions of the complex conjugate of the pump field. Solution of the equations for the weighting coefficients of the expansion, which depend on the time t and on the longitudinal coordinate z, permits calculation of the growth rate of the Stokes wave and yields the conditions for the observation of the PC effect.

1.1. Fundamental equations

Assume that a pump wave

$$E_p \exp[i(\omega t - k_p z)] + \text{c.c.}$$

is incident in the $z = 0$ plane on the boundary of a SBS-active medium in which a Stokes wave

$$E_s \exp[i(\omega_s t + k_c z)] + \text{c.c.}$$

excited up from the spontaneous-noise level, propagates in

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the opposite direction. The equations that describe the interaction of these waves take for nonstationary SBS the form

$$\frac{1}{v_p}\frac{\partial E_p}{\partial t}+\frac{\partial E_p}{\partial z}+\frac{i}{2k_p}\Delta_\perp E_p=\frac{i}{2}g_1E_sQ, \quad (1)$$

$$\frac{1}{v_s}\frac{\partial E_s}{\partial t}-\frac{\partial E_s}{\partial z}+\frac{i}{2k_s}\Delta_\perp E_s=\frac{i}{2}g_2E_pQ, \quad (2)$$

$$\frac{\partial Q}{\partial t}+\frac{Q}{T_2}=ig_3E_pE_s^*+f_n, \quad (3)$$

where P_p, E_s, and Q are the complex amplitudes of the pump, Stokes, and hypersound waves; g_i are constants that depend on the material parameters and determine the growth rate $g=g_2g_3(2\pi/cn)$ of the Stokes wave per unit length and unit intensity, c is the speed of light, n is the refractive index of the medium, v_p and v_s are the pump- and Stokes-wave velocities, and f_n is an extraneous noise source that governs the spontaneous-scattering level. Since the frequency shift is small in SBS, we put hereafter $k_p=k_s=k$.

To calculate its growth rate, we neglect the influence of the Stokes wave on the exciting radiation, i.e., we consider SBS in a given pump field. The pump beam, which has in the $z=0$ plane the factorized form

$$E_p(t,\ z=0,\ \mathbf{r}_\perp)=\varphi(t)\varepsilon_n(0,\ \mathbf{r}_\perp)$$

(the functions $\varepsilon_p(0,\mathbf{r}_\perp)$ and $\varphi(t)$ describe the spatial structure and profile of the exciting-radiation pulse[1)]), is determined in the interior of the medium $z>0$ by the solution of Eq. (1) with a zero right-hand side, i.e.,

$$E_p(t,\ z,\ \mathbf{r}_\perp)=\varphi(t-z/v_H)\varepsilon_H(z,\ \mathbf{r}_\perp)$$

and

$$\frac{\partial\varepsilon_p}{\partial z}+\frac{i}{2k}\Delta_\perp\varepsilon_p=0. \quad (4)$$

We assume also that the radiation entering the medium has a Gaussian correlation function

$$\frac{cn}{2\pi}\left\langle\varepsilon_p\left(0,\mathbf{r}_\perp-\frac{\boldsymbol{\rho}}{2}\right)\varepsilon_p^*\left(0,\mathbf{r}_\perp+\frac{\boldsymbol{\rho}}{2}\right)\right\rangle$$
$$=I_p(0)\exp\left[-\frac{\mathbf{r}_\perp^2}{a^2(0)}-\frac{\rho^2}{\rho^2(0)}+i\frac{k}{F}\boldsymbol{\rho}\mathbf{r}_\perp\right] \quad (5)$$

where the Gaussian-beam envelope and the transverse inhomogeneity of the field have respectively radii $a(0)$ and $\rho(0)\ll a(0)$; here $I_p(0)$ is the intensity on the beam axis. The average wavefront curvature in the $z=0$ plane is equivalent to that of a square lens of focal length F. For PC to occur, the dimensions L of the nonlinear medium must exceed substantially the Fresnel diffraction length $z_k=k\rho^2(0)$, of the transverse inhomogeneity of the field. The pump field in the bulk of the medium can therefore be assumed to have a normal distribution. The pump correlation function inside the medium can be obtained by solving Eq. (4) with boundary conditions (5):

$$\frac{cn}{2\pi}\langle\varepsilon_p(z_1,\mathbf{r}_{\perp1})\varepsilon_p^*(z_2,\mathbf{r}_{\perp2})\rangle$$
$$=I_p(0)\varkappa^{-1}\left[\left(1-\frac{z}{F}\right)^2+\frac{z_F^2}{z_g^2}\right]^{-1}$$
$$\times\exp(-\alpha_1\mathbf{r}_{\perp1}^2-\alpha_2\mathbf{r}_{\perp2}^2+2\alpha_3\mathbf{r}_{\perp1}\mathbf{r}_{\perp2}),$$
$$\alpha_{1,2}=\frac{1}{\varkappa}\left[\pm\frac{ik}{2}\frac{z_F-z_{2,1}}{z_g^2}\left(1+\frac{z_g^2}{F^2}\right)\right.$$
$$\left.+\frac{k}{4}\left(\frac{1}{z_k}+\frac{1}{z_0}\right)\right]\left(1+\frac{z_0^2}{F^2}\right), \quad (6)$$
$$\alpha_3=\frac{1}{\varkappa}\frac{k}{4}\left(\frac{1}{z_k}-\frac{1}{z_0}\right)\left(1+\frac{z_g^2}{F^2}\right),$$
$$\varkappa=1+\frac{1}{z_g^2}(z_F-z_1)(z_F-z_2)\left(1+\frac{z_g^2}{F^2}\right)$$
$$-\frac{i}{2}(z_2-z_1)\left(\frac{1}{z_k}+\frac{1}{z_g}\right)\left(1+\frac{z_g^2}{F^2}\right),$$

where $z_F=Fz_g^2/(F^2+z_g^2)$ is plane of the focal neck; $z_g=k\rho(0)a(0)$ is the envelope diffraction-broadening length obtained when the input beam is collimated; $z_k=k\rho^2(0)$, $z_0=ka^2(0)$. From (6) we obtain the average intensity distribution $I_p(z,\mathbf{r}_\perp)$ in the bulk of the medium, and the change of the envelope radius $a(z)$ and of the pump transverse-correlation radius $\rho(z)$ as functions of z:

$$\langle I_p(z,\mathbf{r}_\perp)\rangle=I_p(z)\exp\left[-\frac{\mathbf{r}_\perp^2}{a^2(z)}\right],$$
$$I_p(z)=I_p(0)\left[\left(1-\frac{z}{F}\right)^2+\frac{z^2}{z_g^2}\right]^{-1},$$
$$a^2(z)=a^2(0)\left[\left(1-\frac{z}{F}\right)^2+\frac{z^2}{z_g^2}\right],$$
$$\rho^2(z)=\rho^2(0)\left[\left(1-\frac{z}{F}\right)^2+\frac{z^2}{z_g^2}\right].$$

We introduce an orthogonal system of modes $\varepsilon_j(z,\mathbf{r}_\perp)$, each of which is proportional to $\varepsilon_p^*(z,\mathbf{r}_\perp)$ and has an envelope $f_j(z,\mathbf{r}_\perp)$ that is smooth on the scale ρ of the transverse inhomogeneity of the field, i.e.,

$$\varepsilon_j(z,\ \mathbf{r}_\perp)=f_j(z,\ \mathbf{r}_\perp)\varepsilon_p^*(z,\ \mathbf{r}_\perp).$$

Since the envelope of a reversed Stokes beam formed in the field of a pump beam with Gaussian envelope is likewise nearly Gaussian (but of smaller radius), we choose the zeroth mode in the form

$$\varepsilon_0=f_0\varepsilon_p^*,\qquad f_0(z,\ \mathbf{r}_\perp)=\exp[-{}^1/{}_2\alpha^2\mathbf{r}_\perp^2/a^2(z)],$$

where α is a parameter chosen such that ε_0 is closest to the minimum-gain structure. We define next the succeeding modes ε_j by the recurrence relation[12]

$$\varepsilon_{j+1}=\frac{\langle I_p(z,\mathbf{r}_\perp)\rangle}{I_p(z)}\varepsilon_j-\sum_{k=0}^{j}A_{kj}\varepsilon_k,$$
$$A_{kj}=\frac{1}{P_k}\int d^2\mathbf{r}_\perp\frac{\langle I_p(z,\mathbf{r}_\perp)\rangle}{I_p(z)}\varepsilon_k^*\varepsilon_j, \quad (7)$$

thereby ensuring orthogonality of the modes:

$$\int \varepsilon_j\varepsilon_k^* d^2\mathbf{r}_\perp = P_k\delta_{kj}$$

(i.e., P_k is the power of the k th mode) and the matrix A_{kj} has the tridiagonal form

$$A_{kj} = A_{k,k-1}\delta_{k,j+1} + A_{k,k}\delta_{k,j} + A_{k,k+1}\delta_{k,j-1}. \quad (8)$$

In other words, the modes ε_j are matched with the inhomogeneous local gain $gI_p(z, \mathbf{r}_\perp)$ induced in the medium by the pump beam in such a way that on scattering of any mode the gain profile excites only the three neighboring modes $\varepsilon_{j-1}, \varepsilon_j \varepsilon_{j+1}$. This circumstance simplifies the subsequent calculations substantially.

We express the mode ε_j in terms of ε_0:

$$\varepsilon_j = \sum_{k=0}^{j} b_{j,k} \exp\left[-k\frac{\mathbf{r}_\perp^2}{a^2(z)}\right]\varepsilon_0.$$

Substituting this expression in (7), we obtain for the coefficients the recurrence relation

$$b_{j,k} = b_{j-1,k-1} - A_{j-1,j-1}b_{j-1,k} - A_{j-2,j-1}b_{j-2,k},$$

with $b_{00} = 1$ and $b_{j,k} = 0$ at $k > j$ and $k < 0$. The relation between $A_{j,k}$ and $b_{j,k}$ can therefore be written in the form

$$A_{j,k} = 2\frac{P_p}{P_j}\sum_{n=0}^{j}\sum_{m=0}^{k}\frac{b_{j,n}b_{k,m}}{2+m+n+\alpha^2}, \quad P_p = \int |\varepsilon_p|^2 d^2\mathbf{r}_\perp.$$

It can be easily seen that $A_{jj+1} = 1$, and $A_{jj} = P_{j+1}/P_j$. Using these relations and specifying the value of the parameter α (i.e., defining the mode ε_0), we can construct the entire system of the modes ε_j.

We introduce the variable coordinate $\eta = t + z/v_s$, connected with the Stokes wave; instead of (2) and (3) we have then

$$\hat{L}E_s = -\frac{\partial E_c}{\partial z} + \frac{i}{2k}\Delta_\perp E_s = \frac{i}{2}g_2 E_p Q^*, \quad (9)$$

$$\frac{\partial Q}{\partial \eta} + \frac{Q}{T_2} = ig_3 E_p E_s^* + f_n. \quad (10)$$

We represent the polarization $\mathscr{P}_s = 1/2 ig_2 E_p Q^*$ that causes the excitation and amplification of the Stokes wave as a sum of two orthogonal terms:

$$\mathscr{P}_s = \frac{i}{2}g_2 E_p Q^* = \sum_{j=0}^{N} C_j(z,\eta)\varepsilon_j(z,\mathbf{r}_\perp) + \tilde{\mathscr{P}}_s, \quad (11)$$

with

$$\int d^2\mathbf{r}_\perp \varepsilon_j^* \tilde{\mathscr{P}}_s = 0$$

for all $j \in [0, N]$. The sum in (11) is that polarization term which causes excitation and amplification of the fastest growing structure in the Stokes wave whose front is reversed towards the wave. The part of $\tilde{\mathscr{P}}_s$ orthogonal to the sum causes excitation of all the other Stokes-wave components. We accordingly resolve the field E_s into two components, $E_s = E + \tilde{E}_s$, and define them, using (9), (10), and (11), by the equations

$$\hat{L}E = \sum_{j=0}^{N} C_j\varepsilon_j, \quad (12)$$

$$\hat{L}\tilde{E}_s = \tilde{\mathscr{P}}_s. \quad (13)$$

Multiplying (11) by ε_j and integrating over the cross section, we obtain

$$C_j = \frac{1}{P_j}\int \mathscr{P}_s\varepsilon_j^* d^2\mathbf{r}_\perp = \frac{i}{2}\frac{g_2}{P_j}\int d^2\mathbf{r}_\perp \varepsilon_j Q^* E_p \quad (14)$$

Expressing the solution (10) and (12) in integral form and substituting in (14), we find

$$C_j(z,\eta) = -\varphi(\eta - \nu z)\int_{\nu z}^{\eta} d\eta' \varphi^*(\eta' - \nu z)$$

$$\times \exp\left(-\frac{\eta-\eta'}{T_2}\right)\int_L^z dz' \sum_k B_{jk}C_k(z',\eta')$$

$$-C_j^{(0)}(z,\eta) - \frac{g_1 g_2}{2P_j}\varphi(\eta - \nu z)$$

$$\times \int_{\nu z}^{\eta} d\eta' \varphi^*(\eta' - \nu z)\exp\left(-\frac{\eta-\eta'}{T_2}\right)\int d^2\mathbf{r}_\perp |E_p|^2 \varepsilon_j \tilde{E}_s, \quad (15)$$

where

$$B_{jk} = \frac{g_2 g_3}{2P_j}\int d^2\mathbf{r}_\perp |E_p|^2 \varepsilon_j^*(z,\mathbf{r}_\perp)\hat{G}_{z'}\varepsilon_k(z',\mathbf{r}_\perp'), \quad (16)$$

$$C_j^{(0}(z,\eta) = \frac{g_2 g_3}{2P_j}\int_{\nu z}^{\eta} d\eta' \varphi^*(\eta' - \nu z)\exp\left(-\frac{\eta-\eta'}{T_2}\right)$$

$$\times \int d^2\mathbf{r}_\perp |E_p|^2 \varepsilon_j^* \hat{G}_{zL}^* E(L,\mathbf{r}_\perp') + \frac{ig_2}{2P_j}\varphi(\eta - \nu z)$$

$$\times \exp\left(-\frac{\eta - \nu z}{T_2}\right)\int d^2\mathbf{r}_\perp E_p \varepsilon_j^* Q^*(t=0) + \frac{ig_2}{2P_j}\varphi(\eta - \nu z)\int_{\nu z}^{\eta} d\eta'$$

$$\times \exp\left(-\frac{\eta-\eta'}{T_2}\right)\int d^2\mathbf{r}_\perp E_p f_n^* \varepsilon_j, \quad (17)$$

$$\hat{G}_{zz'} = \int_{-\infty}^{\infty} d^2\mathbf{r}_\perp' \frac{ik}{2\pi(z-z')}\exp\left[-\frac{ik}{2(z-z')}(\mathbf{r}_\perp - \mathbf{r}_\perp')^2\right]\ldots \quad (18)$$

Here $\hat{G}_{zz}$ is the Green operator of the linear parabolic equation, and $\nu = 1/v_p + 1/v_s$. We note that the system (15) and (13), with allowance for (11), is fully equivalent to the initial system (2), (3).

Neglecting in (15) the last term, which is connected with the field $\tilde{E}_s$, we obtain a closed system of equations for the coefficients C_j. Numerical solution of this system enables us to investigate the enhancement of the reversed wave. Just as in the stationary case, this neglect is justified if the growth rate of the gain over the pump longitudinal-correlation length is given by

$$m_k = gI_p(0)k\rho^2(0) \ll 1 \qquad (m_k = gI_p(z)k\rho^2(z) = \text{const}).$$

The change of variables

$$\zeta=\mathrm{arctg}\left[\frac{1+(F/z_g)^2}{(F/z_g)^2}\frac{z}{z_g}-\frac{z_g}{F}\right]+\mathrm{arctg}\frac{z_g}{F},$$
$$C_j(\zeta,\tau)=C_j(z,\eta)\left[\left(1-\frac{z}{F}\right)^2+\left(\frac{z}{z_g}\right)^2\right],\quad \tau=\frac{\eta}{T_2}, \tag{19}$$

in (15) and (17) reduces (15) to a form suitable for computer analysis:

$$C_j(\eta,\tau)=-\varphi(\tau-\mu)\int_\mu^\tau d\tau'\varphi^*(\tau'-\mu)$$
$$\times\exp[-(\tau-\tau')]\int_{\zeta(0)}^{\zeta}d\zeta'\sum_{k=0}^{N}B_{jk}(\zeta,\zeta')C_k(\zeta',\tau')-C_j^{(0)}(\zeta,\tau), \tag{20}$$

$$B_{jk}(\zeta,\zeta')=\frac{m_0}{P_j}\sum_{n=0}^{j}\sum_{m=0}^{k}b_{j,n}b_{k,m}$$
$$\times\left[(2+m+n+\alpha^2)+\left(1+n+\frac{\alpha^2}{2}\right)\left(m+\frac{\alpha^2}{2}\right)\sin^2(\zeta-\zeta')\right]^{-1}, \tag{21}$$

where

$$m_0=gI_p(0)z_g,\quad \mu=\nu\frac{z_g}{T_2}\frac{F^2}{F^2+z_g^2}\left[\mathrm{tg}\left(\zeta-\mathrm{arctg}\frac{z_g}{F}\right)+\frac{z_g}{F}\right], \tag{22}$$

$$C_j^{(0)}(\zeta,\tau)=\int_\mu^\tau d\tau'\varphi^*(\tau'-\mu)\exp[-(\tau-\tau')]\sum_{k=0}^{N}\sigma_k(\zeta(0))$$
$$\times B_{jk}(\zeta,\zeta(0))+\varphi(t-\mu)\exp[-(\tau-\mu)]\sum_{k=0}^{N}\sigma_k'B_{jk}(\zeta,\zeta)$$
$$+\varphi(\tau-\mu)\{1-\exp[-(\tau-\mu)]\}\sum_{k=0}^{N}\sigma_k''B_{jk}(\zeta,\zeta).$$

The coefficients $\sigma_k(L)$, σ_k', σ_k'' determine the components, in the noise source, of the initial and boundary conditions for the chosen modes:

$$f_n^*(\eta,z,\mathbf{r}_\perp)=ig_3E_p^*\sum\sigma_k''\varepsilon_k(z,\mathbf{r}_\perp)+\ldots,$$
$$Q^*(t=0,z,\mathbf{r}_\perp)=ig_3T_2E_p^*\sum\sigma_k'\varepsilon_k(z,\mathbf{r}_\perp)+\ldots, \tag{23}$$
$$E(L,\mathbf{r}_\perp)=\sum\sigma_k(L)\varepsilon_k(L,\mathbf{r}_\perp)+\ldots.$$

Solving (20) and finding $C_j(z,t)$ we can use (12) to express the field I in any plane z:

$$E(z,\mathbf{r}_\perp)=\hat{G}_{zL}E(L,\mathbf{r}_\perp')+\int_L^z dz'\hat{G}_{zz'}\sum_{j=0}^{N}C_j(z',t)\varepsilon_j(z',\mathbf{r}_\perp'). \tag{24}$$

Representing E in the form

$$E=\sum_{j=0}^{N}\sigma_j(z)\varepsilon_j(z,\mathbf{r}_\perp)+E'(z,\mathbf{r}_\perp),\ \int d^2\mathbf{r}_\perp E'\varepsilon_j^*=0, \tag{25}$$

we easily calculate the components of the field E for the modes ε_j:

$$\sigma_j=\frac{1}{P_j}\int E\varepsilon_j^*d^2\mathbf{r}_\perp$$

and determine the form of the Stokes wave reversed towards the pump

$$\sum\sigma_j\varepsilon_j=\sum\sigma_jf_jE_p^*=f(z,\mathbf{r}_\perp)E_p^*(z,\mathbf{r}_\perp),$$

where

$$f(z,\mathbf{r}_\perp)=\sum_{j=0}^{N}\sigma_j(z)f_j(z,\mathbf{r}_\perp)$$

is a function of the transverse coordinates that is slow in terms of $\rho(z)$. The power fE_p of the pump-conjugated beam

$$P_\sigma=\sum_{j=0}^{N}|\sigma_j|^2P_j$$

at gain growth rates typical of the SBS process, is close to the power $P=\int|E|^2d^2\mathbf{r}_\perp$ of the field E (see the quantity $\chi=P_\sigma/P$ in Table I below). The component E' contains waves that are not correlated with the pump and are due to diffraction of the reversed Stokes beam, which is narrower than the pump beam; this component constitutes a small fraction ($1-\chi\approx0.1$) of the field power E.

To explore the conditions for the onset of PC, the power $\tilde{E}_s$ of the Stokes-wave component not conjugate to the pump must be calculated. One source of $\tilde{E}_s$ waves that is not correlated with $\tilde{E}_p{}^*$ is the scattering by the conjugate wave itself from the inhomogeneities of the gain. Their power is proportional to the reversed-structure power $P_6(z,t)$ and to the parameter m_k, so that they can be neglected if $m_k\ll1$.[14]

A contribution to the nonreversed field $\tilde{E}_s$ is made also by waves that are not correlated with the pump and are launched by initial thermal fluctuations. To calculate their growth rate it suffices to consider the gain of a certain component $\tilde{\varepsilon}$ ($\langle\varepsilon_p{}^*\tilde{\varepsilon}\rangle=0$); this component propagates within the confines of the pump wave, is not correlated with the latter, is orthogonal to the modes ε_j, and satisfies the equation $\hat{L}\tilde{\varepsilon}=0$. Representing the polarization $\tilde{\mathscr{P}}_s$ in the form

$$\tilde{\mathscr{P}}_s=\tilde{C}(z,t)\tilde{\varepsilon}(z,r_\perp)+\mathscr{P}_s$$

we obtain for $\tilde{C}(z,t)$, in terms of the variables ζ and τ, an equation similar to one of the equations of (20):

$$\tilde{C}(\zeta,\tau)=-\varphi(\tau-\mu)\int_\mu^\tau d\tau'\varphi^*(\tau'-\mu)$$
$$\times\exp[-(\tau-\tau')]\int_{\zeta(0)}^{\zeta}d\zeta'B\tilde{C}(\zeta',\tau')-\tilde{C}^{(0)}(\zeta,\tau), \tag{26}$$

where $B=m_0/2=\mathrm{const}(\zeta)$, $C^{(0)}$ just as $C_j^{(0)}$ in (20), is determined by the projections of the initial conditions on the structure $\tilde{\varepsilon}$, see (23). Equation (26) has an analytic solution in terms of Bessel functions, from which it follows that the

TABLE I. Calculated values for PC in focused beams.

L	F	α	χ	χ_0	χ_1	χ_2	χ_3
0,2	0,1	0	0,884	0,401	0,457	0,127	0,015
0,2	0,1	3	0,876	0,994	$3{,}25\cdot10^{-3}$	$2{,}69\cdot10^{-3}$	$5{,}66\cdot10^{-4}$

growth rate $\bar{M}$ of the intensity of the waves that are not correlated with the pump can be written in the form $(\nu = 0)$

$$\bar{M}(\zeta,\tau)=\begin{cases} m-2\tau, & \tau<{}^1/_2 m_0\zeta,\\ m_0\zeta, & \tau\geq{}^1/_2 m_0\zeta,\end{cases}$$
$$m=2\left[2m_0\zeta\int_0^\tau|\varphi(\tau')|^2d\tau'\right]^{1/2}. \qquad (27)$$

Analysis of the PC for nonstationary SBS in a light guide likewise reduces to consideration of equations similar to (26), with an integrand kernel independent of the longitudinal coordinate, and having a similar analytic solution. In this case the growth rates $M_{\rm lg}$ of the reversed wave and $\bar{M}_{\rm lg}$ for the uncorrelated components are respectively $(\nu = 0)$

$$M_{\rm lg}=\begin{cases}\sqrt{2}\,m_{\rm lg}-2t_{\rm pul}/T_2, & t_{\rm pul}\leq gI_p(0)LT_2,\\ 2gI_p(0)L, & t_{\rm pul}\geq gI_p(0)LT_2,\end{cases} \qquad (28)$$

$$\bar{M}_{\rm lg}=\begin{cases}m_{\rm lg}-2t_{\rm pul}/T_2, & t_{\rm pul}\leq{}^1/_2 gI_p(0)LT_2,\\ gI_p(0)L, & t_{\rm pul}\geq{}^1/_2 gI_p(0)LT_2,\end{cases} \qquad (29)$$

where

$$m_{\rm lg}=2\left[2gI_p(0)L\frac{1}{T_2}\int_0^{t_{\rm pul}}|\varphi(t')|^2dt'\right]^{1/2}.$$

Here $I_p(0)$ is the pump intensity, uniform over the cross section and constant along z in a light guide of length L. The excess $\delta_{\rm lg} = M_{\rm lg}/\bar{M}_{\rm lg}$ of one growth rate over the other increases with increasing pulse duration from $\sqrt{2}$ to 2 in the stationary SBS regime at $t_{\rm pul} \geqslant gI_p(0)LT_2$. It follows from (28) and (29) that at a given growth rate $\bar{M}_{\rm lg}$ = const (the SBS sets in at a growth-rate threshold $M_{\rm thr} \approx 20$–30) the ratio of the reversed-wave power $P_{\rm rev}$ to the power P_{uc} of the uncorrelated waves decreases with decreasing $t_{\rm pul}$ under nonstationary scattering conditions like

$$P_{\rm rev}/P_{\rm unc}\propto\exp(M_{\rm lg}-\bar{M}_{\rm lg})$$
$$=\exp[(\sqrt{2}-1)\bar{M}_{\rm lg}]\exp[2(\sqrt{2}-1)t_{\rm pul}/T_2], \qquad (30)$$

realization of the SBS effect becomes more difficult when the pump pulse duration is shortened.

When the time of light passage through the medium becomes comparable with the pulse duration, it is incorrect to assume that $\nu = 0$. An analytic solution for $\nu \neq 0$ can be found only for a rectangular pump pulse ($\varphi(t) = 1$, $t \geqslant 0$). The following equations are obtained in that case for the growth rates: at $L \leqslant t_p/2$

$$M_{\rm lg}\approx2\left[4gI_p(0)L\frac{t_{\rm pul}-\nu L}{T_2}\right]^{1/2}-2\frac{t_{\rm pul}-\nu L}{T_2},$$
$$\bar{M}_{\rm lg}\approx2\left[2gI_p(0)L\frac{t_{\rm pul}-\nu L}{T_2}\right]^{1/2}-2\frac{t_{\rm pul}-\nu L}{T_2}; \qquad (31)$$

and at $L > t_{\rm pul}/2$

$$M_{\rm lg}\approx t([4gI_p(0)/\nu T_2]^{1/2}-1/T_2),$$
$$\bar{M}_{\rm lg}\approx t([2gI_p(0)/\nu T_2]^{1/2}-1/T_2). \qquad (32)$$

1.2. Discussion of numerical results

Equations (20) and (26) were solved numerically by iteration for $\nu = 0$ and for various parameters of the problem. An additional criterion for correct computation was the ability to compare the calculated and analytic solutions of Eq. (26). After finding $C_j(z, t)$ and $\tilde{C}(z, t)$, we calculated the coefficients $\sigma_j(z, t)$ and then the powers P and P_σ and the growth rates M and $\bar{M}$ of the conjugate and uncorrelated waves, the ratio $\chi = P_\sigma/P$, and also the relative weight $\chi_j = |\sigma_j|^2P_j/p_\sigma$ of each mode ε_j in the power P_σ. Calculations with $\alpha = 0$, allowing for a sufficient number of modes (N = 4–6), have shown that a reversed Stokes beam, with a well defined ratio of the quantities χ_j which is practically independent of z, is produced independently of the initial conditions (i.e., of σ_j', σ_j'' or $\sigma_j(L)$) only if the length L of the medium exceeds the length of the focal region of the pump beam. The ratio changes with time so as to make the conjugate-wave envelope narrower. The choice of a value $\alpha \neq 0$ can make only one zeroth mode ε_0 predominant in the power P_σ. Table I lists for $m_0 = 32$, $t_{\rm pul}/T_2 = 1$, $\varphi(t) = 1$, the results of calculations at $\bar{M} \approx 25$; the values of χ and χ_j are given for the Stokes wave as it leaves the medium ($z = 0$) at $t = t_{\rm pul}$. The ratio $\chi = p_\sigma/P$ is found to be close to unity. A deviation of χ from unity, however, means that even if the contribution of the component $\tilde{E}_s$ to the scattered field is neglected, the PC in SBS of the focused beams can still not be ideal, but the accuracy is higher than at $\chi \approx 0.9$.

In Fig. 1 are plotted the increases of the growth rates in a focused beam, $\delta_{\rm foc} = M/\bar{M}$, and in a light guide, $\delta_{\rm lg} = M_{\rm lg}/\bar{M}_{\rm lg}$, vs the total growth rate $\bar{M}$ of uncorrelated waves at $t_{\rm pul}/T_2 = 1$ and vs the relative duration $t_{\rm pul}/T_2$ at a constant total growth rate $\bar{M} = 25$, for a rectangular pump pulse. It can be seen that for SBS the difference between the growth rates in a light guide decreases with increasing $\bar{M}$ at constant $t_{\rm pul}$, tending to $\sqrt{2}$, whereas for focused beams both relative growth rates increase and likewise tend to $\sqrt{2}$. The reason is the narrowing of the envelope of the generating structure with increasing growth rate, so that the gain of the

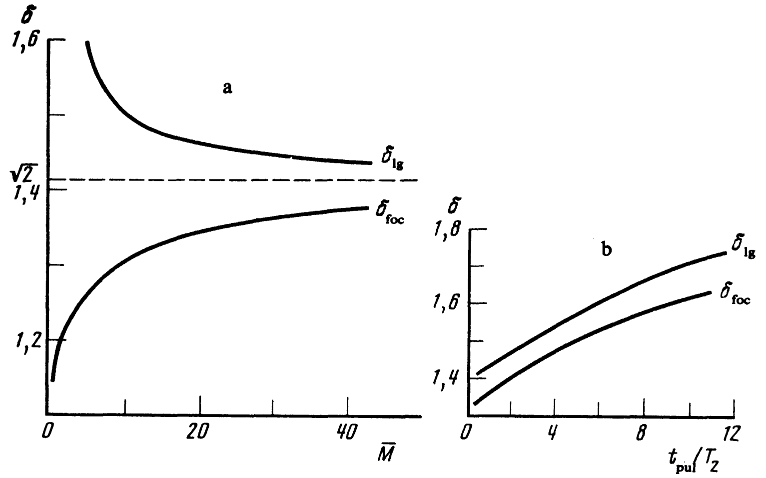

FIG. 1. Excess of the growth rate for SBS in a light guide (δ_{lg}) and in a focused beam (δ_{foc}) as functions of the growth rate $\bar{M}$ at $t_{pul} = T_2$ (a) and on t_{pul}/T_2 at $\bar{M} = 25$ (b).

structure approaches the gain in the light guide. A characteristic feature of the excesses above the growth rates as functions of $T_{pul}/T2$ is that when the latter increases the values of δ_{foc} and δ_{lg} increase monotonically and approach values corresponding to the stationary SBS, viz., $\delta_{lg}^{st} = 2$ and $\delta_{foc}^{st} \approx 1.75$ (at $\bar{M} = 25$).

An excess of M over $\bar{M}$ does not automatically guarantee the presence of PC, since the relative fraction of the pump-correlated structure in the initial Stokes noise wave is very small. For PC to occur it is necessary that the power P_σ of the pump-conjugated wave exceed the power $\tilde{P}_{unc}$ of the components that are not correlated with the pump. Calculating the angle $\tilde{\theta}$ and the aperture $\tilde{a}$ within which effective amplification of waves uncorrelated with the pump takes place, and estimating the relative weight of the stasting reversed-wave in the initial power of the uncorrelated waves, as was done in the analysis of stationary SBS,[11] we obtain the condition for observing the PC effect ($P_\sigma/\tilde{P}_{unc} \gtrsim 1$) in focused beams, in the form

$$\frac{\theta}{\theta_d} \leqslant \left(\frac{\theta}{\theta_d}\right)_{cr} = \frac{\bar{M}}{\sqrt{10}} \exp\left[\frac{\bar{M}}{2}(\delta-1)\right], \qquad (33)$$

where $\theta = 2/k\rho(0)$ and $\theta_d = 2/ka(0)$ are the real and diffractive divergences of the pump beam at the entrance to the medium. Using this condition and the functions $\delta(\bar{M})$ and $\delta(t_{pul}/T_2)$, we can easily separate those θ/θ_d regions in which PC is possible. Figure 2 show plots of the critical ratio $(\theta/\theta_d)_{cr}$ vs the growth rate $\bar{M}$ or vs the relative pulse duration t_{pul}/T_2 ($\varphi(t) = 1$). These plots are found to be close to exponential (for a light guide this follows from (28) and (29)). It can be seen that a small difference between the threshold SBS growth rates determined by the parameters of the medium leads to a substantial difference in $(\theta/\theta_d)_{cr}$.

The inequality (33) states the power condition for the onset of PC, whereas in experiments the criterion for this

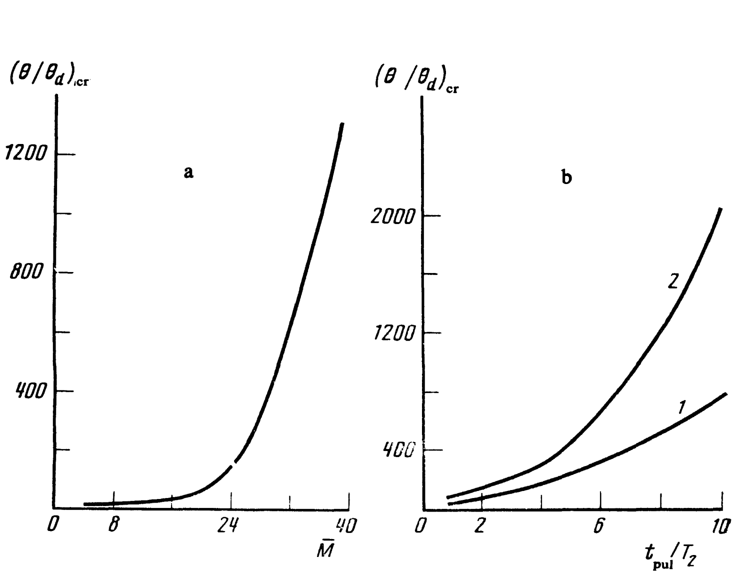

FIG. 2. Critical ratio $(\theta/\theta_d)_{cr}$ of the beam divergences at which the PC shuts off vs the growth rate M at $t_{pul} = T_2$ (a) and vs the relative pump pulse duration t_{pul}/T_2 at two growth rates, $\bar{M} = 20$ (1) and $\bar{M} = 23$ (2).

effect is usually an excess of the reversed-structure energy over the energies of the waves that are uncorrelated with the pump. Calculations show that according to the energy conditions for its onset, the PC terminates at values of $(\theta/\theta_d)_{cr}$ that are approximately 1.3–1.5 times smaller than would follow from (33). The condition that the growth rate be small over the pump longitudinal-correlation length, $m_c \ll 1$, leads to a lower bound on the allowed divergence ratios:

$$\theta/\theta_d > (\overline{M} + 2t_{pul}/T_2)/8\pi t_{pul}/T_2. \quad (34)$$

This restriction becomes significant at $t_{pul}/T_2 < 1$. It follows from comparison of inequalities (33) and (34) that the allowed range of θ/θ_d becomes narrower with increasing pump-pulse duration, so that for SBS it is practically impossible to obtain PC of spatially inhomogeneous light beams at $t_{pul}/T_2 \lesssim 0.2$.

We have also carried out numerical calculations for nonrectangular pump pulses, viz., of Gaussian form and of the form typical of a photoionization CO_2 laser. We found that at equal values of the growth rate $\overline{M}$ the conditions for producing PC depended little on the pulse waveform.

2. EXPERIMENTAL INVESTIGATION OF NONSTATIONARY SBS OF FOCUSED BEAMS

The apparatus used for the investigations is similar to that reported in Ref. 6. The duration of the $\lambda = 1.06$ μm pulse could be varied in the range 0.5–20 ns, the spatial profile of the beam was close to rectangular, with radius $a \approx 0.5$ cm, and its divergence was close to diffractive, $\theta_d = 1.22\lambda/a = 0.26$ mrad. In the investigation of SBS in beams with strong spatial inhomogeneity the parameter θ/θ_d was varied with a set of etched-glass plates that introduced into the beam a divergence $\theta = 3$–40 mrad, and also by varying θ_d to increase the size of the beam spot on the etched plate with the aid of a diverging lens. Corrections were investigated for both small-scale and large-scale aberrations such as astigmatism.

The criterion for the quality of the PC was the parameter W_θ/W, (W and W_θ are respectively the total energy of the reflected radiation and the energy in an angle $\theta = 0.29$ mrad, which is close to the diffraction angle) normalized to its value (≈ 0.8) for the initial pump beam. To determine the PC quality more accurately in individual flashes, the spatial structures of the incident and reflected beams were photographed in the near and far zones.

The SBS-active media used were substances with various hypersound-relaxation times T_2, viz., argon at a pressure $p = 64$ atm ($T_2 \approx 32$ ns, Ref. 15), GLS-1 glass ($T_2 \approx 5$ ns, Ref. 16), acetone ($T_2 \approx 3$ ns, Ref. 17), and carbon tetrachloride ($T_2 \approx 1$ ns, Refs. 17 and 18). The lens focal length F was chosen, on the one hand, to prevent breakdown of the medium, and other hand to make the focal length shorter than the cell. To eliminate the influence of the group delay over the length of the focal neck, the condition $F^2\theta/a \langle ct_{pul}/2$ was met.

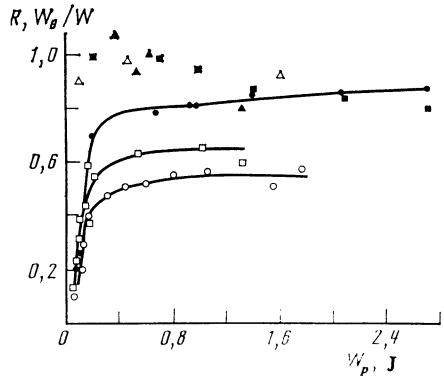

FIG. 3. Reversal parameter W_θ/W(■, ▲, △) and reflection coefficient R (●, □, ○) for SBS of a spatially homogeneous beam in Ar ($p = 64$ atm) vs pump energy at $t_{pul} = 20$ (●, ■), 10 (○, △) and 5 ns (□, ▲).

2.1 Beams with quasihomogeneous intensity distribution

In contrast to Ref. 5, where $t_{pul} \approx T_2$, time-dependent SBS in argon was investigated in the range $t_{pul}/T_2 = 0.15$ to 1. The gas used, having a somewhat smaller stationary growth rate than xenon, has a lower T_2. In the time-dependent case the growth rates become equalized if the pulses are short enough, and furthermore argon has higher optical strength. It is also more suitable for use than polyatomic gases such as SF_6, N_2, and others, since pumping by short pulses excites stimulated Raman scattering in the latter.[19]

Figure 3 shows the measured reflection coefficient $R = W/W_p$ and the reversal parameter W_θ/W for three pulse lengths 20, 10, and 5 ns, and for a spatially homogeneous beam focused by a lens with $F = 110$ cm into a cell 200 cm long. No gas breakdown occurs at this focal length of the lens, as shown by measurements of the balance of the reflected and transmitted energies. It can be seen from the figure that if the pump-energy is slightly above threshold the reversal parameter approaches unity, indicating that high quality of PC has been reached for essentially time-dependent SBS. As the energy is increased, just as for liquids,[6] the accuracy of the WFR decreases somewhat (but not so strongly). For $t_{pul} = 5$ ns and small excess above threshold, the parameter W_θ/W is somewhat larger than unity, apparently because of spatial filtering of the sideband diffraction peaks in the focal region.[20]

Note that when the duration t_{pul} is decreased the threshold energy remains approximately at the same level $W_p \approx 30$ mJ, while the threshold power increases in proportion to t_{pul} in accordance with the theory. This prevented us from effectively reflecting pulses having $t_{pul} < 2$ ns, since the gas breaks down even at $R = 200$ cm, when the length of the focal region becomes comparable with the spatial length of the pulse.

Similar results were obtained also in another nonlinear medium (GLS-1 glass) for 10–/20 ns pulses (see also Ref. 21). When the radiation was focused by a lens of $F = 110$ cm into a sample 63 cm long the reflection coefficient reached $R = 0.9$ at approximately 30 times threshold ($W_P^{thr} \approx 30$ mJ at $t_{pul} = 20$ ns). In the entire energy range, the reversal pa-

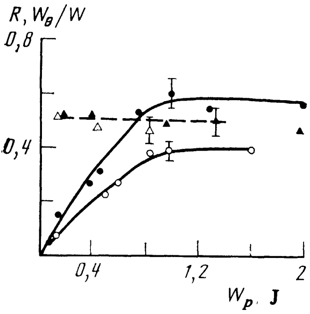

FIG. 4. Reversal parameter W_θ/W (▲, △) and reflection coefficient R(●, ○) vs the pump energy W_p for SBS of a spatially inhomogeneous beam with divergence $\theta = 4$ mrad in Ar at $t_{pul} = 20$ ns (▲, ●) and 5 ns (△, ○).

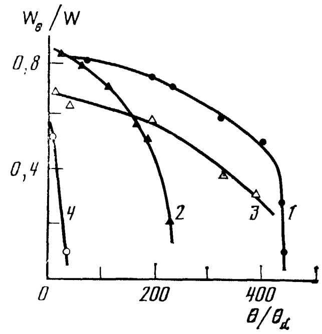

FIG. 5. Reserval parameter W_θ/W vs the relative beam divergence for CCl_4 [$t_{pul}/T_2 = 10$ (1)], GLS-1 glass [$t_{pul}/T_2 \approx 2$ (2)], acetone [($t_{pul}/T_2 \approx 5$ (3)], and Ar [$t_{pul}/T_2 \approx 0.63$ (4)].

rameter W_θ/W was close to unity, larger than for nonlinear liquids such as acetone and CCl_4, especially at high pump values.[6,22] These data offer evidence that the other nonlinear effects that compete with SBS and lower the PC quality are small. Even in glass, however, effective reflection of pulses with $t_{pul} < 5$ ns was impossible, since self-focusing of the radiation produced damage tracks in the bulk of the sample.

The possibility of correcting continuous aberrations, such as the astigmatism introduced into the beam by cylindrical lenses of 0.25, 0.5 and 0.75 diopter, was also investigated for nonstationary SBS in argon ($t_{pul} = 20$ ns). The maximum values obtained in these three cases were $R = 0.8$, 0.7, and 0.5, respectively, for the reflection coefficients and $W_\theta/W = 0.62$, 0.4, and 0.12 for the reversal parameters, close to the corresponding values obtained in the stationary case for distortions of similar type.[22] Note that it is difficult to correct the astigmatism for shorter pulses ($t_{pul} < 5$ ns) because of the group delay on the interaction line, where the amplitude-phase structure of the beam changes substantially.

2.2 PC *of beams with small-scale spatial variations*

Figure 4 shows the dependences of the reflection coefficient and of the reversal parameter W_θ/W on the pump energy for SBS of a spatially inhomogeneous beam with $\theta = 4$ mrad in argon at pulse durations 20 and 5 ns. It can be seen that at $\theta/\theta_d \approx 15$ the reversal parameter attained is quite high, comparable with W_θ/W in the stationary case,[22] if the reflection coefficient is high. The reversal accuracy is practically independent of the excess of the pump power above threshold. When θ/θ_d was tripled the PC practically vanished, and its parameter dropped approximately to 0.1 at a reflection coefficient $R \approx 0.1$. This decrease of the reflection coefficient of strongly divergent radiation can be used for spatial filtering of the narrow component.[20] We verified this by obstructing half the pump beam by an etched plate subtending = 16 mrad. The measurements have shown that 80% of the energy in the reflected radiation had good angular directivity, whereas in the beam incident on the mirroring SBS the fraction of the wide-angle component was 50%. A plot of the reflected-beam structure has shown that the wide-angle component is reflected only in the cone formed by the narrow beam on focusing into the nonlinear medium.

The dependence of the reversal parameter W_θ/W on the excess θ/θ_d, of the pump divergence over the diffractive one was measured for substances with different values of T_2 at equal pulse lengths. The measurement results are shown in Fig. 5. The plots for glass and CCl_4 are similar, but that for acetone is smoother. This may be due to the onset in the acetone of other nonlinear processes that compete with the SBS.[22] It can be seen from Fig. 5 that the PC terminates quite abruptly at some definite value of θ/θ_d.

By assigning the level $W_\theta//W = 0.5$, we can determine the values of $(\theta/\theta d)^{exp}_{cr}$. These are listed in Table II. To compare the experiment with theory we calculated, starting with an estimate of the ambient spontaneous-noise level, the threshold growth rates $\overline{M}$ for each actual medium. Next, assigning simulation parameters consistent with the experimental situation, we determined $(\theta/\theta d)_{cr}$ from the energy

TABLE II. Theoretical and experimental values of the critical divergence ratio for different pulse durations.

Medium	$\frac{t_p}{T_2}$	$\overline{M}$	$\left(\frac{\theta}{\theta_d}\right)^{theor}_{cr}$	$\left(\frac{\theta}{\theta_d}\right)^{exp}_{cr}$
Ar (p=64 atm)	0,6	22	80	20
GLS1	2	22,5	240	190
Acetone	5	19,5	325	275
CCl_4	10	18,5	420	410

condition for observing the PC effect. Table II shows fair agreement between the theoretical and experimental values of $(\theta/\theta_d)_{cr}$.

CONCLUSION

It has thus been shown that there is practically no PC for time-dependent SBS of focused spatially inhomogeneous beams if $t_{pul}/T_2 \lesssim 0.2$. For single-mode beams, the PC accuracy is practically independent of the pulse duration t_{pul}. Measurements at 1.06 μm, however, have shown that the minimum pulse length ($t_{pul}^{min} \approx 1$ ns) that can be reflected by an SBS mirror with PC is determined not by this condition, but by the competition of other nonlinear effects, primarily by breakdown of the optical medium and by self-focusing of the radiation. Special measures are needed to obtain PC of shorter pulses, e.g., shaping of the pulse,[23] realization of zero-threshold reflection,[24] or compression of the pulse.

It follows from the calculation that to improve the PC conditions for nonstationary SBS of focused beams it is better to use a system with a preliminary SBS amplifier or take special measures to raise the threshold of the SBS growth rate. These measures ensure additional preferred enhancement of the reversed Stokes wave.

[1)]A field represented in this form refers to radiation with a stationary wavefront, which is encountered as a rule in many laser-optics problems.

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