# Phenomenon of total external reflection of x rays

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A study is made of the influence of small perturbations of an interface on the total external reflection of x rays. Three small parameters of the problem determine the specific behavior of reflection in the x-ray range: the glancing angle $\alpha$, the permittivity discontinuity $\varepsilon_1 - \varepsilon_2$, and the ratio of the wavelength $\lambda$ to the transverse size of a perturbation. The relationship between these parameters determines the angular dependence of the x-ray reflection coefficient of a plane surface. This angular dependence explains why the height of the irregularities appears greater at low glancing angles. Conditions are found under which x-ray beams are rotated through large angles as a result of propagation along curved interfaces.

## I. INTRODUCTION

The permittivity $\varepsilon$ of all materials differs little from unity, $\varepsilon = 1 - \delta$, in the x-ray range. Therefore, there is a narrow range of glancing angles $\alpha \lesssim \alpha_c = \delta^{1/2}$ where the reflection coefficient of x rays is close to unity. Studies of the angular dependence of the reflection coefficient in this range provide an important and often a unique source of information on the fundamental properties of solids, liquids, and interfaces.[1–3]

The phenomenon of total external reflection underlies a traditional branch of x-ray optics which is being used increasingly in research and physics, astrophysics, and biology.[4] We are speaking here of wavelengths in the range 300 Å $> \lambda >$ 1 Å, so that it is important to consider the irregularities which unavoidably remain on an optical surface after any type of finishing treatment. This applies equally to the surfaces of ideal crystals because, due to reconstruction, their atomic structure may be characterized by a scale much greater than the interatomic distances. Clearly, the future developments and applications of x-ray optics largely depend on whether it will be possible to eliminate surface irregularities on a scale of tens or even a few angstroms or, at any rate, to minimize their influence on the parameters of reflected x rays.

Several reviews and monographs (see, for example, Ref. 5) have been devoted to the theory of reflection of radiation by rough surfaces, but insufficient attention has been given to small glancing angles for which the reflection coefficient of x rays is large. In many cases the range of small angles is outside the scope of a given theory. However, it is at small glancing angles, including those close to the critical value $\alpha_c$, that several features of the reflection coefficient and angular distribution of the scattered radiation are observed.[6–8] They can be explained using different models of a surface layer based on certain assumptions about the distribution of the permittivity in this layer and the nature of the surface irregulaties.[1,9]

Our aim will be to use a very general model of a surface in a study of the specular component of x rays reflected at small glancing angles. In Sec. II this problem will be tackled by perturbation theory. The specular reflection coefficient depends on the relationship between the glancing angle $\alpha$, the wavelength $\lambda$, and the correlation radius of the surface irregularities $a$. If $\alpha \gg (\lambda / a)^{1/2}$, the correlation radius is unimportant and the reflection coefficient after allowance for scattering is given by

$$|R'|^2 = |R|^2 (1 - 4k^2\zeta^2\alpha^2)$$

(where $k = 2\pi/\lambda$ and $\xi$ is the height of irregularities), which follows both from perturbation theory and from the Kirchhoff approximation.[10] If the glancing angle is small, $\alpha \ll (\lambda / a)^{1/2}$ then—as shown below—the dependence of the reflection coefficient on the angle changes to

$$|R'|^2 = |R|^2 \left[ 1 - \frac{4\Gamma(^3/_4)\, k^2\zeta^2}{(ka)^{1/2}} \alpha \right].$$

In both acoustics and radiophysics it is usual to consider media which reflect strongly at all incidence angles. Consequently, the permittivity discontinuity is regarded as larger than any other parameter of the problem. On the other hand, in the x-ray range all materials are characterized by a weak polarizability and the change in the permittivity at the interface is small. We therefore have a new parameter $\delta(a/\lambda)$ which governs the influence of the state of the surface on its reflectivity. A detailed knowledge of the angular dependence of the specular reflection coefficient makes it possible to justify quantitatively the methods used to determine the surface parameters from the x-ray reflection coefficient measurements (see Refs. 2, 11, and 12), and also the requirements in respect of the surfaces intended for use as x-ray optics components.

In Sec. III we shall apply these results to the problem of rotation of an x-ray beam by a concave mirror with whispering-gallery modes. In this case, in contrast to a plane surface, the presence of irregularities generally reduces the reflection coefficient even at the zero glancing angle. The surface of a concave mirror may be regarded as smooth (for a beam glancing along it) if the height $\zeta$ and the correlation radius $a$ of irregularities satisfy one of the two conditions: $\zeta \ll \lambda^{3/4} a^{1/4}$ or $\zeta \ll (\lambda/\delta)(\lambda/a)^{7/4}$.

## II. DERIVATION OF PRINCIPAL FORMULAS

### General relationships

The aim in the present section will be to obtain formulas for the intensity of specularly reflected radiation incident on

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a rough interface between two media. The general approach to problems of this kind can be found in Refs. 5 and 10. We shall use a different variant of the perturbation method (see Ref. 13), which makes it possible to simplify an allowance for the boundary conditions and to consider from a unified standpoint both abrupt and diffuse boundaries, and yields the required result more rapidly.

An ideal interface between media 1 and 2 will be regarded as a one-dimensional inhomogeneous layer the permittivity of which is described by the function $\varepsilon_0(z)$, so that $\varepsilon_0(-\infty)=\varepsilon_1$ and $\varepsilon_0(+\infty)=\varepsilon_2$. The imperfections of the interface are due to the presence of perturbations $\Delta\varepsilon(\mathbf{r})$, the scale of which is limited along the $z$ axis, but is generally not limited (unbounded) in the transverse directions (along the $x$ and $y$ axes).

In the case of an ideal boundary, the field is

$$\psi_0(\mathbf{r})=\psi_0(z)\exp(i\mathbf{q}_0\boldsymbol{\rho}),\quad q_0=k\varepsilon_1^{1/2}\cos\alpha_0, \tag{1}$$

where $\boldsymbol{\rho}=(x,y)$ is a two-dimensional vector in the $z=0$ plane, $\alpha_0$ is the glancing angle, and $\psi_0(z)$ satisfies the equation

$$d^2\psi_0/dz^2+[k^2\varepsilon_0(z)-q_0^2]\psi_0(z)=0. \tag{2}$$

In the presence of a perturbation $\Delta\varepsilon(r)$ the wave field $\psi(r)$ satisfies the equation

$$[\nabla^2+k^2\varepsilon_0(z)]\psi(\mathbf{r})=-k^2\Delta\varepsilon(\mathbf{r})\psi(\mathbf{r}), \tag{3}$$

which can be written in the integral form:

$$\psi(\mathbf{r})=\psi_0(\mathbf{r})-k^2\int G(\mathbf{r},\mathbf{r}')\Delta\varepsilon(\mathbf{r}')\psi(\mathbf{r}')d^3\mathbf{r}', \tag{4}$$

where the Green function is

$$G(\mathbf{r},\mathbf{r}')=-\frac{i}{(2\pi)^2k}\int\frac{y_1(z_<,q)y_2(z_>,q)\exp[i\mathbf{q}(\boldsymbol{\rho}-\boldsymbol{\rho}')]}{\tilde\varepsilon_1^{1/2}(q)+\tilde\varepsilon_2^{1/2}(q)}d^2\mathbf{q}, \tag{5}$$

$$z_<=\min(z,z'),\quad z_>=\max(z,z')\quad \tilde\varepsilon_1(q)=\varepsilon_1-q^2/k^2,$$

$$\tilde\varepsilon_2(q)=\varepsilon_2-q^2/k^2,\quad q=k\varepsilon_1^{1/2}\cos\alpha.$$

In Eq. (5), $y_1(z,q)$ and $y_2(z,q)$ are the solutions of Eq. (2) satisfying the boundary conditions and given by

$$y_1(z,q)=\begin{cases}\exp[-ik\tilde\varepsilon_1^{1/2}(q)z], & z\to-\infty,\\ T^{-1}\left[\dfrac{\tilde\varepsilon_1(q)}{\tilde\varepsilon_2(q)}\right]^{1/2}\{\exp[-ik\tilde\varepsilon_2^{1/2}(q)z]-R\exp[ik\tilde\varepsilon_2^{1/2}(q)z]\}, & z\to+\infty,\end{cases}$$
$$y_2(z,q)=\begin{cases}T^{-1}\{\exp[ik\tilde\varepsilon_1^{1/2}(q)z]+R\exp[-ik\tilde\varepsilon_1^{1/2}(q)z]\}, & z\to-\infty,\\ \exp[ik\tilde\varepsilon_2^{1/2}(q)z], & z\to+\infty,\end{cases} \tag{6}$$

where $R(q)$ and $T(q)$ represent the reflection and transmission coefficients of the waves incident on an ideal boundary from the $z<0$ side.

If the point of observation $z$ lies much further from the interface than the thickness of the transition layer $L$ or the characteristic longitudinal size of the inhomogeneity, then Eq. (4) yields an asymptotic expression valid in the limit $z\to-\infty$:

$$\psi(\boldsymbol{\rho},z)\approx\exp(i\mathbf{q}_0\boldsymbol{\rho})\psi_0(z,q_0)-\int\exp[i\mathbf{q}\boldsymbol{\rho}-ik\tilde\varepsilon_1^{1/2}(q)z]A(\mathbf{q})d^2\mathbf{q}, \tag{7}$$

$$A(\mathbf{q})=-\frac{ik}{(2\pi)^2[\tilde\varepsilon_1^{1/2}(q)+\tilde\varepsilon_2^{1/2}(q)]}\times\int y_2(z,q)e^{-i\mathbf{q}\boldsymbol{\rho}}\Delta\varepsilon(\mathbf{r})\psi(\mathbf{r})d^3\mathbf{r}. \tag{8}$$

We shall use Eq. (7) to calculate the energy flux along the $z$ axis, which intersects a small vertical area located far from the interface:

$$Q=\frac{1}{k}\int_\sigma\operatorname{Im}\psi^*\frac{\partial\psi}{\partial z}d^2\boldsymbol{\rho}=(1-|R|^2)\tilde\varepsilon_1^{1/2}(q_0)\int_\sigma d^2\boldsymbol{\rho}-(2\pi)^2\int|A(\mathbf{q})|^2\operatorname{Re}\tilde\varepsilon_1^{1/2}(q)d^2\mathbf{q}+2(2\pi)^2|R|^2\tilde\varepsilon_1^{1/2}(q_0)\operatorname{Re}\frac{A(\mathbf{q}_0)}{R(\mathbf{q}_0)}. \tag{9}$$

This area $\sigma$ is selected to be sufficiently large to satisfy

$$\int_\sigma e^{i\mathbf{q}\boldsymbol{\rho}}d^2\boldsymbol{\rho}=(2\pi)^2\delta^2(\mathbf{q}).$$

The physical meaning of the terms in Eq. (9) is self-evident: the first term describes the flux created by the unperturbed wave (i.e., by the wave interacting with an ideal boundary); the second and third terms represent the fluxes scattered by inhomogeneities and consisting of two parts: diffuse and specular.

It is clear from Eq. (9) that the diffuse scattering coefficient (i.e., the energy scattered by a surface element, divided by the incident energy) is

$$S=\frac{(2\pi)^2}{\tilde\varepsilon_1^{1/2}(q_0)}\left[\int|A(\mathbf{q})|^2\operatorname{Re}\tilde\varepsilon_1^{1/2}(q)d^2\mathbf{q}\right]\Big/\int_\sigma d^2\boldsymbol{\rho}, \tag{10}$$

and the specular reflection coeffcient, calculated allowing for the scattering, is

$$|R'(q_0)|^2=|R(q_0)|^2-\delta R(q_0),$$
$$\delta R(q_0)=8\pi^2|R(q_0)|^2\operatorname{Re}\frac{A(\mathbf{q}_0)}{R(q_0)}\left(\int_\sigma d^2\boldsymbol{\rho}\right)^{-1}. \tag{11}$$

The expressions (10) and (11) are exact. They are derived making no assumptions about the shape of the interface or the nature of its inhomogeneities (irregularities).

### Model of an interface

We shall assume that an interface is the surface on which an abrupt (in fact, over distances of the order of the atomic spacing) change in the properties of materials takes place. A surface of this kind may appear as a result of processing of amorphous bodies or of cleaving a crystal (model of steps and ledges).

We shall assume that an ideal surface coincides with the $z=0$ plane and a real surface is described by $z=\zeta(\rho)$, where $\zeta(\rho)$ is a random function governing the statistical properties of the surface.

Since we are interested in specular reflection from ultrasmooth surfaces, we shall find the first term of the expansion $\delta R$ [see Eq. (11)] from the height of irregularities $\zeta(\rho)$. It is readily shown that this can be done by retaining only the first perturbation-theory term in Eq. (4) and using the following properties of the permittivity and its correlation functions:

$$\varepsilon(\mathbf{r})=\varepsilon_0(z)+\Delta\varepsilon(\mathbf{r}),\quad \varepsilon_0(z)=\varepsilon_1+\theta(z)\delta\varepsilon,\quad \delta\varepsilon=\varepsilon_2-\varepsilon_1,$$

$$\Delta\varepsilon(\mathbf{r})=[\theta(z-\zeta(\rho))-\theta(z)]\delta\varepsilon$$

$$=-\delta\varepsilon\sum_{m=0}^{\infty}(-1)^m\frac{\zeta^{m+1}(\rho)}{(m+1)!}\delta^{(m)}(z), \tag{12}$$

$$\overline{\Delta\varepsilon(\mathbf{r})}\approx\delta\varepsilon\frac{\overline{\zeta^2}}{2}\delta'(z),$$

$$\overline{\Delta\varepsilon(\mathbf{r})\Delta\varepsilon(\mathbf{r}')}\approx(\delta\varepsilon)^2\delta(z)\delta(z')\overline{\zeta^2(\rho-\rho')},$$

where $\theta(z)$ is the Heaviside step function, $\delta(z)$ is the Dirac delta function, and

$$\overline{\zeta^2(\rho-\rho')}=\overline{\zeta^2}\chi(\rho-\rho'),\quad \chi(0)=1 \tag{13}$$

is the correlation function of the surface irregularities, which we shall assume to be isotropic. From now on we shall omit the averaging sign (bar) of $\zeta^2$.

Thus, assuming (in accordance with the above discussion) that $\psi(\mathbf{r})=\psi_0(\mathbf{r})$ applies on the right-hand side of Eq. (4), we shall substitute the result in Eq. (8) and average over the surface irregularities using the relationships in Eq. (12). Then, the average amplitude becomes

$$\overline{A(\mathbf{q}_0)}=2\left(\frac{k\zeta}{2\pi}\right)^2R(q_0)\tilde{\varepsilon}_1^{1/2}(q_0)\operatorname{Re}F(\mathbf{q}_0), \tag{14}$$

where

$$F(\mathbf{q}_0)=\frac{1}{(2\pi)^2}\int[\tilde{\varepsilon}_1^{1/2}(q)-\tilde{\varepsilon}_2^{1/2}(q)]\chi(\mathbf{q}-\mathbf{q}_0)d^2\mathbf{q}+\tilde{\varepsilon}_2^{1/2}(q_0),$$
$$\chi(\mathbf{q})=\int e^{i\mathbf{q}\rho}\chi(\rho)d^2\rho. \tag{15}$$

Finally, substituting Eq. (14) into Eq. (11), we find the expression for the specular reflection coefficient of a rough surface:

$$|R'(q_0)|^2=|R(q_0)|^2[1-4k^2\zeta^2\tilde{\varepsilon}_1^{1/2}(q_0)\operatorname{Re}F(\mathbf{q}_0)]. \tag{16}$$

According to Eq. (15) the function $F(\mathbf{q}_0)$ depends on the optical constants $\varepsilon_1$ and $\varepsilon_2$ of a material, on the angle of incidence, and on the correlation function of the heights of irregularities.

We must point out directly that if the correlation radius of the surface is very large (in comparison with a quantity we shall define later), then in Eq. (15) we have $\chi(\rho)\approx1$ and $\chi(\mathbf{q})=(2\pi)^2\delta^2(\mathbf{q})$ and Eq. (16) reduces to the frequently employed simple expression for the reflectivity of a rough surface:

$$|R'(q_0)|^2=|R(q_0)|^2(1-4k^2\zeta^2\varepsilon_1^{1/2}\sin^2\alpha), \tag{17}$$

which does not contain the correlation radius of the surface irregularities or the ratio of the optical constants $\varepsilon_1$ and $\varepsilon_2$. In fact, it follows from Eq. (16) that both these factors play a role. We shall discuss later this topic in detail.

### Grazing incidence

We shall assume that the correlation function of the heights of irregularities on the surface given by Eq. (13) is Gaussian:

$$\chi(\rho)=\exp(-\rho^2/a^2), \tag{18}$$

where $a$ is the correlation radius, and we shall integrate with respect to angles in Eq. (15):

$$F(q_0)=\frac{a^2}{2}\int_0^\infty q[\tilde{\varepsilon}_1^{1/2}(q)-\tilde{\varepsilon}_2^{1/2}(q)]I_0\left(\frac{qq_0a^2}{2}\right)\times\exp\left(-\frac{q^2+q_0^2}{2}a^2\right)dq. \tag{19}$$

In optical and x-ray experiments the correlation radius is considerably greater than the radiation wavelength: $a\gg\lambda$, so that we can use in Eq. (19) the asymptotic representation of a modified Bessel function with large values of the argument. We then have

$$\operatorname{Re}F(q_0)=\frac{a}{2(\pi q_0)^{1/2}}\left\{\int_0^{k\varepsilon_1^{1/2}}q^{1/2}\left(\varepsilon_1-\frac{q^2}{k^2}\right)^{1/2}\times\exp\left[-\frac{(q-q_0)^2}{4}a^2\right]dq\right.$$
$$\left.-\int_0^{k\varepsilon_2^{1/2}}q^{1/2}\left(\varepsilon_2-\frac{q^2}{k^2}\right)^{1/2}\exp\left[-\frac{(q-q_0)^2}{4}a^2\right]dq\right\}$$
$$+\operatorname{Re}\left(\varepsilon_2-\frac{q^2}{k^2}\right)^{1/2}. \tag{20}$$

The reflection coefficient of x rays differs from zero only for small glancing angles $\alpha$ which is smaller than or of the order of the critical value. The following conditions are satisfied in this range:

$$\varepsilon_1>\varepsilon_2,\quad a\gg\lambda,\quad \alpha\ll1,\quad \alpha^2<2[1-(\varepsilon_2/\varepsilon_1)^{1/2}], \tag{21}$$

so that the quantity $\operatorname{Re}F(q_0)$ and, therefore, the specular reflection coefficient depend only on one parameter:

$$\mu=\frac{ak\varepsilon_1^{1/2}}{4}\alpha^2=\frac{\pi\varepsilon_1^{1/2}a}{2\lambda}\alpha^2. \tag{22}$$

In fact, we shall bear in mind that $\operatorname{Re}(\varepsilon_2-q^2/k^2)^{1/2}=0$ and substituting the variables $q=k\varepsilon_1^{1/2}u$ and $q=k\varepsilon_2^{1/2}u$, we shall rewrite Eq. (20) in the form

$$\operatorname{Re}F(q_0)=\frac{ak\varepsilon_1}{2\pi^{1/2}}\left[\int_0^1[u(1-u^2)]^{1/2}\times\exp\left[-\frac{k^2a^2\varepsilon_1}{4}\left(1-\frac{\alpha^2}{2}-u\right)^2\right]du\right.$$
$$-\left(\frac{\varepsilon_2}{\varepsilon_1}\right)^{5/4}\int_0^1[u(1-u^2)]^{1/2}$$
$$\left.\times\exp\left\{-\frac{k^2a^2\varepsilon_2}{4}\left[\left(\frac{\varepsilon_1}{\varepsilon_2}\right)^{1/2}\left(1-\frac{\alpha^2}{2}\right)-u\right]^2\right\}du\right]. \tag{23}$$

We shall initially assume that $ka(\varepsilon_1 - \varepsilon_2) \gg 1$. The integrands in Eq. (23) have then sharp maxima (stationary points), because $k^2a^2\varepsilon_1/4 \gg 1$, but in the second integral the maximum is outside the range of integration and the integral itself is exponentially small. In the remaining integral we must allow for the contribution of the stationary point $u = 1 - \alpha^2/2$ and of the nearby point $u = 1$. We shall assume that $u = 1 - \alpha^2(1+y)/2$ and simplify the integrand using the conditions (21):

$$\mathrm{Re}\, F(q_0) = \frac{ak\varepsilon_1}{4\pi^{1/2}}\alpha^3 J(\mu), \quad J(\mu) = \int_{-1}^{\infty}(1+y)^{1/2}e^{-\mu^2y^2}dy. \tag{24}$$

Substituting Eq. (24) into the general formula (16), we obtain the following expression for the specular reflection coefficient of a round surface when the glancing angle is less than the critical value

$$|R'|^2 = |R|^2 - \delta R = |R|^2[1 - 4k^2\zeta^2\varepsilon_1^{1/2}\alpha^2\Phi(\mu)], \tag{25}$$

$$\Phi(\mu) = \frac{\mu J(\mu)}{\pi^{1/2}} = \begin{cases} 1 - \dfrac{1}{16\mu^2} + \dots, & \mu \gg 1 \\[2ex] \dfrac{1}{2(\pi\mu)^{1/2}}\left[\Gamma\left(\dfrac{3}{4}\right) + \dfrac{\mu}{2}\Gamma\left(\dfrac{1}{4}\right) + \dots\right], & \mu \ll 1 \end{cases}$$

It is clear from Eq. (25) that the contribution of the scattering by surface irregularities to the specular component of the reflection coefficient is described by Eq. (17) only in the case of relatively large glancing angles $\alpha > 2(ak\varepsilon_1^{1/2})^{-1/2}$. If $\alpha \ll 2(ak\varepsilon_1^{1/2})^{-1/2}$, the contribution of the scattering to the specularly reflected radiation is proportional to the glancing angle:

$$\delta R_{\alpha\to 0} = |R|^2 \frac{4\varepsilon_1^{1/4}\Gamma(3/4)}{\pi^{1/2}}\frac{(k\zeta)^2}{(ka)^{1/2}}\alpha. \tag{26}$$

In other words, it seems that the effective height of surface irregularities rises for infinitesimally small glancing angles:

$$\zeta_{\mathrm{eff}}^2 = \zeta^2\varepsilon_1^{-1/4}\,\Gamma(3/4)/\alpha(\pi ak)^{1/2}.$$

This effect had been observed[8] in the course of a study of diffuse scattering of radiation of wavelengths $\lambda \approx$ 2–12 Å.

It should be pointe dout that Eq. (26) does not contain the permittivity of the material from which the mirror is made, in view of the condition $ka(\varepsilon_1 - \varepsilon_2) \gg 1$ that justifies dropping of the second term from Eq. (23). Physically, this corresponds to the approximation of a totally reflecting (soft or hard) surface, which is used in radiophysics and acoustics when the permittivity discontinuity can be regarded as infinitely large.[5]

The low polarizability of matter in the x-ray range makes the permittivity discontinuity very small: $\varepsilon_1 - \varepsilon_2 \ll 1$, so that the opposite condition $ka(\varepsilon_1 - \varepsilon_2) \ll 1$ may be satisfied. The second term on the right-hand side of Eq. (23) should be retained. Obviously, it contributes only when integration is carried out in the vicinity of the boundary point $u = 1$. We can include this point by combining the integrands in Eq. (23) taking out the factor

$$1 - (\varepsilon_2/\varepsilon_1)^{5/4}\exp[-k^2a^2(\varepsilon_1 - \varepsilon_2)^2/16\varepsilon_1].$$

Transformations and the substitution of the result into the general formula (16) readily shows that if $ka(\varepsilon_1 - \varepsilon_2) \ll 1$, we must distinguish two cases

$$\left[\frac{5(\varepsilon_1 - \varepsilon_2)}{4\varepsilon_1}\right]^{1/2} < \frac{ka}{4}\frac{\varepsilon_1 - \varepsilon_2}{\varepsilon_1^{1/2}} \ll 1,$$

$$\delta R = |R|^2 \frac{1}{4\pi^{1/2}}\Gamma\left(\frac{3}{4}\right)(k\zeta)^2(ka)^{3/2}\frac{(\varepsilon_1 - \varepsilon_2)^2}{\varepsilon_1^{3/4}}\alpha \tag{27}$$

and

$$\frac{ka}{4}\frac{\varepsilon_1 - \varepsilon_2}{\varepsilon_1^{1/2}} < \left[\frac{5(\varepsilon_1 - \varepsilon_2)}{4\varepsilon_1}\right]^{1/2} \ll 1,$$

$$\delta R = |R|^2 \frac{5}{\pi^{1/2}}\Gamma\left(\frac{3}{4}\right)(k\zeta)^2(ka)^{-1/2}\frac{\varepsilon_1 - \varepsilon_2}{\varepsilon_1^{1/4}}\alpha. \tag{28}$$

A comparison of Eqs. (27) and (28) with Eq. (26) shows that a reduction in the permittivity discontinuity $\varepsilon_1 - \varepsilon_2$ weakens the influence of the scattering by surface irregularities on the specular reflection. In other words, we can say that an x-ray beam characterized by a high penetrating power bypasses the surface irregularities without scattering. A similar conclusion was reached in Ref. 14 in connection with the study of x-ray waveguides.

It therefore follows that there are two regimes of reflection of a grazing x-ray beam by a plane rough surface. When the glancing angle is large ($\mu > 1$), the influence of the surface imperfections on the specular reflection is governed, in accordance with the Rayleigh formula (17) (see Ref. (10), only by the square of the ratio of the "apparent height of the irregularities" $\zeta \sin\alpha$ to the wavelength. In the case of small glancing angles ($\mu < 1$) the correction for the surface roughness becomes proportional to $\alpha$ (and not to $\alpha^2$) and, moreover, it begins to depend on the correlation size $a$ and on the ratio of the permittivities $\varepsilon_1$ and $\varepsilon_2$ [see Eqs. (26)–(28)]. For example, in the case of Vitreosil and Spectrosil quartz glasses the height of surface irregularities is $\zeta \approx 10$ Å, and the correlation length is $a \approx 100\mu$ (Ref. 12). Then, according to Eq. (25), the transition from one reflection regime to another at the wavelength of $\lambda = 10$ Å occurs for glancing angles $\alpha \approx 0.1°$.

## III. INFLUENCE OF SCATTERING ON THE PROPAGATION OF X RAYS ALONG A BENT INTERFACE

It was shown in Ref. 15 that under certain conditions a beam of soft x rays may tarvel along a bent interface without significant intensity losses. These conditions are satisfied in the wavelength intervals $\lambda < 20$ Å and $\lambda > 50$ Å. This effect may be used to rotate synchrotron radiation and other x-ray-source beams through large angles,[1)] to construct filters of short wavelengths, and also to make x-ray radiation concentrators.[18,19] Consequently, we have to consider the role of microirregularities. The special feature of the problem considered here is that the influence of microirregularities remains important even at zero glancing angle because of multiple reflections. We shall consider this problem in greater detail employing the results reported in Sec. II.

We shall assume that a narrow beam of x rays is incident on a concave cylindrical interface between two media. At low glancing angles $\alpha$ the propagation of such a beam can be regarded as consisting of consecutive reflections, so that the total reflection coefficient is

$$|R(\alpha, \varphi)|^2=|R(\alpha)|^{\varphi/\alpha}, \tag{29}$$

where $\varphi$ is the angle supported by the mirror and equal to the angle of rotation of the beam and $|R(\alpha)|^2$ is the single-reflection coefficient which is given by Eq. (26) after allowance for the scattering by surface inhomogeneities [it is assumed that $ka(\varepsilon_1-\varepsilon_2)\gg 1$]. Since at low glancing angles $\alpha$ the number of reflections $\varphi/2\alpha$ is large, it follows that Eq. (29) can be transformed to

$$|R(\alpha,\varphi)|^2=\exp\left[-\frac{\varphi}{2\alpha}(1-|R|^2)\right] \times\exp\left[-4\varphi\left(\frac{k^3\zeta^4}{a}\right)^{1/2}\mu^{1/2}\Phi(\mu)\right]. \tag{30}$$

The first factor of Eq. (30) is the reflection coefficient of an ideally smooth concave surface, which can reach a few tens of percent for the angles of rotation $\varphi\approx\pi/2$ and the glancing angles $\alpha<\alpha_c$ (Refs. 15 and 18). The second factor in Eq. (30) allows for the reduction in the reflection coefficient due to the scattering. It is clear from Eq. (30) that in the case of a grazing beam ($\alpha=0$) the condition for an ideal concave surface is of the form $k^3\zeta^4<a/4\varphi$, whereas for glancing angles of the order of the total external reflection angle ($\alpha\sim\alpha_c$), we have $k\zeta<(2\alpha_c\varphi)^{-1/2}$.

We shall now consider an example. A mirror with surface irregularities of height 60 Å rotates a $\lambda=200$ Å x-ray beam through an angle $\varphi=\pi/2$ and the mirror can be regarded as smooth if the correlation radius is $a\gtrsim 1$ $\mu$. It should be pointed out that only the specular component is allowed for and the diffuse reflection is effectively ignored. Therefore, in reality we have to ensure a surface smoothness needed to observe rotation of x rays through large angles and these requirements may in fact be less stringent, since the diffusely scattered component is also rotated by a concave mirror.

## IV. CONCLUSIONS

We have developed a theory of total external reflection of x rays from a rough interface. The angular dependence of the reflection coefficient is governed by the permittivity of the reflecting material in the x-ray range and also by the characteristics of the surface, which are the height of the irregularities and the correlation radius.

The results allow us to explain qualitatively the phenomenon of an increase in the height of irregularities, observed on reflection at small angles, and to formulate the criteria of smoothness of the surfaces to be used in x-ray optics characterized by single and multiple reflection. A determination is also reported of the conditions which must be satisfied by a concave interface for the observation of the effects of propagation and rotation of x-ray beams through large angles.

We shall note in conclusion that the currently available surfaces with irregularities and microinclusions are unsuitable for the realization of the full physical potential of x-ray optics materials[8,20] and this limits the use of the spherical model in scientific and technical applications.

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[1)]Concave mirrors are characterized by a high reflection coefficient in a wide range of photon energies, in contrast to selective multilayer structures, which are being investigated and developed rapidly in recent years.[16,17]


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