# Parity nonconservation effects in electric conductors: current in a magnetic field

L. N. Labzovskiĭ

*A. A. Zhdanov Leningrad State University*
(Submitted 22 April 1985)
Zh. Eksp. Teor. Fiz. **89**, 1921–1932 (December 1985)

It is shown that allowance for the weak interaction between the conduction electrons and the lattice nuclei in an electric conductor can lead to the appearance in the conductor of a current of strength proportional to the magnetic field intensity and direction perpendicular to the directions of the magnetic induction **B** and electron spin $\langle \mathbf{s} \rangle$ vectors: $\mathbf{i} = \sigma_B [\langle \hbar^{-1}\mathbf{s}\rangle \times \mathbf{B}]$. Here the spins of the electrons in the conductor are assumed to be polarized in a direction different from the direction of the vector **B** (a ferromagnet with high coercivity). The Kubo theory of linear response to an external field is used; the weak interaction is taken into account with the aid of the Salam-Weinberg Hamiltonian. The use of a simple model leads to the relation $\sigma_B \approx 10^{-16}\sigma_E$ (for heavy metals, e.g., Pt), where $\sigma_E$ is the normal electrical conductivity. A possible experimental setup in which a magnetized conductor is placed in an electromagnet is discussed, and the interfering factors are evaluated. The Hall effect, which attends the flow of current in a conductor located in a magnetic field, is considered with the aid of the Boltzmann kinetic equation.

**1.** The parity nonconservation effects due to the interaction between weak neutral currents of electrons and nuclei have been extensively discussed in the last decade.[1] These effects were first observed in atomic experiments,[1] and have been theoretically investigated for atoms and molecules in the gaseous and liquid phases,[1,2] superconductors,[1] helical crystals,[3] semiconductors,[4] and dielectrics.[5]

In the present paper we consider the possible phenomenon of electric-current flow in a conductor located in an external constant magnetic field when allowance is made for the weak interactions. We show that, in the case when the electron spins are oriented (i.e., in ferromagnets), there arises a current with density

$$\mathbf{i}=\sigma_B[\langle \hbar^{-1}\mathbf{s}\rangle \times \mathbf{B}], \tag{1}$$

where $\langle \mathbf{s} \rangle$ is the mean value of the electron spin, **B** is the magnetic induction, and $\sigma_B$ is the coefficient of proportionality, which plays the role of conductivity. The relation (1) is similar in form to Ohm's law:

$$\mathbf{i}=\sigma_B\mathbf{E}, \tag{2}$$

where **E** is the electric field intensity and $\sigma_E$ is the electrical conductivity. The vectors **i** and **E** are polar, while **s** and **B** are axial; therefore, Eq. (1) can hold only when spatial parity is not conserved.

In Sec. 2 of the present paper we derive an expression for the force that acts on a conduction electron in a magnetic field with allowance for the weak interaction with the lattice. Two possible approaches are available for the description of the phenomenon of electrical conduction in the kinetic regime: we can use either the Boltzmann kinetic equation or the Kubo theory of linear response to an external perturbation. Each of these approaches has its own advantages, and we shall use both. To begin with, in Sec. 3 we briefly state the principles of the Kubo theory. In Sec. 4, using this theory and a simple model, we derive expressions for the current in both an electric and a magnetic field with allowance for the weak interactions. In Sec. 5 we show that the model considered leads directly to the simplest expression for the electrical conductivity, namely, the Drude formula. In Sec. 6 we obtain the basic expression for the conductivity in a magnetic field, and estimate the magnitude of the proposed effect. In Sec. 7 we consider a possible experimental setup. Finally, in Sec. 8, using the Boltzmann equation, we describe the Hall effect, which attends the flow of current in a magnetic field.

**2.** The effective nonrelativistic Hamiltonian for the weak interaction between neutral electron currents and the nuclei in a crystal has the form[1]

$$\hat{H}_w=\frac{1}{m}Q\mathbf{s}\Big[\hat{\mathbf{p}},\sum_{\mathbf{a}}\delta(\mathbf{x}-\mathbf{a})\Big]_+, \tag{3}$$

where $Q \equiv GgZ\hbar^2/2^{1/2}c^2$, $G \approx 10^{-5}(1/m_p)^2$, $m$ is the electron mass, $m_p$ is the proton mass, $Z$ is the nuclear charge (in units of the electron charge $e$), $c$ is the velocity of light, $g$ is a dimensionless constant (for heavy atoms in the Salam-Weinberg theory $g \approx -0.75$), $\hat{\mathbf{p}} = -i\hbar\nabla$, **s** is the electron spin, and $[\ldots]_+$ is the anticommutator.[1)] The summation in (3) is over all the lattice vectors; for simplicity we consider a lattice with one atom in the unit cell, and assume that in a cell with $\mathbf{a} = 0$ the coordinate of the atomic nucleus $\mathbf{R} = 0$. We thus limit ourselves to the consideration of lattices possessing a center of inversion.

In the presence of an external magnetic field, the expression (3) should, on the basis of gauge-invariance considerations, be replaced by

$$\hat{H}_w=\frac{1}{m}Q\mathbf{s}\Big[\Big(\hat{\mathbf{p}}+\frac{e}{c}\mathbf{A}\Big),\sum_{\mathbf{a}}\delta(\mathbf{x}-\mathbf{a})\Big]_+, \tag{4}$$

where **A** is the vector potential of the external field. The Hamiltonian for a conduction electron in an electric and magnetic fields can be written with the required accuracy in the form[2)]

$$\hat{H}=\frac{1}{2m}\hat{\mathbf{P}}^2+e(V+\varphi)+\hat{H}_s, \tag{5}$$

 0038-5646/85/121108-06$04.00  

where $V$ is the lattice potential and $\varphi$ is the scalar potential of the external field,

$$\hat{H}_s = \frac{e}{mc}(\mathbf{Bs}), \quad \hat{\mathbf{P}} = \mathbf{p} + \frac{e}{c}\mathbf{A} + 2Q\mathbf{s}\sum_{\mathbf{a}} \delta(\mathbf{x}-\mathbf{a}).$$

To compute the velocity $\hat{\mathbf{v}}$ of, and the force $\mathbf{F}$ acting on, the electron, we use the formulas

$$\hat{\mathbf{v}} = \dot{\mathbf{x}} = \frac{i}{\hbar}[\hat{H}, \mathbf{x}]_-, \tag{6}$$

$$\hat{\mathbf{F}} = m\ddot{\mathbf{x}} = -\frac{m}{\hbar^2}[\hat{H}, [\hat{H}, \mathbf{x}]_-]_-, \tag{7}$$

where $[\ldots]_-$ denotes the commutator. The evaluation of the commutators yields

$$\hat{v}_\mu = \frac{1}{m}\hat{P}_\mu, \tag{8}$$

$$\hat{F}_\mu = \frac{i}{2\hbar m}[\hat{\mathbf{P}}^2, \hat{P}_\mu]_- - e\nabla_\mu(V+\varphi) + \frac{2ieQ}{mc\hbar}B_\nu[s_\nu, s_\mu]_- \sum_{\mathbf{a}} \delta(\mathbf{x}-\mathbf{a}). \tag{9}$$

The first term in (9) gives, when allowance is made for the relation

$$\frac{\partial A_\mu}{\partial x_\nu} - \frac{\partial A_\nu}{\partial x_\mu} = \varepsilon_{\nu\mu\lambda}B_\lambda$$

($\varepsilon_{\nu\mu\lambda}$ is the unit antisymmetric tensor), the Lorentz force[3)]

$$\hat{F}_\mu^{(L)} = \frac{e}{c}[\hat{\mathbf{v}}\times\mathbf{B}]_\mu, \tag{10}$$

where $\hat{v}_\mu$ is defined by (8). The second term represents the usual electrostatic force

$$\hat{F}_\mu^{(e)} = e(\mathbf{E}+\mathbf{E}_{cr})_\mu, \tag{11}$$

where $\mathbf{E} = -\nabla\varphi$ is the intensity of the external electric field and $\mathbf{E}_{cr} = -\nabla V$ is the intensity of the internal crystal field. Finally, the third term in (9) reduces, when allowance is made for the relation $[s_\nu, s_\mu]_- = i\hbar\varepsilon_{\nu\mu\lambda}s_\lambda$, to the expression

$$\hat{F}_\mu^{(w)} = \frac{2eQ}{mc}[\mathbf{B}\times\mathbf{s}]_\mu \sum_{\mathbf{a}} \delta(\mathbf{x}-\mathbf{a}), \tag{12}$$

which plays a major role below.

**3.** In order to see how the current (1) arises, let us use the linear response theory developed by Kubo, and used normally to describe irreversible processes.[6,7] The basic equation in this theory is the equation where

$$i\hbar\frac{\partial\hat{\rho}(t)}{\partial t} = [\hat{H}, \hat{\rho}]_-, \tag{13}$$

where $\hat{\rho}(t)$ is the density-matrix operator and $\hat{H}$ is the Hamiltonian of the system. It is assumed that the Hamiltonian has the form

$$\hat{H} = \hat{H}_0 + \lambda(t)\hat{H}', \tag{14}$$

where $\hat{H}_0$ is the Hamiltonian of the unperturbed system, $\hat{H}'$ is the perturbation operator, and $\lambda(t)$ is the switching-on function.

The problem can be formulated as follows: at some moment of time $t_0$ the system is in a thermal-equilibrium state, which is described by the density matrix

$$\hat{\rho}_0 = \frac{1}{Z_0}\exp\left(-\frac{\hat{H}_0}{kT}\right), \tag{15}$$

where $Z_0 = \mathrm{Sp}\exp(-\hat{H}_0/kT)$ is the partition function. At the subsequent moments of time $t > t_0$ the perturbation (external field) is switched on, and there arises a nonequilibrium state that leads to the appearance of a current. The correction to an arbitrary physical quantity, corresponding to the operator $\hat{A}$, can then be computed with the aid of perturbation theory, using the equilibrium operator $\hat{\rho}_0$.

For the correction $\hat{\rho}_1$ to the density matrix we obtain from (13) the equation

$$i\hbar\frac{\partial\rho_1}{\partial t} = [\hat{H}_0, \hat{\rho}_1]_- + \lambda(t)[\hat{H}', \rho_0]_-, \tag{16}$$

the solution to which can be represented in the form

$$\hat{\rho}_1(t) = -\frac{i}{\hbar}\int_{t_0}^{t} dt' \exp\left\{-\frac{i}{\hbar}\hat{H}_0(t-t')\right\}\lambda(t') \times [\hat{H}'\hat{\rho}_0]_- \exp\left\{\frac{i}{\hbar}\hat{H}_0(t-t')\right\}. \tag{17}$$

The change that occurs in a physical quantity $\hat{A}$ when the field is switched on (the linear response of the system to the perturbation) can be written as[6–8]:

$$\mathrm{Sp}\,\hat{\rho}_1\hat{A} = -\frac{i}{\hbar}\int_{t_0}^{t} dt'\lambda(t')\langle\langle[\hat{A}(t), \hat{H}'(t')]_-\rangle\rangle. \tag{18}$$

Here $\hat{A}(t)$ and $\hat{H}'(t')$ are the operators in the interaction representation,

$$\langle\langle\hat{A}\rangle\rangle \equiv \mathrm{Sp}\,\hat{A}\hat{\rho}_0 = \sum_s W_s\langle s|\hat{A}|s\rangle; \tag{19}$$

$$W_s = \frac{1}{Z_0}\exp\left(-\frac{E_s}{kT}\right),$$

and the summation in (19) is over the complete set of eigenfunctions of the operator $\hat{H}_0$.

**4.** Let us consider with the aid of (18) the response of the system to external electric and magnetic fields, which we shall assume to be stationary (the entire time dependence is contained in the function $\lambda(t)$). Since the Lorentz force (10) does not give rise to a current, let us take the last two terms in (5) as the perturbation Hamiltonian $\hat{H}'$. Then the operator $\hat{H}'$ has, in the second-quantization representation, the form

$$\hat{H}' = \int\hat{n}(\mathbf{x})\varphi(\mathbf{x})d\mathbf{x} - \frac{1}{c}\int\hat{\mathbf{j}}^{(s)}(\mathbf{x})\mathbf{A}(\mathbf{x})d\mathbf{x}. \tag{20}$$

Here $\hat{\mathbf{n}}(x)$ is the charge-density operator:

$$\hat{n}(\mathbf{x}) = -e\hat{\Psi}^+(\mathbf{x})\Psi(\mathbf{x}), \tag{21}$$

and $\hat{\mathbf{j}}^{(s)}(\mathbf{x})$ is the spin-current density operator[9]:

$$\hat{\mathbf{j}}^{(s)}(\mathbf{x}) = -\frac{e\hbar}{m}\mathrm{rot}\,\hat{\Psi}^+(\mathbf{x})\mathbf{s}\hat{\Psi}(\mathbf{x}), \tag{22}$$

where $\hat{\Psi}(\mathbf{x})$ is the second-quantized wave function.

The linear response of the electron system in a conductor[4)] to an external field—the electric current—is obtained from the formula (18) if we set in it

$$\hat{A} = \hat{\mathbf{j}} = \int\hat{\mathbf{j}}(\mathbf{x})d\mathbf{x},$$

where $\hat{\mathbf{j}}(\mathbf{x})$ is the current operator. According to the results obtained in Sec. 2, we should set in the commutator containing the first term in (20) $\hat{\mathbf{j}} = \hat{\mathbf{j}}^{(0)}$, with

$$\hat{\mathbf{j}}^{(0)}(\mathbf{x}) = -\frac{ie\hbar}{2m}[(\nabla\hat{\Psi}^{(+)}(\mathbf{x}))\hat{\Psi}(\mathbf{x}) - \hat{\Psi}^{+}(\mathbf{x})(\nabla\hat{\Psi}(\mathbf{x}))]. \quad (23)$$

We then obtain the usual electric current

$$I_\mu^{(e)} = -\frac{i}{\hbar}\int_{t_0}^{t}\lambda(t')dt'\int d\mathbf{x}\int d\mathbf{x}'\langle\langle[\hat{j}_\mu^{(0)}(\mathbf{x}t), \hat{n}(\mathbf{x}'t')]_-\rangle\rangle\varphi(\mathbf{x}'). \quad (24)$$

We must set in the commutator containing the second term in (2) $\hat{\mathbf{j}} = \hat{\mathbf{j}}^{(w)}$, where

$$\hat{\mathbf{j}}^{(w)}(\mathbf{x}) = -\frac{2eQ}{m}\hat{\Psi}^{(+)}(\mathbf{x})\mathbf{s}\sum_{\mathbf{a}}\delta(\mathbf{x}-\mathbf{a})\hat{\Psi}(\mathbf{x}). \quad (25)$$

Thus, the following current arises in a magnetic field:

$$I_\mu^{(M)} = \frac{i}{\hbar}\int_{t_0}^{t}\lambda(t')dt'\int d\mathbf{x}\int d\mathbf{x}'\langle\langle[\hat{j}_\mu^{(w)}(\mathbf{x}t), \hat{j}_\nu^{(s)}(\mathbf{x}'t')]_-\rangle\rangle A_\nu(\mathbf{x}'). \quad (26)$$

In a real situation there is always some finite relaxation time $\tau$ (e.g., the interval between two collisions of an electron with the lattice) during which the evolution in time of the state is determined by the interaction with the external field.[5)] In this case equilibrium in the system is established by the collisions, and the presence of the current is due to small deviations from the equilibrium configuration. Such a situation is described by the kinetic theory. In particular, Eqs. (1) and (2) are kinetic equations.

The rigirous transition from Eqs. (24) and (26), which were derived from quantum mechanics, to the kinetic equations, which are time irreversible, is, in principle, a complicated problem. Various approaches to this problem are extensively discussed and summarized in, for example, Ref. 7. In the present paper, for this transition, we use the simplest model of the irreversible process. We shall assume that each collision momentarily switches off and immediately switches on again the external field. In such a situation the time integration in (24) and (26) should be carried out over the period of growth of the current: from $t_0 = 0$ to $t = \tau$. In this case the relaxation mechanism is considered to be extraneous with respect to the system in question, and is not included in the quantum-mechanical description of the process.[6)]

We make the assumption, and this is the major simplifying assumption that we shall use, that the operators in (24) and (26) are time independent. This means that we assume $\exp(i\hat{H}_0\tau) \approx 1$. We know from the theory of the electrical conductivity of metals that to the conduction electrons corresponds a state band of width $\Delta E_T \approx kT$ in the vicinity of the Fermi surface. This result is a consequence of the Pauli principle.[10] Accordingly, only the contribution of the "diffusion band" $\Delta E_T$ at the Fermi surface remains when the averaging (19) over the states is carried out in (24) and (26). Using the estimates given in, for example, Ref. 10 for the relaxation time $\tau$ in the heavy metals, we find that $\hbar^{-1}\Delta E_T\tau \approx 1$ at $T \approx 3\times10^2$ K. Thus, the condition $\exp(i\hat{H}_0\tau) \approx 1$ is, strictly speaking, fulfilled only in the "kinetic limit," when $\tau\to0$. We shall nevertheless use this condition, for this will enable us to obtain a simple relation between $\sigma_B$ and $\sigma_E$.

Now the integrands in (24) and (26) do not depend on $t$ and $t'$, and the current $I_\mu$ depends linearly on $t$. The current averaged over the interval between two collisions is clearly equal to

$$\bar{I}_\mu = {}^1/_2[I_\mu(t=0) + I_\mu(t=\tau)]. \quad (27)$$

In the approximation being used, the computation of $\bar{I}_\mu$ is equivalent to integration over $t$ from 0 to $\frac{1}{2}\tau$ in (24), (26):

$$\bar{I}_\mu^{(e)} = -\frac{i\tau}{2\hbar}\int d\mathbf{x}\int d\mathbf{x}'\langle\langle[\hat{j}_\mu^{(0)}(\mathbf{x}), \hat{n}(\mathbf{x}')]_-\rangle\rangle_F\varphi(\mathbf{x}') \quad (28)$$

$$\bar{I}_\mu^{(M)} = \frac{i\tau}{2\hbar c}\int d\mathbf{x}\int d\mathbf{x}'\langle\langle[\hat{j}_\mu^{(w)}(\mathbf{x}), \hat{j}_\nu^{(s)}(\mathbf{x}')]_-\rangle\rangle_F A_\nu(\mathbf{x}'), \quad (29)$$

where the symbol $\langle\langle\ldots\rangle\rangle_F$ indicates that the summation over the states is carried out within the limits of the "diffusion band" at the Fermi surface.

**5.** Let us, to begin with, consider the normal current in an electric field. Going over in (28) to the matrix elements in the coordinate representation, we obtain

$$\bar{I}_\mu^{(e)} = -\frac{i\tau e^2}{2\hbar}\sum_{s(F)}W_s([\hat{v}_\mu^{(0)}\ \varphi]_-)_{ss} = \frac{e^2\tau}{2m}E_\mu\sum_{s(F)}W_sN_s, \quad (30)$$

where $\hat{\mathbf{v}}^{(0)} = m^{-1}\hat{\mathbf{p}}$, $\mathbf{E}$ is the intensity of the external field, $N_s$ is a normalization integral, and $s(F)$ denotes averaging over the "diffusion band".

Let us normalize the conduction-electron wave functions to the volume $V$ of the conductor. For example, in band theory the $\psi_s$ are the Bloch functions[11]:

$$\psi_s(\mathbf{x}) = \psi_{\alpha\mathbf{k}}(\mathbf{x}) = N^{-1/2}e^{i\mathbf{k}\mathbf{x}}u_{\alpha\mathbf{k}}(\mathbf{x}), \quad (31)$$

where $\mathbf{k}$ is the quasimomentum, $\alpha$ is the band number, and $N$ is the number of unit cells in the volume $V$. We assume that the functions $u_{\alpha\mathbf{k}}$ are normalized in the unit-cell volume $V_0 = V/N$:

$$\int_{V_0}u_{\alpha\mathbf{k}}^*(\mathbf{x})u_{\beta\mathbf{k}}(\mathbf{x})d\mathbf{x} = \delta_{\alpha\beta}. \quad (32)$$

Then

$$N_s = \int\psi_{\alpha\mathbf{k}}^*(\mathbf{x})\psi_{\alpha\mathbf{k}}(\mathbf{x})d\mathbf{x} = \frac{1}{N}\sum_{\mathbf{a}}\int_{V_0}u_{\alpha\mathbf{k}}^*(\mathbf{x}+\mathbf{a})u_{\alpha\mathbf{k}}(\mathbf{x}+\mathbf{a})d\mathbf{x}, \quad (33)$$

where the summation is over all the lattice vectors. Using the periodicity property of the Bloch functions, i.e., the fact that $u_{\alpha\mathbf{k}}(\mathbf{x}+\mathbf{a}) = u_{\alpha\mathbf{k}}(\mathbf{x})$, and the condition (32), we obtain $N_s = 1$

The quantity (30) is the current due to one electron in the entire volume of the conductor. The current density in the conductor is obtained by multiplying (30) by the electron concentration $n$. Taking into account here the fact that the conduction-electron concentration is equal to

$$n_0 = nn_F,$$

where

$$n_F = \sum_{s(F)}W_s,$$

we write

$$i_\mu^{(e)}=n\bar{I}_\mu^{(e)}=\frac{e^2\tau n_0}{2m}E_\mu. \quad (34)$$

Thus, we obtain for the electrical conductivity $\sigma_E$ the well-known expression, i.e., the Drude formula,

$$\sigma_E=e^2\tau n_0/2m. \quad (35)$$

**6.** Let us now go over to the investigation of the current in a magnetic field with allowance for the weak interactions. Evaluating the commutator in (29), and going over to the matrix elements in the coordinate representation, we obtain

$$\bar{I}_\mu^{(M)}=-\frac{e^2\tau Q}{mc}\sum_{s(F)}W_s\Big(\sum_{\mathbf{a}}\delta(\mathbf{x}-\mathbf{a})\Big)_{ss}[\mathbf{B}\langle\mathbf{s}\rangle]_\mu, \quad (36)$$

where $\langle\mathbf{s}\rangle$ is the mean value of the electron spin. We assume here that we are dealing with a magnetized sample in which the spin polarization is complete and is the same in all the states.[7)]

Let us estimate the magnitude of the proposed effect. The computation of the matrix element in (36) with the Bloch wave functions (31) leads to the result

$$\Big(\sum_{\mathbf{a}}\delta(\mathbf{x}-\mathbf{a})\Big)_{ss}=\frac{1}{N}\int u_{\alpha\mathbf{k}}^*(\mathbf{x})\sum_{\mathbf{a}}\delta(\mathbf{x}-\mathbf{a})u_{\alpha\mathbf{k}}(\mathbf{x})\,d\mathbf{x}$$

$$=\int_{V_0}u_{\alpha\mathbf{k}}^*(\mathbf{x})\delta(\mathbf{x})u_{\alpha\mathbf{k}}(\mathbf{x})\,d\mathbf{x}=|u_{\alpha\mathbf{k}}(0)|^2. \quad (37)$$

Denoting the weight of the *ns* wave in the Bloch function $u_{\alpha\mathbf{k}}$ by $\gamma_n$, and using in the case of heavy atoms the statistical-model estimate for the wave function on a nucleus,[9] we obtain[8)]

$$|u_{\alpha\mathbf{k}}(0)|^2\approx\sum_n\varkappa_n\gamma_n^2RZa_0^3, \quad (38)$$

where $a_0$ is the Bohr radius, $\varkappa_n=4/n^3$ is a numerical factor stemming from the normalization of the *ns* function, and $R$ is the relativistic enhancement factor (e.g., for Pt, $R\approx8.0$ (Ref. 1)). Then

$$\sum_{s(F)}W_s\Big(\sum_{\mathbf{a}}\delta(\mathbf{x}-\mathbf{a})\Big)_{ss}\approx\varkappa RZa_0^3n_F, \quad (39)$$

where $\varkappa\equiv\Sigma_n\varkappa_n\gamma_n^2$. Writing the expression for the current density similarly to (34) in the form

$$i_\mu^{(M)}=n\bar{I}_\mu^{(M)}=\sigma_B[\langle\hbar^{-1}\mathbf{s}\rangle\mathbf{B}]_\mu, \quad (40)$$

we find from (36)–(39) that

$$\sigma_B=\sigma_E2^{-1/2}Ggm^2\varkappa R\alpha^3Z^2, \quad (41)$$

where $\alpha=e^2/\hbar c$ is the fine structure constant. Taking into account the fact that $2^{-1/2}Gm^2\approx3\times10^{-12}$, and assuming in the case of a heavy atom (e.g., Pt, $Z=78$), for which the dominant contribution to (38) is made by the 5*s* and 6*s* states, that $\varkappa R\approx10^{-2}$, we obtain $\sigma_B\approx10^{-16}\sigma_E$.

We must further investigate the question of the possibility of the effect's being imitated by the impurities and the lattice defects (dislocations). The effect of the defects and impurities manifests itself in the fact that the space-inversion symmetry of the potential $V(x)$ is violated in some unit cells not as a result of weak interactions. Evidently, the participation of the spin-orbit interaction is also necessary for the appearance of the imitating effect.

The expression for the spin-orbit interaction operator has the form[11]

$$\hat{H}_{so}=-\frac{1}{2m^2c^2}[\mathbf{s}\nabla V]\mathbf{p}, \quad (42)$$

and in an external magnetic field we have, when allowance is made for gauge invariance,

$$\hat{H}_{so}=-\frac{1}{2m^2c^2}[\mathbf{s}\nabla V]\Big(\hat{\mathbf{p}}+\frac{e}{c}\mathbf{A}\Big). \quad (43)$$

Now the momentum $\hat{\mathbf{P}}$ in (5) should be replaced by the expression

$$\hat{\mathbf{P}}=\hat{\mathbf{p}}+\frac{e}{c}\mathbf{A}-\frac{1}{2mc^2}[\mathbf{s}\times\nabla V],$$

and the current operator $\hat{\mathbf{j}}^{(w)}$, by the corresponding operator $\hat{\mathbf{j}}^{(so)}$. Then within the framework of the approximations being used we obtain

$$\bar{I}_\mu^{(so)}=\frac{1}{4}\frac{\sigma_E\delta}{m^2c^3}\sum_s W_s[(\nabla V)_{ss}[\langle\mathbf{s}\rangle\times\mathbf{B}]]_\mu, \quad (44)$$

where $\delta$ is the relative concentration of the distorted lattice sites. But the quantity $(\nabla V)_{ss}=(\mathbf{E}_{cr})_{ss}$ is the average intensity of the crystal field acting on a conduction electron, and should be equal to zero. Thus, the current (44) does not occur.

**7.** Let us now discuss a possible experimental setup. As can be seen from (40), the magnitude of the effect depends directly on the magnetic-induction strength $B$, which it is advantageous to make as large as possible. In this case it is necessary that the magnetic field does not reorientate the spins of the magnetized conductor; as follows from (40), the direction of the polarization $\langle\mathbf{s}\rangle$ should not coincide with the direction of the vector $\mathbf{B}$. Therefore, it is necessary to use ferromagnets with a broad hysteresis loop and a large coercive force $H_c$ (i.e., precisely those that are used as permanent magnets). We can take, for example, the Pt-Co alloy, for which $H_c\approx10^3$ Oe and the remanence $B_r\approx10^3$ G (generally speaking there exist materials for which $H_c\approx10^5$ Oe.)[14]

As the source of the external magnetic field we can use either a permanent magnet or an electromagnet, i.e., a current carrying coil (see Fig. 1). Ohm's law for the closed circuit with the current $\mathbf{i}_1$ assumes the form

$$\mathbf{i}_1=\sigma_E(\mathbf{E}_1+\mathbf{E}_{\rm ext}^{\rm eff}), \quad (45)$$

where $\mathbf{E}_1$ is the intensity of the field associated with the current $\mathbf{i}_1$ and the quantity

$$\mathbf{E}_{\rm ext}^{\rm eff}=\frac{\sigma_B}{\sigma_E}\Big[\Big\langle\frac{1}{\hbar}\mathbf{s}\Big\rangle\mathbf{H}_{\rm ext}\Big] \quad (46)$$

plays the role of the intensity of a field produced by extraneous forces. The quantity $\mathbf{H}_{\rm ext}$ is then the intensity of the "extraneous" magnetic field[9)] (in the situation considered in Fig. 1 $\mathbf{H}_{\rm ext}=\mathbf{H}_2$). The current strength in the closed circuit is determined as usual by only the strength of the extraneous electromotive force $U_{\rm ext}^{\rm eff}$:

$$I_1=R^{-1}U_{\rm ext}^{\rm eff}, \quad (47)$$

where $R$ is the total resistance in the circuit. The Joule law

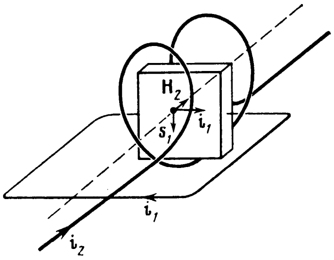


FIG. 1.

also has the usual form:

$$Q_1 = R^{-1}(U_{\text{стр}}^{\text{eff}})^2, \tag{48}$$

where $Q_1$ is the heat released in the closed circuit with the current $\mathbf{i}_1$.

Taking account of the estimate given above for $\sigma_B$, we obtain in the case of a field with $H_{\text{ext}} \approx H_c \approx 10^3$ Oe the estimate $E_{\text{ext}}^{\text{eff}} \approx 10^{-13}$ esu $\approx 3\times 10^{-11}$ V/cm. Let the conductor be a $1\times 1$ cm square platinum plate of thickness 1 mm. Then from the formula $R = l/\sigma_E S$, where $l = 1$ cm is the length, $S = 0.1$ cm$^2$ is the transverse cross section of the conductor, and $\sigma_E \approx 10^5\ \Omega^{-1} - \text{cm}^{-1}$ ($T = 3\times 10^2$ K), we obtain $R \approx 10^{-4}\ \Omega$; $U_{\text{ext}}^{\text{eff}} = E_{\text{ext}}^{\text{eff}} l \approx 10^{-11}$ $V$; and $I_1 \approx 10^{-7}$ A. Taking account of the fact that the conductivity increases by approximately a factor of $10^4$ when the temperature is lowered to $T \approx 10$ K (Ref. 10), at $T \approx 10$ K we obtain $R \approx 10^{-8}\ \Omega$; $U_{\text{ext}}^{\text{eff}} \approx 10^{-11}$ V; and $I_1 \approx 10^{-3}$ A.

We should bear in mind that, in experiments with an electromagnet, the instability of the current $\mathbf{i}_2$ (see Fig. 1) and the attendant variation of the magnetic flux linked with the loop carrying the current $\mathbf{i}_1$ lead to the imitation of the effect in question. In this case the induction emf $U_{\text{ind}}$ is given by the formula

$$U_{\text{ind}} = c^{-1} d\Phi/dt, \tag{49}$$

where $\Phi$ is the magnetic flux linked with the loop. Let the instability of the field $H_2$ be equal to $dH_2/dt$, let $S' = 10$ cm$^2$ be the area of a turn of the coil, and let the entire magnetic flux $\Phi' = S'B_2$ produced by the coil go through the loop with the current $i_1$: $\Phi' = \Phi$. Then

$$U_{\text{ind}}(\text{B}) \approx 10^{-7} \frac{dH_2}{dt}\left(\frac{\text{Oe}}{\text{sec}}\right).$$

If we want to exclude spurious effects (i.e., if we want to ensure that $U_{ind} < U_{\text{ext}}^{\text{eff}}$), then, as follows from (49), we should fulfill the condition $dH_2/dt < 10^{-4}$ Oe/sec in the case when $H_{\text{ext}} \approx 10^3$ Oe.

Using the well-known formula $H_2 = 4\pi n I_2$, where $n$ is the number of turns per unit length of the coil, and setting $n = 10$ cm$^{-1}$, we obtain the condition for the stability of the current $I_2$: $dI_2/dt < 10^{-6}$ A/sec (the current in the coil should be $I_2 \approx 10$ A for it to produce a field of intensity $H_2 \approx 10^3$ Oe). Here it should be remembered that the loop with the current $\mathbf{i}_1$ can be geometrically positioned so as to diminish the spurious effect.

**8**. Besides the drift corresponding to the current (1), the electrons in the external magnetic field also undergo displacement under the action of the Lorentz force, which leads, as in the case of a normal electric current, to the Hall effect. To investigate this effect, it is convenient to go over to another method of describing kinetic phenomena, namely, to the Boltzmann equation.

We can, in accordance with the expressions (12), (40), and (41), asume that the electrons are acted upon by an effective force

$$\mathbf{F}_{\text{eff}} = -\frac{e}{\hbar}\frac{\sigma_B}{\sigma_E}[\langle \mathbf{s}\rangle \times \mathbf{B}],$$

"diffused" throughout the volume of the conductor. Let us, as usual, write the Boltzmann equation for the distribution function $f_{\mathbf{k}}$ for the electrons with quasimomentum $\mathbf{k}$ in the form

$$\left(\frac{\partial f_{\mathbf{k}}}{\partial t}\right)_{\text{sc}} + \left(\frac{\partial f_{\mathbf{k}}}{\partial t}\right)_{\text{dif}} + \left(\frac{\partial f_{\mathbf{k}}}{\partial t}\right)_{\text{field}} = 0, \tag{50}$$

where the first term takes into account the scattering processes; the second term, the diffusion process; and the third term, the effect of the external fields. Assuming the deviation from equilibrium to be small,[10)] we set $f_{\mathbf{k}} = f_{\mathbf{k}}^0 + g_{\mathbf{k}}$, where $f_{\mathbf{k}}^0$ is the distribution function for the equilibrium state. Then

$$\left(\frac{\partial f_{\mathbf{k}}}{\partial t}\right)_{\text{dif}} = -\mathbf{v}_{\mathbf{k}} \nabla f_{\mathbf{k}} = 0. \tag{51}$$

The collision term has, in the usual approximation,[15] the form

$$(\partial f_{\mathbf{k}}/\partial t)_{\text{sc}} = -g_{\mathbf{k}}/\tau, \tag{52}$$

where $\tau$ is the relaxation time. Finally,

$$\left(\frac{\partial f_{\mathbf{k}}}{\partial t}\right)_{\text{field}} = -\frac{d\mathbf{k}}{dt}\nabla_{\mathbf{k}} f_{\mathbf{k}} = \frac{e}{\hbar}\left(\mathbf{E}_{\text{ext}} + \frac{1}{c}[\mathbf{v}_{\mathbf{k}} \times \mathbf{B}_{\text{self}}] + \frac{1}{c}[\mathbf{v}_{\mathbf{k}} \times \mathbf{H}_{\text{ext}}] + \frac{\sigma_B}{\sigma_E}[\langle \mathbf{s}\rangle \times \mathbf{H}_{\text{ext}}]\right)\nabla_{\mathbf{k}} f_{\mathbf{k}}, \tag{53}$$

where $\mathbf{B}_{\text{self}}$ is the magnetic induction of the magnetic self-field of the conductor and $\mathbf{E}_{\text{ext}}$ and $\mathbf{H}_{\text{ext}}$ are the intensities of the extraneous electric and magnetic fields. Under the experimental condition in our case $\mathbf{E}_{\text{ext}} = 0$, and, according to the footnote in Sec. 7, $\mathbf{H}_{\text{ext}} = \mathbf{B}_{\text{ext}}$.

Taking account of the fact that $\nabla_{\mathbf{k}} E_{\mathbf{k}} = \hbar \mathbf{v}_{\mathbf{k}}$, we can write

$$\nabla_{\mathbf{k}} f_{\mathbf{k}} = \nabla_{\mathbf{k}} f_{\mathbf{k}}^0 + \nabla_{\mathbf{k}} g_{\mathbf{k}} = \hbar \mathbf{v}_{\mathbf{k}} \frac{\partial f_{\mathbf{k}}^0}{\partial E_{\mathbf{k}}} + \nabla_{\mathbf{k}} g_{\mathbf{k}}. \tag{54}$$

Then, discarding the terms of higher order in smallness, we obtain from (51), (52), and (53), the Boltzmann equation in the form

$$\frac{e}{\hbar}\frac{\sigma_B}{\sigma_E}\mathbf{v}_{\mathbf{k}}\frac{\partial f_{\mathbf{k}}^0}{\partial E_{\mathbf{k}}}[\langle \mathbf{s}\rangle \times \mathbf{H}_{\text{ext}}] = \frac{g_{\mathbf{k}}}{\tau} - \frac{e}{\hbar c}([\mathbf{v}_{\mathbf{k}} \times \mathbf{H}_{\text{ext}}] + [\mathbf{v}_{\mathbf{k}} \times \mathbf{B}_{\text{self}}])\nabla_{\mathbf{k}} g_{\mathbf{k}}. \tag{55}$$

Let us hereafter simplify the situation by limiting ourselves to free electrons, for which $\hbar\mathbf{k} = m\mathbf{v}$, and, as usual, seek the solution to (55) in the form

$$g = e\tau \frac{\partial f^0}{\partial E}(\mathbf{va}). \tag{56}$$

For the vector $\mathbf{a}$ we obtain the equation

$$\frac{1}{\hbar}\frac{\sigma_B}{\sigma_E}[\langle s\rangle\times H_{ext}]v=(va)-\alpha_0([v\times H_{ext}]+[v\times B_{self}])a, \qquad (57)$$

where $\alpha_0 = e\tau/mc$. That solution to (57) which is obtained through a number of identity transformations has the form (for brevity, we set $\mathbf{H}\equiv\mathbf{H}_{ext}$, $\mathbf{B}\equiv\mathbf{B}_{self}$ and $\boldsymbol{\lambda}\equiv\hbar^{-1}(\sigma_B/\sigma_E)\langle\mathbf{s}\rangle$):

$$\mathbf{a}=(1+\alpha_0^2(\mathbf{H}+\mathbf{B})^2)^{-1}\{[\boldsymbol{\lambda}\times\mathbf{H}]+\alpha_0[\mathbf{H}\times[\boldsymbol{\lambda}\times\mathbf{H}]]+\alpha_0^2(\mathbf{H}+\mathbf{B})(\mathbf{B}[\boldsymbol{\lambda}\times\mathbf{H}])\}. \qquad (58)$$

Substituting the values of the constants into the expression for $\alpha_0$, we find that $\alpha_0 H\approx 10^6 c^{-1}\tau$ for $H\approx 10^3$ Oe. Using the previous estimates for $\tau$, we see that the inequality $\alpha_0 H\ll 1$ is satisfied at all temperatures. Then, neglecting the terms containing $(\alpha_0 H)^2$ and $(\alpha_0 B)^2$, we obtain in place of (58) the expression

$$\mathbf{a}=[\boldsymbol{\lambda}\times\mathbf{H}]+\alpha_0[\mathbf{H}\times[\boldsymbol{\lambda}\times\mathbf{H}]]. \qquad (59)$$

Using for the current the expression[15]

$$\mathbf{i}=-e\int \mathbf{v}_\mathbf{k} g_\mathbf{k}\, d\mathbf{k}, \qquad (60)$$

and substituting (56) into it, we obtain in the free-electron model, which corresponds to the case of an isotropic solid, the expression

$$\mathbf{i}=\sigma_E\mathbf{a}, \qquad (61)$$

where

$$\sigma_E=-\frac{e^2\tau}{3}\int\frac{\partial f^0}{\partial E}v^2 d\mathbf{k}. \qquad (62)$$

The expression (62) for the electrical conductivity is, in principle, more exact than the formula (35). We shall assume, however, that the ratio $\sigma_B/\sigma_E$ is given as before by the formula (41). Then

$$\mathbf{i}=\frac{1}{\hbar}\sigma_B[\langle s\rangle\times\mathbf{H}_{ext}]+\frac{1}{\hbar}\sigma_B\alpha_0[\mathbf{H}_{ext}\times[\langle s\rangle\times\mathbf{H}_{ext}]]. \qquad (63)$$

The first term in the formula (63) duplicates (40), while the second describes the Hall effect, which in the present case consists in the fact that the current acquires a component in the direction parallel to the vector $\langle\mathbf{s}\rangle$. It can be seen that this component is small compared to the first term in (63), since it contains the small parameter $\sigma_0 H$. Notice that, up to the terms of the order of $(\alpha_0 H)^2$, the field $B_{self}$ exerts no influence on the current.

The author thanks V. G. Gorshkov, V. F. Ezhov, M. G. Kozlov, A. N. Moskalev, V. V. Flambaum, and I. B. Khriplovich for a discussion. The author is also grateful to Yu. E. Lozovik for valuable consultations.

[1]In (3) we have discarded the nuclear-spin-dependent part of the interaction. In the first place, this part makes, in the case of heavy atoms, a minor contribution (approximately $1/Z$ of the contribution made by (3)). In the second place, the observation of the corresponding effect requires the polarization of the nuclear spins.

[2]This convenient form of the Hamiltonian was suggested by V. V. Flambaum and I. B. Khriplovich.

[3]Here we neglect the terms containing the derivatives of the $\delta$ function. This is equivalent to the replacement of the contact interaction (3) with the lattice nuclei by an effective interaction "diffused" throughout the crystal, in the same way as is done in the description of the parity nonconservation effects in superconductors.[1]

[4]We consider each electron separately within the framework of the single-particle approximation.

[5]In this sense there is no difference between the stationary external field and any electron-lattice interaction that can be considered to be sufficiently weak, e.g., the interaction $\hat{H}_w$

[6]Although the growth of the current constitutes a departure from equilibrium, the picture is repeated in each time interval $(t_0, \tau)$, and the current-flow process is, on the whole, a stationary process.

[7]Like Stoner, we prefer to regard as directly polarized the conduction-electron spins, and not the atomic-electron spins. Here it is a question of convenience, whereas in actual fact the situation is apparently an intermediate one.[10,12]

[8]It is known that ferromagnetism is explained by the presence of $d$ and $f$ electrons in the atoms forming the lattice. It should be remembered, however, that the degree of hybridization (i.e., of mixing of the states with different orbital angular momenta) when we go over from an isolated atom to a lattice can be as high as 50% (Ref. 13). Thus, the Bloch functions for the conduction electrons always contain contributions from the $s$ waves.

[9]Generally speaking, the magnetic induction, i.e., the microfield acting on an electron in the conductor, should enter in formula (1). But since we are dealing with an already magnetized sample whose magnetization does not change in the presence of the $\mathbf{H}_{ext}$ field, we can assume that the current is determined by the quantity $\mathbf{H}_{ext}$.

[10]We also make the simplifying assumption that there are no temperature gradients and that the medium is homogeneous: $f_\mathbf{k}$ does not depend on $\mathbf{x}$.


[1]I. B. Khriplovich, Nesokhranenie chetnosti v atomnych yavleniyakh (Parity Nonconservation in Atomic Phenomena), Nauka, Moscow, 1981.

[2]V. L. Varentsov, V. G. Gorshkov, V. F. Ezhov, M. G. Kozlov, L. N. Labzovskiĭ, and V. N. Fomichev, Pis'ma Zh. Eksp. Teor. Fiz. **36**, 141 (1982) [JETP Lett. **36**, 175 (1982)].

[3]I. B. Khriplovich, Pis'ma Zh. Eksp. Teor. Fiz. **35**, 424 (1982) [JETP Lett. **35**, 524 (1982)].

[4]E. L. Al'tshuler, E. L. Ivchenko, A. N. Moskalev, G. E. Pikus, and R. M. Ryndin, Zh. Eksp. Teor. Fiz. **85**, 349 (1983) [Sov. Phys. JETP **58**, 203 (1983)].

[5]E. L. Al'tshuler, A. N. Moskalev, V. A. Ryzhov, R. M. Ryndin, P. G. Sil'vestrov, V. N. Fomichev, and I. B. Khriplovich, Preprint No. 911, Institute of Nuclear Physics, Leningrad, 1983.

[6]R. Kubo, J. Phys. Soc. Jpn. **12**, 570 (1957) [Russ. Transl., in: Voprosy kvantovoĭ teorii neobratimykh protsessov] (Problems of the Quantum Theory of Irreversible Processes), IL, Moscow, 1961.

[7]G. V. Chester, Rep. Prog. Phys. **26**, 411 (1963).

[8]V. L. Bonch-Bruevich and S. V. Tyablikov, Metod funktsiĭ Grina v statisticheskoĭ mekhanike (The Green Function Method in Statistical Mechanics), GIFML, Moscow, 1961 (Eng. Transl., North-Holland, Amsterdam. 1962).

[9]L. D. Landau and E. M. Lifshitz, Kvantovaya mekhanika (Quantum Mechanics), Nauka, Moscow, 1974 (Eng. Transl., Pergamon, Oxford, 1977).

[10]S. V. Vonsovskiĭ and M. I. Katsnel'son, Kvantovaya fizika tverdogo tela (Quantum Physics of Solids), Nauka, Moscow, 1983.

[11]E. M. Lifshitz and L. P. Pitaevskiĭ, Statisticheskaya fizika (Statistical Physics), Vol. 2, Nauka, Moscow, 1978 (Eng. Transl., Pergamon, Oxford, 1980).

[12]T. Moriya, J. Magn. & Magn. Mater. **14**, 1 (1979).

[13]C. A. Coulson, Valence, Clarendon Press, Oxford, 1961 (Russ. Transl., Mir, Moscow, 1965).

[14]C. Kittel, Introduction to Solid State Physics, Wiley, New York 1956 (Russ. Transl., GIFML, Moscow, 1962).

[15]J. Ziman, Principles of the Theory of Solids, Cambridge University Press, Cambridge, 1964 (Russ. Transl., Mir, Moscow, 1966)


Translated by A. K. Agyei