Existence criterion and properties of deeply penetrating Rayleigh waves in crystals

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A study is made of surface Rayleigh sound waves in highly anisotropic hexagonal, tetragonal, and cubic crystals. The anisotropy of the velocity of bulk transverse vibrations in the uniaxial (tetragonal and hexagonal) crystals is described by introducing a parameter η which goes over to the familiar elastic anisotropy parameter of the cubic crystal for $C_{11} = C_{33}$. The dependence on this parameter is analyzed for the main characteristics of the surface Rayleigh waves (both the ordinary and generalized). It is shown that a necessary and sufficient condition for the existence of deeply penetrating Rayleigh waves is that this anisotropy parameter differ appreciably from unity, i.e., that the velocity of the bulk transverse vibrations in the sagittal plane be highly anisotropic. The surface waves are studied in the case of degenerate roots of the characteristic equation for the bulk vibrations. The contribution of capillary effects to the velocity and penetration depth of surface waves is taken into account, and it is pointed out that the role of these effects is greater in highly anisotropic media than in isotropic solids.

Two of the most important characteristic of surface sound waves from the standpoint of applications are their velocity and penetration depth. The possibility of varying these characteristics in an isotropic solid is very limited: the velocity of the surface wave is nearly the same as that of the bulk transverse wave, and the penetration depth is of the order of the wavelength.[1] Anisotropic media, on the other hand, have a much broader spectrum of these main characteristics of the surface waves—the penetration depth can be either of the order of the wavelength or it can be considerably larger, and the velocity can be either very close to that of a bulk transverse wave in the given direction or it can be considerably smaller. In particular, deeply penetrating and slow Rayleigh waves can propagate in highly anisotropic crystals[2–5] and in crystals near structural phase transitions[6–8] due to a softening of acoustic phonons (ferroelastic, ferroelectric, magneto-orientational, and other transitions). In other words, the bulk elastic properties of the crystals have an important influence on the main parameters of surface sound waves. In this paper we formulate a criterion for the existence of deeply penetrating Rayleigh waves and analyze the connection between the main characteristics of these waves and the bulk elastic properties of the crystals.[1)] We also investigate the influence of capillary effects on the velocity and penetration depth of surface waves in highly anisotropic crystals. We analyze the transition from the ordinary to the generalized Rayleigh waves on a change in the anisotropy parameter for the case of waves on a high-symmetry cut of a uniaxial crystal and elucidate the connection between this transition and the properties of the isofrequency surface of the bulk transverse vibrations.

1. PROPERTIES OF SURFACE RAYLEIGH WAVES IN HIGHLY ANISOTROPIC UNIAXIAL CRYSTALS

Let us consider a uniaxial (tetragonal or hexagonal) crystal with a (001) boundary plane and a wave propagation direction [100]. In such a geometry the properties of the Rayleigh waves in hexagonal and tetragonal crystals are identical, and the displacement in the waves is of the form

$$U_z=(A_1 \exp \gamma_1 kz+A_2 \exp \gamma_2 kz)\exp\{i(kx-\omega t)\},$$
$$U_x=i(A_1\Gamma_1 \exp \gamma_1 kz+A_2\Gamma_2 \exp \gamma_2 kz) \exp\{i(kx-\omega t)\}. \quad (1)$$

Here $\mathbf{U}$, k, and ω are the displacement vector, the wave number, and the frequency of the wave, the z axis is directed along the outward normal to the crystal, γ_1 and γ_2 are the eigenvalues of the biquadratic characteristic equation for the bulk vibrations

$$C_{55}C_{33}\gamma^4-\gamma^2\left[C_{33}\left(C_{11}-\rho\frac{\omega^2}{k^2}\right)+C_{55}\left(C_{55}-\rho\frac{\omega^2}{k^2}\right)\right.$$
$$\left.-(C_{13}+C_{55})^2\right]+\left(C_{11}-\rho\frac{\omega^2}{k^2}\right)\left(C_{55}-\rho\frac{\omega^2}{k^2}\right)=0, \quad (2)$$

(where C_{ik} are the moduli of elasticity and ρ is the density of the crystal), and Γ_1 and Γ_2 are the eigenvectors for the bulk equations of motion:

$$\Gamma_i=\frac{\rho(\omega/k)^2+C_{33}\gamma_i^2-C_{55}}{\gamma_i(C_{13}+C_{55})} \quad (i=1,2). \quad (3)$$

We use the following boundary conditions for $z=0$ with allowance for capillary effects[10,11]:

$$\sigma_{zz}=g\frac{\partial^2 U_z}{\partial x^2}-\rho_s\ddot{U}_z,$$
$$\sigma_{zx}=(g+h)\frac{\partial^2 U_x}{\partial x^2}-\rho_s\ddot{U}_x, \quad (4)$$

where σ_{ik} is the stress tensor, $g_{\mu\nu}=g\delta_{\mu\nu}$ is the surface-tension tensor ($\mu,\nu = 1,2$ label the coordinate axes in the tangent plane), $h_{11}=h$ is one of the excess surface moduli, and ρ_s is the excess surface mass. With allowance for boundary conditions (4), the dispersion relation for a Rayleigh wave in the nondegenerate case ($\gamma_1 \neq \gamma_2$) is of the form

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$$[C_{55}k^2-\rho\omega^2]^{1/2}[(C_{11}C_{33}-C_{13}^2)k^2-C_{33}\rho\omega^2]$$
$$-\rho\omega^2[C_{33}C_{55}(C_{11}k^2-\rho\omega^2)]^{1/2}$$
$$=\{(\rho_s\omega^2-gk^2)[C_{33}C_{55}(C_{11}k^2-\rho\omega^2)]^{1/2}$$
$$+C_{33}[\rho_s\omega^2-(g+h)k^2][C_{55}k^2-\rho\omega^2]^{1/2}\}(\gamma_1+\gamma_2), \quad (5)$$

where $(\gamma_1+\gamma_2)$ is determined from (2) with the aid of Viète's theorem. When γ_1 and γ_2 are real and different, surface wave (1) is an ordinary wave; when γ_1 and γ_2 are complex, it is a generalized wave. The transition from one kind of wave to the other occurs when $\gamma_1=\gamma_2$ (the degenerate case): the solution for the surface wave in this case is no longer of form (1) and will be discussed below. We note that in the case $C_{33}=C_{11}$, $C_{13}=C_{12}$ Eq. (5) describes a Rayleigh wave in a cubic crystal in the geometry under study (with allowance for capillary effects).

We introduce the anisotropy parameter for a uniaxial crystal

$$\eta=[(C_{11}C_{33})^{1/2}-C_{13}]/2C_{55}. \quad (6)$$

As in a cubic crystal, this parameter describes the anisotropy in the xz plane of a bulk transverse wave polarized in this plane. According to the conditions for elastic stability,[12] the parameter η can vary from zero to infinity. For $\eta\approx 1$ the crystal is slightly anisotropic, and for values of η substantially different from unity the crystal is highly anisotropic. Examples of highly anisotropic crystals are layered and chain-like compounds (graphite, GaS, GaSe, etc.), solid helium in the hexagonal close-packed phase, paratellurite TeO_2, and crystals near a proper ferroelastic transition due to a softening of one of the bulk transverse sound velocities ($RbMnCl_2$, Nb_3Sn, V_3Si, etc.).

Let us follow the evolution of the main characteristics of the Rayleigh waves on changes in the anisotropy parameter (6). In the case $\eta\gg 1$ a Rayleigh wave (in neglect of capillary effects) is of the form

$$U_x=U_0\left[i\delta^{1/2}\frac{C_{13}}{C_{33}}\exp\left\{\frac{C_{11}}{C_{33}}\delta^{1/2}kz\right\}-i\delta^{1/2}\exp\left\{\frac{kz}{\delta^{1/2}}\right\}\right]$$
$$\times\exp\{i(kx-\omega t)\},$$
$$U_z=U_0\left[\exp\left\{\frac{C_{11}}{C_{33}}\delta^{1/2}kz\right\}-\delta\frac{C_{13}}{C_{33}}\exp\left\{\frac{kz}{\delta^{1/2}}\right\}\right]\exp\{i(kx-\omega t)\}, \quad (7)$$

where

$$\delta=\frac{C_{55}C_{33}}{C_{11}C_{33}-C_{13}^2}=\frac{C_{55}}{(C_{11}C_{33})^{1/2}-C_{13}}\frac{C_{33}}{(C_{11}C_{33})^{1/2}+C_{13}}\sim\frac{1}{\eta}\ll 1. \quad (8)$$

As in an isotropic solid ($\eta\approx 1$), this wave (7) contains two components which are exponentially damped with depth in the crystal. However, while for $\eta\approx 1$ the two components have comparable amplitudes and comparable semiaxes of the polarization ellipses and penetrate into the medium to a depth of the order of the wavelength, for $\eta\gg 1$ one of the components becomes predominant, has an almost linear polarization (normal to the surface), and penetrates into the crystal to a depth considerably greater than the wavelength. In the limit $\eta\to\infty$ the surface Rayleigh wave thus approaches an SV bulk wave propagating in the same direction.

It is known that near-surface distortions (capillary effects) make it possible for a purely shear surface wave, close to an SH bulk wave, to propagate on the surface of a solid in addition to the Rayleigh wave.[13,14] In other words, a surface shear wave is extremely sensitive to capillary effects. It turns out that in highly anisotropic crystals the Rayleigh wave also exhibits a heightened sensitivity to capillary effects.

For $\eta\gg 1$ the dispersion relation for surface sound waves with allowance for capillary effects is of the form

$$\omega^2=\frac{C_{55}}{\rho}k^2\left\{1-\delta^2\frac{C_{11}}{C_{33}}\left[1-2\left(\frac{g}{C_{55}}-\frac{\rho_s}{\rho}\right)\frac{k}{\delta^{1/2}}\right]\right\}, \quad (9)$$

and the penetration depth determined by the smaller of the roots for γ in Eq. (2) is given by

$$l=\frac{1}{\gamma_1 k}=\frac{1}{k}\frac{C_{33}}{C_{11}}\frac{1}{\delta^{3/2}}\left[1-\left(\frac{g}{C_{55}}-\frac{\rho_s}{\rho}\right)\frac{k}{\delta^{1/2}}\right]^{-1}. \quad (10)$$

Analysis of (9) and (10) shows that in this case the role of capillary effects is markedly greater than in the case $\eta\approx 1$. In fact, the corrections to the frequency and penetration depth of the surface wave contain in the denominator the small quantity $C_{55}a\delta^{1/2}/g\ll 1$ (for $g\sim C_{11}a\sim C_{33}a$, where a is the interatomic distance). The presence of this enhancing factor means that in highly anisotropic crystals the domain of applicability of the methods of the linearized theory of elasticity, which does not incorporate spatial dispersion, is considerably narrower than in isotropic crystals: Instead of the restriction $ak\ll 1$ in the isotropic case, for $\eta\gg 1$ we get the condition $ak\ll(1/\eta)^{3/2}\ll 1$.

Let us now investigate the structure of the Rayleigh wave in the other limiting case of a highly anisotropic crystal: $\eta\ll 1$. In this case the roots of Eq. (2) are complex conjugates, and the generalized Rayleigh wave is of the form

$$U_z=U_0(\exp\gamma_1kz-f\exp\gamma_2kz)\exp\{i(kx-\omega t)\},$$
$$U_x=\frac{U_0}{d}(\exp\gamma_1kz+f\exp\gamma_2kz)\exp\{i(kx-\omega t)\}, \quad (11)$$
$$f=e^{-2i\varphi},\quad \operatorname{tg}\varphi=\frac{C_{11}b}{d^3}\left\{C_{55}\frac{d^2-1}{d^2+1}+\frac{C_{11}}{1+d^2}\left(1+\frac{C_{55}}{C_{11}}\right)\right\}^{-1},$$

where $\gamma_1=b\eta-(d-c\eta)i$, $\gamma_2=b\eta+(d-c\eta)i$,

$$b^2=\frac{d^4}{(1+d^2)^3}\left(\frac{C_{55}}{C_{33}}+2d^2+\frac{C_{55}}{C_{11}}\right)\left(1+\frac{C_{55}}{C_{11}}\right)-\frac{(1-d^2)^2}{(1+d^2)^2}\frac{C_{55}}{C_{33}},$$
$$c=\frac{d^3}{1+d^2}\left(1+\frac{C_{55}}{C_{11}}\right),\quad d^2=\left(\frac{C_{11}}{C_{33}}\right)^{1/2}.$$

[In the case $d=1$ wave (11) goes over to a deeply penetrating generalized Rayleigh wave in a highly anisotropic cubic crystal[4,5]]. For $\eta\to 0$ the penetration depth of the Rayleigh wave goes to infinity, and the period of the oscillations with depth goes to λ/d, where λ is the wavelength, i.e., the period of the oscillations is considerably smaller than the penetration depth, and the generalized character of the surface wave is most clearly expressed. In this limit the surface wave under study is actually a superposition of two purely bulk transverse waves polarized in the sagittal plane and propagating at angles of $\pm\theta=\operatorname{arccot}d$ to the z axis. For $\eta\ll 1$ the dispersion relation of the surface sound wave with allowance

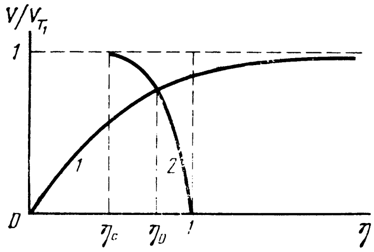

FIG. 1. Velocity of Rayleigh waves in uniaxial crystals as a function of the anisotropy parameter η of Eq. (6) (curve 1), and the solution corresponding to the degenerate case (curve 2).

for capillary effects is of the form

$$\omega^2=\frac{4d^2}{1+d^2}\,\eta\,\frac{C_{55}}{\rho}\left\{1-\Delta+\frac{bk}{2C_{55}}\left[g+\frac{(g+h)}{d^2}\right]\right\}k^2, \qquad (12)$$

where

$$\Delta=\frac{\eta d^2}{(1+d^2)^2}\left(d^4\frac{C_{55}}{C_{11}}+2d^2+\frac{C_{55}}{C_{11}}\right)\approx\eta.$$

In this case also the influence of capillary effects on the velocity (and penetration depth) of the Rayleigh wave is stronger than in the case of an isotropic solid. For studying surface waves in highly anisotropic crystals it is thus necessary to take the spatial dispersion or the discreteness of the lattice into account even in the region of comparatively long wavelengths: for $\eta\gg1$, this is necessary starting at $ak\sim\eta^{-3/2}\ll1$, and for $\eta\ll1$ starting at $ak\sim\eta\ll1$.

In both cases considered the Rayleigh wave is slow (its velocity is determined by a small parameter) and deeply penetrating. These features are due to the noticeable difference of the anisotropy parameter (6) from unity. This circumstance enables one to formulate an existence criterion for Rayleigh waves having these properties. Deeply penetrating Rayleigh waves propagate in crystals which are characterized by strong anisotropy in the sagittal plane of the velocity of bulk transverse waves polarized in this plane. This conclusion is based on consideration of the properties of Rayleigh waves with an *xz* sagittal plane in uniaxial (and cubic) crystals, but it is valid for other sagittal planes as well. For example, in the case of an *xy* plane the anisotropy parameter for the velocity of bulk transverse waves polarized in this plane is

$$\eta'=(C_{11}-C_{12})/2C_{66}. \qquad (13)$$

For tetragonal and cubic crystals this parameter can differ appreciably from unity upon the softening of one of the transverse velocities: $V_{T1}=(C_{66}/\rho)^{1/2}$ or $V_{T2}=[(C_{11}-C_{12})/2\rho]^{1/2}$. In the presence of such a softening, a Rayleigh wave (ordinary or generalized) propagating in the [100] direction on the (010) plane or in the [110] direction on the $(1\bar{1}0)$ plane will be slow and deeply penetrating. In the case of a hexagonal crystal, anisotropy parameter (13) [unlike parameter (6)] is identically equal to unity. Consequently, the velocity of a bulk transverse wave polarized in the *xy* plane is isotropic, and deeply penetrating Rayleigh waves with an *xy* sagittal plane do not arise. (The softening of the transverse velocity $V_{T1}=V_{T2}$ leads only to a slowing of the velocity of the Rayleigh wave.)

2. DEGENERATE CASE AND THE TRANSITION FROM ORDINARY TO GENERALIZED RAYLEIGH WAVES IN UNIAXIAL CRYSTALS

In the previous section we showed that in tetragonal (and cubic) and hexagonal crystals, a Rayleigh wave along the [100] direction on the same high-symmetry boundary surface (001) can, depending on anisotropy parameter (6), be either ordinary (for $\eta\gg1$) or generalized (for $\eta\ll1$). The solution [normalized to $V_{T1}=(C_{55}/\rho)^{1/2}$] of dispersion relation (5) as a function of the anisotropy parameter η is shown by curve 1 in Fig. 1. This curve describes both the ordinary and generalized Rayleigh waves in the given geometry. The transition from one kind of wave to the other occurs when $\eta=\eta_0$, corresponding to equality of the roots of characteristic equation (2). The condition $\gamma_1=\gamma_2$ (the degenerate case) leads to the following equation:

$$(\rho V^2)^2\left(\frac{1}{C_{55}}-\frac{1}{C_{33}}\right)^2$$
$$-2\rho V^2\left[\left(\frac{1}{C_{55}}+\frac{1}{C_{33}}\right)\left[\frac{C_{11}}{C_{55}}+\frac{C_{55}}{C_{33}}-\frac{(C_{13}+C_{55})^2}{C_{33}C_{55}}\right]\right.$$
$$\left.-2\left(\frac{1}{C_{33}}+\frac{C_{11}}{C_{33}C_{55}}\right)\right]$$
$$+\left[\frac{C_{11}}{C_{55}}+\frac{C_{55}}{C_{33}}-\frac{(C_{13}+C_{55})^2}{C_{33}C_{55}}-2\left(\frac{C_{11}}{C_{33}}\right)^{1/2}\right] \qquad (14)$$
$$\times\left[\frac{C_{11}}{C_{55}}+\frac{C_{55}}{C_{33}}-\frac{(C_{13}+C_{55})^2}{C_{33}C_{55}}+2\left(\frac{C_{11}}{C_{33}}\right)^{1/2}\right]=0.$$

Curve 2 in Fig. 1 describes the dependence on the anisotropy parameter η of the root (corresponding to a real value $\gamma_1=\gamma_2=\gamma$) of Eq. (14). This curve exists in the interval from $\eta=1$ to $\eta=\eta_c<1$. At the point $(C_{11}C_{33})^{1/2}-C_{13}=2C_{55}$ (at which $\eta=1$), Eq. (14) has a root of zero, and in the case

$$(C_{13}+C_{55})^2+C_{33}(C_{55}-C_{11})=0 \qquad (15)$$

(which corresponds to $\eta=\eta_c$) the root corresponds to the bulk transverse velocity $V_{T1}=(C_{55}/\rho)^{1/2}$ in the given direction. In the degenerate case the solution for a surface wave should be sought in the form

$$U_x=(A_1+A_2z)\exp\{\gamma kz-i(kx-\omega z)\},$$
$$U_z=(B_1+B_2z)\exp\{\gamma kz-i(kx-\omega t)\}. \qquad (16)$$

Using the bulk equations of motion $\rho\ddot{U}_i=\partial\sigma_{ik}/\partial x_k$ and boundary conditions (4), one can show that a nontrivial solution for a surface wave of the form (16) exists only at one point—the point of intersection of the Rayleigh-wave branch (curve 1 in Fig. 1) with the branch $\gamma_1=\gamma_2$ (curve 2). Thus for Rayleigh waves propagating on high-symmetry faces along high-symmetry directions, the transition from the ordinary to the generalized waves on a change in the anisotropy parameter occurs at the point of degeneracy of the roots of the characteristic equation for the bulk vibrations. The value η_0 corresponding to the point of the transition from the ordinary to the generalized Rayleigh waves and the value η_c corresponding to the extreme point of curve 2 in Fig. 1 are of order unity.[2)]

We shall now show that the presence of a point $\eta=\eta_c$

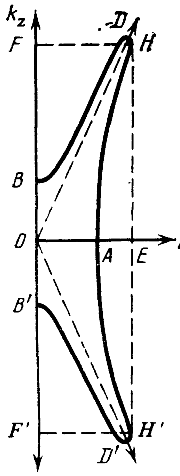

FIG. 2. Cross section of an isofrequency surface for bulk transverse vibrations polarized in the xz plane, in the limit $\eta \ll 1$.

is related to features of the bulk vibrational characteristics of the crystal. Equation (2) for the bulk vibrations can be used to study the question of whether the k_x, k_z cross section of an isofrequency surface of the bulk transverse vibrations polarized in the xz plane is convex. This cross section is onconvex with respect to the [100] direction in the given uniaxial crystals under the condition

$$(C_{13}+C_{55})^2+C_{33}(C_{55}-C_{11})>0.$$

Equality of the left-hand side to zero corresponds to the condition for the transition from a convex to a nonconvex cross section and coincides with the definition of the extreme point $\eta=\eta_c$ on curve 2 of Fig. 1 [see Eq. (15)]. Since for $\eta<\eta_c$ the corresponding cross section is nonconvex, the surface wave in the given geometry should clearly be a generalized wave. As we see from Fig. 1, the Rayleigh wave is also a generalized wave for $\eta_0<\eta<\eta_c$, i.e., nonconvexity of the corresponding cross section of the bulk transverse vibrations polarized in the sagittal plane is a sufficient condition for the existence of generalized Rayleigh waves.

The geometric interpretation of the connection between nonconvexity of the corresponding cross section of the isofrequency surface of the bulk vibrations and the main characteristics of the surface waves is particularly transparent in the case of highly anisotropic crystals. Figure 2 shows, for the case $\eta \ll 1$, the shape of the $k_x>0$ part of the cross section of an isofrequency surface of the bulk vibrations polarized in the xz plane. The lengths of the segments $OB=OB'=OA$ and $OD=OD'$ are inversely proportional to the velocities of the bulk transverse sound propagating in the given direction. In the limiting case $\eta \to 0$ the cross section is stretched out along the straight lines OD and OD', directed at an angle $\theta=\arctan(C_{33}/C_{11})^{1/4}$ to the z axis ($OD \gg OA$). Here the bulk transverse velocities are as follows: along the direction OA the velocity is

$$V_{T1}=(C_{55}/\rho)^{1/2},$$

while along the direction OD it is

$$V_{T2}=[4C_{55}\eta/\rho(1+d^2)(1+1/d^2)]^{1/2}.$$

The line HE corresponds to a surface wave of the given frequency, segment OE is equal to the wave number k of the transverse wave and is inversely proportional to its velocity, and segment OF determines the period of the oscillations of the surface wave with depth. Since in highly anisotropic crystals the separation of the surface wave from the boundary of the continuous spectrum is small (the waves are deeply penetrating), the line EH actually touches the cross section near the point D. In this case we see from Fig. 2 that $OE=OF\tan\theta$ and $OE=OD\sin\theta$, i.e., we get the following expressions for the quantity Im γ which determines the period of the oscillations (with depth) and for the velocity V_s of the surface wave:

$$\operatorname{Im}\gamma=\cot\theta=\left(\frac{C_{11}}{C_{33}}\right)^{1/4},$$

$$V_s^2=\frac{V_{T2}^2}{\sin^2\theta}=V_{T2}^2\left[1+\left(\frac{C_{11}}{C_{33}}\right)^{1/2}\right]$$
$$=\frac{4C_{55}}{\rho}\eta\left[1+\left(\frac{C_{33}}{C_{11}}\right)^{1/2}\right]^{-1}. \qquad (17)$$

The same values of the main parameters of the generalized surface wave are obtained from an analytical treatment of (11) and (12) in the limit $\eta \to 0$. In addition, Fig. 2 clearly illustrates that the surface wave in this limit is actually a superposition of two extreme[15] bulk waves, OD and OD', propagating at an angle θ to the z axis. In the other limiting case of a highly anisotropic crystal, $\eta \gg 1$, the cross section is stretched out along the coordinate axes. Here $OA \gg OD$, and segment OF, which determines the period of the oscillation with depth, goes to zero—the Rayleigh wave is ordinary.

Since in highly anisotropic crystals, both for $\eta \gg 1$ and for $\eta \ll 1$, the surface waves are deeply penetrating (unlike the $\eta \approx 1$ case), it is of interest to examine the overall picture of how the damping coefficient and the period of the oscillation with depth depend on the anisotropy parameter. Figure 3 shows how the damping constants γ_1 and γ_2 for an ordinary Rayleigh wave and $\gamma=\gamma' \pm i\gamma''$ for a generalized Rayleigh wave depend on the parameter η over its entire range of variation. As we see in Fig. 3, in both limiting cases ($\eta \ll \eta_0 \sim 1$ and $\eta \gg \eta_0$) the surface Rayleigh wave is characterized by small values of γ (or γ'), i.e., the wave is deeply penetrating. At the point $\eta=\eta_0$ of the transition from the ordinary to the generalized surface waves the penetration depth is minimum. We also note that as η decreases from η_0 to zero the increase in the penetration depth is accompanied

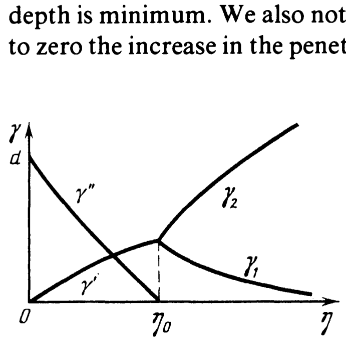

FIG. 3. Damping coefficients for Rayleigh waves versus the anisotropy parameter η, $d=(C_{11}/C_{33})^{1/4}$.

by a decrease in the period of the oscillation with depth to its minimum value of $\lambda(C_{33}/C_{11})^{1/4}$.

In an analogous way one can study a Rayleigh wave (having the same sagittal plane xz) propagating on a (100) boundary in the [001] direction. Expressions (7)–(10) for $\eta \gg 1$ and (11) and (12) for $\eta \ll 1$ carry over to such a wave if x is replaced by z. Upon such a replacement the moduli C_{13} and C_{33} remain unchanged, while C_{11} and C_{33} exchange places, i.e., the definition of anisotropy parameter (6) remains unchanged. For example, in the case of a generalized deeply penetrating Rayleigh wave for $\eta \ll 1$, we obtain the following expressions for γ'', which determines the period of the oscillation with depth (along the x axis), and for the velocity V_s of the surface wave:

$$\gamma''=\left(\frac{C_{33}}{C_{11}}\right)^{1/4}, \quad V_s^2=4\frac{C_{55}}{\rho}\eta\left[1+\left(\frac{C_{11}}{C_{33}}\right)^{1/2}\right]^{-1}. \quad (18)$$

Result (18) can also be obtained in a geometric analysis of the cross sections of an isofrequency surface (see Fig. 2) with allowance for the fact that the length of segment OF in this case is proportional to the wave number and inversely proportional to the velocity of the surface wave, and segment OE determines the period of the oscillation with depth.

CONCLUSION

We have investigated ordinary and generalized surface Rayleigh waves in highly anisotropic tetragonal and hexagonal crystals and formulated a criterion for the existence of deeply penetrating Rayleigh waves. Deeply penetrating Rayleigh waves propagate in crystals characterized by strong anisotropy in the sagittal plane of the velocity of bulk transverse waves polarized in this plane. On the basis of this criterion we have shown how the period of the oscillation with depth and the velocity of a generalized Rayleigh wave in a highly anisotropic crystal are related to features of the shape of the isofrequency surface of the bulk transverse vibrations. For uniaxial crystals we have introduced an anisotropy parameter $\eta = [(C_{11}C_{33})^{1/2} - C_{13}]/2C_{55}$ for the velocity of bulk transverse vibrations (polarized in the xz plane) and analyzed the transition from the ordinary to the generalized Rayleigh waves on a change in this parameter for waves in the [100] direction on a (001) boundary plane and in the [001] direction on a (100) plane. We have described the form of the surface wave at the point of the transition which occurs at the point of degeneracy of the roots of the characteristic equation for the bulk vibrations. We have noted an increase in the influence of capillary effects on the velocity and penetration depth of the Rayleigh waves as the anisotropy of the bulk elastic properties of the crystal becomes stronger.

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[1)]Such a connection was analyzed numerically for a series of slightly anisotropic crystals in Ref. 9.

[2)]For example, in the case of small bulk transverse velocities $C_{55} \ll C_{11}$, $(C_{11}C_{33})^{1/2} - C_{13} \ll C_{11}$ one can obtain analytically $\eta_0 = \frac{2}{9}(3 + 1/d^2)$, $\eta_c = \frac{1}{4}(2 + 1/d^2)$, where $d^2 = (C_{11}/C_{33})^{1/2}$.

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