Energy distribution of electrons interacting with dispersion-free phonons and with radiation

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An analysis is made of a steady-state distribution of nondegenerate free electrons which absorb radiation and then emit spontaneously a cascade of dispersion-free optical phonons, and of the transient process which establishes such a distribution. It is shown that monochromatic radiation induces relaxational oscillations of the electron distribution, and that the period and decrement of such oscillations depend strongly on the ratio of the radiation Ω and phonon ω_0 frequencies (it is assumed that $\Omega/\omega_0 = p + \omega$, where $p = 1, 2, \ldots; 0 < \omega < 1$). For an integral phonon-photon resonance ($\omega = 0$) there is no transient process at all. In the case of fractional resonances ($\omega = M/N$; M and N are integers) with low values of N the relaxation process is short-lived; on increase in N, the oscillation period rises proportionally to N and the decay time proportionally to N^2, i.e., the transient process becomes increasingly longer. An allowance for a weak energy relaxation in the case of a quasielastic interaction with acoustic phonons may have a considerable influence on the transient process and on the steady-state distribution. It is shown that peaks of a distribution of the Kumekov-Perel' type are obtained not only near an integral resonance, but also for small offsets from fractional resonances when a system of N quasiequidistant peaks appears in the distribution. A description is given of a possible way of controlling the localization of such peaks in the case of arbitrary radiation intensities by utilizing additional radiation with the opposite sign of the frequency offset relative to an integral or a fractional resonance.

1. INTRODUCTION

Absorption of infrared radiation by free carriers (we shall consider specifically electrons) at low temperatures when the optical phonons are frozen out is accompanied (when $\Omega > \omega_0$, where Ω is the radiation frequency and ω_0 is the optical phonon frequency) by rapid emission of optical phonons by a photoelectron and by dropping of this electron to the passive range of energies ($\varepsilon < \hbar\omega_0$). Under intense photoexcitation conditions the electron distribution is far from equilibrium and it exhibits a number of special features which will be studied below.

The energy transfer involving electrons in the passive region (Fig. 1a) is described by a difference equation with a given dimensionless transition energy $\omega = \Omega/\omega_0 - [\Omega/\omega_0]$; here and later the square brackets in similar expressions will denote the integer part; energy will always be measured in units of $\hbar\omega_0$. Two cases are possible: commensurable ($\omega = M/N$ is a rational irreducible fraction) and incommensurate (ω is irrational). In the former case (Fig. 1b) the electrons initially localized at an energy $\varepsilon' < 1$ return to the previous state after N transitions. The electrons then cross N "levels" in the passive region and become distributed diffusely between them in the course of their drift because of the probabilistic nature of the transitions. At high values of N the drift time governing one cycle of passage through the passive region increases proportionally to N and the time for the spreading between all the levels is proportional to N^2. For this reason we can expect radiation to induce relaxational oscillations of the electron distribution with a period and decrement dependent strongly on the ratio of Ω to ω_0. The commensurate case corresponding to relatively low values of N will also be called by us a fractional photon-phonon resonance of order N.

In the limit $N \to \infty$ (i.e., on transition to the incom-

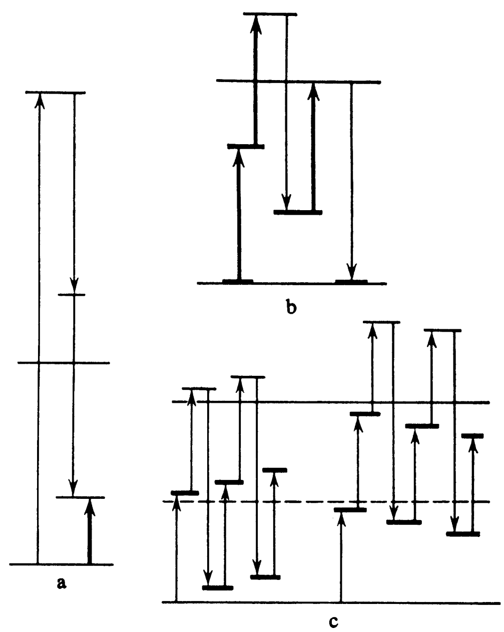

FIG. 1. Energy scheme of electron transitions: a) absorption of a photon of frequency $\Omega = (2 + \omega)\omega_0$ followed by emission of two phonons in a cascade; b) transitions in the passive region in the case of a commensurate photon-phonon resonance at $\omega = 2/3$; c) evolution of an electron in the case of positive and negative offsets from a half-integral resonance.

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mensurate case) the concept of discrete "levels" loses its meaning and a smooth (averaged over a small energy interval) steady-state distribution[1] is established. The transient process can then be described by selecting conveniently (Fig. 1c) the rational part of the transition energy ω with a relatively small denominator N, writing down

$$\omega = M/N + \omega', \quad |\omega'| \ll 1/N, \tag{1}$$

where ω' is the "offset" (detuning) energy of a photon-phonon resonance of order N. A quasisteady electron distribution is established rapidly between N intervals in the passive region. Further evolution of this distribution is governed by slow (determined by transitions in steps of ω') drift and diffusion within intervals of width N^{-1}.

In the case of a small offset from a resonance the distribution may exhibit qualitative singularities due to the fact that the direction of the offset drift is governed by the sign of ω'. In the presence of two radiations (resonances of the order of N_1 and N_2 with offsets ω_1' and ω_2', where $\omega_1'\omega_2' < 0$) a distribution between N intervals is rapidly established (here, N is the lowest common multiple of N_1 and N_2) and the drift fluxes may, for some ratio of the intensities of the radiations, be balanced out[1)] at a point ε_s. If the sign of the drift velocity at the energy ε_s changes on increase in ε from positive to negative, the electrons become bunched near $\varepsilon_s + l/N$, where $l = 0, 1, \ldots, N-1$, forming a system of quasiequidistant peaks. However, if the velocity corresponding to ε_s rises, then the electron density at these points is low and the electrons collect at the boundaries of the intervals at the energies l/N, where $l = 0, 1, \ldots, N$. A similar system of N peaks at the points $\varepsilon_s + l/N$ appears if the drift due to radiation characterized by a positive offset is compensated by a negative drift due to quasielastic relaxation in the passive region (for example, in the case of scattering by acoustic phonons). A peak of this kind is considered in Ref. 2 for the case of an offset from an integral resonance ($\omega = 0$ or 1).

These singularities are clearly pronounced if the frequency of spontaneous emission of optical phonons ν_0 is much higher than the frequency of energy relaxation in the passive region ν_{qe}, which is governed by the scattering on acoustic phonons or by the electron-electron scattering. The frequency of phototransitions ν_r is limited by the condition

$$\nu_0 \gg \nu_r \gg \nu_{qe}, \tag{2}$$

so that in the passive range of energies the distribution is far from equilibrium because of $\nu_r \gg \nu_{qe}$ and the density of electrons in the active region ($\varepsilon > 1$) is low because of the strong inequality on the left-hand side. If phototransitions assisted by optical phonons predominate, the condition $\nu_0 \gg \nu_r$ is equivalent to an allowance for just one-photon transitions in the electric field $\mathbf{E}\cos\Omega t$ of the radiation, i.e., it is equivalent to the condition of smallness of the parameter

$$\gamma = (eE/\Omega)^2/6m\hbar\Omega \ll 1, \tag{3}$$

where m is the electron effective mass.

Certain semiconductors satisfy the inequality $\nu_0 \gg \nu_{qe}$ by a large margin (3–4 orders of magnitude) at helium temperatures (see, for example, Ref. 3). The requirement $\nu_r > \nu_{qe}$ can be satisfied using high-power (megawatt) far-infrared lasers that have been developed recently.[4]

We shall use the exact transport equation for electrons in the field of an optical wave to obtain an approximate transport equation for electrons in the passive region which allows for the rapid emission of a cascade of optical phonons (Sec. 2). We shall then solve the problem of relaxation of the electron distribution in the passive region when radiation is turned on abruptly (Sec. 3). We shall then consider the approximation of a small offset in the case of integral and fractional phonon-photon resonances (Sec. 4) and investigate the case of compensation of drift fluxes leading to systems of peaks in the electron distribution (Sec. 5). In Sec. 6 we shall consider the influence of quasielastic energy scattering on the distributions obtained earlier. Ways of observing experimentally these results will be considered in the Conclusions.

2. TRANSPORT EQUATION FOR ELECTRONS IN THE PASSIVE REGION

The kinetics of electrons in a homogeneous high-frequency electric field of monochromatic laser radiation is described by the distribution function of the canonical momenta $f(\mathbf{p}, t)$ (Refs. 5 and 6). In the one-photon approximation corresponding to $\gamma \ll 1$ the anisotropy of $f(\mathbf{p}, t)$ is weak. In the case of the electron energy distribution function $f(\varepsilon, t)$, which varies slowly (compared with the radiation period $2\pi/\Omega$) in time, we obtain the following transport equation which allows for the spontaneous emission of optical phonons $\hat{I}_{\rm opt}$, quasielastic energy relaxation $\hat{I}_{qe}$, and one-photon stimulated emission and absorption of radiation $\hat{I}_r$:

$$\varepsilon^{1/2}\frac{\partial f(\varepsilon,t)}{\partial t} = [\hat{I}_{opt} + \hat{I}_r + \hat{I}_{qe}] f(\varepsilon, t). \tag{4}$$

Here, $\varepsilon^{1/2}$ is the dimensionless density of states for the assumed quadratic dispersion law,

$$\hat{I}_{opt} f(\varepsilon, t) = \nu_0(\varepsilon+1) f(\varepsilon+1, t) - \nu_0(\varepsilon) f(\varepsilon, t),$$

$$\nu_0(\varepsilon) = \begin{cases} \dfrac{1}{\tau_{do}}[\varepsilon(\varepsilon-1)]^{1/2}, & do \\[2ex] \dfrac{1}{2\tau_{po}} \ln \left| \dfrac{\varepsilon^{1/2} + (\varepsilon-1)^{1/2}}{\varepsilon^{1/2} - (\varepsilon-1)^{1/2}} \right|, & po \end{cases} \tag{5}$$

is the emission frequency of optical phonons in the case of the deformation (do) and polarization (po) mechanisms of the electron-phonon scattering [it should be pointed out that the frequency $\nu_0(\varepsilon)$ is defined in an unconventional manner: usually[3] the scattering frequency is understood to be $\nu_0(\varepsilon)/\varepsilon^{1/2}$], and

$$\begin{aligned} \hat{I}_r f(\varepsilon, t) &= \nu_r(\varepsilon, \varepsilon+1+p+\omega) f(\varepsilon+1+p+\omega, t) \\ &- \nu_r(\varepsilon, \varepsilon-1+p+\omega) f(\varepsilon, t) + \nu_r(\varepsilon, \varepsilon+1-p-\omega) \\ &\times f(\varepsilon+1-p-\omega, t) - \nu_r(\varepsilon, \varepsilon-1-p-\omega) f(\varepsilon, t) \end{aligned} \tag{6}$$

is the change in the number of electrons as a result of one-photon transitions assisted by optical phonons (do or po). If phototransitions involve the scattering of momentum by interaction with impurities, then instead of Eq. (6) we can use

$$\hat{I}_r f(\varepsilon, t) = \nu_r(\varepsilon, \varepsilon+p+\omega)[f(\varepsilon+p+\omega, t) - f(\varepsilon, t)] + \nu_r(\varepsilon, \varepsilon-p-\omega)[f(\varepsilon-p-\omega, t) - f(\varepsilon, t)]. \qquad (7)$$

The phototransition frequency $\nu_r(\varepsilon, \varepsilon')$ is given by the expressions

$$\nu_r(\varepsilon, \varepsilon') = \gamma \frac{\theta(\varepsilon)\theta(\varepsilon')}{p+\omega} \begin{cases} \tau_{do}^{-1}(\varepsilon\varepsilon')^{1/2}(\varepsilon+\varepsilon'), & do, \\ \tau_{po}^{-1}(\varepsilon\varepsilon')^{1/2}, & po, \\ 4\tau_{im}^{-1} \ln\left|\dfrac{\varepsilon^{1/2}+\varepsilon'^{1/2}}{\varepsilon^{1/2}-\varepsilon'^{1/2}}\right|, & im, \end{cases} \qquad (8)$$

where $\theta(\varepsilon) = 1$ for $\varepsilon > 0$ and $\theta(\varepsilon) = 0$ for $\varepsilon < 0$; the times τ_{do} and τ_{po} are defined by Eq. (5) and τ_{im} represents the momentum relaxation time for an electron of energy $\hbar\omega_0$ in the case of scattering by impurities. Phototransitions assisted by acoustic phonons are ignored compared with those governed by Eqs. (6) and (7).

In the active energy range ($\varepsilon > 1$) the function $f(\varepsilon, t)$ is found from the difference equation derived from Eq. (4) if we retain only $\hat{I}_{\text{opt}} f$ and one-photon inflow of electrons from the passive region ($0 < \varepsilon < 1$). The function $f(\varepsilon, t)$ is assumed to vary slowly with time ($t \gg \tau_{do}, \tau_{po}$) and quasielastic relaxation as well as multiphoton processes are ignored on the strength of Eq. (2). The solution of this difference equation gives $f(\varepsilon, t)$ for the active region in terms of the distribution function in the passive region and then it determines the transfer of electrons from the active to the passive region because of a cascade of optical phonon emissions. This gives the following closed equation for the passive region

$$\varepsilon^{1/2} \frac{\partial f(\varepsilon, t)}{\partial t} = [\hat{I}_r^{(p)} + \hat{I}_{qe}] f(\varepsilon, t), \quad 0 \leqslant \varepsilon \leqslant 1, \qquad (9)$$

$$\hat{I}_r^{(p)} f(\varepsilon, t) = -\nu_r(\varepsilon, \varepsilon+p+\omega) f(\varepsilon, t) + \theta(\omega-\varepsilon)\nu_r(\varepsilon+1-\omega, \varepsilon+1+p) \times f(\varepsilon+1-\omega, t) + \theta(\varepsilon-\omega)\nu_r(\varepsilon-\omega, \varepsilon+p) f(\varepsilon-\omega, t). \qquad (10)$$

Equation (10) and all the expressions given below apply to phototransitions involving the participation of an impurity; in the case of transitions assisted by phonons, Eq. (10) should be modified by replacing p with $p - 1$. Equation (10) is derived for the case of phototransitions to the active region ($p \geqslant 1$ applies in the case of impurity scattering denoted by *im* and $p \geqslant 2$ corresponds to the phototransitions assisted by optical phonons). In the $p = 0$ (*im* scattering) and $p = 1$ (*do* and *po* scattering) cases, there is no inflow to the range $\varepsilon > \omega$ so that the last term is missing from Eq. (10). In the $p = 0$ case the phototransitions accompanied by optical phonon emission are forbidden by the law of conservation of energy.

The initial condition for Eq. (9) can be any initial distribution, including an equilibrium Maxwellian distribution at the bottom of the passive region ($T \ll 1$). Quasielastic relaxation of the energy in Eqs. (4) and (9) is generally described by a differential operator

$$\hat{I}_{qe} f(\varepsilon, t) = \frac{\partial}{\partial \varepsilon}\left[\varepsilon \nu_{qe}(\varepsilon)\left(T(\varepsilon)\frac{\partial f}{\partial \varepsilon} + f(\varepsilon, t)\right)\right]. \qquad (11)$$

[As before, the density of states $\varepsilon^{1/2}$ is "introduced" into $\nu_{qe}(\varepsilon)$.] If throughout the passive region the quasielastic relaxation process is due to the scattering by acoustic phonons (*da*) under conditions of their constant distribution (which is assumed in the estimates obtained below), then $T(\varepsilon) = T$ and $\nu_{qe}(\varepsilon) = \nu_{qe}(1)$. It should be pointed out that Eq. (11) is valid also in the presence of a heating static electric field **F**; we then have

$$T(\varepsilon) = T + e^2F^2D(\varepsilon)/\varepsilon^{1/2}\nu_{qe}(\varepsilon),$$

where $D(\varepsilon)$ is the diffusion coefficient.

The substitution of Eq. (11) into Eq. (9) makes the latter a differential-difference equation and we have to formulate the boundary conditions at the limits of the passive region where Eq. (9) is defined. These boundary conditions are the normalizability condition $f(\varepsilon, t)$ in the interval (0, 1) as well as the condition

$$f(1, t) \sim O(\gamma), \qquad (12)$$

which appears because of the postulated strong inequalities of Eq. (2).

3. RELAXATIONAL OSCILLATIONS OF THE ELECTRON DISTRIBUTION WHEN RADIATION IS SWITCHED ON

We shall now consider the evolution of the distribution $f(\varepsilon, t)$ when monochromatic radiation of constant intensity is switched on at the moment $t = 0$. The relationship between $f(\varepsilon, t)$ and the initial distribution $f(\varepsilon, 0)$ can be conveniently written in the form

$$n(\varepsilon, t) = \varepsilon^{1/2} f(\varepsilon, t) = \int_0^1 d\varepsilon' \, G_t(\varepsilon, \varepsilon') \varepsilon'^{1/2} f(\varepsilon', 0), \qquad (13)$$

which contains the Green function describing the electron density G_t. This function is defined by the equation

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$$\frac{\partial}{\partial t} G_t(\varepsilon, \varepsilon') = -\beta(\varepsilon, \varepsilon+p+\omega) G_t(\varepsilon, \varepsilon') + \theta(\omega-\varepsilon)\beta(\varepsilon+1-\omega, \varepsilon+1+p) \times G_t(\varepsilon+1-\omega, \varepsilon') + \theta(\varepsilon-\omega)\beta(\varepsilon-\omega, \varepsilon+p) G_t(\varepsilon-\omega, \varepsilon'),$$

$$G_{t=0}(\varepsilon, \varepsilon') = \delta(\varepsilon-\varepsilon'), \qquad (14)$$

$$\beta(\varepsilon, \varepsilon') = \nu_r(\varepsilon, \varepsilon')/\varepsilon^{1/2}.$$

Replacing the continuous variable ε with a discrete one

$$\varepsilon_k = \varepsilon' + k\omega - [\varepsilon' + k\omega], \quad k = 0, 1, 2, \ldots, \qquad (15)$$

we can represent the Green function by a series

$$G_t(\varepsilon, \varepsilon') = \sum_{k=0}^{\infty} G_k(t) \delta(\varepsilon - \varepsilon_k). \qquad (16)$$

In the case of $G_k(t)$ we can derive from Eq. (14) the following chain of equations

$$\frac{dG_0(t)}{dt} = -\beta_0 G_0(t), \quad G_0(0) = 1, \qquad (17)$$

$$\frac{dG_k(t)}{dt}=-\beta_k G_k(t)+\beta_{k-1}G_{k-1}(t),\quad G_k(0)=0,\quad k=1,2,\ldots,$$
$$\beta_k=\beta(\varepsilon_k,\ \varepsilon_k+p+\omega),\quad k=0,1,2,\ldots,$$

which is integrated step by step, so that the exact expression can be obtained for Eq. (16).

At high radiation intensities the qualitative features of the general case are retained in the $\beta_k \approx \beta_0$ approximation (which is valid if $p \gg 1$); we then find that

$$G_k(t)=\frac{(\beta_0 t)^k}{k!}e^{-\beta_0 t}. \tag{18}$$

According to Eq. (18), $G_k(t)$ first rises, passes through a maximum at $t_{\max} \approx k/\beta_0$, and then falls; the maximum value of Eq. (18) is [it should be noted that the right-hand side of Eq. (19) is valid when $k \gg 1$]:

$$G_k(t_{\max})=\frac{k^k e^{-k}}{k!}\approx\frac{1}{(2\pi k)^{1/2}}. \tag{19}$$

Thus, electrons which start from the origin of a discrete k axis drift along this axis at a constant (if $\beta_k \approx \beta_0$) velocity and spread diffusely around a drifting maximum of the distribution. The width of the packet is in the same ratio to the total distance of drift along the k axis as $k^{-1/2}=(\beta_0 t)^{-1/2}$.

However, it is interesting to consider the distribution not along the k axis but in the interval (0, 1) of the ε axis which is crossed repeatedly by an electron during its k-axis drift. An explicit expression for $G_t(\varepsilon,\varepsilon')$ is obtained simply in the commensurate case when $\omega = M/N$ and the function (15) is periodic: $\varepsilon_{k+N}=\varepsilon_k$. In this case, we find that Eq. (16) transforms into a sum over N levels:

$$G_t(\varepsilon,\varepsilon')=\sum_{k=0}^{N-1}\mathcal{G}_k(t)\delta(\varepsilon-\varepsilon_k),$$
$$\mathcal{G}_k(t)=\sum_{l=0}^{\infty}G_{k+lN}(t). \tag{20}$$

The sum $\mathcal{G}_k(t)$ can be calculated directly [after Laplace transformation of $\exp(\beta_0 t)\mathcal{G}_k(t)$ we obtain a geometric progression] and we then find that

$$\mathcal{G}_k(t)=N^{-1}\left\{1+\sum_{l=1}^{N-1}\exp(-\gamma_l(N)t)\cos[\varpi_l(N)t-k\varphi_l]\right\},$$
$$\gamma_l(N)=\beta_0\left(1-\cos\frac{2\pi l}{N}\right),\quad \varpi_l(N)=\beta_0\sin\frac{2\pi l}{N},\quad \varphi_l=\frac{2\pi l}{N}. \tag{21}$$

If $N=1$ (integral resonance) there is no transient process at all and the initial distribution is of the steady-state type. If $N=2$ (half-integral resonance), monotonic relaxation with a damping decrement $\gamma_1(2)=2\beta_0$ takes place. If $N \geqslant 3$, the relaxation becomes oscillatory and the term with $l=1, N-1$ decreases more slowly so that the largest damping decrement is $\beta_0[1-\cos(2\pi/N)]$. The period of such relaxational oscillations is $2\pi/\beta_0\sin(2\pi/N)$. We can see that if $N \gg 1$, then switching on of radiation results in weakly damped (with a decrement $2\pi^2\beta_0/N^2$) relaxational oscillations (period N/β_0) of the distribution of electrons in the passive region. The Q factor of the oscillatory process rises on increase in N. A steady-state distribution of the particles between N levels is uniform if $\beta_k \approx \beta_0$.

Equations (18)–(21), obtained on the assumption that $\beta_k=\beta_0$, become much more cumbersome when this assumption is not obeyed, so that we shall give only the expressions for the steady-state Green function

$$G_{t\to\infty}(\varepsilon,\varepsilon')=\left(\sum_{k=0}^{N-1}\beta_k^{-1}\right)^{-1}\sum_{k=0}^{N-1}\beta_k^{-1}\delta(\varepsilon-\varepsilon_k). \tag{22}$$

In the case of two radiations of frequencies $p_1+\omega_1$ and $p_2+\omega_2$, the right-hand side of the equation for the Green function contains terms analogous to Eq. (14) and characterized by phototransition rates $\beta_{\rm I}$ and $\beta_{\rm II}$ (in the approximation $p_{1,2} \gg 1$), which are governed by the radiation powers. The Green function can therefore be described conveniently by a double series

$$G_t(\varepsilon,\varepsilon')=\sum_{k_1,k_2=0}^{\infty}G_{k_1k_2}(t)\delta(\varepsilon-\varepsilon_{k_1k_2}),$$
$$\varepsilon_{k_1k_2}=\varepsilon'+k_1\omega_1+k_2\omega_2-[\varepsilon'+k_1\omega_1+k_2\omega_2], \tag{23}$$

where $G_{k_1k_2}(t)$ represents a chain of equations which is analogous to Eq. (17) and is defined on a discrete k_1k_2 plane. Such equations can be solved using the Green functions (18) ($G^{(\rm I)}$ and $G^{(\rm II)}$ contain the coefficients $\beta_{\rm I}$ and $\beta_{\rm II}$, respectively):

$$G_{k_1k_2}(t)=G_{k_1}^{(\rm I)}(t)G_{k_2}^{(\rm II)}(t). \tag{24}$$

In the doubly commensurate case when $\omega_1=M_1/N_1$ and $\omega_2=M_2/N_2$, we can use the periodicity of $\varepsilon_{k_1k_2}$ in the k_1k_2 plane to obtain the Green function on the ε axis:

$$G_t(\varepsilon,\varepsilon')=\sum_{k_1,k_2=0}^{N_1-1,N_2-1}\mathcal{G}_{k_1k_2}(t)\delta(\varepsilon-\varepsilon_{k_1k_2}),$$
$$\mathcal{G}_{k_1k_2}(t)=\mathcal{G}_{k_1}^{(\rm I)}(t)\mathcal{G}_{k_2}^{(\rm II)}(t), \tag{25}$$

where $\mathcal{G}^{(\rm I,II)}$ are given by expressions analogous to Eq. (20) and containing parameters of the first or second radiation. In the passive region there are now N levels (N is the smallest common multiple of N_1 and N_2) and the relaxation in time between these levels is found by superposition of the transient processes described above and involving monochromatic radiations. A steady-state distribution corresponds to a uniform occupancy of these levels, so that

$$G_{t\to\infty}=N^{-1} \tag{26}$$

This pattern can be generalized in a natural manner to the case of a large number of independent radiations.

4. APPROXIMATION OF A SMALL OFFSET

In the above analysis of a fractional transient process in the case of a photon-phonon resonance of order N, we have used three time scales: the time for a transition between neighboring levels β_0^{-1}, the time for drift across the interval (0, 1), i.e., the oscillation period N/β_0, and the time for diffuse spreading of the initial distribution between all the levels N^2/β_0. Hence, we can see that a steady-state distribution is never established in the limit $N \rightarrow \infty$. However, in practice such very large values of N are not of interest because there are always mechanisms that ensure an energy indeterminacy $\delta\varepsilon$, so that only the average number of particles in this interval is physically meaningful. Therefore, a finite-difference analysis in the preceding section is meaningful for $N < (\delta\varepsilon)^{-1}$, whereas for $N > (\delta\varepsilon)^{-1}$ we have to use the approximation of a small offset from a frequency of some fractional resonance with a sufficiently small denominator N. The offset ω' is introduced by the relationship (1) and it should be selected so as to satisfy the requirement of smallness of the change in the electron density in the energy interval

$$|\omega'\partial \ln n(\varepsilon,\ t)/\partial\varepsilon| \ll 1. \tag{27}$$

We shall begin with the case of a small offset from an integral resonance, when $\omega' = \omega \ll 1$ (positive offset) or $\omega' = \omega - 1,\ 1 - \omega \ll 1$ (negative offset). When the inequality of Eq. (27) is obeyed, the operator $\hat{I}_r^{(p)}$ describing phototransitions in Eq. (9) [see Eq. (10)] can be expanded so that in the absence of quasielastic scattering $n(\varepsilon, t)$ is given by the equation

$$\frac{\partial}{\partial t} n(\varepsilon, t) \approx \frac{\partial}{\partial \varepsilon}\left[D_r \frac{\partial}{\partial \varepsilon} n(\varepsilon, t) - v_r(\varepsilon) n(\varepsilon, t)\right],$$
$$v_r(\varepsilon) = \omega' \begin{cases} \beta(\varepsilon, \varepsilon+p), & \omega'>0, \\ \beta(\varepsilon, \varepsilon+p+1), & \omega'<0, \end{cases} \quad D_r \approx \beta_0 \omega'^2/2, \tag{28}$$

when allowance is made for the energy dependence of the radiative drift velocity $v_r(\varepsilon)$ (giving rise to fundamentally new effects under flux competition conditions discussed below) and for the radiative diffusion coefficient D_r depending smoothly on ε applies when $p \gg 1$.

The boundary condition to Eq. (28), supplementing the normalization, will be the quasiperiodicity requirement

$$v_r(0) n(0,\ t) = v_r(1) n(1,\ t), \tag{29}$$

according to which the interval (0, 1) closes to form a ring, by analogy with the discrete pattern in the preceding section.[2)] Equation (29) can be derived formally by considering the exact difference equation in the interval $(0, \omega')$ [or $(\omega, 1)$ if $\omega' < 0$] and assuming that the contribution of this region to the normalization is small.

The Green function of the problem (28)–(29) describing the evolution of a δ-like distribution at the moment $t = 0$ is described by the following expression in the case of a constant velocity v_r:

$$G_t(\varepsilon, \varepsilon') = (4\pi D_r t)^{-1/2} \sum_{n=-\infty}^{\infty} \exp\left[\frac{-(\varepsilon-\varepsilon'-n-v_r t)^2}{4D_r t}\right]. \tag{30}$$

It is clear from the series (30) that in the case of short times $t \ll D_r/4$ the distribution drifts along the ε axis (and the direction of the drift is the same as the sign of the offset), spreading slowly, and crossing periodically the interval (0, 1) in a time $|v_r|^{-1}$. Such oscillations are described by the function ($4D_r t \ll 1$):

$$G_t(\varepsilon, \varepsilon') \approx (4\pi D_r t)^{-1/2} \exp\left\{\frac{-(\varepsilon-\varepsilon'-v_r t-[\varepsilon-\varepsilon'-v_r t])^2}{4D_r t}\right\}. \tag{31}$$

The damping decrement is governed by the asymptote of G_t obtained for long times [by expanding Eq. (30) as a Fourier series]

$$G_t(\varepsilon,\ \varepsilon') \approx 1 + 2\exp(-\pi^2 D_r t) \cos \pi\varepsilon \cos \pi(\varepsilon' + v_r t) + O(\exp(-4\pi^2 D_r t)) \tag{32}$$

and it is equal to $\pi^2 D_r$. These results are in agreement with the above analysis of the commensurate case.

We shall now consider an offset of a half-integral photon-phonon resonance ($\omega = 1/2 + \omega', |\omega'| \ll 1/2$) when the expansion of the difference operator $\hat{I}_r^{(p)} f$ for the distribution $n(\varepsilon, t)$ which varies in the half-intervals (0, 1/2) and (1/2, 1) gives a system of two differential equations. Introducing functions which vary in the interval (0, 1/2)

$$\bar{n}(\varepsilon,\ t) = {}^1/_2[n(\varepsilon,\ t) + n(\varepsilon + {}^1/_2,\ t)], \tag{33}$$
$$\tilde{n}(\varepsilon,\ t) = n(\varepsilon,\ t) - n(\varepsilon + {}^1/_2,\ t),$$

we obtain for them the following equations (we are assuming that $p \gg 1$, so that the coefficients v_r and D_r are constant):

$$\frac{\partial}{\partial t}\tilde{n}(\varepsilon, t) = -2\beta_0 \tilde{n}(\varepsilon, t) - \frac{\partial}{\partial \varepsilon}\left[\left(D_r \frac{\partial}{\partial \varepsilon} - v_r\right)\tilde{n}(\varepsilon, t)\right], \tag{34}$$

$$\frac{\partial}{\partial t}\bar{n}(\varepsilon, t) = \frac{\partial}{\partial \varepsilon}\left[\left(D_r \frac{\partial}{\partial \varepsilon} - v_r\right)\bar{n}(\varepsilon, t)\right]. \tag{35}$$

It follows from Eq. (34) that after times greater than $1/2\beta_0$ the populations of the half-intervals equalize ($\tilde{n}$ approaches zero exponentially) and we have to consider then the evolution of the distribution $\bar{n}(\varepsilon, t)$ within a half-interval by supplementing Eq. (35) with the boundary conditions specifying periodicity of $\bar{n}(\varepsilon, t)$ in the interval (0, 1/2). This condition is derived by analogy with Eq. (29) from the requirement of the smallness of the contribution of a narrow range of energies near $\varepsilon \approx 1/2$ where the differential expansion of the difference operator is invalid. The relaxation $\bar{n}(\varepsilon, t)$ is described by a Green function analogous to Eq. (30) and applicable to the half-interval (0, 1/2) and the present results differ from those in the case of an offset from a whole resonance only by halving of the oscillation period [v_r in Eq. (35) does not change, but the interval decreases] and by an increase of the offset-induced damping decrement by a factor of 4.

The same results can be obtained by investigating offset one-third resonances, etc. However, an increase in the multiplicity of a resonance N reduces the oscillation period of $\bar{n}(\varepsilon, t)$ to $1/Nv_r$, whereas the time needed to establish a uniform occupancy of N levels (i.e., rapid decay of $\tilde{n}$) deduced from Eq. (21) is $N^2/2\pi^2\beta_0$. Hence, the offset must obey the condition

$$|\omega'| \ll 2\pi^2/N^3, \tag{36}$$

if the method considered above is to be valid.

5. ENERGY DISTRIBUTION IN THE CASE OF COMPENSATION OF DRIFT FLUXES

In the case of two radiations which have opposite offsets relative to an integral resonance or relative to fractional resonances M_1/N_1 and M_2/N_2 (in the latter case one of the resonances may be integral), the situation becomes complicated because of the competition between drift fluxes mentioned in the Introduction. Then, the steady-state distribution does not always tend to a quasiuniform state and discrete peaks may appear in the distribution. In the general case of compensation of the fluxes, a system of peaks appears in N intervals, where N is the lowest common multiple of N_1 and N_2. We can describe these peaks if we allow for the dependences of $\beta_{\rm I}(\varepsilon, \varepsilon+p_1)$ and $\beta_{\rm II}(\varepsilon, \varepsilon+p_2)$ on ε, which we have avoided deliberately above by going to the limit $p_{1,2}\to\infty$.

We shall consider the vicinity of a integral resonance when $n(\varepsilon, t)$ is given by Eq. (28) and

$$v_r(\varepsilon)=\omega_1'\beta_{\rm I}(\varepsilon, \varepsilon+p_1)+\omega_2'\,\beta_{\rm II}(\varepsilon, \varepsilon+p_2),\qquad D_r=(\beta_{\rm I}\omega_1'^2+\beta_{\rm II}\omega_2'^2)/2. \tag{37}$$

If $p_1=p_2$, the drift due to the two radiations is equivalent to the drift due to one of them ($\beta_{\rm I}$) subject to an effective offset $\omega'=\omega_1'+(\beta_{\rm II}/\beta_{\rm I})\omega_2'$. If by selecting $\beta_{\rm II}/\beta_{\rm I}$ (where $\omega_1'\omega_2'<0$), we balance out the effective offset, then at each point in the passive region we annul the drift as in the case of an exact integral resonance. However, we now have $D_r\neq0$, so that the initial distribution gradually diffuses and becomes smeared out over the passive region in a drift-free manner.

If $p_1\neq p_2$, the situation changes radically because by a suitable selection of the ratio of the radiation intensities[3)] we can suppress the drift only at the "rest point" $\varepsilon=\varepsilon_s$, where

$$v_r(\varepsilon_s)=0. \tag{38}$$

If the velocity $v_r(\varepsilon)$ at the rest point changes from positive to negative on increase in ε, then the drift on either side of ε_s is directed toward that rest point and it compensate diffusion from ε_s to the periphery. In such a situation we can expect a peak of the distribution near ε_s and this peak is similar to that discussed in Ref. 2. The solution of the steady-state equation (28) with the parameters (37) gives a Gaussian peak (and later we shall use C to denote the normalization constant)

$$n(\varepsilon)=C\exp\left\{-\frac{\varkappa_s}{2D_r}(\varepsilon-\varepsilon_s)^2\right\},\quad \varkappa_s=\left|\frac{dv_r(\varepsilon)}{d\varepsilon}\right|_{\varepsilon=\varepsilon_s} \tag{39}$$

This expression is valid if

$$2|\omega_{1,2}'|\ll[2D_r/\varkappa_s]^{1/2}\ll1, \tag{40}$$

so that a peak is fairly narrow and the boundary conditions are unimportant. The left-hand inequality in Eq. (40) ensures that the distribution (27) is smooth. The transient process of the establishment of such a peak is monotonic and it is characterized by a damping decrement $\sim\varkappa_s$.

However, if the velocity $v_r(\varepsilon)$ near ε_s changes from negative to positive on increase in ε, the drift on both sides of the rest point is directed to the edges of the passive region where electrons collect. Since $v_r(\varepsilon=0;1)\neq0$, it follows that the widths of the resultant "half-peaks" can be estimated from Eq. (28) to be $D_r/v_r(\varepsilon=0;1)\sim|\omega_{1,2}'|$ (it is assumed that $|\omega_1'+\omega_2'|\sim|\omega_{1,2}'|$), so that determination of the peak profile requires an analysis of the difference equation at the edges of the passive region. The solution is fairly cumbersome and it will not be given here. It should be noted simply that variation of the parameters of the radiations can alter only the width of a "half-peak" given above, whereas in the case of Eq. (39) the distribution may be shifted by radiation within the limits of the passive region.

The characteristic features of the mechanism of the compensation of fluxes in the case of fractional resonances can be seen in the simplest example of two radiations, one of which is offset from an integral resonance p_1 and the other from a half-integral resonance $p_2+\frac{1}{2}$. The distribution of the particles between the half-intervals $n(\varepsilon, t)$ and $n(\varepsilon+\frac{1}{2}, t)$ (we now have $0<\varepsilon<\frac{1}{2}$) can be described conveniently by introducing $\bar n(\varepsilon, t)$, as in Eq. (33), as well as

$$\tilde n(\varepsilon,t)=2\frac{n(\varepsilon,t)\beta(\varepsilon,\varepsilon+p_2+{}^1/_2)-n(\varepsilon+{}^1/_2,t)\beta(\varepsilon+{}^1/_2,\varepsilon+p_2+1)}{\beta(\varepsilon,\varepsilon+p_2+{}^1/_2)+\beta(\varepsilon+{}^1/_2,\varepsilon+p_2+1)}, \tag{41}$$

which obeys a system of equations similar to but more cumbersome than Eqs. (34) and (35). In the absence of an offset, the quantity $\tilde n(\varepsilon, t)$ introduced by Eq. (41) rapidly vanishes (after a time $1/2\beta$) because for an arbitrary energy ε of a half-interval the steady-state numbers of arrivals and departures become equal:

$$n(\varepsilon)\beta(\varepsilon,\ \varepsilon+p_2+{}^1/_2)=n(\varepsilon+{}^1/_2)\beta(\varepsilon+{}^1/_2,\ \varepsilon+p_2+1). \tag{42}$$

After a long time we can expect relaxation of $\bar n(\varepsilon, t)$ in a half-interval, which is described by Eq. (28) with an effective drift velocity [with the diffusion coefficient still given by Eq. (37)]

$$\begin{aligned} v_r(\varepsilon)=\{&[\omega_1'\beta(\varepsilon,\ \varepsilon+p_1)\\ &+\omega_2'\beta(\varepsilon,\ \varepsilon+p_2+{}^1/_2)]\beta(\varepsilon+{}^1/_2,\ \varepsilon+p_2+1)\\ &+[\omega_1'\beta(\varepsilon+{}^1/_2,\ \varepsilon+p_1+{}^1/_2)\\ &+\omega_2'\beta(\varepsilon+{}^1/_2,\ \varepsilon+p_2+1)]\beta(\varepsilon,\ \varepsilon+p_2+{}^1/_2)\}\\ &\times[\beta(\varepsilon,\ \varepsilon+p_2+{}^1/_2)+\beta(\varepsilon+{}^1/_2,\ \varepsilon+p_2+1)]^{-1}. \end{aligned} \tag{43}$$

The problem of $\bar n(\varepsilon, t)$ differs from the case of an integral photon-phonon resonance only by the fact that a half-interval is considered and the energy dependence $v_r(\varepsilon)$ is more complex. Depending on the slope of $v_r(\varepsilon)$, we find that $\bar n(\varepsilon)$ neither has a peak $\varepsilon_s'[v_r(\varepsilon_s')=0]$ at the rest point, analogous to Eq. (39), or narrow half-high peaks at the limits of the interval $(0, \frac{1}{2})$. The distribution over half-intervals $n(\varepsilon)$ and $N(\varepsilon+\frac{1}{2})$ are found from the definition of $\bar n(\varepsilon)$ and the relationship (42). The energy dependences of the phototransition rates β in Eq. (42) modify somewhat the shape of the peaks and an extremum of a peak [in the case of a falling dependence $v_r(\varepsilon)$] shifts away from the points ε_s' and $\varepsilon_s'+\frac{1}{2}$.

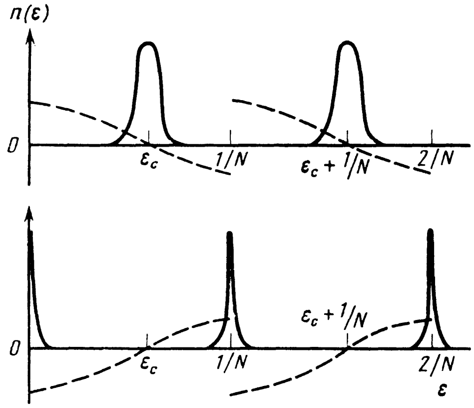

FIG. 2. System of peaks which appear under conditions of compensation of drift fluxes [the dashed curves represent the energy dependence of the drift velocity $v_r(\varepsilon)$].

If $v_r(\varepsilon)$ rises, electrons collect in narrow regions near the points $\varepsilon = 0, \frac{1}{2}, 1$.

A similar analysis can be made of the general case of two radiations which are offset relative to fractional resonances. Once again after the stage of fast relaxation between N levels described by Eqs. (22)–(25), there remains only the equation for

$$\bar{n}(\varepsilon, t) = N^{-1}\sum_{l=0}^{N-1} n(\varepsilon + l/N, t),$$

which describes relaxation in the interval $(0, N^{-1})$. Only the relationships between $\bar{n}(\varepsilon)$ and the distributions between the levels $n(\varepsilon + l/N)$, as well as an expression for the average drift velocity analogous to Eq. (43), are more cumbersome. This is due to the fact that an allowance for the energy dependence of β requires a solution of the problem (22)–(25) of the distribution between N levels when $\beta_k \neq \text{const}$. We shall not give the relevant formulas because the resultant pattern of peaks (Fig. 2) is qualitatively similar to the cases discussed above. The conditions which must be satisfied by the radiation intensities and by $\omega'_{1,2}$ are now more stringent because they have to be satisfied within the interval $1/N$. For this reason there may be a change in the system of peaks (and a change in their number), and a change from peaks to a smooth distribution when the pump intensity is varied. Such a situation is described below (at the end of Sec. 6) for the simpler case of competition between radiation and acoustic fluxes.

6. INFLUENCE OF QUASIELASTIC DRIFT AND DIFFUSION

In Secs. 3–5 the influence of the operator $\hat{I}_{qe}f$ on the relaxation process and steady-state pattern has been ignored completely. Here we shall allow for this influence subject to the following assumption: the rate of quasielastic energy relaxation, governed by the frequency $\nu_{qe}(\varepsilon)$ in Eq. (11), is low in the sense of the condition (2), but at the same time sufficiently high to compete with the offset effects, i.e., it makes a significant contribution to $v_r(\varepsilon)$ and D_r in Eqs. (28) or (35).

We shall begin by allowing for the quasielastic transport in the case of an offset from an integral resonance. Supplementing the right-hand side of Eq. (28) by the addition of $\hat{I}_{qe}f$ in the form of Eq. (11) and restricting the analysis to the case when $T(\varepsilon) = T$, we once again obtain Eq. (28) but with renormalized coefficients: $v_r(\varepsilon)$ and D_r should now be replaced by the following expressions

$$v(\varepsilon) = v_r(\varepsilon) - \varepsilon^{1/2}\nu_{qe}(\varepsilon) + T\nu_{qe}(\varepsilon)/2\varepsilon^{1/2}, \qquad D(\varepsilon) = D_r + T\nu_{qe}(\varepsilon)\varepsilon^{1/2}. \tag{44}$$

If quasielastic transport is due to acoustic phonons with a uniform distribution, then $\nu_{qe}(\varepsilon) \propto \varepsilon$ and the corrections which are then introduced are largest for $\varepsilon = 1$, whereas they disappear in the limit $\varepsilon \to 0$. In addition to the renormalization of $v(\varepsilon)$ and $D(\varepsilon)$, an allowance for quasielastic transport is related to the need to satisfy the boundary condition (12).

We shall be interested only in sufficiently large values of $v_r(\varepsilon) = \omega'\beta$, when there is a real competition between two types of drift. We shall distinguish qualitatively two cases: $|\omega'| \gg T$ and $|\omega'| \ll T$. If $|\omega'| \gg T$, then throughout the passive range we have $D(\varepsilon) \approx D_r$, i.e., only the radiative contribution to the diffusion process remains. Consequently, the quasielastic correction to $v(\varepsilon)$ is always negative and in the case of a positive offset from an integral resonance in the range

$$0 < \omega'\beta(1, 1+p) < \nu_{qe}(1) \tag{45}$$

there are peaks in the distribution because of the compensation of $v_r(\varepsilon)$ by the quasielastic drift (without the need for a second radiation beam). These are the Kumekov-Perel' peaks.[2] For the values of $\omega'\beta(1, 1+p)$ outside the range (45) the steady-state distribution is

$$n(\varepsilon) \approx C/|v(\varepsilon)|. \tag{46}$$

It should be pointed out that if the offset is positive, then $v(\varepsilon)$ is minimal for $\varepsilon = 1$, so that the distribution described by Eq. (46) exhibits a monotonic rise in the passive region, i.e., it always exhibits an inversion.

The approximate distribution (46) does not satisfy the condition (12). We can show that the exact distribution $n(\varepsilon)$ differs significantly from Eq. (46) only in a narrow layer near $\varepsilon = 1$, where $n(\varepsilon)$ decreases rapidly from $C/|v(1)|$ to 0. The width of this layer is small not only compared with the characteristic scales of Eq. (46), but also compared with the offset $|\omega'|$. Consequently, we can obtain the dependence $n(\varepsilon)$ for this case by ignoring the "offset" equation (28) and solving the exact difference equation. A similar need arises also when dealing with the lower limit of the passive region at $\varepsilon = 0$.

On the whole, in the case when $|\omega'| \gg T$ [for $|\omega'|\beta(1,1+p) \gtrsim \nu_{qe}(1)$] we can conclude that the condition (12) introduces no significant corrections to the distribution $n(\varepsilon)$ obtained subject to the quasiperiodicity condition (29) (when v_r is replaced with v) both in the range of smooth distributions and in the range of peaks.

In the other limiting case of $|\omega'| \ll T$ almost throughout the passive region we can ignore the contribution of D_r to $D(\varepsilon)$ since the diffusion process is of thermal nature. In the case of a positive offset from an integral resonance, the solution of Eq. (28) subject to Eq. (44) and to the boundary condition (12) is

$$n(\varepsilon)=C\int_{\varepsilon}^{1}\frac{d\varepsilon'}{D(\varepsilon')}\exp\Big[-\int_{\varepsilon}^{\varepsilon'}d\varepsilon''\frac{v(\varepsilon'')}{D(\varepsilon'')}\Big]. \qquad (47)$$

If $v(1)>0$, it follows from Eq. (47) that

$$n(\varepsilon)\approx\frac{C}{v(\varepsilon)}\Big[1-\exp\Big(-\int_{\varepsilon}^{1}d\varepsilon'\frac{v(\varepsilon')}{D(\varepsilon')}\Big)\Big]; \qquad (48)$$

the solution (48) differs from that given by Eq. (46) in a layer of the order of $D(1)/v(1)$ which shrinks on increase in the offset. As long as $\omega'^2 \ll T\nu_{qe}(1)/\beta(1,1+p)$, the thickness of this layer exceeds ω', and, therefore, we can use Eq. (48).

If $v(1)<0$, the solution (47) gives the distribution in the form of a peak at $\varepsilon=\varepsilon_s$ [where $v(\varepsilon_s)=0$] and then in the limit $1-\varepsilon_s \gg T$ we have

$$n(\varepsilon)\approx C\exp\Big\{-\Big|\int_{\varepsilon}^{\varepsilon_s}d\varepsilon'\frac{v(\varepsilon')}{D(\varepsilon')}\Big|\Big\}. \qquad (49)$$

An analysis of the solution (47) allows us to follow readily the change from the distribution (49) to the Maxwellian distribution at the bottom of the passive region when $\omega'\beta(1, 1+p)$ and the energy ε_s are both reduced.

In the case of negative offsets ($\omega'<0$) from an integral resonance the solution (47), which predicts only cooling of an electron gas and its progressive localization at $\varepsilon=0$ on increase in $|\omega'\beta(0,p)|$, becomes rapidly invalid. The error resulting from the use of Eq. (47) in the case of negative offsets is due to the fact that near the upper limit of the passive region in a layer $(1-\omega', 1)$ Eq. (28) is no longer valid [because of the function $\theta(\omega-\varepsilon)$ in Eq. (10)]. The difference equation which then has to be solved for the ω' layer is readily shown to reduce to

$$D(1)\frac{d^2n}{d\varepsilon^2}\approx-\beta(0,p)\,n(\varepsilon-1+\omega') \qquad (50)$$

for $n(1)=0$ [i.e., the condition (12) must be applied to the solution of Eq. (50)]. Outside the ω' layer the solution analogous to Eq. (47) becomes

$$n(\varepsilon)\approx C_1\exp\Big(-\int_0^{\varepsilon}d\varepsilon'\frac{|v(\varepsilon')|}{D(\varepsilon')}\Big)+\frac{C}{|v(\varepsilon)|}\Big[1-\exp\Big(-\int_0^{\varepsilon}d\varepsilon'\frac{|v(\varepsilon')|)}{D(\varepsilon')}\Big)\Big], \qquad (51)$$

i.e., it consists of a "cooled" solution given by the first term on the right-hand side of Eq. (51) and a smooth distribution close to Eq. (46) and represented by the second term. The solution of Eq. (50) obtained in the form of Eq. (51) gives the relationship between C_1 and C:

$$\frac{C}{|v(1)|}\approx\frac{\beta(0,p)}{D(1)}\int_0^{\omega'}d\varepsilon\,(\omega'-\varepsilon)\,n(\varepsilon) \qquad (52)$$

$$\approx C_1\frac{\beta(0,p)\,\omega'^2}{\nu_{qe}(1)T}J(\varkappa), \qquad (52')$$

where $\varkappa=T\nu_{qe}(1)/\beta(0,p)|\omega'|^{1/2}$. The formula (52') and the explicit form of $J(\varkappa)$ are obtained from Eq. (52) by the replacement of $n(\varepsilon)$ with the first term on the right-hand side of Eq. (51) subject to Eq. (44). If $\varkappa\ll1$, the factor $J(\varkappa)\sim J(0)$ is of the order of 1, and the quantity $C_1|\omega'|J(\varkappa)$ is close to the total number of "cooled" electrons $\mathfrak{N}_1$ [i.e., of the electrons described by the first term on the right-hand side of Eq. (51)]. The left-hand side of Eq. (52) is close to the total number of electrons $\mathfrak{N}_2$ in the smooth part of the distribution. We can conclude from Eq. (52) that if $\varkappa\ll1$, then

$$\mathfrak{N}_2\sim\mathfrak{N}_1\frac{\beta(0,p)\,|\omega'|}{\nu_{qe}(1)T}, \qquad (53)$$

so that in a narrow range of offsets when

$$\beta(0,p)|\omega'|/\nu_{qe}(1)T\ll1 \qquad (54)$$

the distribution is dominated by the "cooled" electrons and in the case of a strong but opposite inequality the distribution assumes the form given by Eq. (46) and the contribution of the "cooled" component can be ignored at the bottom of the passive region.

An allowance for the quasielastic transport in the case of a positive offset from a fractional resonance of order N also gives rise to a system of N peaks located at the points ε_s+l/N, where $l=0,1,...,N-1$.

The system of peaks exists in the range of intensities

$$\frac{1}{N^{1/2}}\sum_{l=1}^{N-1}l^{1/2}\nu_{qe}\Big(\frac{l}{N}\Big)<\omega'\beta_0<\frac{1}{N^{1/2}}\sum_{l=1}^{N}l^{1/2}\nu_{qe}\Big(\frac{l}{N}\Big), \qquad (55)$$

and we have $\varepsilon_s=0$ at the lower limit of the interval (55), whereas $\varepsilon_s=1/N$ applies at the upper limit. The width of the interval (55) is comparable with the width of the interval (45) for an integral resonance, but it is shifted in the direction of higher radiation powers. A comparison of the width of the peak with the width of the interval N^{-1} yields a strong inequality which governs the possibility of detection of the peaks in the case of an offset from an Nth order resonance

$$(\omega'+2T)^{1/2}\ll1/N, \qquad (56)$$

so that as N increases, the temperature has to be lowered and the offset reduced [and, consequently, the radiation intensity has to be increased—see Eq. (55)].

We shall consider also the case $\omega=1/N+\omega'$ when $\omega'\ll1/N$. In this case an increase in the radiation intensity should first produce a wide ($\sim N^{-1/2}$) single peak which gradually passes through the whole passive region, and this is followed by a smooth distribution which splits into a system of N peaks at still higher radiation intensities. These peaks cross intervals of width N^{-1} and go over to the smooth distribution. Similar transitions between smooth distributions and systems with different numbers of peaks may occur in the case of offsets from fractional resonances of different orders or in the case when the drift fluxes created by two radiations are balanced out.

7. CONCLUSIONS

The investigated features of the transient process and the steady-state distribution of electrons in a radiation field alter a number of transport phenomena. We shall now consider some possibilities of experimental observation of the predicted results.

a) Relaxational oscillations of the electron distribution give rise to oscillations of the μ-conductivity, i.e., the photocurrent has a damped high-frequency component. We can also observe the dependence of the steady-state μ-photoconductivity on the intensity of frequency of the pump radiation.

b) In addition to a change in the electrical conductivity as a result of a redistribution of electrons within the passive region, there is also a change in the diffusion coefficient and this can be deduced from the photo-emf which appears due to a spatial gradient of the electron density (as described in Ref. 7).

c) When carriers of both signs are present in a sample, their nonequilibrium distribution (due to infrared-radiation pumping) may be identified not only from a change in the density because of the energy dependence of the recombination, but also directly using hot luminescence (see, for example, Ref. 8).

d) A redistribution of electrons in the passive region is in principle detectable also on the basis of submillimeter luminescence when intraband transitions take place. Since among the above distributions there are several variants of inversion behavior, the spectral dependence of the luminescence may be nonmonotonic.

These effects should occur in electronic semiconductors (InSb, GaAs, etc.) for typical parameters $\hbar\omega_0 = 20$–40 meV, $\nu_0 \sim 10^{12}$–10^{13} sec^{-1}, and $\nu_{qe} \lesssim 10^9$ sec^{-1} at helium temperatures. The radiation intensities needed to satisfy Eq. (2) correspond to the megawatt range of CO_2 laser radiation which can be tuned in a range 9.2–10.7 μ sufficient for the observation of the above-mentioned strong spectral dependences. We can also use a submillimeter laser in the range $\sim 30\,\mu$ when the necessary intensity of the radiation is an order of magnitude less [see Eq. (3)].

The dissipation of energy by acoustic vibrations is assumed in estimating ν_{qe} and this predominates in compensated materials. However, if the scattering of electrons by one another (or by carriers with the opposite sign) predominates, we must allow for such scattering.

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[1)]Such a compensation of the drift velocity occurs "on the average" for the whole system of N intervals, because rapid transitions with energies M_1/N_1 and M_2/N_2 take place and these impose a distribution in a system of intervals.

[2)]It should be pointed out that the condition (29) is in conflict with the boundary condition (12). This is due to the fact that, without allowance for the operator $\hat{I}_{qe}f$, Eq. (9) is a difference equation describing carrier transitions only in the passive region without crossing the boundaries of this region. This property of Eq. (9) is retained also when the differential equation (28) is used. The need for the condition (12) is related solely to the allowance for nonradiative transport described by $\hat{I}_{qe}f$.

[3)]In the case when several radiations are present (or when several phototransition mechanisms are active), we can encounter several rest points in the interval (0, 1). The resultant distribution is obtained by combination of all the cases discussed above.

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