Orbital angular momentum in ³He-A

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The intrinsic angular momentum in the A phase of superfluid ³He is found in terms of the response to the angular velocity of rotation. It is shown that in the weak-coupling approximation at an arbitrary temperature and with allowance for the Fermi-liquid renormalization the intrinsic angular momentum is small in accordance with the smallness of the asymmetry in the distribution of particles and holes.

The question of the orbital angular momentum of superfluid ³He-A has stimulated increased interest since the time of the discovery of superfluid ³He. The A phase is a superfluid Fermi liquid, the Cooper pairs of which are in a state with orbital angular momentum $L = 1$, $L_z = 1$ (the latter is the component of the orbital angular momentum along the selected direction l) and with pair spin $S = 1$, $S_z = 0$. The order parameter is (see Ref. 1)

$$A_{\alpha i} = \Delta(T) V_\alpha (\Delta_i' + i\Delta_i''). \tag{1}$$

Here $\Delta(T)$ is the modulus of the order parameter, **V** is the unit spin vector, and Δ' and Δ'' are unit orthogonal vectors whose vector product $\Delta' \times \Delta'' = l$ specifies the direction of the orbital angular momentum. Each pair in the liquid possesses orbital angular momentum $\hbar l$, and the total orbital angular momentum of a vessel containing N atoms of ³He, at $T = 0$, is

$$\mathscr{L} \sim N\hbar/2. \tag{2}$$

This angular momentum is formed as a result of the macroscopic motion of the centers of mass of the pairs, i.e., is expressed in terms of the current density as

$$\mathscr{L} = \int [\mathbf{r} \times \mathbf{j}] dV.$$

At the same time (see Ref. 2 and the references therein), within the volume of the liquid there is almost complete cancellation of the orbital motion of the pairs, and the orbital angular momentum of unit volume of the liquid, containing n_s atoms of the superfluid component (i.e., the local orbital angular momentum corresponding to the intrinsic rotation of the pairs) turns out to be equal not to $(n_s/2)\hbar l$ but to a quantity considerably smaller in order of magnitude:

$$\mathbf{L}_{int}^0 \sim \frac{n_s \hbar}{2} \left(\frac{T_c}{\varepsilon_F}\right)^2 \ln \frac{\varepsilon_F}{T_c} \mathbf{l}. \tag{3}$$

The intrinsic angular momentum for an arbitrary temperature was first found in papers of Volovik[3] and Cross.[4,5]

In all the indicated papers[2-5] the magnitude of the intrinsic angular momentum has essentially been established from the current density. The procedure of separating out in the current density the term

$$\mathbf{j}^{0L} = {}^1/_2 \operatorname{rot} \mathbf{L}_{int}^0, \tag{4}$$

corresponding to the intrinsic angular momentum, is not fully defined, since besides the term (4) the current density

$$j_i^0 = \rho_{ij}^s (v_j^s - v_j^n) + \frac{1}{2} C_{ij}^0 (\operatorname{rot} \mathbf{l})_j$$

$$+ \frac{1}{4} \cdot \frac{1}{3} \frac{\mathrm{Sp}\,\rho^s}{\rho} [\nabla \rho \times \mathbf{l}]_i + j_i^{0L} \tag{5}$$

also contains the term C_{ij}^0 (curl l)$_j$. Here $\mathbf{j}^0$ is the current without Fermi-liquid corrections, and the coefficients in (5) are equal to

$$\rho_{ij}^n = 3\rho T \sum_{\omega,p} \hat{p}_i \hat{p}_j \frac{\omega^2 - E^2}{(\omega^2 + E^2)^2},$$

$$E^2 = \xi^2 + \Delta\Delta^+, \quad \omega = \pi T(2n+1),$$

$$\rho_{ij}^s = \rho - \rho_{ij}^n = \rho_\parallel l_i l_j + (\delta_{ij} - l_i l_j) \rho_\perp^s, \quad C^0 = \frac{\rho_\parallel^s}{2} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix};$$

ρ_{ij}^n and ρ_{ij}^s are the tensors of the density of the superfluid and normal component, respectively. Thus, Cross's approach,[4,5] which does not give rise to any doubts for $T \to T_c$ (see the review in Ref. 6 and the more detailed account in Ref. 7) is not so obvious in the intermediate range of temperatures, in which allowance for the Fermi-liquid renormalization of the current prevents us from separating out a contribution to the current in the form (4) of an exact curl of an intrinsic angular momentum.

On the other hand, the generalization of the method of Ref. 2 to nonzero temperatures with Fermi-liquid corrections taken into account leads to the result that the intrinsic angular momentum of the A phase of ³He turns out to be small not to the extent that the asymmetry $\sim (T_c/\varepsilon_F)^2 \ln(\varepsilon_F/T_c)$ in the distribution of the particles and holes is small, to the extent that of the anisotropy $F_1(\rho_\perp^n - \rho_\parallel^n)/\rho^n$ of the normal-density tensor is small. Here,

$$\rho_{ij}^n = \rho_\parallel^n l_i l_j + (\delta_{ij} - l_i l_j) \rho_\perp^n,$$

and F_1 is the parameter of Fermi-liquid theory. As a result, according to Refs. 8 and 9, in the dynamical equations for the vector l the inertial term associated with such a large magnitude of the internal angular momentum turns out,

near the temperature of the A–B transition, to be of the order of several percent of the frictional torque arising as a result of the orbital viscosity. Therefore, in Ref. 8, an experiment to measure the intrinsic angular momentum has even been proposed. We should say immediately that we do not share the optimism of the authors of Ref. 8, since, as shown in Refs. 10 and 11, in the dynamical equations there appears a dynamical intrinsic orbital angular momentum, in the calculation of which the contribution from the change in the distribution function of the excitations drops out; therefore, the inertial term does not contain a Fermi-liquid renormalization, and consequently remains small in accordance with the smallness of $(T_c/\varepsilon_F)^2 \ln(\varepsilon_F/T_c)$. Moreover, in the present paper it is shown that even the static intrinsic angular momentum with Fermi-liquid renormalization taken into account remains small in accordance with the smallness of $(T_c/\varepsilon_F)^2 \ln(\varepsilon_F/T_c)$.

To find L_{int} we shall regard superfluid ^{3}He formally as a charged Fermi liquid with p-pairing in an external magnetic field. We obtain a gauge-invariant expression for the energy $\mathscr{F}$ (quadratic in the gradients) of such a liquid with allowance for Fermi-liquid renormalization at an arbitrary temperature. If we make use of the well known analogy between superfluid and superconducting systems and assume that the vector-potential field is equivalent to the normal-velocity field:

$$\mathbf{A} = \frac{mc}{|e|}\mathbf{v}^n,$$

while the magnetic field corresponds to the angular velocity $\boldsymbol{\Omega} = 1/2\,\mathrm{curl}\,\mathbf{v}^n$ of rotation of the normal component, the intrinsic angular momentum can be found as the response of the system to the angular velocity of rotation:

$$L_{int} = -\delta\mathscr{F}/\delta\Omega. \tag{6}$$

The calculations we have made (see the Appendix) show that the gradient energy contains not only the terms found by Cross,[4] in which, of course, it is necessary to make the Galilean-invariant replacement $\mathbf{v}^s \to (\mathbf{v}^s - \mathbf{v}^n)$ (see also Ref. 12, in which, in addition to (4), terms proportional to gradients of the density ρ are taken into account), but also a term linear in the angular velocity of rotation:

$$\mathscr{F}^L = -\int L_{int}\Omega dV. \tag{7}$$

The quantity L_{int} is nonzero only because of the asymmetry of the particles and holes, and for the A phase is equal to

$$L_{int} = 1\frac{\rho}{4m}\frac{N_1}{N_0}\left(1 + \frac{1}{3}F_1\frac{\rho_\perp^n}{\rho}\right)^{-1}\left(\frac{\Delta(T)}{\varepsilon_F}\right)^2\ln\frac{\varepsilon_F}{T_c} \tag{8}$$

(m is the mass of the ^{3}He atom), which for $F_1 = 0$ coincides with the result of Cross.[4] Here N_0 and N_1 are the coefficients in the expansion of the density of states for one spin direction: $N(\xi) = N_0 + N_1(\xi/\varepsilon_F)$. The corresponding contribution to the current is obtained by varying (7) with respect to $\mathbf{v}^n$:

$$\mathbf{j}^L = -\delta\mathscr{F}^L/\delta\mathbf{v}^n = {}^1\!/_2\,\mathrm{rot}\,L_{int}. \tag{9}$$

Thus, the spontaneous orbital angular momentum in ^{3}He-A

is a well defined quantity, which appears in the energy and in the current in the form of the terms (7) and (9), respectively.

The authors are grateful to G. E. Volovik for suggesting that we fill out the details in the question of the spontaneous orbital angular momentum.

APPENDIX

Using the gradient expansion of the Gor'kov equations we shall find the Green functions of the system, after which we shall obtain a microscopic expression for the current and for the free energy. As already stated above, using the analogy between superfluid and superconducting systems we can consider a superconductor with p-pairing, placed in a magnetic field. To go over to the case of ^{3}He we make the replacement

$$\mathbf{v}^n = (|e|/cm)\mathbf{A}. \tag{A.1}$$

In the following we shall use atomic units ($|e| = \hbar = c = 1$).

The Poisson-brackets method used by Cross[13] to obtain the free energy is not suitable in the present case, since in this method the gauge invariance is broken for the small nonlocal terms responsible for the appearance of the spontaneous orbital angular momentum. Therefore, we shall use the method proposed by Eilenberger,[14] having generalized it to the case of p-pairing and for the presence of Fermi-liquid renormalizations. The Gor'kov equations have the form

$$\left\{i\omega + \mu(\mathbf{r}) - \frac{1}{2m^*}\left(\mathbf{k} + \frac{1}{i}\frac{\partial}{\partial\mathbf{r}} + \frac{1}{2i}\left[\mathbf{H}\times\frac{\partial}{\partial\mathbf{k}}\right]\right)^2\right.$$
$$\left. - \delta\varepsilon(\mathbf{k},\mathbf{r})\right\}\bar{G}(\mathbf{k},\mathbf{r},\omega)$$
$$- \bar{\Delta}(\mathbf{k}+\mathbf{O},r)\bar{F}^+(\mathbf{k},\mathbf{r},\omega) = 1,$$

$$\left\{-i\omega + \mu(\mathbf{r}) - \frac{1}{2m^*}\left(\mathbf{k}+\mathbf{O} + \frac{1}{2i}\left[\mathbf{H}\times\frac{\partial}{\partial\mathbf{k}}\right]\right)^2\right. \tag{A.2}$$
$$\left. - \delta\varepsilon^\tau(-\mathbf{k},\mathbf{r})\right\}\bar{F}^+(\mathbf{k},\mathbf{r},\omega)$$
$$+ \bar{\Delta}^+\left(\mathbf{k} + \frac{1}{i}\frac{\partial}{\partial\mathbf{r}}, r\right)\bar{G}(\mathbf{k},\mathbf{r},\omega) = 0,$$

where

$$\mathbf{O} = \frac{1}{i}\frac{\partial}{\partial\mathbf{r}} - 2\mathbf{A}(\mathbf{r}), \quad \omega = (2n+1)\pi T, \quad \mathbf{H} = \mathrm{rot}\,A,$$

and $\mu(\mathbf{r})$ is the chemical potential. Here we have used the barred Green functions

$$\bar{G}(\mathbf{k},\mathbf{r}) = \int \exp[-i\mathbf{k}(\mathbf{r}-\mathbf{r}')]\bar{G}(\mathbf{r},\mathbf{r}')d^3(\mathbf{r}-\mathbf{r}'), \tag{A.3}$$

$$\bar{F}^+(\mathbf{k},\mathbf{r}) = \int \exp[-i\mathbf{k}(\mathbf{r}-\mathbf{r}')]\bar{F}^+(\mathbf{r},\mathbf{r}')d^3(\mathbf{r}-\mathbf{r}'),$$

which are related to the usual Green functions by

$$\bar{G}(\mathbf{r},\mathbf{r}') = G(\mathbf{r},\mathbf{r}')\exp\left[i\int_{\mathbf{r}'}^{\mathbf{r}}\mathbf{A}(\mathbf{x})d\mathbf{x}\right],$$

$$\bar{F}^+(\mathbf{r},\mathbf{r}') = F^+(\mathbf{r},\mathbf{r}')\exp\left[i\int_{\mathbf{r}'}^{\mathbf{r}}\mathbf{A}(\mathbf{x})d\mathbf{x}\right] \tag{A.4}$$

A. V. Balatskiĭ and V. P. Mineev   1196

(the integration is taken over the straight line connecting $\mathbf{r}$ and $\mathbf{r}'$). Below, for brevity of writing, we omit the dependence of the functions on the frequency. Under the gauge transformation

$$\mathbf{A} \to \mathbf{A} + \nabla\chi,$$

$$\bar{G} \to \bar{G}, \quad \bar{F}^+ \to \bar{F}^+ \exp[2i\chi(\mathbf{r})], \tag{A.5}$$

it is not difficult to verify that the gauge invariance (A.5) is preserved for the solutions of (A.2) in any order of the expansion in the slow gradients $-i\partial/\partial\mathbf{r}$ and $\mathbf{O}$. The order parameter

$$\Delta = -3g \int \frac{d\Omega_{\mathbf{k}'}}{4\pi} \frac{\mathbf{k}\mathbf{k}'}{k_F^2} T \sum_\omega \bar{F}(\mathbf{k}',\mathbf{r}), \tag{A.6}$$

$$\Delta(\mathbf{k},\mathbf{r}) = i\sigma_\alpha\sigma_y A_{\alpha i}k_i/k_F, \tag{A.7}$$

where $A_{\alpha i}$ is the matrix of the order parameter (1) of the A phase, and $\sigma_\alpha = (\sigma_x,\sigma_y,\sigma_z)$ are the Pauli matrices. The Fermi-liquid corrections $\delta\varepsilon(\mathbf{k},\mathbf{r})$ are determined in the standard way:

$$\delta\varepsilon(\mathbf{k},\mathbf{r}) = \sum_{\mathbf{k}'} f(\mathbf{k},\mathbf{k}')\delta n(\mathbf{k}',\mathbf{r})$$

$$\delta n(\mathbf{k}',\mathbf{r}) = T \sum_\omega \delta G(\mathbf{k}',\mathbf{r}) \tag{A.8}$$

($\delta\varepsilon^T$ is the transposed matrix), and δG is the deviation of the Green function from the equilibrium Green function. As in Ref. 4, in $f(\mathbf{k},\mathbf{k}')$ we shall retain only the term with $l=1$:

$$f(\mathbf{k},\mathbf{k}') = f_1(\hat{\mathbf{k}},\hat{\mathbf{k}}') = N_0^{-1} F_1(\hat{\mathbf{k}},\hat{\mathbf{k}}'). \tag{A.9}$$

Following Ref. 14, we can expand (A.2) in a series in powers of $\partial/\partial\mathbf{r}$, $\mathbf{O}$, and $\mathbf{A}$:

$$\bar{G} = G_0 + G_1 + G_2 + \ldots, \quad \bar{F}^+ = F_0^+ + F_1^+ + F_2^+ + \ldots. \tag{A.10}$$

Writing the expansion (A.10) in matrix form, we obtain

$$\mathscr{G}_0 = D^{-1}E^+ \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \tag{A.11a}$$

$$\mathscr{G}_1 = -D^{-1}E^+ R_1\mathscr{G}_0, \tag{A.11b}$$

$$\mathscr{G}_2 = -D^{-1}E^+ (R_1\mathscr{G}_1 + R_2\mathscr{G}_0). \tag{A.11c}$$

Here,

$$\mathscr{G}_i = \begin{pmatrix} G_i \\ F_i^+ \end{pmatrix}, \quad i = 0,1,2,\ldots, \tag{A.12}$$

$$E = \begin{pmatrix} \varepsilon & -\Delta \\ \Delta^+ & \varepsilon^* \end{pmatrix}, \tag{A.13}$$

$$D = EE^+ = |\varepsilon|^2 + \Delta\Delta^+, \tag{A.14}$$

$$\varepsilon = i\omega + \mu - k^2/2m^* = i\omega - \xi, \tag{A.15}$$

$$R_1 = \begin{pmatrix} -\dfrac{\mathbf{k}}{im^*}\dfrac{\partial}{\partial\mathbf{r}} - \delta\varepsilon(\mathbf{k},\mathbf{r}) & -\dfrac{\partial\Delta}{\partial\mathbf{k}}\mathbf{O} \\ -i\dfrac{\partial\Delta^+}{\partial\mathbf{k}}\dfrac{\partial}{\partial\mathbf{r}} & -\dfrac{\mathbf{k}}{m^*}\mathbf{O} + \delta\varepsilon(\mathbf{k},\mathbf{r}) \end{pmatrix}, \tag{A.16}$$

$$R_2 = \begin{pmatrix} -\dfrac{1}{2m^*}\left(\dfrac{1}{i}\dfrac{\partial}{\partial\mathbf{r}}\right)^2 - \dfrac{1}{2m^*i}\mathbf{k}\left[\mathbf{H}\times\dfrac{\partial}{\partial\mathbf{k}}\right] & -\dfrac{1}{2}\dfrac{\partial^2\Delta}{\partial k_i\partial k_j}O_iO_j \\ -\dfrac{1}{2}\dfrac{\partial^2\Delta^+}{\partial k_i\partial k_j}\dfrac{\partial^2}{\partial r_i\partial r_j} & -\dfrac{1}{2m^*}\mathbf{O}^2 - \dfrac{1}{2im^*}\mathbf{k}\left[\mathbf{H}\times\dfrac{\partial}{\partial\mathbf{k}}\right] \end{pmatrix}. \tag{A.17}$$

The current density can be expressed in terms of the Green function:

$$\mathbf{j}(\mathbf{r}) = \sum_k \frac{\mathbf{k}}{m}\frac{1}{2}\operatorname{Sp} T \sum_\omega G_1(\mathbf{k},\mathbf{r}) + \text{h.c.} \tag{A.18}$$

Alternatively, recalling (A.8), from (A.18) we obtain

$$\delta\varepsilon(\mathbf{k},\mathbf{r}) = mk_F^{-2} f_1 k_i j_i. \tag{A.19}$$

Substituting into (A.18) the solution (A.11b), we obtain the expression for the current with allowance for the Fermi-liquid renormalization:

$$j_i = (1 + \tfrac{1}{3}F_1\rho_{ij}^n/\rho)^{-1}j_i^0, \tag{A.20}$$

where $\mathbf{j}^0$ is the current (5), i.e., the current without Fermi-liquid corrections.

To obtain the free energy we shall use the method (generalized to p-pairing) of integration over the coupling constant (see, e.g., Ref. 15). We have

$$\mathscr{F} - \mathscr{F}(\lambda = 0)$$

$$= -\int d^3r\left\{\frac{1}{2}\operatorname{Sp} T \sum_{\omega,\mathbf{k}}\int_0^1 d\lambda \Delta(\mathbf{k},\mathbf{r})F_2^+(\mathbf{k},\mathbf{r},\lambda) + \text{h.c.}\right.$$

$$\left. + \frac{1}{2}\frac{m}{m^*}\frac{1}{3\rho}F_1(\mathbf{j})^2\right\}, \tag{A.21}$$

Here $F_2^+(\mathbf{k},\mathbf{r},\lambda)$ is the function from (A.11c), in which the replacement $\Delta \to \lambda\Delta$ has been made.

Concerning the gradient energy $\mathscr{F}$ it is necessary to make the following remark. In Refs. 13 and 4 $\mathscr{F}$ was calculated by integrating the function G_2 over the frequency, as first suggested in Ref. 14. If we proceed by this method, the current density (A.18), with allowance for the small terms arising on account of the asymmetry in the distribution of the particles and holes, is not equal to the variation $\delta\mathscr{F}/\delta\mathbf{A}$. The reason for this is not clear to us; we note only that, as shown in Ref. 16, in the A phase it is necessary to exercise care with the frequency dependences. The calculation from (A.21), as we shall show, is free of this discrepancy.

We shall write F_2^+ from (A.11c) in the form of three terms, of zeroth, first, and second order, respectively, in the Fermi-liquid corrections:

$$F_2^+ = F_2^{0+} + F_2^+(\delta\varepsilon) + F_2^+(\delta\varepsilon\delta\varepsilon). \qquad (A.22)$$

Then, retaining terms of each type only in leading order in Δ/ε_F, we have

$$\mathscr{F}(F_2^{0+})$$

$$= - \int d^3r \frac{1}{2} \mathrm{Sp} \left\{ - \frac{T}{m^{*2}} \sum_{\omega,k} \int_0^1 d\lambda\lambda \frac{\varepsilon^2}{D_\lambda^3} \Delta(\mathbf{k}\mathbf{O})(\mathbf{k}\mathbf{O})\Delta^+ \right.$$

$$- \frac{T}{im^*} \sum_{\omega,k} \int_0^1 d\lambda\lambda \frac{\Delta\varepsilon}{D_\lambda}(\mathbf{k}\mathbf{O})\frac{\varepsilon}{D_\lambda}\frac{\partial\Delta^+}{\partial k_i}\frac{\partial G^0}{\partial r_i} - \frac{T}{im^*} \sum_{\omega,k} \int_0^1 d\lambda\lambda \frac{\varepsilon\Delta}{2D_\lambda^2}$$

$$\left. \times \left(\mathbf{k}\left[\mathbf{H}\frac{\partial}{\partial\mathbf{k}}\right] \right) \Delta^+ + \text{h.c.} \right\}, \qquad (A.23)$$

where $D_\lambda = \omega^2 + \xi^2 + \lambda^2\Delta\Delta^+$. Integrating over λ, we obtain

$$\mathscr{F}(F_2^{0+}) = \mathscr{F}_s^0 \frac{m}{m^*} + \int d^3r \left\{ \frac{1}{2}(\nabla_\alpha\varphi - 2A_\alpha) \right.$$

$$\times \frac{1}{4}\cdot\frac{1}{3}\frac{\mathrm{Sp}\,\rho_s}{2}[\nabla\rho\times\mathbf{l}]_\alpha \frac{m}{m^*}$$

$$\left. - \frac{\dot{L}_{in}\mathbf{l}\mathbf{H}}{2}\frac{m}{m^*} \right\}, \qquad (A.24)$$

where the first term is the energy found by Cross.[4] The spontaneous orbital angular momentum arises from the last term in (A.23).

Concerning the second term in (A.23) it is necessary to make the following remark. The origin of the terms with $\nabla\rho$ is associated (see Ref. 17) with the coordinate dependence of the potential $\mu(\mathbf{r})$ near the walls of the vessel with ^{3}He-A, where, on account of the influence of the walls, the modulus of the order parameter is reduced in a layer of thickness $\sim\psi$ on the walls, leading to a density change $\nabla\rho/\rho \sim (\Delta/\varepsilon_F)^2\xi^{-1}$. A further decrease of the density, to zero, occurs over atomic distances, and, of course, lies beyond the limits of applicability of the gradient expansion. Therefore, the term with $\nabla\rho$ in the energy and in the current describes the extent to which the density of ^{3}He inside the vessel exceeds the density at distances of the order of the interatomic distance from the surface of the vessel. In accordance with the philosophy developed in Ref. 2, this term gives that contribution to the total (integral) orbital angular momentum of the vessel which arises on account of the near-wall current:

$$\frac{\vec{\mathscr{L}}}{V} \sim \mathbf{l}\rho_s \frac{\delta\rho}{\rho}\frac{\hbar}{m} \sim \frac{\hbar}{m}\mathbf{l}\rho_s\left(\frac{\Delta}{\varepsilon_F}\right)^2.$$

If we formally regard the result of the gradient expansion as valid up to the boundary (i.e., $\delta\rho = \rho$), the term with $\nabla\rho$ gives

$$\frac{\vec{\mathscr{L}}}{V} \sim \frac{\hbar}{m}\mathbf{l}\rho_s\frac{\delta\rho}{\rho} \approx \frac{\hbar}{m}\mathbf{l}\rho_s,$$

as can be easily seen by integrating the second term in (A.24) by parts. It would be incorrect, however, to identify this quantity with the intrinsic angular momentum of the liquid, as was done, e.g., in Ref. 9, since, although $\delta\rho$ does coincide with ρ, $\nabla\rho$ is concentrated in the near-wall layer. Therefore, the second term gives the orbital angular momentum only from the near-wall region. This angular momentum for a uniform texture $\mathbf{l}$ coincides, to within a coefficient, with the angular momentum of the superfluid current flowing in a nonuniform texture.[7]

We consider now the terms linear in $\delta\varepsilon$:

$$F_2^+(\delta\varepsilon) = \frac{E^2 - 3\omega^2}{m^*D^3}\delta\varepsilon(\mathbf{k}\mathbf{O})\Delta^+ + \delta\varepsilon\frac{\varepsilon^2}{D^2}\frac{\partial\Delta^+}{\partial k}\frac{\nabla}{i}G^0$$

$$+ \frac{\varepsilon}{D}\frac{\partial\Delta^+}{\partial k}\frac{\nabla}{i}\left(\delta\varepsilon\left(\frac{-\omega^2 + E^2}{D^2}\right)\right). \qquad (A.25)$$

Substituting (A.25) into (A.21) and proceeding as in the derivation of (A.24), with allowance for (A.19) we obtain

$$\mathscr{F}(F_2^+(\delta\varepsilon)) = \int d^3r \left\{ \frac{m}{m^*}\frac{1}{3\rho}F_i j_i (j_i^0 - j_i^{0L}) \right.$$

$$\left. + \frac{m}{m^*}\frac{1}{3\rho}F_i\left(\frac{L_{in}^0\mathbf{l}}{2}\mathrm{rot}\,\mathbf{j}\right) \right\}. \qquad (A.26)$$

Finally, we consider

$$F_2^+(\delta\varepsilon\,\delta\varepsilon) = (\delta\varepsilon\,\delta\varepsilon)\frac{\Delta^+}{D^3}(3\omega^2 - E^2), \qquad (A.27)$$

whence

$$\mathscr{F}(F_2^+(\delta\varepsilon\delta\varepsilon)) = \int d^3r \left\{ \frac{1}{2}\frac{m}{m^*}\frac{1}{3\rho}F_i j_i (j_i - j_i^0) \right\}. \qquad (A.28)$$

Here it is necessary to make the following remark: In the integration over λ we assumed that $\delta\varepsilon$ does not depend on λ, inasmuch as in the Gor'kov equations (A.2) $\delta\varepsilon$ plays the role of an external potential, and only in the equations for the current is it found self-consistently, as it should be in Fermi-liquid theory. According to (A.21) we should subtract the contribution corresponding to $\lambda = 0$, i.e., in the integration over λ the lower limit $\lambda = 0$ (corresponding to $\Delta = 0$) should be discarded. After this, from (A.27) it is not difficult to obtain (A.28), whence, summing (A.28), (A.26), and (A.24) and taking into account the last term in (A.21), we obtain

$$\mathscr{F} = \mathscr{F}(F_2^{0+})$$

$$+ \int d^3r \left\{ \frac{m}{m^*}\frac{1}{3\rho}F_i\left(\frac{1}{2}j_i j_i^0 - j_i j_i^{0L} + \frac{L_{in}^0\mathbf{l}}{2}\mathrm{rot}\,\mathbf{j} \right) \right\}. \qquad (A.29)$$

Varying (A.29) with respect to the vector potential gives the expression (A.20) for the current, while varying it with respect to the magnetic field $\mathbf{H} = \mathrm{curl}\,\mathbf{A}$ gives the spontaneous orbital angular momentum (8) with allowance for Fermi-liquid renormalization. Finally, from (A.29) it can be seen that in the energy there is a term of the form (7), corresponding to the energy of interaction of the angular momentum and the field.

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1198   Sov. Phys. JETP **62** (6), December 1985

A. V. Balatskiĭ and V. P. Mineev   1198

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Translated by P. J. Shepherd