# Bremsstrahlung in atom-atom collisions

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It is shown that in the collision of a fast atom with a target atom when the frequencies are on the order of the potentials or higher, there arises bremsstrahlung comparable in intensity with the bremsstrahlung emitted by an electron with the same velocity in the field of the target atom. The mechanism by which bremsstrahlung is produced in atom-atom collisions is elucidated. Results of specific calculations of the bremsstrahlung spectra are given for $\alpha$ particles and helium atoms colliding with xenon.

## INTRODUCTION

Buĭmistrov and Trakhtenberg[1] demonstrated the important role of the internal degrees of freedom of the target atom in the bremsstrahlung of an electron in a hydrogen atom. Subsequently the bremsstrahlung process has been studied in more complex atoms.[2–8] Important features of bremsstrahlung were found, which are related to the dynamical polarization of the target atom during the collision. For example, in Ref. 4 it was shown that in the frequency region $\omega \gg I$, where I is the ionization potential of the atom, the radiation of an electron in an atom actually occurs at the nucleus. In Ref. 7 it was noted that the radiation which arises in collision between fast heavy charged particles can be of the same order as the bremsstrahlung which occurs when electrons collide with an atom, whereas it was previously believed that it is reduced by a ratio $(m_e/M)^2$, where $m_e$ and $M$ are the masses of the electron and the heavy particle. In Refs. 2–3 and 5–8 bremsstrahlung was studied in the photon-frequency region $\omega \gtrsim I$, where taking into account the polarizability of the target atom makes a very substantial contribution to both the differential and total cross sections for the process.

In all of the studies mentioned, the incident particle was assumed to be charged and without internal degrees of freedom. In the present work we study the bremsstrahlung produced when a fast atom collides with a target atom and show that the deformation of their electron shells in the scattering process gives rise to bremsstrahlung of the same intensity as that of a charged particle colliding with an ion or even of an electron in the same target with the same incident velocity. Here, as in all studies of the series cited above, we treat the bremsstrahlung without including ionization of the atoms, which is generated completely by the neutral atoms themselves because the ionized electrons do not radiate. Because of the large number of particles involved in producing the bremsstrahlung (the atomic nuclei and the electrons bound to them), the solution of this problem in the general case is rather complicated. However, for a collision between fast atoms the problem is simplified and permits a relatively simple description.

The mechanism of atomic bremsstrahlung is as follows. In the collision process each of the atoms is polarized and acquires a dipole moment. When these induced atomic dipole moments and consequently also the total dipole moment of the system rotate, bremsstrahlung is omitted. We will show that when fast atoms collide, bremsstrahlung over a wide range of frequencies and velocities of the incident particle results mainly at distances comparable with the atomic dimensions. This permits us to assume that the polarization and dipole moment of each of the atoms arise as a consequence of the displacement in opposite directions of the electron shell and the nucleus in the static field of the other nucleus, screened by electrons.

For photon frequencies $\omega \gg I$ we can assume that the electrons of the two atoms radiate as if they were. However, when free electrons collide, radiation of dipole quanta is impossible, and therefore bremsstrahlung in this region of $\omega$ arises as the result of acceleration of the electrons of each of the atoms in the field of the other nucleus.

In this paper we show that when two identical atoms collide there is no dipole bremsstrahlung. The reason for this is that the induced dipole moments of the atoms completely cancel each other out so that the total moment of the system vanishes. Therefore in a pair identical atoms bremsstrahlung arises only as a consequence of corrections associated with relativistic effects.

The bremsstrahlung mechanism described appears also when an ion collides with an atom. An ion-atom collision and the atom-atom collision discussed above differ qualitatively, because in the system consisting of the incident ion and the atom has a long-range Coulomb field the ion, which causes the bremsstrahlung intensity increase when the collision occurs at large distances.

The discussion in this paper is arranged according to the following plan. In Section 1 we obtain the amplitude of bremsstrahlung in atom-atom collisions. In Section 2 we study the bremsstrahlung cross section. We discuss the angular distribution of unpolarized photons, which in the region $\omega R_{at}/v_1 \ll 1$, where $v_1$ is the velocity of the incident atom and $R_{at}$ is the size of the atom, coincides with the angular distribution of the radiation of a uniformly rotating dipole, which agrees with the qualitative picture of the phenomenon given above. We study the polarization of bremsstrahlung and obtain the bremsstrahlung spectrum $d\sigma/d\omega$, for which we find an especially simple formula in the region $\omega \gg 1$. We bring out the role of nuclear recoil in the production of bremsstrahlung. In Section 3 we give the results of a numerical calculation and compare the bremsstrahlung spectra arising when a helium atom or its ion (an $\alpha$ particle) collides

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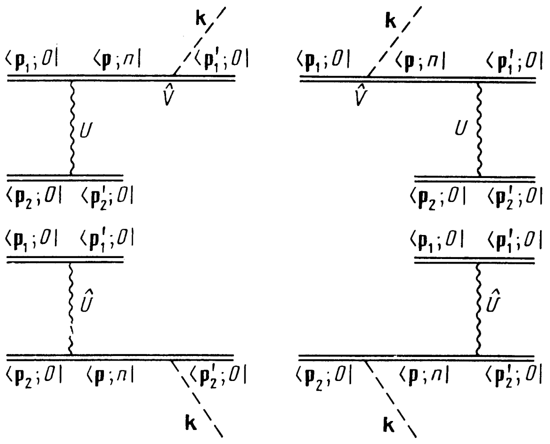


FIG. 1. Diagram representation for the amplitude of bremsstrahlung in atom-atom collisions.

with a xenon atom and the bremsstrahlung spectrum of an electron colliding with a xenon atom.

## 1. AMPLITUDE OF BREMSSTRAHLUNG

Let us consider the collision between a pair of fast but nonrelativistic atoms leading to emission of a photon with momentum **k** and polarization **e**, in which both at the beginning and at the end of the process both atoms are in the ground state. The momenta of the translational motion of the first and second atoms of the pair in the initial state will be designated respectively as $p_1$ nd $p_2$, and those in the final state $p_1'$ and $p_2'$. To describe the bremsstrahlung from fast atoms we shall use the Born approximation, i.e., we shall consider the process to first order in the interatomic interaction $U(\{\mathbf{r}_i\};\{\mathbf{r}_k\})$. The potential $U$ is the sum of all possible Coulomb interactions between the particles making up the first and second atoms, and $\{\mathbf{r}_i\}$ and $\{\mathbf{r}_k\}$ are the coordinates of these particles. It is also sufficient to discuss the radiation process to first order in the interaction $\hat{V}$ of the atoms with the radiation field.

The Feynman diagrams which describe the amplitude of the bremsstrahlung of fast particles are shown in Fig. 1. The double lines describe atoms, the dashed lines represent a radiated photon with momentum **k** and polarization **e**, and the wavy lines represent the interaction $U$ between atoms. The point of emission of a photon corresponds to the interaction $\hat{V}$ of the atom with the radiation field. We represent the wave functions of the moving atoms in the form

$$|\mathbf{p};\,n\rangle = e^{i\mathbf{pR}}\Psi_n(\{\mathbf{r}_i-\mathbf{R}\}), \tag{1}$$

where **p** is the momentum of the translational motion. **R** is the radius vector of the center of inertia, and $\Psi_n(\{\mathbf{r}_i-\mathbf{R}\})$ is the wave function of the $n$th state, which depends on the relative coordinates of the internal motion of the electrons and the nucleus.

In Fig. 1 the states of the atoms are denoted by two letters in correspondence with Eq. (1). A summation which includes the discrete and continuous spectra of the atom is carried out over the intermediate states ($\mathbf{p}$;$n$). The amplitude shown graphically in Fig. 1 describes the radiation emitted by the nucleus and by the electrons of each of the atoms in the static field of the other one during the collision.

We shall integrate the matrix elements for radiation of the photon and for the scattering process over the coordinates of their centers of mass. These matrix elements enter as factors in the desired amplitude. First let us consider the matrix element for radiation of a photon by a moving atom:

$$\langle\mathbf{p}';m|\hat{M}|\mathbf{p};n\rangle = \langle\mathbf{p}';m|-\sum_{i=1}^{N+1}\frac{e_i}{m_ic}e^{-i\mathbf{kr}_i}(\mathbf{e}\hat{\mathbf{p}}_i)|\mathbf{p};n\rangle. \tag{2}$$

Here the summation is carried out over all particles making up the atom, including the nucleus; $\hat{\mathbf{p}}_i$, $e_i$ and $m_i$ are the momentum operator, charge, and mass of the $i$th particle of the atom, and $c$ is the velocity of light. Using the wave functions of the moving atom (1), we arrive at the following expression:

$$\langle\mathbf{p}';\,m|\hat{M}|\mathbf{p};\,n\rangle = -(2\pi)^3\delta(\mathbf{p}-\mathbf{p}'-\mathbf{k})M_{mn}(\mathbf{k};\,\mathbf{e};\,\mathbf{p}), \tag{3}$$

$$M_{mn}(\mathbf{k};\,\mathbf{e};\,\mathbf{p}) = \langle m|\sum_{i=1}^{N+1}e_ie^{-i\mathbf{kr}_i}|n\rangle\frac{(\mathbf{ep})}{Mc} + \langle m|\sum_{i=1}^{N+1}\frac{e_i}{m_ic}e^{-i\mathbf{kr}_i}(\mathbf{e}\hat{\mathbf{p}}_i)|n\rangle.$$

Here $\delta(\mathbf{p}-\mathbf{p}'-\mathbf{k})$ expresses the conservation of momentum during the photon emission, and $M$ is the mass of the atom. In the system $\mathbf{p}=0$, $M_{mn}$ ($\mathbf{k}$,$\mathbf{e}$,0) is the matrix element for radiation of a photon in the center-of-mass system of the atom.

In the dipole nonrelativistic approximation ($\omega R_{\rm at}/c\ll1$, $v/c\ll1$, where $v=p/M$), which is valid even for frequencies comparable with the ionization potential of the inner shells, the function $M_{mn}$ ($\mathbf{k}$,$\mathbf{e}$,$\mathbf{p}$) is simplified:

$$M_{mn}(\mathbf{e},\,\mathbf{p}) = \delta_{mn}\sum_{i=1}^{N+1}e_i\frac{(\mathbf{ep})}{Mc} + i\frac{\omega_{mn}}{c}(\mathbf{ed}_{mn}), \tag{4}$$

where $\mathbf{d}_{mn}$ is the matrix element of the dipole moment of the atom and $\omega_{mn}=\varepsilon_m-\varepsilon_n$ is the frequency of the transition between states $m$ and $n$.

Similarly, separating the coordinates of the centers of mass of the atoms in the scattering matrix element and integrating over them, we can transform it to the form

$$\langle\mathbf{p}_1',m_1';\mathbf{p}_2',m_2'|\hat{N}|\mathbf{p}_1,m_1;\mathbf{p}_2,m_2\rangle = (2\pi)^3\delta(\mathbf{p}_1+\mathbf{p}_2-\mathbf{p}_1'-\mathbf{p}_2') \times\frac{4\pi}{Q_2^2}S^{(1)}_{m_1'm_1}(\mathbf{Q}_1)S^{(2)}_{m_2'm_2}(\mathbf{Q}_2). \tag{5}$$

The subscripts and superscripts 1 and 2 in Eq. (5) and below denote the assignment of the various quantities to the first or second atom respectively. The function $S_{mm'}(\mathbf{Q})$ which describes the distribution of the charge of the electrons and nucleus inside the atom is defined by the relation

$$S_{m'm}(\mathbf{Q}) = \langle m'|\sum_{i=1}^{N+1}e_ie^{-i\mathbf{Qr}_i}|m\rangle, \tag{6}$$

where the momentum transfer is $\mathbf{Q}=\mathrm{p}-\mathrm{p}'$. The amplitude (5) is similar to the amplitude of the scattering of two charged structureless particles. Here the role of the charges in Eq. (5) is played by the functions $S$.

As will be shown below, to describe bremsstrahlung it is actually sufficient to consider the function $S_{m'n}(\mathbf{Q})$ in the limit of small momentum transfers $QR_{at} \ll M/Zm_e$ ($Z$ is the nuclear charge and $m_e$ is the electron mass, where it is somewhat simplified:

$$S_{m'm}(\mathbf{Q}) = Z\delta_{m'm} - \langle m' | \sum_{i=1}^{N} e_i \exp(-i\mathbf{Q}\mathbf{r}_i) | m\rangle + i\mathbf{Q}\mathbf{d}_{m'm}\frac{Zm_e}{M}. \qquad (7)$$

Equation (7) was obtained from (6) by expanding the exponential

$$Z\exp(-i\mathbf{Q}\mathbf{r}_{N+1}) \approx Z - i\mathbf{Q}\mathbf{r}_{N+1}, \quad \mathbf{r}_{N+1} = -\sum_{i=1}^{N}\frac{m_i\mathbf{r}_i}{M},$$

which contains the displacement of the nucleus in the center-of-mass system of the atom, expressed in terms of the coordinates of its $N$ electrons.

We shall represent the analytic expression for the bremsstrahlung amplitude in atom-atom collisions in the form of a sum of two terms:

$$F = F_1 + F_2, \qquad (8)$$

where $F_1$ describes the radiation of a photon by the first atom (the first two diagrams of Fig. 1), and $F_2$ describes the radiation of a photon by the second atom (the third and fourth diagrams of Fig. 1). Using Eqs. (3) and (5), integrating over the momenta of the translational motion of the atom $\mathbf{p}$ in the intermediate state and summing over its excited states, we obtain the following expression for $F_1$, written in atomic units $\hbar = m_e = e = 1$:

$$F_1 = -\frac{4\pi}{Q_2^2} S_{00}^{(2)}(\mathbf{Q}_2) \sum_n \left\{ \frac{M_{0n}^{(1)}(\mathbf{k};\mathbf{e};\mathbf{p}_1')S_{n0}^{(1)}(\mathbf{q}_1)}{\omega-\omega_{n0}+p_1'^2/2M_1-(\mathbf{p}_1'+\mathbf{k})^2/2M_1} - \frac{S_{0n}^{(1)}(\mathbf{q}_1)M_{n0}^{(1)}(\mathbf{k};\mathbf{e};\mathbf{p}_1)}{\omega+\omega_{n0}-p_1^2/2M_1+(\mathbf{p}_1-\mathbf{k})^2/2M_2} \right\}. \qquad (9)$$

Here and below we shall not write out the delta function which describes conservation of momentum in the collision, and $\mathbf{q}_1 = \mathbf{p}_1 - \mathbf{p}_1 - \mathbf{k}$. In Eq. (9) the factor in front of the summation sign is the Fourier transform of the static atomic potential of the second atom, and the expression in curly brackets is the dynamical reaction of the first atom to the external perturbation produced by the second atom.

The polarization of the atoms is particularly strong in the region $\omega \gtrsim I$, where the bremsstrahlung process is well described in the dipole approximation. Substituting (4) into (9), and omitting in the energy-dependent denominators the terms which contain as a factor the small parameters $v_1/c \ll 1$, $Q/Mv_1 \ll 1$, or $\omega/Mv_1^2 \ll 1$, we obtain the following expression for the amplitude $F_1$ of dipole bremsstrahlung:

$$F_1(\omega;\mathbf{Q}_2;\mathbf{e}) = \frac{4\pi}{Q_2^2}\frac{\omega}{c}(\mathbf{e}\mathbf{Q}_2)G_1(\omega;\mathbf{Q}_2)S_2(\mathbf{Q}_2),$$

$$G_1(\omega;\mathbf{Q}_2) = -\sum_{i=1}^{N+1} e_i\frac{S_1(\mathbf{Q}_2)}{M_1\omega^2} + A_1(\omega;\mathbf{Q}_2), \qquad (10)$$

$$S_1(\mathbf{Q}) \equiv S_{00}^{(1)}(\mathbf{Q}), \quad S_2(\mathbf{Q}) \equiv S_{00}^{(2)}(\mathbf{Q}_2).$$

Here the quantity $A_1$, which is defined by the following equality:

$$\mathbf{Q}A_1(\omega;\mathbf{Q}) \equiv -i\sum_n \frac{2\omega_{n0}}{\omega^2-\omega_{n0}^2} S_{0n}^{(1)}(\mathbf{Q})\mathbf{d}_{n0}, \qquad (11)$$

is the generalized polarizability of the first atom. For a spherically symmetric atom it follows from (11) that $A_1(\omega; \mathbf{Q})$ depends only on the absolute value of $\mathbf{Q}$, which will be used subsequently in calculating the bremsstrahlung cross section. For neutral atoms, and also for heavy ions with mass $M \gg m_e$ we have $G_1(\omega; \mathbf{Q}) \approx A(\omega; \mathbf{Q})$. In the region of small momentum transfers, $A(\omega; Q)$ goes over to the ordinary dipole dynamical polarizability of the atom $\alpha_d(\omega)$, since for $QR_{at} \ll 1$

$$S_{n0}(\mathbf{Q}) = \sum_{i=1}^{N+1} e_i\delta_{n0} - i\mathbf{Q}\mathbf{d}_{n0}, \quad \mathbf{d}_{00} = 0.$$

The analytic expression for $F_2$ in (8) is obtained from $F_1$ by permutating the indices 1 and 2. Taking this into account, we find from (8) the following expression for the bremsstrahlung amplitude in atom-atom collisions:

$$F(\omega;\mathbf{Q};\mathbf{e}) = -\frac{4\pi\omega}{c}\frac{(\mathbf{e}\mathbf{Q})}{Q^2}\{G_1(\omega;\mathbf{Q})S_2(\mathbf{Q}) - G_2(\omega;\mathbf{Q})S_1(\mathbf{Q})\},$$

$$\mathbf{Q} \equiv \mathbf{Q}_1 = -\mathbf{Q}_2. \qquad (12)$$

In derivation of (12) we used the condition $\mathbf{Q}_2 = -\mathbf{Q}_1 + \mathbf{k} \approx -\mathbf{Q}_1$, which follows from the relation $Q_2 \geqslant \omega/v_1 \gg \omega/c = \mathrm{k}$, where $v_1$ is the velocity before the collision of the first atom in the rest system of the second atom.

In the region $\omega \gg I$, neglecting $\omega_n 0 \sim I$ in comparison with $\omega$ in the energy-dependent denominators in (9), and also taking into account that $M \gg m_e$, we obtain a simple expression for the bremsstrahlung amplitude (8):

$$F(\omega,\mathbf{Q},\mathbf{e}) = \frac{4\pi}{Q^2}\frac{(\mathbf{e}\mathbf{Q})}{\omega c}\{Z_2W_1(\mathbf{Q}) - Z_1W_2(\mathbf{Q})\}. \qquad (13)$$

Here $W(\mathbf{Q})$ is the form factor of the atom. For the function $S(\mathbf{Q})$ in derivating (13) we used the approximation $S(\mathbf{Q}) = Z - W(\mathbf{Q})$, which follows from (7) for the condition $W(Q) \ll (Zm_e/M)QR_{at}$, which is satisfied in the region of momenta comparable with atomic momenta which is of interest here. As follows from Eqs. (12) and (13), there is no dipole bremsstrahlung when identical atoms collide: $F(\omega; \mathbf{Q}; e) = 0$, and therefore the bremsstrahlung in this case is determined by the relativistic correction to the amplitude found.

## 2. BREMSSTRAHLUNG CROSS SECTION

Calculating the differential cross section corresponding, to the bremsstrahlung amplitude (12), we shall take as unit polarization vectors the vectors

$$\mathbf{e}_1=[\mathbf{p}_1, \mathbf{k}]/|[\mathbf{p}_1, \mathbf{k}]|, \quad \mathbf{e}_2=[\mathbf{k}, \mathbf{e}_1]/k,$$

where the square brackets denote a vector product, and we shall introduce the polarization matrix of the photon density, expressed in terms of the Stokes parameters $\xi_i$ ($i = 1, 2, 3$). Integrating $d^3\sigma/d\mathbf{k}d\mathbf{p}_1'd\mathbf{p}_2'$ over the momenta of the atoms in the final state, we obtain the following expression:

$$\frac{d\sigma}{d\omega\, d\Omega_{\mathbf{k}}}(\xi_i)=\frac{1}{2\pi c^3}\frac{\omega^3}{v_1^2}\int\limits_{\omega/v_1}^{2\mu v_1}\frac{dQ}{Q}\Big\{1+\cos^2\theta+\xi_3\sin^2\theta$$

$$+\left(\frac{\omega}{v_1 Q}\right)^2(1-3\cos^2\theta-3\xi_3\sin^2\theta)\Big\}$$

$$|G_1(\omega;Q)S_2(Q)-G_2(\omega;Q)S_1(Q)|^2. \qquad (14)$$

Here $\theta = \mathbf{k}\hat{\mathbf{p}}_1$ and $\mu = M_1M_2/(M_1 = M_2)$. The upper and lower limits of integration over $Q$ are established from conservation of energy and momentum.

The parameter $\xi_3$ characterizes the degree of linear polarization of the photon along the vectors $\mathbf{e}_1$ and $\mathbf{e}_2$; the probability of polarization along $e_1$ is $1/2(1 + \xi_3)$, and that along $\mathbf{e}_2$ is $1/2(1 - \xi_3)$. Thus, the resulting bremsstrahlung is found to be linearly polarized. Setting $\xi_3 = 0$ in (14) and multiplying by two, we obtain the cross section averaged over photon polarization $d\sigma/d\omega d\Omega_k$

Equation (14) shows that to calculate cross sections for the collision of two neutral atoms it is necessary to know the amplitude in the region $QR_{\rm at} \sim 1$, for outside this region the integrand falls off rapidly. On the other hand, when even one of the particles of the pair is an ion, the integrand rises as $1/Q$ in the region $QR_{\rm at} \ll 1$ if this region of momentum transfer is allowed kinematically, i.e., when $Q_{\rm min}R_{\rm at} = \omega R_{\rm at}/v_1 \ll 1$. This can easily be seen from the following relation:

$$S(Q)=Z-N+O(Q^2R_{\rm at}^2),$$

$$G(\omega;\ Q)=\alpha_d(\omega)[1+O(Q^2R_{\rm at}^2)], \quad QR_{\rm at}\ll 1.$$

The difference for small $Q$ is due to the Coulomb long-range interaction in ion-atom and ion-ion collisions, which does not exist in collisions between neutral atoms. The long-range interaction of ions is felt in the region $Q \sim Q_{\rm min}$ (here $Q_{\rm min} R_{\rm at} \ll 1$), since in this case there is a large contribution to the bremsstrahlung spectrum from events which occur at large distances between the colliding particles. When the condition $\omega R_{\rm at}/v_1 \gtrsim 1$ is satisfied, the bremsstrahlung originates at distances comparable with the atomic dimensions, and the cross sections (14) for neutral atoms and charged ions are found to be of the same order.

In the region $R_{\rm at}/v_1 \ll 1$ the cross section (14) is simplified: in the expression in curly brackets we can neglect terms containing the parameter $(\omega/v_1Q)^2$, because in the region $QR_{\rm at} \sim 1$ which is most important for the formation of bremsstrahlung of neutral atoms this term is small $(\omega/v_1Q)^2 \sim (\omega R_{\rm at}/v_1)^2 \ll 1$.

In addition, for neutral atoms the lower integration limit in (14) can be approximately set equal to zero, since the integrand becomes extremely small in the region $Q \sim Q_{\rm min}$. Then

$$\frac{d\sigma}{d\omega\, d\Omega_{\mathbf{k}}}(\xi_i)=\frac{\omega^3}{2\pi v_1^2c^3}J(\omega)\{1+\cos^2\theta+\xi_3\sin^2\theta\},$$

where the integral in Eq. (14), calculated between limits of zero and infinity, is a function of $\omega$ characterizing a given pair of atoms. From this formula we obtain the angular distribution of unpolarized photons $\{1 + \cos^2\theta\}$. It coincides with the angular distribution of the radiation of a dipole rotating in the plane of scattering and averaged over the possible orientations of this plane.

In the case of an ion-atom or ion-ion collision for frequencies $R_{\rm at}/v_1 \ll 1$ the angular distribution of unpolarized photons $\{1 + \cos^2\theta\}$ is obtained form (14) in the so-called logarithmic approximation, for which it is necessary to expand the integrand in powers of $Q$ and to integrate it between limits $Q_{\rm min}$ and $Q \sim 1/R_{\rm at}$ and then to separate only the leading logarithmic term of this expansion.

Averaging (14) over polarizations and then integrating over $d\Omega_{\mathbf{k}}$, we obtain the differential cross section for bremsstrahlung as a function of frequency, i.e., the bremsstrahlung spectrum:

$$\frac{d\sigma}{d\omega}=\frac{16}{3}\frac{1}{c^3}\frac{\omega^3}{v_1^2}\int\limits_{\omega/v_1}^{2\mu v_1}\frac{dQ}{Q}|G_1(\omega;Q)S_2(Q)-G_2(\omega;Q)S_1(Q)|^2. \qquad (15)$$

The remarks made above regarding the features of the calculation and the possible simplification of the cross section $d\sigma/d\omega d\Omega_{\mathbf{k}}$ in the region $\omega R_{\rm at}/v_1 \ll 1$ apply equally to Eq. (15).

In the region $\omega \gg I$ the bremsstrahlung spectrum corresponding to the amplitude (13) is particularly simple:

$$\frac{d\sigma}{d\omega}=\frac{16}{3}\frac{1}{c^3}\frac{1}{v_1^2}\frac{1}{\omega}\int\limits_{\omega/v_1}^{\infty}\frac{dQ}{Q}|Z_1W_2(Q)-Z_2W_1(Q)|^2. \qquad (16)$$

The upper integration limit in (16) has been replaced by infinity. The integration in the region $Q \gg Q_{\rm max}$ does not make an appreciable contribution to the bremsstrahlung spectrum, since in this region, as over the entire region $QR_{\rm at} \gg 1$, the integrand falls off rapidly.

The structure of (16) is quite clear. In the region $\omega \gg I$ the bremsstrahlung spectrum is formed only as the result of acceleration of the electrons of each of the atoms in the field of the other nucleus. The bremsstrahlung amplitude of quasifree electrons belonging to one of the atoms, for example the first in the nucleus of the second, is proportional to the factor $Z_2W_1(Q)$, which is analogous to the product of the two charges in the amplitide of bremsstrahlung of two structureless charged particles. Therefore the complete amplitude of bremsstrahlung in collision of two atoms is proportional to $Z_1W_2(Q) - Z_2W_1(Q)$. The minus sign in this expression arises as the result of the fact the dipole moments of the atoms have been oriented in opposite directions in the collision process.

Recoil of the nuclei in the scattering process has been taken into account in the cross sections (14) and (15) in the functions $S_{n0}^{(1)}(Q)$ and $S_{n0}^{(2)}(Q)$ (see the third term in Eq. (7)), which enter into these formulas both explicitly for $n = 0$ and implicitly in terms of the definition of the generalized polarizabilities. However, in calculation of these cross sections, recoil can be neglected since it becomes appreciable only for rather large $Q$ when $W(Q) \lesssim QR_{\rm at}(Zm_e/M)$, and for the bremsstrahlung spectrum of atoms values $QR_{\rm at} \sim 1$ are most important.

In bremsstrahlung processes in which the momentum transfer is fixed, in the region of large $Q$ the role of recoil is very important. In fact in the polarizability $A(\omega;\,Q)$ the main contribution is from values of $S_{no}(Q)$ with energy $\varepsilon_n \sim I$. It follows from Eq. (7) that in the region $W(Q) \ll (Zm_e/M)QR_{\rm at}$ the contribution of recoil (the third term in Eq. (7)) to $S_{no}(Q)$ for $n \neq 0$, $\varepsilon_n \sim I$ and consequently also to $A(\omega;\,Q)$ will be the main contribution. Considering the cross section in just this region, we arrive at the following result:

$$\frac{d\sigma}{d\omega\,dQ}=\frac{16}{3}\frac{1}{c^3}\frac{\omega^3}{v_1^2}\frac{Z_1^2Z_2^2}{Q}\left|\frac{\alpha_d^{(1)}(\omega)}{M_1}-\frac{\alpha_d^{(2)}(\omega)}{M_2}\right|^2,$$

$$QR_{\rm at} \ll \frac{M}{Zm_e},\qquad QR_{\rm at} \gg \frac{M}{Zm_e}W(Q). \qquad (17)$$

Integrating the cross section (17) over the momentum transfer $Q$ in the region determined in (17), we see that the expression obtained here contains a small parameter $(Z_1Z_2/M_{1(2)}) \ll 1$. This demonstrates the possibility of neglecting recoil in calculation of the cross sections (14) and (15).

The spectrum (15) in the case of an incident structureless charged particle goes over to the expression obtained in Refs. 5 and 6 in discussion of bremsstrahlung in a many-electron atom with inclusion of its dynamical polarizability. Indeed, setting in (15) for the target atom

$$G_2(\omega;\,Q)=A(\omega;\,Q),\quad S_2(Q)=Z-W(Q)$$

(the recoil of the nucleus can be neglected) and for the incident particle

$$G_1(\omega;\,Q)=-1/M\omega^2,\quad S(Q)=-1,$$

where $M$ is the mass of this particle, we arrive at the well known result:

$$\frac{d\sigma}{d\omega}=\frac{16}{3}\frac{1}{c^3}\frac{\omega^3}{p^2}\int_{\omega/v}^{2p}\frac{dQ}{Q}\left|\frac{Z-W(Q)}{\omega^2}-MA(\omega;\,Q)\right|^2. \qquad (18)$$

Here the first term within the absolute value signs describes the radiation of the incident particle in the field of the neutral atom, and the second describes the radiation of the atom polarized by the incident particle in the course of the collision.

## 3. BREMSSTRAHLUNG IN COLLISION OF HELIUM WITH XENON

As was mentioned above, the bremsstrahlung of heavy charged particles in an atom turns out to be of the same order as the bremsstrahlung of an electron in the same target. For the bremsstrahlung spectra arising when a helium atom an $\alpha$ particle, and an electron collide with a xenon atom we shall demonstrate that the intensities of radiation when neutral atoms and charged particles scatter are of the same order over a wide region of characteristic atomic frequencies. The results of a numerical calculation of the bremsstrahlung spectra of an $\alpha$ particle and a helium atom on a xenon atom have been given briefly in a previous note.[9]

To calculate the bremsstrahlung spectra of a helium atom and an $\alpha$ particle scattering on a xenon atom we shall use Eq. (15), while to calculate the bremsstrahlung spectrum of an electron we shall use Eq. (18). In (15) it is not necessary to take into account the recoil of nuclei in the frequency region studied and the velocity region $\omega R_{\rm at}/v \lesssim 1$, as follows from the discussion given in the preceding section. The values of the He form factor necessary for the calculations were taken from Ref. 10. The xenon form factor in the Hartree-Fock approximation and the generalized polarizability of xenon $A^{\rm Xe}(\omega;Q)$ were taken from Ref. 8, where the bremsstrahlung spectrum of an electron is an xenon atom was calculated. The generalized polarizability of helium in the region of frequencies above the threshold for ionization of the $4d^{10}$ subshell of xenon which we are discussing here can be represented in the form $A^{\rm He}(\omega;\,Q) \approx -{\rm W}(Q)/\omega^2$. For frequencies $\omega \gtrsim I({\rm xe},\,4d^{10})$ the relation $|A^{\rm He}(\omega;Q)|^2 \ll |A^{\rm Xe}(\omega;Q)|^2$ holds. However, since in the region $QR_{\rm at} \sim 1$ we have

$$(Z^{\rm He}-W^{\rm He}(Q))\,{\rm Re}\,A^{\rm Xe}(\omega;\,Q) \sim (Z^{\rm Xe}-W^{\rm Xe}(Q))\,{\rm Re}\,A^{\rm He}(\omega;\,Q),$$

these two terms interfere in the spectrum (16). The nature of the interference is determined by the signs of the generalized polarizabilities. Since ${\rm Re}A^{\rm He}(\omega;\,Q) < 0$, consequently for ${\rm Re}A^{\rm Xe}(\omega;\,Q) < 0$ (the region $\omega \gtrsim 7.8$ Ry in Fig. 2) the real parts of the polarizabilities of the atoms in the bremsstrahlung spectrum (16) interfere destructively, while for ${\rm Re}A^{\rm Xe}(\omega;\,Q) > 0$ (the region $\omega \lesssim 7.8$ Ry in Fig. 2) they interfere constructively. There actually is no interference of the imaginary parts of the polarizabilities in the spectrum, since in the frequency region we are considering ${\rm Im}A^{\rm He}(\omega;\,Q) \approx 0$. The imaginary part ${\rm Im}A^{\rm Xe}(\omega;\,Q)$ is very large for $\omega \gtrsim I({\rm Xe},\,4d^{10})$ and when the real parts of the polarizabilities interfere destructively it basically describes the behavior of the bremsstrahlung spectrum of atoms.

Results of the calculation of the bremsstrahlung spectra of helium, $\alpha$ particles, and electrons in xenon atoms for $v = 5v_{\rm at}$ are given in Fig. 2, where we also show the results from calculating the bremsstrahlung spectrum of He in Xe without taking into account the polarizability of He (Ref. 9) ($v_{\rm at} = e^2/\hbar$ is the characteristic velocity of an electron in the

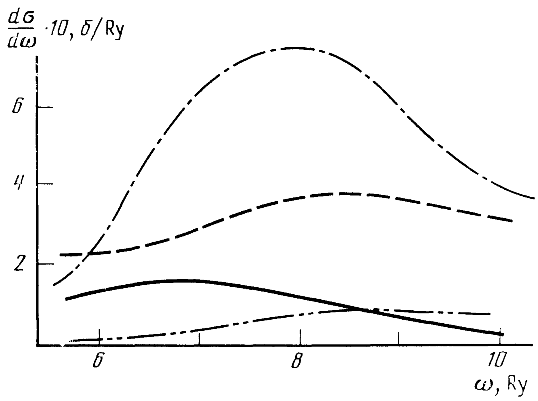


FIG. 2. Bremsstrahlung spectra in Xe: of an electron (dashed curve), of an $\alpha$ particle (dot-dash line), of helium without taking into account its polarizability (dash double-dot line), and of helium including the polarizability of both atoms (solid line) for a velocity of incidence $v_1 = 5v_{\rm at}$.

atom). The selected velocity $v$ is close to the lower limit of applicability of the approximation described. The radiation spectra shown in Fig. 2 exhibit broad maxima. They arise as the result of the fact that in the generalized polarizability $|A^{Xe}(\omega;\ Q)|^2$ as a function of frequency in the region $\omega \gtrsim I(\mathrm{Xe}, 4d^{10})$ there is a broad maximum.[6]

In the example we are considering (see Fig. 2) above the ionization threshold of the $4d^{10}$ subshell of Xe for frequencies $\omega \sim 8$ Ry and $v \sim 5v_{at}$ the relation $Q_{min} R_{at} \sim 1$ is satisfied, and consequently the intensities of bremsstrahlung of helium atoms, $\alpha$ particles, and electrons in Xe are of the same order, as follows from the discussion given in Section 2.

## CONCLUSIONS

In this article we have discussed the polarization mechanism by which bremsstrahlung is produced in collisions of atoms, which leads to very substantial intensities of the radiation in the continuum.

We can expect that as incident velocity decreases the mutual deformation of the atoms will rise, giving rise to an increase of the intensity of the bremsstrahlung they emit. As in the case of electron bremsstrahlung in an atom when the target polarization is included in the collision process,[7] at relativistic velocities the retardation in the interaction between the atoms should enhance the effectiveness of their polarization mechanism of radiation.

In addition to this process in which bremsstrahlung is produced without changing the internal state of the atom at the end of the collision, photon emission results also from reactions accompanied by ejection of electrons or by transition of atoms to an excited state. The relative role of these processes in the total cross section for bremsstrahlung is being studied at the present time.[11]

It is also of interest to investigate the bremsstrahlung produced in collisions of atoms which do not have spherical symmetry, for example, in the case in which the quadrupole moment of the atom is nonzero. Since this creates a long-range field falling off as $1/r^3$, we can expect the intensity of bremsstrahlung in processes involving such atoms to increase.

The conclusion drawn in our work that the intensity of bremsstrahlung in neutral atom scattering is significant can be qualitatively generalized to collisions involving molecules. To calculate the bremsstrahlung cross section in this case it is necessary to take into account both the nonspherical static distribution of the molecular charge in the field of which the motion of the other molecule occurs, and also the molecular (vibrational and rotational) degrees of freedom, which should appear in the polarizability especially strongly for $\omega$ greater than or of the order of their excitation energy.


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