Weak quantum effects in strong electromagnetic fields

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The influence of a strong electromagnetic field on quantum effects of the weak type which are of interest in astrophysical applications is considered. A model external electromagnetic field in the form of a constant crossed E and B fields is considered. Expressions are obtained for the probabilities of the processes and they are investigated numerically. The dependence of the probabilities on the parameters characterizing the external field and the particle energies is considered.

The interaction between relativistic particles and strong electromagnetic fields gives rise to a number of important physical effects which include, for example, the emission of photons by electrons, pair production, etc.[1] Under certain conditions, external electromagnetic fields can appreciably influence even processes due to weak interactions, as was noted earlier in a number of papers.[1,2] The discovery of strong fields in astrophysics renders urgent the problem of investigating the effects of external electromagnetic fields on the dynamics of quantum processes in astrophysical applications too, since it is well known that a number of both electromagnetic[3,4] and weak processes[4,5] play an important role in astrophysics. Thus, for example, the processes of neutrino emission can influence the evolution of stars.[5] We note that near pulsars the magnetic fields reach the order of 10^{12}–10^{14} G (Ref. 6) and in accordance with certain estimates[7] in the internal regions of pulsars these fields can even be of the order of 10^{17}–10^{18} G. Fields of such strengths can influence quantum participate processes even fairly heavy particles.

A number of papers have already noted the importance in astrophysics of the processes of neutrino production in reactions of the type

$$p+p\rightarrow d+e^{+}+\nu_e$$

and others,[5,8] β-decays of nuclei,[5] processes of the type[9]

$$e^{-}+{}^{A}X_{Z}\rightarrow{}^{A}X_{Z-1}+\nu_e,$$

etc. Such reactions play an important role in the formation of neutrino fluxes from stars.[5]

Reference 8 notes the significant role of processes of the type

$$p\rightarrow n+e^{+}+\nu_e \qquad (1)$$

in view of the hypothesis about the origin of elements formed in the central regions of stars. Processes of type (1) can also occur through the interaction of high-energy protons with strong electromagnetic fields, for example, the fields near pulsars. As in (1), we can consider reactions of the type

$$p\rightarrow N^{*0}+e^{+}+\nu_e, \qquad (2)$$

where N^{*0} is the spin-parity $(3/2)^{+}$ isobar.[10] It has recently been reported[11] that in the neighborhood of certain nuclei the magnetic fields due to the motion of the nucleons in the nucleus and their magnetic moments reach strengths on the order of 10^{16}–10^{17} G. The presence of such fields can affect the dynamics of processes such as the capture of μ^{-} mesons, β decay,[11,12] and also the decays of hypernuclei.[13]

We therefore think that estimates of the influence of external electromagnetic fields on weak processes in models of fields that admit detailed investigation will be useful for understanding the dynamics of similar effects in fields of a more complicated type. We make such estimates for processes (1) and (2). As a model of the external field convenient to select the constant crossed field,[1,2] for which the wave functions of the charged particles are well known.[1] if the external field is modeled by the crossed field, estimates of its influence on processes of type (1) can be made as in Refs. 1 and 14. We use the usual current-current scheme for the Lagrangian of the weak interaction,[15] taking into account in the expression for the weak hadronic current only the contribution of the form factors f_1 and g_1 and omitting the induced form factors. For the charged particles we use the wave functions given in Ref. 1.

After somewhat lengthy calculations for the probability of processes (1) we obtain

$$W_{p\rightarrow ne^{+}\nu_e}(\chi)=\frac{G^2m^6c}{32\pi^{7/2}}\int_0^\infty\frac{du}{(u+1)^2}\int_{\lambda_1}^\infty d\lambda J^2K_{p\rightarrow ne^{+}\nu_e}(\chi), \qquad (3)$$

where

$$K_{p\rightarrow ne^{+}\nu_e}(\chi)=(|f_1|^2+|g_1|^2)h_1+2\,\mathrm{Re}\,f_1g_1^{*}h_2+2(|f_1|^2-|g_1|^2)\lambda_1^{1/2}h_3. \qquad (4)$$

For h_1 we have

$$h_1=[2\lambda(\mu+1-\lambda)t_{+}-(\mu-1+\lambda)(1-\mu+\lambda)d_{-}]\int_z^\infty\Phi(y)\,dy-2(\chi u)^{2/3}\left[2\lambda\frac{2u^2+2u+1}{u(u+1)}t_{+}-2\lambda d_{+}+\frac{(1+\mu+\lambda)}{u(u+1)}d_{-}\right]\Phi'(z),$$

$$h_2=[2\lambda(1+\mu-\lambda)t_{-}+(\mu-1+\lambda)(1-\mu+\lambda)d_{-}]\int_z^\infty\Phi(y)\,dy-2(\chi u)^{2/3}\left\{2\lambda\frac{2u^2+2u+1}{u(u+1)}t_{-}-8\lambda\left(1-\frac{2}{3}J\right)+2\frac{u+1}{u}\left[\mu+\left(\frac{u}{u+1}\right)^2\right]d_{-}+\left[4\lambda-2(1+\mu+\lambda)\frac{u^2+u+1}{u(u+1)}+\frac{1-\mu+\lambda}{u}-\frac{\mu-1+\lambda}{u+1}\right]d_{-}\right\}\Phi'(z), \qquad (5)$$

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$$h_3=(\mu-1+\lambda)\int_z^\infty \Phi(y)\,dy+2(\chi u)^{2/3}[u(u+1)]^{-1}\Phi'(z),$$

$$z=(\chi u)^{-2/3}[\mu(u+1)+\lambda u(u+1)-u].$$

In (3)–(5) the following notation is used:

$c=1/p_0,\quad \lambda_1=m_n^2/m^2,\quad \mu=m'^2/m^2,\quad \chi=e[(F_{\mu\nu}P_\nu)^2]^{1/2}m^{-3},$

m, m_n, and m' are, respectively, the masses of the proton, the neutron, and the electron, $F_{\mu\nu}$ is the tensor of the electromagnetic field,[1] p is the 4–momentum of the proton, and Φ and Φ' are the Airy function and its derivative,[1]

$$t_\pm=1\pm{}^1/_3J,\quad d_\pm=1\pm{}^4/_3J,\quad J=(\lambda-\lambda_1)/2\lambda. \qquad (6)$$

In the derivation of Eqs. (3)–(5), invariant integration was carried out with respect to the momenta of the neutral particles.[2]

We note that the probabilities of the processes (1) in the external fields were considered previously by Zharkov[16] for the case of the S- and P-versions of the weak interaction and under certain other simplifying assumptions. The use of the realistic V–A form of weak interaction significantly complicates the structure of the probability of the processes (1), as can be seen from expressions (3)–(5).

To analyze the processes (2), we use the same form of the weak hadronic current as in Ref. 10, leaving only the form factors F_1 and G_1 and omitting the induced form factors. After making the necessary calculations for the pobability of the processes (2) we obtain expression (3) if we substitute

$$J^2K_{p\to ne^+\nu}\to{}^2/_3N_{p\to N^{*0}e^+\nu},$$

where

$$N_{p\to N^{*0}e^+\nu_e}(\chi)$$

$$=(|F_1|^2+|G_1|^2)H_1+\mathrm{Re}\,F_1G_1^*H_2+(|F_1|^2-|G_1|^2)\lambda_1^{1/2}H_3. \qquad (7)$$

The quantities H_i in this case have the form

$$H_1=\left[(\lambda-1-\mu)b_1+\frac{1}{2}(\lambda+1-\mu)(1-\mu-\lambda)b_2\right]\int_z^\infty\Phi(y)\,dy$$

$$-2(\chi u)^{2/3}\left\{2\lambda(c_1-\lambda_1^{-1}L)-b_1\left[2+\frac{1}{u(u+1)}\right]+\frac{1}{2}b_2\right.$$

$$\left.\times\frac{1+\mu+\lambda}{u(u+1)}\right\}\Phi'(z)+2(\chi u)^{4/3}\frac{c_1}{u(u+1)}\Phi(z),$$

$$H_2=[2(\lambda-1-\mu)\tau_1+(\lambda+\mu-1)(\lambda+1-\mu)c_2]\int_z^\infty\Phi(y)\,dy$$

$$+4(\chi u)^{2/3}\left[2\lambda\lambda_1^{-1}L-3A+\tau_1\frac{u^2+u+1}{u(u+1)}+c_2\lambda\right.$$

$$\left.+\frac{1}{2}c_2\frac{\lambda+1+\mu}{u(u+1)}\right]\Phi'(z)-8(\chi u)^{4/3}\frac{c_2+\lambda_1^{-1}L}{u(u+1)}\Phi(z),$$

$$H_3=(1-\mu-\lambda)\tau_2\int_z^\infty\Phi(y)\,dy-2(\chi u)^{2/3}$$

$$\times\left[\frac{\tau_2}{u}+\frac{1}{u+1}(I-N-2\lambda_1^{-1}A)\right]\Phi'(z). \qquad (8)$$

The value of z in (8) is the same as in (5),

$$b_1=3A-2\lambda\lambda_1^{-1}L,\quad b_2=N-3B+4\lambda_1^{-1}L+2\lambda\lambda_1^{-1}Q,$$

$$c_1=N-2B+2\lambda_1^{-1}L+\lambda\lambda_1^{-1}Q,\quad c_2=B-N,$$

$$\tau_1=A+N\lambda-I\lambda_1,\quad \tau_2=I-3N+2\lambda_1^{-1}A+2\lambda\lambda_1^{-1}B, \qquad (9)$$

$$A=\frac{1}{3}I\left[\lambda_1-\lambda\left(\frac{\lambda+\lambda_1}{2\lambda}\right)^2\right],\quad N=\frac{\lambda+\lambda_1}{2\lambda}I,$$

$$B=\frac{1}{3}I\left[-\frac{\lambda_1}{\lambda}+4\left(\frac{\lambda+\lambda_1}{2\lambda}\right)^2\right],\quad L=\frac{\lambda+\lambda_1}{2\lambda}A,$$

$$Q=\frac{\lambda+\lambda_1}{2\lambda^2}(B\lambda-2A),\quad I=\frac{1}{2\lambda}[(\lambda+\lambda_1)^2-4\lambda_1\lambda]^{1/2}.$$

The notation used in (8) and (9) is the same as that used in writing down Eqs. (3)–(5).

We draw attention to the fact that the probabilities of processes in which spin 3/2 particles take part are determined not only by $\Phi'(z)$ and integrals of $\Phi(y)$, as for the processes with the participation of particles of lower spin,[1] but also by quantities containing the Airy function $\Phi(z)$ as a factor. Here one sees the probabilities of the quantum processes in the external electromagnetic field depend on the spin characteristics of the particles which participate in them.[1]

It can be seen from expressions (3)–(5) and (7)–(9) that the probabilities of the processes (1) and (2) have a fairly complicated structure. Using the properties of the functions Φ nad Φ' we can investigate the asymptotic behavior of the probabilities (3) of the processes (1) and (2) for small and large χ. Thus, for $\chi\ll 1$, the probabilities (3) are exponentially small.[16] In the opposite case $\chi\gg 1$ the probabilities exhibit a characteristic $\chi^2\ln\chi$ dependence on this parameter.[1] However, this limiting case is not of interest since in this region of variation of χ the probabilities of the processes (1) and (2) will also be determined, to a considerable extent, by the behavior of the form factors of the weak hadron current.[16] The analysis of the probabilities (3) for values of χ of order unity is therefore of greater interest.[1,14] In just this region of variation of χ the nonlinear dependence of the probabilities of the quantum processes on the parameters characterizing the field and particle energies is the most significant.[1]

Using the tables of Ref. 17 for the Airy functions and their derivatives, we carried out a numerical analysis of the probabilities of the processes (1) and (2) which are determined by expressions (3)–(9).

We note, first of all, that expressions (3)–(5), together with the effects (1), also describe decays of the type $\Sigma^\pm\to\Lambda+e^\pm+\nu$ occurring with relatively low energy release in the external field. The influence of the external field on these processes is of interest[18] because the effect of the field on the particle decays with low energy release increases, as noted in Ref. 2. In this case, as shown in Ref. 2, $\chi\delta^{-2}$, becomes the effective parameter determining the effect of the field on the decay, where $\delta=(1-\mu-\lambda_1)/2\lambda_1$.

Numerical investigation showed that the probabilities of the decays $\Sigma^\pm\to\Lambda+e^\pm+\nu$ increase significantly in comparison with their values in the absence of a field for values of χ as small as 10^{-4} to 10^{-3}. Such values of χ are associated with electromagnetic fields of order 10^{16} to 10^{17}

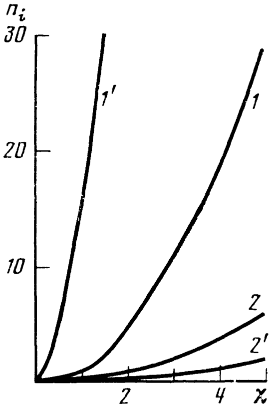

FIG. 1. The probabilities of the processes $p \to n + 1^+ + \nu$ and $p \to N^{*0} + 1^+ + \nu$ as functions of χ for the crosssed field. Curve 1 is calculated for $p \to n + e^+ + \nu_e$ with $f_1/g_1 = 0.8$ (Ref. 15), and 2, for $p \to n + \mu^+ + \nu_\mu$. The curves with primes relate to the similar processes $p \to N^{*0} + l^+ + \nu$ with $F_1/G_1 = 0.22$.[10]

G, which, as has already been indicated, exist near nuclei. Owing to this, this effect can be significant for different weak processes occurring in heavy nuclei, for example, the capture of μ^- mesons, decays of hypernuclei, etc.

Figure 1 gives the results of the calculations of the probabilities W_i of processes (1) and (2) as a function of χ for the crossed field normalized as $n_i = W_i/W_i^0$. For reaction (1)

$$W^0_{p\to ne^+\nu_e} = G^2 |g_1|^2 m^6 c/2\pi^3,$$

for (2), $W^0_{p\to N^{*0}e^+\nu_e}$ is obtained by the substitution $g_1 \to G_1$. Processes (1) and (2) with muon production were also considered in order to estimate the dependence on the charged-particle mass. For processes (1) and (2) with electron production a very rapid increase of the probabilities is observed as χ increases. This is due to the significant (for small μ) contribution of the region of small u and large λ to the integrals (3) which determine probabilities (1) and (2), as can be seen from the expression for z in (5). Because of this, the probabilities for the muonic processes (1) and (2) for these values of χ are significantly lower.

Thus, the dynamics of the reactions (1) and (2) in the external fields is characterized by a strong dependence of the probabilities of these processes on the mass of the resulting charged particles. At the same time, their behavior in different ranges of χ for the reactions (1) and (2) is significantly different. We note that a kinematic factor also plays an important role here. Thus, as the estimates show, to observe the effect of the external electromagnetic fields on hyperon decay processes of the type

$$Y \to N + \pi, \tag{10}$$

where Y denotes the decaying hyperon and N the nucleon, and on the inverse processes

$$N \to Y + \pi, \tag{11}$$

fields one or two orders of magnitude larger than for processes of type (1) for the same initial particle energies are necessary. We note that these estimates can be of interest both for astrophysical applications[3–5] and, for the study of the possible manifestation of a similar effect[13] in the decays of heavy hypernuclei.

When ultrarelativistic particles interact with strong electromagnetic fields the following processes, due to the weak neutral currents, are also possible:

$$\pi^\pm \to \pi^\pm + \nu + \tilde{\nu}, \quad p \to p + \nu + \tilde{\nu}, \quad p \to N^{*+} + \nu + \tilde{\nu}.$$

The presence of very strong electromagnetic fields near the pulsars can therefore cause, as a result of their interaction with such particles, specific astrophysical mechanisms in which weak neutral currents manifest themselves.

This examination of the effect of external electromagnetic fields on the dynamics of weak processes is therefore of interest for the analysis of a fairly wide range of problems, important from the point of view of astrophysical applications[3–5,8,9] and also in investigating the dynamics of quantum effects in strong electromagnetic fields.[1]

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