Resonance splitting of bands of quasicharacteristic radiation in electrons channeling in crystals with a superlattice

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The spontaneous radiation of electrons in planar and axial channeling has been investigated theoretically in crystals with a superlattice under resonance conditions when the energy difference of two levels of the transverse motion in the averaged potential of the channel is approximately equal to a multiple of $2\pi\hbar c/l$, where l is the period of the superlattice along the channel axis. It is shown that the total energy levels of the electrons are split under the action of the spatially periodic perturbation. This changes the spectral and angular characteristics of the radiation substantially. In particular, resonance splitting of individual bands of the quasicharacteristic radiation is predicted. The magnitude of this splitting is estimated. It is established that there is no observable splitting in the channeling of positrons.

INTRODUCTION

Following the predictions by Kumakhov[1,2] in 1976 of intense x-rays and γ radiation in channeling of relativistic electrons and positrons in single crystals, a large number of theoretical and experimental papers have been devoted to this phenomenon; a review can be found in Refs. 3 and 4. As estimates show, at sufficiently high particle energies ($\gtrsim 100$ MeV) a classical description of the motion of the channeled particles is applicable. At lower energies, when there are only a few levels of the transverse motion, the classical description becomes clearly inapplicable. In this case experiments devoted to studying the radiation of channeled particles detect bands of quasicharacteristic radiation which appear distinctly on the background of a broad spectrum; these bands arise from spontaneous transitions between discrete energy levels of the transverse motion.[5–7]

In channeling a particle experiences coherent scattering by different atoms located along the channel axis. As a result the action of the combined potential of the atoms of the crystal usually can to a good approximation be replaced in lowest order by the action of a potential averaged along the direction of the axis. Here it is important that the ratio of the time for a particle to travel a distance equal to the period of the averaged potential is small compared to the period of the transverse oscillations. Some features of the radiation when the discreteness of the atomic planes is taken into account have been discussed in Refs. 8–11.

The extensive production of various semiconductor multilayer superlattices by methods of molecular epitaxy (see for example Ref. 12 and references cited therein) permits the properties of particle channeling in crystals with a superlattice to be discussed. Chu *et al.*[13] observed experimentally an increase in the probability of de-channeling of ions in layered superlattices of $GaP/GaAs_xP_{1-x}$ when the superlattice period coincided with the wavelength of the transverse oscillations. Ikezi *et al.*[14] considered theoretically the radiation of electrons in axial channeling in multilayer crystals. However, the authors of that work overlooked the important fact that the levels of the total energy of a channeled particle are multiply degenerate. This degeneracy, which does not appear under ordinary conditions, is partially lifted under the influence of a large-scale spatially periodic perturbation of the channel potential in which the difference of the energies of two levels of the transverse motion is equal to a multiple of $2\pi\hbar c/l$, where l is the period of the superlattice along the channel axis.

In the present work we shall show that the resonance splitting of the energy levels of a channeled particle under the influence of a spatially periodic perturbation gives rise to a substantial change of the spectral and angular characteristics of the radiation. In particular, one should observe a resonance splitting of the series of bands of quasicharacteristic radiation in planar and axial channeling of electrons. The magnitude of the splitting has been estimated. It is noted that the effect of resonance splitting of the bands is characteristic for electrons and should not be present in channeling of relativistic positrons.

1. RADIATION IN PLANAR CHANNELING OF ELECTRONS

The quantum theory of spontaneous radiation of channeled particles in single crystals was developed in Refs. 15 and 16. We shall consider first the planar channeling of electrons in crystals with a superlattice. Let the xz plane be the plane of channeling and the z axis be directed along the channel axis. We shall consider the motion of an electron in a potential $U(x) + V(x)W(z)$. Here $W(z)$ is a function periodic along the z axis with a period l, $W(z+1) = W(z)$; $U(x)$ is the averaged continuous potential of the channel, i.e.,

$$\int_0^l W(z)\,dz=0.$$

The form of the function $V(x)$ depends on how the uniform potential of the channel is distorted. For example, if only the channel axis is periodically deformed, then $V(x) = \nabla_x U(x)$,[14] and if the channel axis remains straight, but the channel potential is periodically modulated in amplitude along the z axis, then in the simplest case $V(x) = C + U(x)$, where C is some constant. Other cases are also possible, however. We shall assume that the potential

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$V(x)W(z)$ can be taken into account on the basis of perturbation theory. As will be shown below, this assumption is fulfilled well for real experimental situations.

The stationary states of an electron channeled in a potential $U(x)$ are described by wave functions[4,16]

$$\psi_{i,p}^{(0)}=\frac{1}{L^{1/2}}\varphi_i(x)\exp\left(\frac{i}{\hbar}pz\right), \tag{1}$$

where L is a normalization constant, p is the momentum projection on the z axis, and the functions $\varphi_i(x)$, which describe the states of the transverse motion satisfy the Schrödinger equation with a relativistic mass, which is a consequence of the Dirac equation to within $O(U^2/c^2p^2)$:

$$\left[-\frac{\hbar^2}{2m_0\gamma}\frac{d^2}{dx^2}+U(x)\right]\varphi_i=\mathscr{E}_i\varphi_i, \tag{2}$$

where γ is the relativistic factor, the subscript i numbers the states of the transverse motion,

$$\mathscr{E}_i=E_{i,p}^{(0)}-c(m_0^2c^2+p^2)^{1/2}$$

is the energy of the transverse motion, and $E_{i,p}^{(0)}$ is the total energy of the particle.

It is obvious that in the absence of the perturbation potential $V(x)W(z)$ the energy eigenvalue $E_{i,p}^{(0)}$ corresponds to the set of eigenfunctions $\psi_{i,p}^{(0)}$, $\psi_{f,p'}^{(0)}$,..., where $p'=p+\hbar\omega_{if}/c$ and we have used the notation $\hbar\omega_{if}=\mathscr{E}_i-\mathscr{E}_f$. Thus, each energy level $E_{i,p}^{(0)}$ turns out to be multiply degenerate. The action of the perturbation operator $V(x)W(z)$ under certain resonance conditions partially lifts the degeneracy.

For this purpose we shall consider matrix elements of the operator $V(x)W(z)$ calculated with the wave functions (1):

$$\langle i,p|VW|f,p'\rangle=V_{if}\sum_{\alpha=1}^{\infty}(b_\alpha\delta_{p,p'+2\pi\alpha\hbar/l}+b_\alpha^*\delta_{p,p'-2\pi\alpha\hbar/l}), \tag{3}$$

where

$$V_{if}=\int_l\varphi_i^*(x)V(x)\varphi_f(x)\,dx,$$

$$b_\alpha=\frac{1}{l}\int_0^l W(z)\exp\left(-i\frac{2\pi\alpha z}{l}\right)dz.$$

We shall consider the case of resonance, in which for some $\alpha=\tilde{\alpha}$ the following inequality is satisfied:

$$|E_{i,p}^{(0)}-E_{f,p+q}^{(0)}|<|V_{if}b_{\tilde{\alpha}}|, \tag{4}$$

where $q=2\pi\tilde{\alpha}\hbar/l$ (for definiteness we shall assume that $\mathscr{E}_i>\mathscr{E}_f$). In this case as the correct wave functions in lowest order we must take linear combinations of the functions (1), and the energy eigenvalues must be found from the corresponding secular equation.[17]

The left side of the inequality (4) obviously vanishes for the condition

$$l=2\pi\tilde{\alpha}c/\omega_{if}. \tag{5}$$

In other words, the difference in the energies of two levels of the transverse motion in the averaged potential of the channel must be a multiple of $2\pi\hbar c/l$, where l is the period of the superlattice in the direction of the channel axis. In order to estimate the order of magnitude of l, we shall assume that $\hbar\omega_{if}\sim 100$–10 nm. Therefore the necessary value of the period corresponds to real semiconductor superlattices.[12]

As is well known,[3,4] the transverse motion of a channeled electron is highly anharmonic, and this permits observation of separate bands in the spectrum of quasicharacteristic radiation. Therefore we shall assume that the variation in the distance between the transverse energy levels exceeds the value $|V_{if}b_{\tilde{\alpha}}|$ and that inequality (4) is satisfied only for the pair of levels of the transverse motion energy i and f. Then in the two-level approximation the correct normalized wave functions of the zero approximation have the form

$$\psi_{i,p}^{(1,2)}=\frac{1}{2^{1/2}}\left\{\left[1\pm\frac{M_{if}}{(M_{if}^2+\Lambda_{if}^2)^{1/2}}\right]^{1/2}\psi_{i,p}^{(0)}\right.$$

$$\left.\pm e_{if}\left[1\mp\frac{M_{if}}{(M_{if}^2+\Lambda_{if}^2)^{1/2}}\right]^{1/2}\psi_{f,p+q}^{(0)}\right\}, \tag{6}$$

where

$$e_{if}=(V_{fi}b_{\tilde{\alpha}}^*/V_{if}b_{\tilde{\alpha}})^{1/2};$$

the dimensionless quantity M_{if} characterizes the amount of detuning from the exact resonance,

$$M_{if}=(E_{i,p}^{(0)}-E_{f,p+q}^{(0)})/\hbar\omega_{if},$$

and

$$\Lambda_{if}=2|V_{if}b_{\tilde{\alpha}}|/\hbar\omega_{if} \tag{7}$$

is a measure of the magnitude of the splitting of the selected energy levels for the resonance (5).

The energy values corresponding the functions (6) are

$$E_{i,p}^{(1,2)}=\frac{1}{2}[E_{i,p}^{(0)}+E_{f,p+q}^{(0)}\pm\hbar\omega_{if}(M_{if}^2+\Lambda_{if}^2)^{1/2}]. \tag{8}$$

In Eqs. (6) and (8) the superscripts 1 and 2 correspond to the upper and lower signs. In a similar way we can express $\psi_{f,p}^{(1,2)}$ in terms of $\psi_{f,p}^{(0)}$ and $\psi_{i,p-q}^{(0)}$, and we can express $E_{f,p}^{(1,2)}$ in terms of $E_{f,p}^{(0)}$ and $E_{i,p-q}^{(0)}$.

Using the wave functions $\psi_{i,p}^{(1,2)}$, $\psi_{f,p}^{(1,2)}$ and the energy values $E_{i,p}^{(1,2)}$, $E_{f,p}^{(1,2)}$, it is straightforward to calculate the properties of the quasicharacteristic radiation near the resonance condition (5). The spectral and angular dependence of the spontaneous radiation power for a transition between the discrete levels a and b is given by the formula[16,18]

$$\frac{d^2I_{ab}}{d\Omega\,d\omega}=\frac{e^2\omega^2}{2\pi c}\sum_{\lambda=1}^{2}|\alpha_{ab}\beta_\lambda|^2\delta\left(\frac{E_{a,p}-E_{b,p'}}{\hbar}-\omega\right), \tag{9}$$

where

$$\alpha_{ab}=\frac{c}{E_{a,p}}\int\exp(-i\boldsymbol{\varkappa}\mathbf{r})\psi_{b,p'}^*\hat{\mathbf{p}}\psi_{a,p}\,d\mathbf{r};$$

here $\hbar\varkappa$ and β_λ are the momentum and polarization of the photon; $\omega=c|\varkappa|$.

Depending on the form of the perturbation operator, two different situations are possible, which are shown schematically in Figs. 1 and 2. The first situation occurs when the channel potential perturbation operator is such that the resonance condition (4) or (5) is satisfied for the two closest levels of the transverse energy, i and $f\equiv i-1$, and the second

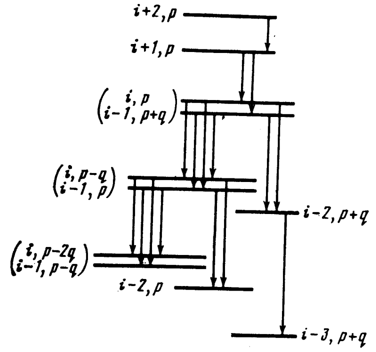

FIG. 1. Diagram of a series of the most intense spontaneous transitions of a channeled electron in the case $V(x) = \nabla_x U(x)$. The resonance condition (4) is satisfied for transverse energy levels i and $i-1$. The transverse-energy-level numbers and the longitudinal momentum of the electron have been indicated. In parentheses we have given the quantum numbers of the energy levels split under the action of a spatially periodic channel-potential perturbation.

situation occurs when two further removed levels fall into resonance, say, i and $f \equiv i-2$. In Fig. 1 we have shown a series of levels of the electron energy and the most intense transitions, in the dipole approximation, between them in the first case, when theperturbation operator $V(x)W(z)$ connects neighboring levels of the transverse energy (i.e., $V_{i,i-1} \neq 0$ and the inequality (4) is satisfied). This case occurs, for example, when $V(x) = \nabla_x U(x)$. Splitting of the electron energy levels leads to an appreciable change of the spectral and angular characteristics of the radiation. For example, for the transitions $|i^{(1,2)}\rangle \to |f^{(1,2)}\rangle (|f\rangle \equiv |i-1\rangle$; see Fig. 1) in the dipole approximation we obtain from (6)–(9).

$$\frac{d^2 I_{if}}{d\Omega\, d\omega} = B_{if} \frac{e^2 |x_{if}|^2 \omega^2 \omega_{if}'^2}{8\pi c^3} \Big\{ \sin^2\zeta + \cos^2\zeta \Big[\frac{\cos\theta - \beta_\parallel [1+\sin^2\theta\, (\varepsilon (M_{if}^2+\Lambda_{if}^2)^{1/2} - M_{if})]}{1-\beta_\parallel \cos\theta} \Big]^2 \Big\} \times \delta[\omega_{if}(1+\varepsilon (M_{if}^2+\Lambda_{if}^2)^{1/2} - M_{if}) - \omega(1-\beta_\parallel \cos\theta)], \quad (10)$$

where ex_{if} is the matrix element of the dipole moment of the transition, ζ is the azimuthal angle of the radiation, θ is the angle between $\varkappa$ and the z axis, and $\beta_\parallel = p/m_0\gamma c$. Values of the dimensionless quantities B_{if} and ε are given in Table I, where for brevity we have used the notation $\Delta = M_{if}/(M_{if}^2 + \Lambda_{if}^2)^{1/2}$.

As follows from Eq. (10) for the transition $|i\rangle \to |f\rangle (\equiv |i-1\rangle)$, for exact resonance (5) three maxima appear in the frequency spectrum of the radiation:

$$\tilde{\omega}_{if} = \omega_{if}(1+\varepsilon\Lambda_{if})/(1-\beta_\parallel) \quad (11)$$

($\varepsilon = 0, \pm 1$) instead of one. The dimensionless quantity Λ_{if} which characterizes the splitting, as is shown by estimates given in Section 3, under certain conditions can be fairly large and accessible for experimental observation.

Thus, in the case considered (Fig. 1, Table I), when the resonance condition (4) is satisfied for the two closest levels of the transverse energy, i and $i-1$, in the spectral distribution of the intensity of radiation the band corresponding to the transition $|i\rangle \to |i-1\rangle$, will be split into three lines, and the bands corresponding to the transitions $|i+1\rangle \to |i\rangle$ and $|i-1\rangle \to |i-2\rangle$ will be split into two lines. The bands corresponding to all other transitions, as can be seen from Fig. 1, remain unsplit. This fact is directly related to the assumption that the magnitude of the variation of the transverse energy level separations for the electron in the channel exceeds the magnitude of the resonance splitting Λ_{if}. It will be shown below that this assumption is well satisfied for real semiconductor superlattices. As can be seen from Fig. 1, when the value of $|M_{if}|/\Lambda_{if}$ increases the intensities are redistributed between the individual lines of the split bands. Obviously, detuning from the exact resonance (5) will reduce the chances for experimental observation of the resonance splitting effect.

It should be mentioned that if the radiative width of the energy levels is taken into account, the δ-function in Eq. (10) must be replaced by a Lorentz curve, as was done, for example in Ref. 4. In addition, the criterion for the approach used above to be applicable is that the ratio of the half-sum of the widths of the levels i and f to the inverse time of flight of the particle over a distance equal to the superlattice period be small.

In Fig. 2 we have shown the other possible situation, in which the channel-potential perturbation operator $V(x)W(z)$ connects energy levels which are spaced further apart (than in Fig. 1), between which a spontaneous transition is impossible in the dipole approximation. This case occurs, for example, when $V(x) = C + U(x)$. Then $V_{i,i-2} \neq 0$, and if $\omega_{i,i-2} = 2\pi\tilde{\alpha}c/l$, then, as can be seen from Fig. 2, each of the four bands corresponding to the transitions $|i+1\rangle \to |i\rangle$, $|i\rangle \to |i-1\rangle$, $|i-1\rangle \to |i-2\rangle$, $|i-2\rangle \to |i-3\rangle$ is found to split into two lines. The bands corresponding to all other transitions are unsplit. The absolute value fo the splitting $|V_{i,i-2} b_{\tilde{\alpha}}|$ for all four bands is the same. For the transitions indicated, the formula for the spectral and angular dependence of the radiation power is similar to Eq. (10) with the corresponding values of ω_{if}, a coefficient 2 in front of the formula, and $\varepsilon = \pm 1/2$.

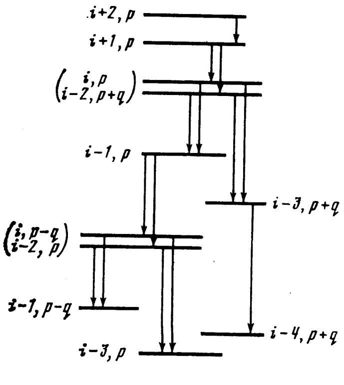

FIG. 2. Diagram of a series of the most intense spontaneous transitions of a channeled electron in the case of a perturbation potential $U(x)W(z)$. The resonance condition (4) is satisfied for transverse energy levels i and $i-2$.

2. RADIATION IN AXIAL CHANNELING

The results of the previous section are easy to relate to the case of axial channeling of electrons in a crystal with a

TABLE I. Values of B_{if} and ε for a number of transitions.

Transition	B_{if}	ε	Transition	B_{if}	ε
$\left\{ \begin{array}{l} \lvert i^{(1)}\rangle \to \lvert f^{(1)}\rangle \\ \lvert i^{(2)}\rangle \to \lvert f^{(2)}\rangle \end{array} \right.$	$1-\Delta^2$	0	$\left\{ \begin{array}{l} \lvert i+1\rangle \to \lvert i^{(1)}\rangle \\ \lvert f^{(2)}\rangle \to \lvert f-1\rangle \end{array} \right.$	$2(1+\Delta)$	$-1/2$
$\lvert i^{(1)}\rangle \to \lvert f^{(2)}\rangle$	$(1+\Delta)^2$	1	$\left\{ \begin{array}{l} \lvert i+1\rangle \to \lvert i^{(2)}\rangle \\ \lvert f^{(1)}\rangle \to \lvert f-1\rangle \end{array} \right.$	$2(1-\Delta)$	$1/2$
$\lvert i^{(2)}\rangle \to \lvert f^{(1)}\rangle$	$(1-\Delta)^2$	-1			

superlattice. In this case the channel-potential perturbation operator has the form $V(\rho)W(z)$, where ρ is the modulus of the two-dimensional radius vector in the xy plane. Then using the usual approach[16] to the axial-channeling case, we obtain the following formula for the spectral and angular dependence of the dipole radiation power for the condition of exact resonance (5):

$$\frac{d^2I_{if}}{d\Omega\, d\omega}=\frac{e^2|r_{if}|^2\omega^2\omega_{if}^2}{8\pi c^3(1-\beta_\parallel\cos\theta)^2}\Big\{(1-\beta_\parallel\cos\theta)^2-\frac{1}{2}\sin^2\theta[1-\beta_\parallel^2 -2\varepsilon\Lambda_{if}(\beta_\parallel-\cos\theta)-\varepsilon^2\Lambda_{if}^2\sin^2\theta]\Big\} \times\delta[\omega_{if}(1+\varepsilon\Lambda_{if})-\omega(1-\beta_\parallel\cos\theta)], \quad (12)$$

where $r_{if}=\int\varphi_i{}^*\varphi_f\rho^2d\rho$, and the values of the remaining quantities can be understood from comparison of Eq. (12) with Eq. (10). Dipole transitions are possible only when the angular-momentum projection on the z axis changes by unity.

As in the case of planar channeling, two basic cases are possible. The first case occurs when the perturbation operator connects states with different parity. It arises, for example, when $V(\rho)=\nabla_x U(\rho)$. We shall not give the diagrams of the energy levels and allowed transitions between them or the values of ε, since these are easily obtained by analogy with the previous section.

3. ESTIMATION OF THE MAGNITUDE OF THE RESONANCE SPLITTING OF THE LEVELS

In the present section we present results of calculations of the magnitude of the resonance splitting Λ_{if} (7) for planar channeling of electrons. For definiteness the estimates were made for the first harmonic of the spatially periodic perturbation ($\tilde{\alpha}=1$). As is well known, for electrons the averaged potential of a plane can be represented accurately in the form

$$U(x)=-U_0\,\mathrm{ch}^{-2}(x/b).$$

Here the functions $\varphi_i(x)$ which describe the transverse oscillations are expressed in terms of ultraspherical polynomials.[19] In Tables II and III we give values of the resonance splitting Λ_{if} and the quantities associated with them for a number of the lowest levels of the transverse energy. We considered the two cases shown in Figs. 1 and 2. Numerical estimates were made for typical values $U_0=20$ eV, $b=0.03$ nm,[4] and total energy $E=50$ MeV. The form of the function $W(z)$ depends on the specific situation, but in many cases a good approximation (for alternating layers of identical thickness) is

$$W(z)=a_0,\quad nl\leqslant z<(n+1/2)l,$$
$$W(z)=-a_0,\quad (n+1/2)l\leqslant z<(n+1)l,$$

where a_0 is a constant characterizing the amplitude of distortion of the uniform channel potential and n is an integer. In Table II data are given for the perturbation potential $\nabla_x U(x)W(z)$. For estimates we took the value $a_0=0.002$ nm.[14] We have also used the gamma function $\Gamma(s)$ and

$$s=\left(\frac{1}{4}+\frac{2b^2EU_0}{\hbar^2c^2}\right)^{1/2}-\frac{1}{2}.$$

In Table III we give data for the perturbation potential $U(x)W(z)$; for estimates we took $a_0=0.1$.

The values of Λ_{if} given in Tables II and III are less than the magnitude of the variation in the evergy level spacing of the transverse motion $\delta=\omega_{i,i-1}/\omega_{i+1,i}-1$. We also carried out calculations of the value of the resonance splitting for axial channeling of electrons. For an axis potential $U(\rho)=-\alpha/\rho$ with $\alpha=1$ nm · eV and $E=4$ MeV (Ref. 4) the values of Λ_{if} fell in the range 0.05–0.25 for the series of lowest levels of the transverse energy. Thus, the estimates show that with reasonable values of the parameters, the conditions for the two-level approximation to be applicable are satisfied for treatment of electron channeling in crystals with a supperlattice.

CONCLUSIONS

In the present work we have investigated the spontaneous radiation of electrons in channeling in crystals with a

TABLE II. Parameters characterizing the splitting in planar channeling of electrons in a perturbation potential $\nabla_x U(x)W(z)$.

i	f	$\lvert\langle i\lvert\nabla_x U\rvert f\rangle\rvert\cdot\left\{\frac{U_0 s}{b}\frac{[2(s-1)]^{1/2}}{s+1}\times\left[\frac{\Gamma(s+1/2)}{\Gamma(s+1)}\right]^2\right\}^{-1}$	$U_0=20$ eV, $b=0.03$ nm, $E=50$ MeV, $a_0=0.002$ nm: $2\lvert V_{if}b_1\rvert$, eV	$\frac{2\lvert V_{if}b_1\rvert}{\hbar\omega_{if}}$
1	0	1	0,730	0,145
2	1	$\frac{[2(s-2)]^{1/2}(s-5/4)}{(s-1/2)^{3/2}}$	0,774	0,185
3	2	$\frac{[3(s-2)(s-3)]^{1/2}(s^2-3s+13/8)}{(s-1/2)^2(s-3/2)}$	0,661	0,200

TABLE III. Parameters characterizing the splitting in planar channeling of electrons in a perturbation potential $U(x)W(z)$.

i	f	$\lvert\langle i \mid U \mid f\rangle\rvert \cdot \left[\frac{U_0}{s+1/2}\left(\frac{s-2}{2s-1}\right)^{1/2}\right]^{-1}$	$U_0 = 20$ eV, $b = 0.03$ nm, $E = 50$ MeV, $a_0 \doteq 0.1$ nm: $2\lvert V_{if}b_1\rvert$, eV	$\frac{2\lvert V_{if}b_1\rvert}{\hbar\omega_{i,\,i-1}}$
2	0	1	0,228	0,054
3	1	$\left[\frac{3(s-3)}{s-2}\right]^{1/2}$	0,346	0,105
4	2	$\left[\frac{6(s-1/2)(s-4)}{(s-1)(s-3/2)}\right]^{1/2}$	0,405	0,166

superlattice under the resonance conditions (4), when the energy difference of two levels of the transverse motion is close to a multiple of $2\pi\hbar c/l$. Under such conditions large-scale spatially periodic perturbation of the channel potential, called a superlattice, partially lifts the degeneracy and causes splitting of a number of levels of the total channeled particle energy, which in turn leads to splitting of a number of bands of the quasicharacteristic radiation. The form of the channel-perpturbation potential can be deduced from the nature of the band splitting (into two or three lines). The magnitude of the splitting Λ_{if} will depend both on the form of the resonance perturbation operator and on the specific wave functions of the transverse motion.

The results of numerical calculations presented above show that with appropriate choice of the experimental conditions the magnitude of the splitting can be quite large: $\Lambda_{if} \approx 0.1$–0.2. For experimental observation of the phenomenon considered in the present work, it is necessary to create conditions under which the half-width of the line of quasi-characteristic radiation will be less than the magnitude of the splitting. From this point of view planar channeling of electrons is apparently more appropriate than axial channeling. For example, in Refs. 5 and 7 for planar channeling of electrons with energy $E \approx 55$ MeV in silicon and diamond the half-width of the individual lines of the radiation was less than 0.1 of the line frequency.

In channeling of positrons the anharmonicity of the transverse oscillations is small, so that the observed radiation spectrum is one unsplit band.[4] It is quite obvious that a periodic perturbation of the channel potential cannot lead to an observable splitting of this band. The correct wave functions were calculated to lowest order and the corresponding energy values of a channeled positron were evaluated in the $2N$-level approximation, where N was chosen from the condition $2\Lambda_{if} \approx N\delta$; here $\delta = \omega_{i,i-1}/\omega_{i+1,i} - 1$. It was found that even if Λ_{if} is much greater than the nonequidistance of the initial transverse energy levels, the variation in separation of the renormalized levels still will not exceed δ. The action of the spatially periodic perturbation of the channel potential produces some broadening of the radiation band, leaving it unsplit.

In conclusion it must be mentioned that the results we have obtained differ qualitatively from those of Ref. 14, in which the splitting of the channeled-particle energy levels under the action of a periodic perturbation was not taken into account and, in essence, a situation far from resonance was considered.

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