Influence of the electronic-magnon relaxation rate on the damping of nuclear spin waves in antiferromagnets

A. V. Andrienko, V. I. Ozhogin, V. L. Safonov, and A. Yu. Yakubovskiĭ

I. V. Kurchatov Institute of Atomic Energy

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Experiments on the antiferromagnets $CsMnF_3$ and $CsMnCl_3$ have revealed anomalies in the behavior of the relaxation rate γ_n of nuclear spin waves (NSWs); these anomalies find a natural explanation when the contribution to γ_n due to relaxation of the electronic spin waves (ESWs) is taken into account. Direct confirmation of such a contribution comes from the presence of magnon-phonon peaks in the relaxation of NSWs with wave vectors corresponding to the crossing points of the ESW spectrum with the spectra of transverse or longitudinal acoustic waves in $CsMnF_3$. In addition, a hexagonal anisotropy of the threshold for the parametric excitation of NSWs has been observed in $CsMnF_3$ on rotation of the magnetic field in the basal plane of the crystal. This anisotropy reproduces the corresponding angular dependence of the ESW excitation threshold. For pump frequencies in the interval $\nu_p = 760$–790 MHz (in which the hardness of the parametric excitation is maximum) there is a resonant enhancement of the contribution to the NSW relaxation from the "longitudinal" magnon-phonon peak. It is conjectured that the anomalies observed in this frequency interval are of a dislocation nature. The contribution $\gamma_n^{(e)}$ of the electronic subsystem to the NSW relaxation rate is calculated theoretically, and it is shown that there is a region of parameters in which the width of the NSW spectrum is governed mainly by $\gamma_n^{(e)}$.

INTRODUCTION

Systems with several degrees of freedom typically exhibit coupled oscillations involving the simultaneous participation of two or more subsystems. The normal modes of the coupled oscillations can differ noticeably form the original "pure" modes; in addition to the change in the spectra of the oscillations there are changes in the relaxation parameters. For example, the normal coupled oscillations become damped if even just one of the consituent pure modes is damped. In this paper we study the relaxation of the coupled oscillations for the particular example of the system of electronic and nuclear spins in antiferromagnets.

In weakly anisotropic antiferromagnets at low temperatures the dynamic hydperfine interaction leads to a strong intermixing of the oscillations of the electronic (e) and nuclear (n) spins. As a result, the spectra of the original pure modes (electronic magnons and nuclear magnetic resonances) repel each other; here the spectrum of the electronic spin waves (ESWs) remains practically unchanged, while the structure of the spectrum in the region of the NMR frequencies changes substantially: collective nuclear-electronic excitations, the so-called nuclear spin waves (NSWs), arise. It is noteworthy that the spatial dispersion in the NSW spectrum (ω_{nk}) is entirely "transformed in" from the electronic subsystem, and the nuclear subsystem itself becomes paramagnetic at liquid-helium temperatures ($\langle I \rangle / I \sim 1\%$, I is the nuclear spin). The NSW spectrum was first calculated by De Gennes *et al.*[1] The possibility that the NSW damping is renormalized because of the finite width ($\Delta\omega_{ek}$) of the ESW spectrum was discussed by Turov and Kuleev.[2] On the basis that study it can be concluded that $\Delta\omega_{ek}$ has a small effect on the relaxation of nuclear magnons. Our experimental results, however, indicate that the ESW damping has an appreciable effect on the NSW relaxation rate. Moreover, in a number of cases the damping introduced from the electronic branch of the spectrum turns out to be the main source of the NSW relaxation. The necessity of taking this additional broadening of the NSW spectrum into account was pointed out by the authors in an earlier paper.[3]

CALCULATION OF THE RELAXATION OF COUPLED OSCILLATIONS

In this section we consider the problem of finding the linewidths of two coupled oscillations. The most systematic method of solving this problem is to seek the line shapes of the new normal modes of the system with allowance for the coupling by proceeding from the known line shapes of the original pure modes. Recall that the shape of the resonance line $f_1(\Omega)$ is the normalized weight function according to which the statistical-average parameters of the oscillation, the moments $\mathscr{M}_n$, are determined[4]:

$$\mathscr{M}_n = \int_{-\infty}^{+\infty} (\Omega-\omega_1)^n f_1(\Omega)\, d\Omega; \tag{1}$$

here ω_1 is the eigenfrequency of the oscillator and $n = 1, 2, \dots$. For example, for a harmonic oscillator with no damping

$$f_1(\Omega) = \delta(\Omega-\omega_1), \quad \mathscr{M}_n = 0. \tag{2}$$

If an interaction is turned on between two oscillators with known eigenfrequencies ω_1 and ω_2, and line shapes $f_1(\Omega)$ and $f_2(\Omega)$ the line shape of any new normal mode of this coupled system can be represented in the form

$$\tilde{f}(\Omega) = \iint_{-\infty}^{+\infty} \delta[\Omega-\tilde{\omega}(\Omega_1, \Omega_2)] f_1(\Omega_1) f_2(\Omega_2)\, d\Omega_1\, d\Omega_2,$$

where $\tilde{\omega}(\Omega_1, \Omega_2)$ is the frequency of the corresponding normal mode as obtained from the characteristic equation. This expression reflects the fact that the line shape of the normal mode is formed with allowance for the interactions between the different spectral components of the original oscillations, taken with the corresponding weight factors. For symmetric $f_1(\Omega)$ and $f_2(\Omega)$ one can easily obtain from formula (2) an expression for the moments of the normal modes:

$$\tilde{\mathscr{M}}_n=\iint_{-\infty}^{+\infty} [\tilde{\omega}(\Omega_1,\Omega_2)-\tilde{\omega}(\omega_1,\omega_2)]^n f_1(\Omega_1) f_2(\Omega_2)\, d\Omega_1\, d\Omega_2. \quad (3)$$

Expressions (2) and (3) give the general solution of the problem of finding the linewidths of the normal modes of two coupled oscillations or, in other words, the renormalization of the relaxation rates of two coupled oscillators, since the relaxation rate γ is directly proportional to the half-width of the resonance line at half-height. For example, the half-width of a Lorentzian line is expressed in terms of the moments as[4]

$$\Delta\omega=\left(\frac{12}{\pi^2}\mathscr{M}_2\frac{\mathscr{M}_2^2}{\mathscr{M}_4}\right)^{1/2}. \quad (4)$$

If the frequencies of the interacting oscillations are rather widely spaced ($\omega_1 \gg \omega_2$), formula (3) simplifies to

$$\tilde{\mathscr{M}}_n \approx \iint_{-\infty}^{+\infty}\left[\frac{\partial\tilde{\omega}(\Omega_1,\Omega_2)}{\partial\Omega_1}(\Omega_1-\omega_1)+\frac{\partial\tilde{\omega}(\Omega_1,\Omega_2)}{\partial\Omega_2}(\Omega_2-\omega_2)\right]^n$$
$$\times f_1(\Omega_1) f_2(\Omega_2)\, d\Omega_1\, d\Omega_2 \approx \sum_{m=0}^{n}\frac{n!}{m!(n-m)!}$$
$$\times\left[\frac{\partial\tilde{\omega}(\omega_1,\omega_2)}{\partial\omega_1}\right]^{n-m}\mathscr{M}_{n-m}^{(1)}\left[\frac{\partial\tilde{\omega}(\omega_1,\omega_2)}{\partial\omega_2}\right]^{m}\mathscr{M}_m^{(2)}. \quad (5)$$

Just such a situation is realized in the system of coupled electronic-nuclear spin oscillations in weakly anisotropic antiferromagnets. Here as the original pure modes we can take:

1) electronic magnons of the quasiferromagnetic branch of the spectrum, with a frequency

$$\omega_{ek}=\frac{g\mu_B}{\hbar}[H(H+H_D)+H_\Delta^2+(\alpha k)^2]^{1/2}, \quad (6)$$

where H is the external magnetic field, H_D is the Dzyaloshinskiĭ field, $H_\Delta^2 \propto T^{-1}$ is a parameter characterizing the static hyperfine interaction, $g=2$ is the spectroscopic factor, μ_B is the Bohr magneton, and α is the inhomogeneous exchange constant;

2) the free precession of the nuclear spins of magnetic atoms having an NMR frequency

$$\omega_n=A\langle S\rangle/\hbar; \quad (7)$$

where A is the hyperfine interaction constant and $\langle S\rangle \approx S$ is the electronic spin.

If we neglect the transverse (dynamic) part of the hyperfine interaction, which couples the circulating components of the electronic and nuclear spins, the oscillations mentioned above are normal modes. Here the ESW has a Lorentzian line shape with a half-width $\Delta\omega_{ek}/2\pi \gtrsim 0.1$ MHz at $\omega_{ek}/2\pi \sim 10$ GHz (i.e., a Q of $\lesssim 10^5$). At the same time the line shape of the free precession of the nuclear spins can be regarded as a δ function, since its width, which is due to the longitudinal fluctuations of the spins of the electron shells,[5] is negligible, being estimated as $\Delta\omega_n/2\pi \lesssim 0.1$ Hz (i.e., at $\omega_n/2\pi \sim 0.5$ GHz the Q is $\gtrsim 5\cdot 10^9$).

Allowance for the transverse part of the hyperfine interaction leads to an intermixing of the oscillations of the electronic and nuclear spins. Here the spectrum of the normal "quasi-electronic" modes remains practically unchanged[1,6]:

$$\tilde{\omega}_{ek}=\omega_{ek}(1+\zeta), \qquad \zeta=\frac{1}{2}(g\mu_B H_\Delta/\hbar\omega_{ek})^2(\omega_n/\omega_{ek})^2 \lesssim 10^{-2}. \quad (8)$$

The relaxation rate of the ESWs also remains practically unchanged, since the contribution from ESW and NSW interaction processes is small.[7]

At the same time, however, in the region of the NMR frequencies there is a substantial restructuring: Nuclear spin waves arise, with a spectrum

$$\omega_{nk}=\omega_n[1-(g\mu_B H_\Delta/\hbar\omega_{ek})^2]^{1/2}. \quad (9)$$

In addition, the Lorentzian line shape of the ESWs is "transformed" to the NSW branch. From formulas (4) and (5) we easily obtain an expression for the rate of the NSW relaxation introduced from the electronic branch:

$$\gamma_n^{(e)}=\frac{d\omega_{nk}}{d\omega_{ek}}\gamma_e,$$
$$\frac{d\omega_{nk}}{d\omega_{ek}}=\frac{(1-\xi^2)^{3/2}}{\xi}\frac{\hbar\omega_n}{g\mu_B H_\Delta}, \qquad \xi=\frac{\omega_{nk}}{\omega_n}; \quad (10)$$

here γ_e is the relaxation rate of the pure ESWs. This contribution adds to those of the other NSW relaxation processes. It should be noted that the foregoing descussion and the formulas obtained are valid when the dynamic shifts of the NMR frequency are not too small, namely, for $\omega_n - \omega_{n0} \gg \Delta\omega_{SN}$, where $\Delta\omega_{SN}/2\pi \sim 1$ MHz is the Suhl-Nakamura linewidth of the NMR[1)] (this linewidth stems from the indirect transverse interaction between nuclear spins).

In concluding this section we note that the renormalization of the relaxation properties of interacting oscillations is usually described by a model method in which the equations of motion are supplemented by phenomenological relaxation terms.[9,10] This method of calculating the damping of coupled oscillations is not always satisfactory, however, since in a number of cases it leads to nonphysical results. This is most simply demonstrated in terms of the Hamiltonian variables a_j^*, a_j ($j=1, 2$). In this case the equations of motion are of the form (the equations for a_j^* are the complex conjugates):

$$i(d/dt+\gamma_1)a_1=\omega_1 a_1+\mathscr{A}a_2^*+\mathscr{B}a_2,$$
$$i(d/dt+\gamma_2)a_2=\omega_2 a_2+\mathscr{A}a_1^*+\mathscr{B}^*a_1,$$

where $\mathscr{A}$ and $\mathscr{B}$ are the coefficients of the bilinear form in the interaction Hamiltonian, and γ_1 and γ_2 are the relaxation parameters of the original pure modes. For $\omega_1 \gg \omega_2$, $\gamma_1 \ll \omega_1$ and $\gamma_2=0$ the characteristic equations give

$$\tilde{\omega}_2^2=\omega_2^2-2\frac{\omega_2}{\omega_1}(|\mathscr{A}|^2+|\mathscr{B}|^2),$$

$$\tilde{\gamma}_2=\left[\left(1+2\frac{\omega_2}{\omega_1}\right)|\mathscr{B}|^2-\left(1-2\frac{\omega_2}{\omega_1}\right)|\mathscr{A}|^2\right]\frac{\gamma_1}{\omega_1^2}.$$

It is easy to see that for

$$|\mathscr{A}|/|\mathscr{B}|>1+2\omega_2/\omega_1$$

the quantity $\tilde{\gamma}_2$ becomes negative, i.e., the physical meaning of relaxation is lost.

Turov and Kuleev[2] calculated the renormalization of the relaxation in the coupled electronic-nuclear system of an antiferromagnet by adding a dissipative term to the Landau-Lifshitz equation. They obtained a conversion coefficient for the transformation of the electronic relaxation into nuclear relaxation that differed by a factor of $\omega_n/\omega_{ek} \ll 1$ from that implied by a calculation based on the line shapes (10).

EXPERIMENTAL RESULTS AND DISCUSSION

Parametric NSWs were excited by a parallel microwave pump over a wide range of frequencies ($\omega_{nk}/2\pi$ = 300–600 MHz. The measurements were made on single-crystal samples of the easy-plane antiferromagnets $CsMnF_3$ and $CsMnCl_3$. The experimental apparatus and the technique used to measure the relaxation rate of parametric NSWs have been described elsewhere.[11] We chose the given crystals as objects of study because the relaxation of both the ESWs[12–17] and the NSWs[3,11,18] has been studied in detail, and the ESW relaxation in $CsMnF_3$ exhibits a number of clear features which, owing to the renormalization, should also be reflected in the NSW relaxation. These features include, first of all, two magnon-phonon peaks on the curve of the ESW relaxation rate as a function of the wave vector, due to the crossing of the ESW spectrum with the longitudinal and transverse acoustic branches,[12] and also the presence of hexagonal anisotropy of the threshold for parametric excitation of ESWs.[14] The relative increase in the ESW relaxation rate at $T \approx 2$ K in the regions of the crossings with the transverse and longitudinal acoustic branches is ~30% and ~3%, respectively.[12] If the renomalization of the NSW relaxation occurs as discussed above, then these peaks (in any case, the stronger of them) should also be reflected in the NSW relaxation. We note that the crossing of the NSW spectrum with the acoustic branches occurs at very small values $k \sim 10^4$ cm^{-1} and, consequently, cannot mask the expected effect. As is seen from (9), for ω_{nk} = const the frequency ω_{ek} of the ESWs with which the parametric NSWs interact changes with temperature T in such a way that $(g\mu_B H_\Delta/\hbar\omega_{ek})^2 = 1-\xi^2$ = const. Since the crossing point of the ESW and phonon spectra is determined from the condition $v_S k^* = \omega_{ek}^*$ (where v_S is the sound velocity), under the experimental conditions we have (H_Δ/k^*) = const, i.e., $k^* \propto T^{-1/2}$. The exact expression is

$$k^*=\frac{H_\Delta}{(1-\xi^2)^{1/2}}\frac{g\mu_B}{\hbar v_S}. \tag{11}$$

Figure 1 shows the k dependence of the NSW relaxation rate in $CsMnF_3$. In addition to a rapid growth of the relaxation at large k (i.e., small H) due to scattering of the NSWs by domain walls, one sees a relaxation peak corresponding to the crossing of the ESW spectrum with the transverse acoustic branch. The temperature dependence of the position of the peak (Fig. 2) is described well by formula (11).

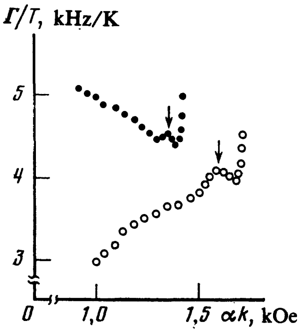

FIG. 1. Behavior of the NSW relaxation in $CsMnF_3$ in the coordinates $(\Gamma/T\alpha k)$ at a pump frequency ν_p = 1022 MHz at two temperatures: 4.23 K (filled dots) and 3.03 K (open dots). The arrows indicate the positions of the transverse magnon-phonon peak.

The relative increase ($\approx 5\%$) in the NSW relaxation rate at the peak is in approximate correspondence with the renormalization ($\approx 8\%$) expected from formula (10) for NSWs of such a frequency. The reason why the second magnon-phonon peak corresponding to the longitudinal phonons is not seen in Fig. 1 is that the weak coupling of the ESWs with longitudinal phonons should make the amplitude of this peak about an order of magnitude smaller than that of the observable "transverse" peak, i.e., smaller than the measurement error of the NSW relaxation rate. As the pump frequency was decreased, however, we found a narrow frequency region ν_p = 760–790 MHz in which the longitudinal magnon-phonon peak in the NSW relaxation could be reliably observed (Fig. 3), and the amplitude of the peak at ν_p = 775 MHz was about an order of magnitude larger than would be expected on the basis of renormalization for NSWs with a frequency of 387 MHz. The temperature dependence of the position of this peak is shown in Fig. 2.

In addition to the magnon-phonon peak (H = 0.8–1 kOe; Fig. 3) we see near the maximum pump field a step corresponding to $\tilde{k} \approx k^*/2 \approx 0.5 \cdot 10^4$ cm^{-1}. Since $\tilde{k}$ is independent of the NSW frequency and the temperature, it can

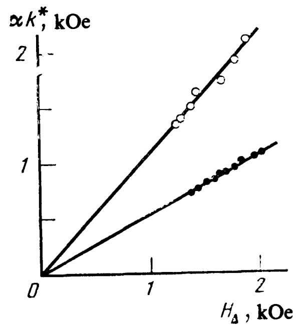

FIG. 2. Temperature dependence of the positions of the transverse (open dots, ν_p = 1022 MHz) and longitudinal (filled dots, ν_p = 775 MHz) magnon-phonon peaks in the NSW relaxation of $CsMnF_3$, in the coordinates $(\alpha k^*, H_\Delta \propto T^{-1/2})$, in which the theoretical curves should be straight lines passing through the origin [Eq. (11)].

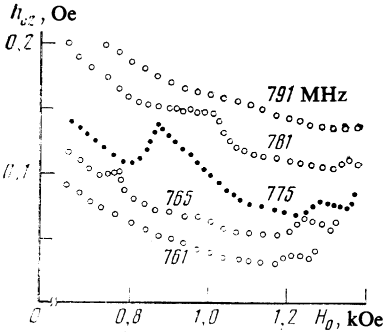

FIG. 3. Threshold field h_{c2} versus H_0 in $CsMnF_3$ at $T = 1.86$ K for different pump frequencies. The scale along the ordinate corresponds to the curve for $\nu_p = 775$ MHz. For clarity the other curves have been shifted along the ordinate with no change in scale, since the values of h_{c2} in a field $H_0 = 1.0$ kOe are the same for all the frequencies to an accuracy of $\pm 10\%$.

be assumed that this step is due to a size effect with a characteristic parameter $l = 2\pi/\tilde{k} \approx 1.2 \cdot 10^{-4}$ cm. It is possible that the enhancement of the magnon-phonon peak is also due to this effect, as it is observed for a NSW wavelength of $\lambda = l/2$.

As we have said, the slope of the straight lines in Fig. 2 is determined from (11). The experimental values we have used for the parameters of the ESW and phonon spectra are: $\alpha = (0.95 \pm 0.1) \cdot 10^{-5}$ kOe · cm;[13] $v_S^{\parallel} = (4.60 \pm 0.03) \cdot 10^5$ cm/sec; $v_S^{\perp} = (2.33 \pm 0.03) \cdot 10^5$ cm/sec;[19] $H_\Delta^2 = (6.4 \pm 0.2)/T$ kOe^2/K.[20] We easily find the ratios $k^*_{\text{exp}}/k^*_{\text{theo}} = 1.2 \pm 0.2$ and 1.0 ± 0.15, respectively, for the longitudinal and transverse magnon-phonon peaks.

Another distinct feature of the ESWs in $CsMnF_3$ is a marked hexagonal anisotropy of the parametric-excitation threshold[2)] h_{c1} on rotation of the external field **H** in the basal plane of the crystal.[14] Since h_{c1} characterizes the ESW relaxation at nearly thermal occupation numbers (and it is the influence of the linewidth of thermal ESWs that is responsible for the renormalization of the NSW relaxation), one can expect the presence of hexagonal anisotropy for both parametric-excitation thresholds (h_{c1} and h_{c2}) for NSWs.

Figure 4 shows how the threshold fields for the parametric excitation of NSWs in $CsMnF_3$ depends on the direction of **H** in the basal plane of the crystal. We see that there is a sharp hexagonal anisotropy of both threshold fields, and the relative increase in the threshold h_{c2} at the peaks (~ 1.7) corresponds well to the value expected on the basis of (10) and the results of Ref. 14. We note that the absolute increase in the thresholds h_{c1} and h_{c2} at the peaks is approximately the same (~ 0.015 Oe); this is probably evidence that the intrinsic anisotropy in the NSW system is weak.

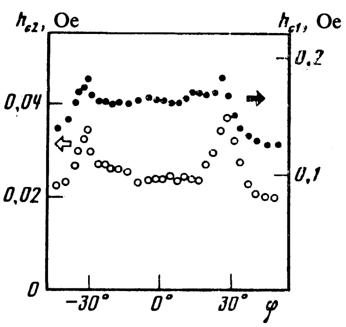

FIG. 4. Threshold fields h_{c1} and h_{c2} for the parametric excitation of NSWs in $CsMnF_3$ versus the direction of **H** in the basal plane of the crystal ($\varphi = 0$ corresponds to the two-fold axis of the crystal): $T = 1.95$ K, $H_0 = 0.94$ kOe, $\nu_p = 784$ MHz.

There is yet another important circumstance. When the field **H** is in the directions corresponding to the maxima of h_{c1} and h_{c2} in Fig. 4, the longitudinal magnon-phonon peak introduced into the NSW relaxation from the electronic branch of the spectrum is an order of magnitude weaker in any case than for the other directions of **H**. This is evidence of the anisotropic character of the mechanism responsible for the enhancement of the renormalization of the longitudinal magnon-phonon peak for pump frequencies in the range $\nu_p = 760$–790 MHz.

Let us now turn from the resonance features to the total NSW relaxation, which can be determined to a large extent by the width of the ESW spectrum. It is known that the ESW relaxation in antiferromagnets with a strong hyperfine interaction at low temperatures ($T \lesssim 2$ K) and in weak magnetic fields ($H \lesssim 2$ kOe) is governed by the elastic scattering of ESWs by fluctuations of the longitudinal component of the nuclear magnetization. This mechanism was first studied theoretically by Woolsey and White,[21] and the contribution of this process to the ESW relaxation in $CsMnF_3$ and $CsMnCl_3$ was detected experimentally in Ref. 17. With allowance for renormalization (10) the contribution of this process to the NSW relaxation can be written

$$\gamma_n^{(e)} = \frac{(1-\xi^2)^2}{\xi}(V_0^{1/3}k)\frac{\omega_n}{8\pi}\frac{T}{\Theta_N}\frac{J_0}{k_B\Theta_N}, \tag{12}$$

where V_0 is the volume of the cell, $J_0 \equiv g\mu_B H_E/S$ is the exchange constant, and $\Theta_N \equiv g\mu_B \alpha / V_0^{1/3} k_B$ is a quantity of the order of the Néel temperature.

In this region of the external parameters the NSW relaxation without allowance for the renormalization due to the ESWs (we denote the corresponding relaxation rate by γ_R) is also due to elastic scattering of the nuclear magnon by fluctuations of the longitudinal component of the nuclear magnetization. This damping mechanism was proposed by Richards,[22] who calculated the corresponding contribution to the relaxation rate γ_R in the limiting case $\omega_n - \omega_{nk} \ll \omega_n$. A calculation for arbitrary ω_{nh} is given in Ref. 23:

$$\gamma_R = \xi^3 (V_0^{1/3}k)\frac{\omega_n}{8\pi}\frac{T}{\Theta_N}\frac{J_0}{k_B\Theta_N}. \tag{13}$$

We see from expressions (12) and (13) that $\gamma_n^{(e)}$ and γ_R have the same linear dependence on T and k but substantially different dependences on the NSW frequency. The total NSW relaxation is given by the sum

$$\gamma = \gamma_n^{(e)} + \gamma_R.$$

Figure 5 shows the measured NSW relaxation rate in $CsMnF_3$ and $CsMnCl_3$ as a function of the NSW frequency in the coordinates $(\Gamma/T, \xi^3)$, where $\Gamma \equiv \gamma/2\pi$. This choice of

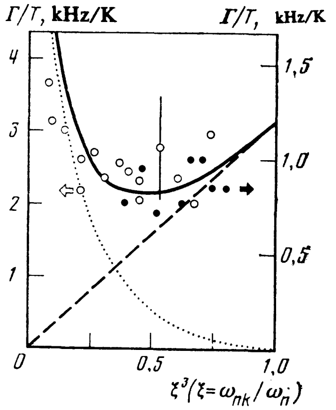

FIG. 5. Frequency dependence of the NSW relaxation for $CsMnF_3$ (open dots, left-hand scale) and $CsMnCl_3$ (filled dots, right-hand scale) in the coordinates $(\Gamma/T, \xi^3)$ for $\alpha k = 1$ kOe. The dashed line is the theoretical dependence from (13), the dotted curve is the contribution to the NSW relaxation due to the ESW damping (12), and the solid curve is the sum of these curves (14); $\xi = \omega_{nk}/\omega_n$.

coordinates enables us to isolate the increments to the relaxation process γ_R (13) represented by the dashed straight line. We see that this process alone does not describe the behavior of the NSW relaxation rate over the entire frequency range. The dotted curve in Fig. 5 shows the frequency dependence of the contribution $\gamma_n^{(e)}$ (12) introduced into the NSW relaxation from the electronic subsystem. The solid curve corresponds to the s

$$\frac{\gamma_R+\gamma_n^{(e)}}{2\pi}[\text{kHz}] = A_R\left[\xi^3 + \frac{(1-\xi^2)^2}{\xi}\right] T[\text{K}]\alpha k[\text{kOe}]. \qquad (14)$$

Table I gives the calculated values A_R^{theo} and the values A_R^{exp} obtained from the data shown in Fig. 5. The analogous quantities for the antiferromagnet $MnCo_3$ (according to the data of Ref. 24) are also given.

By taking into account the contribution introduced in the NSW relaxation from the ESW branch one can obtain a good description of the experimental frequency dependence of the NSW relaxation rate in $CsMnF_3$ and $CsMnCl_3$ at low temperatures without resorting to any adjustable parameters. The linear dependence of the relaxation rate on k and T at all frequencies has been verified previously.[11,18] We note that the relaxation process proposed by Richards[22,23] is the main relaxation channel of the nuclear magnons only near the upper boundary of NSW frequencies ($\omega_n - \omega_{nk} \ll \omega_n$), while in the rest of the frequency range one must take into account the renormalization of the NSW relaxation due to the ESW relaxation process calculated by Woolsey and White.[21] Moreover, at large dynamic frequency shifts the contribution introduced from the electronic branch becomes the main source of NSW relaxation, and the term γ_R (13) can be neglected altogether.

Another ESW relaxation channel, which becomes the main channel at high temperatures or in stronger magnetic fields, is the coalescence of two electronic magnons of the quasi-ferromagnetic branch of the spectrum into an electronic magnon Ω_{ek} of the quasi-antiferromagnetic branch[25,26]: $\omega_{e\mathbf{k}} + \omega_{e\mathbf{q}} = \Omega_{e(\mathbf{k}+\mathbf{q})}$. With allowance for renormalization (10) the expression for the NSW relaxation rate due to this three-magnon process is of the form

$$\gamma_n^{(3m)} = \frac{3}{8\pi I(I+1)}\frac{(1-\xi^2)^2}{\xi}\frac{g\mu_B H^2}{\hbar\alpha k}\frac{k_B T^4}{\hbar\omega_n\Theta_N^3}\operatorname{sh} x_k I_{3m},$$

$$I_{3m} = \int_{x_+}^{x_-} dx\,(x+x_k)^2/\operatorname{sh} x \operatorname{sh}(x+x_k), \qquad (15)$$

$$x_k = \hbar\omega_{ek}/2k_BT, \quad x_\pm = \hbar(\omega_{e0}^2+\omega_\pm^2)^{1/2}/2k_BT,$$

$$2\omega_\pm = [(\eta+1)(\eta-3)]^{1/2}\omega_{ek} \pm (\eta-1)g\mu_B\alpha k/\hbar, \quad \eta = (\Omega_{e0}/\omega_{e0})^2,$$

where $\hbar\Omega_{e0}/g\mu_B \approx 41$ kOe.[27]

Unfortunately, in the experimental parameter region ($1.5 < T \leqslant 4.2$ K; $H < 2$ kOe; 600 MHz $< \nu_p <$ 1200 MHz) one cannot distinguish the interval in which $\gamma_n^{(3m)}$ would be the main contribution, since at the upper limit of T and ν_p (where $\gamma_n^{(3m)}$ is expected to be its largest) there is a substantial contribution to the NSW relaxation from the following coalescence processes: 1) a nuclear magnon and a phonon into a phonon $[\gamma_{n2ph} \propto (1-\xi^2)^2 T^5/k]$ (Refs. 3, 11, 23); 2) a nuclear magnon and an electronic magnon into an electronic magnon $[\gamma_{n2m} \propto (1-\xi^2)^2 H^2 T^5/k]$ (Refs. 3, 11, 28). We therefore processed the experimental data (42 points) on the NSW relaxation in $CsMnF_3$ after subtraction of contribution (14) according to the formula

$$\frac{\gamma_n}{2\pi}[\text{kHz}] = (A_{n2ph} + A_{n2m}H^2)(1-\xi^2)^2 T^5/\alpha k + A_n^{(3m)}\frac{(1-\xi^2)^2}{\xi}H^2T^4\operatorname{sh} x_k I_{3m}/\alpha k.$$

Here the units are: T [K], αk [kOe], H [kOe]. As a result, we obtained the following values of the parameters: $A_{n2ph} = (3.5 \pm 1.1)\cdot 10^{-2}$, $A_{n2m} = (0.7 \pm 0.6)\cdot 10^{-2}$, $A_n^{(3m)} = (1.0 \pm 0.8)\cdot 10^2$, with a minimum sum of the squares of the relative deviations $\chi^2 = 3$. A theoretical estimate gives $A_n^{(3m)} \approx 1.7\cdot 10^2$.

It is of interest to carry out the inverse procedure—to use the known contribution $\gamma_n^{(3m)}$ to the NSW relaxation to estimate [with allowance for (10)] the relaxation rate γ_{3m} of the electronic magnons. One easily finds

$$\frac{\gamma_{3m}}{2\pi}[\text{kHz}] = (0{,}75 \pm 0{,}6)\,10^4\,\frac{H^2T^3\operatorname{sh} x_k I_{3m}}{\omega_{ek}\alpha k}. \qquad (17)$$

Here H [kOe]; T [K]; αk [kOe]; ω_{ek} [GHz]. A contribution $\gamma \propto H^2$ to the ESW damping in $CsMnF_3$ has been ob-

TABLE I.

Crystal	V_0, 10^{-22} cm³	$\omega_n/2\pi$, MHz	J_0/k_B, K	α, 10^{-5} kOe · cm	A_R^{theo}	A_R^{exp}
$CsMnF_3$	0.84	666	18,8	0.95±0,1	2,7±1	3,2±0,8
$CsMnCl_3$	1,4	568	37,6	1,3±0.3	3,0±1,6	1,2±0,3
$MnCO_3$	0.49	640	17,2	0.78±0.1	2,5±1	2,3±0,7

served experimentally,[13,15] with a relaxation rate $\gamma \approx 2\pi \cdot 100$ kHz at $T=1.7$ K, $H=2$ kOe, and $\omega_{ek} = 2\pi \cdot 10$ GHz. A calculation of γ_{3m} according to (17) in this case gives $\gamma_{3m} = 2\pi \cdot (6\text{–}60)$ kHz, in order-of-magnitude agreement with the experimental data.

Let us conclude by returning to the discussion of a possible physical mechanism which would explain the nature of the aforementioned anomalies in the threshold for the parametric process in $CsMnF_3$. The most important fact, we believe, is that the enhancement of the magnon-phonon peak and the maximum hardness $(h_{c1}/h_{c2} - 1)$ of the parametric excitation of NSWs are observed in the same range of pump frequencies $\nu_p = 760\text{–}790$ MHz, and that the value of the hardness, the amplitude of the magnon-phonon peak, and the anisotropy of the threshold $h_{c2}(\varphi)$ differ markedly from sample to sample; this is unmistakable evidence that these phenomena are not inherent to the ideal crystal but are due to defects in the sample.

We believe that all the anomalies mentioned are of a dislocation nature. This conclusion is supported by the following considerations: The hexagonal anisotropy of the threshold h_{c2} apparently means that the magnon relaxation rate depends on the direction of the magnon wave vector $\mathbf{k}$.[14] At the same time, the dislocations have a tendency to align along certain crystallographic directions[29] and can, in principle, lead to anisotropy of h_c. Alignment of the dislocations can also explain the anisotropy of the enhancement of the magnon-phonon peak. The frequency range in which the anomalies in the NSW relaxation are observed[30] and the dimension $l \sim 10^{-4}$ cm (see Fig. 3) are also characteristic of dislocations. Finally, a study[31] of the nuclear spin echo in $CsMnF_3$ has revealed an appreciable dislocation contribution to the relaxation of NSWs with $k \lesssim 10^4$ cm^{-1}.

CONCLUSIONS

1. In analyzing the experimental results on the relaxation of any of the branches of coupled oscillations one must take into account the renormalization of the damping due to the finite width of the orginal pure branches of the spectrum. This applies not only to mixed oscillations of electronic and nuclear spins but also to other coupled oscillations (e.g., magnetoelastic).

2. Allowance for the renormalization of the NSW damping due to the ESW branch results in a good description of the experimental behavior of the NSW relaxation rate over a wide range of ν_p, k, and T. For NSWs with frequencies $\omega_{nk} \lesssim 0.6\omega_n$ at $T < 2$ K the width of the spectrum is actually governed by the renormalization, while the contribution from process (13) can be neglected.

3. One can in principle obtain information on the relaxation of coupled oscillations of both branches by studying the width of only one of them. For example, by measuring the NSW relaxation rate one can tell about the damping of the ESWs as well.

4. The combination of features in the behavior of the NSW relaxation in $CsMnF_3$ in a narrow range of pump frequencies $\nu_p = 760\text{–}790$ MHz finds a natural explanation in the interaction of magnons with dislocations.

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1) For $\omega_n - \omega_{n0} \sim \Delta\omega_{SN}$ the dispersion in the spectrum of nuclear-electronic oscillations vanishes, and the NMR line shape becomes approximately Gaussian.[8]

2) Recall that for a "hard" excitation of magnons the parametric instability is characterized by two longitudinal amplitudes of the microwave field: h_{c1} and h_{c2} (excitation at h_{c1} and quenching at h_{c2}, $h_{c1} > h_{c2}$). We shall call $(h_{c1}/h_{c2} - 1)$ the "hardness" of the parametric excitation.

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