Alternating-current-induced decay of zero-voltage states of Josephson junctions

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The effect of ac current $I_1 \cos \Omega t$ on the rate of tunneling decay of zero-voltage states of Josephson junctions was investigated in the frequency range $V \gg \Omega \gg \omega$, where V is the potential-barrier height and ω the plasma Josephson frequency. It is shown that the effective ac current amplitude that determines the change of the decay rate contains a factor $\exp(\Omega\tau_s)$, where τ_s is the characteristic below-barrier motion time. For junctions with low viscosity $\eta \ll \omega$, the contribution linear in the ac current to the argument of the tunneling exponential is calculated in the entire range of temperatures T and dc currents I_0 lower than the critical current I_c. For $I_c - I_0 \ll I_c$ it is found, in particular, that $\tau_s \approx 1.177\omega^{-1}$. The dissipative effects increase τ_s in accordance with the deceleration of the particle motion below the barrier. For strong dissipation, when $\eta \gg \omega$, it is found that $\tau_s = \eta\omega^{-2}$ at $I_c - I_0 \ll I_0$.

Zero-voltage states of Josephson junctions correspond to the minima of the junction energy $V(\varphi)$ as a function of the phase difference between the two superconductors. At sufficiently low temperatures the finite lifetime of the zero-voltage states is due to macroscopic quantum tunneling through the potential barrier.[1–8] We have previously shown[9] that for particle tunneling through a potential barrier in the presence of an alternating field $\mathscr{E} \cos \Omega t$ the effective field amplitude that determines the barrier transparency is exponentially enhanced compared with $\mathscr{E}$, so that $\mathscr{E}_{\rm eff} = \mathscr{E} \exp(\Omega\tau_s)$, where τ_s is the characteristic time of below-barrier motion. In this article we consider the simulation of tunneling decays by an alternating perturbation as applied to Josephson junctions, whose properties are determined by the critical current I_c, the capacitance C, and the shunting resistance R.

We shall distinguish below between junctions with weak dissipation ($\eta \ll \omega$) and those with strong dissipation ($\eta \gg \eta$), where $\eta = 1/RC$ is the viscosity coefficient and $\omega = (2eI_c/C)^{1/2}$ is the plasma Josephson frequency. Estimates show that at $\eta \ll \omega$ we have in order of magnitude $\tau_s \sim \omega^{-1}$, and in the opposite limiting case $\tau_s \sim \eta\omega^{-2}$. In the case of weak dissipation the argument of the tunneling exponential should be an oscillating function of the direct current through the junction, of the alternating-current frequency, and of the temperature, in accordance with the possibility of resonant absorption of ac energy in the Josephson junction.

The effect of an alternating current of the decay rate of zero-voltage states was investigated in experiment,[10] but the zero-voltage states decayed because of activation processes. Such processes were investigated theoretically at zero alternating current in Ref. 11.

1. DECAY OF ZERO-VOLTAGE STATES IN THE CASE OF WEAK DISSIPATION

Assume that an alternating current of amplitude I_1 and frequency Ω flows through the current in addition to the direct current I_0, so that

$$I(t) = I_0 + I_1 \cos \Omega t.$$

Let the alternating current be small, $I_1 \ll I_c$, and let the direct current not exceed the critical value $I_0 < I_c$. Neglecting the tunneling between neighboring minima of the potential, the junction voltage is subject only to small oscillations about the zero value. With tunneling taken into account zero-voltage states decay with lifetimes $\sim D^{-1}$, where D is the probability of tunneling through the potential barrier. In the static case D can be obtained by quantum mechanics if $\eta = 0$, and if η is arbitrary one can use the methods of the recently developed theory of quantum tunneling in the presence of dissipation.[2–4] Our aim is to generalize the indicated results to include alternating current.

The equation of motion for the phase difference φ is known,

$$\frac{d^2\varphi}{dt^2} + \eta \frac{d\varphi}{dt} + \omega^2(\cos\varphi - k_0 - k_1 \cos\Omega t) = 0, \tag{1}$$

where $k_0 \equiv I_0/I_c$ and $k_1 \equiv I_1/I_c$. For this adiabatic description to be valid, it is necessary that the ac frequency be lower than the characteristic relaxation times in the superconductor; we assume therefore that $\Omega \ll T_c$.

With dissipation neglected, Eq. (1) corresponds to the Lagrangian

$$\mathscr{L} = \frac{V}{2\omega^2}\left(\frac{d\varphi}{dt}\right)^2 + V(-\sin\varphi + k_0\varphi + k_1\varphi\cos\Omega t),$$

where $V = I_c/2e$. As before,[9] we represent the quasiclassical tunneling probability in the exponential approximation as

$$D = \exp(-A), \qquad A = -i\int_C \mathscr{L}\,dt,$$

where A is the classical action and the integration is along the contour C in the complex time plane (Fig. 1).[4] We note that in the quantum-mechanical problem the location of the contour C as $\mathrm{Re}\, t \to -\infty$ is determined by the initial energy of the particle.[9] We shall assume below that the tunneling is from a thermodynamic-equilibrium state, so that as $\mathrm{Re}\, t \to -\infty$ the contour C is at a distance $i/2T$ from the real axis.

Regarding the alternating current as a small perturbation, we write the action in the form $A = A_0 + A_1$, where A_0 is the action at $I_1 = 0$ and A_1 is linear in I_1. This expansion is

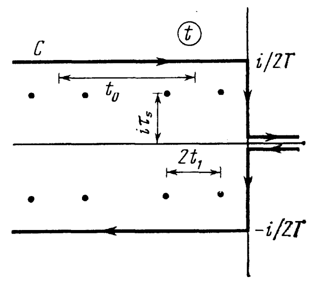

FIG. 1. Contour of integration in Eqs. (3), (13), and (14), and locations of the singular points of the $\varphi(t)$ trajectory.

valid so long as $A_1 \ll A_0$. Smallness of A_1 compared with A_0 does not mean at all that the effect of the alternating current on D is small. On the contrary, the quasiclassical approximation is applicable only if $A_1 \gg 1$, so that the alternating-current amplitude is bounded from below by the condition that D change, say, by one or two orders.

We write for A_0 and A_1 the expressions

$$A_0=-i\int_C \left[\frac{V}{2\omega^2}\left(\frac{d\varphi}{dt}\right)^2-V(\varphi)\right]dt, \qquad V(\varphi)=V(\sin\varphi-k_0\varphi), \tag{2}$$

$$A_1=-iVk_1\int_C \varphi(t)\cos\Omega t\,dt. \tag{3}$$

It is important that both A_0 and A_1 are determined by the same function $\varphi(t)$ given by the solution of (1) at $\eta=0$ $k_1=0$. The integral (3) constitutes a Fourier transform along the contour C. In the limit as $\Omega\to\infty$ the asymptote of the Fourier integral is determined by the singularities of the function $\varphi(t)$. In our case the contour C must be shifted in the direction Re $t\to-\infty$, where the alternating current can be regarded as adiabatically turned off. This means that we need consider only the $\varphi(t)$ singularities located inside C. Their distance from the real axis will determine the degree of exponential enhancement of the alternating-current amplitude.

We must therefore study the singularities of the solution of the unperturbed problem. In the absence of alternating current, the function $\varphi(t)$ is implicitly given by

$$\omega t=(V/2)^{1/2}\int [E-V(\varphi)]^{-1/2}\,d\varphi. \tag{4}$$

The type of the solution $\varphi(t)$ becomes clear from Fig. 2, where the turning points are determined by the condition $V(\varphi_{1,2,3})=E$. The energy E must be obtained from the condition that the time of below-barrier motion between points φ_2 and φ_1 is $i/2T$. We get then from (4)

$$\frac{\omega}{T}=(2V)^{1/2}\int_{\varphi_2}^{\varphi_3}[V(\varphi)-E]^{-1/2}\,d\varphi. \tag{5}$$

The last relation determines the temperature dependence of the energy $E(T)$.

The singular points of the trajectory correspond to those instants of the complex time at which $V(\varphi)$ becomes infinite. It follows from (4) that these singularities are logarithmic, so that near the singularities we have

$$\varphi_{1,2}(t)=\pm 2i\ln\{\omega[t+t_0(N+{}^1/_2)-i\tau_s\pm t_1]\},$$
$$\varphi_{3,4}(t)=\mp 2i\ln\{\omega[t+t_0(N+{}^1/_2)+i\tau_s\pm t_1]\}. \tag{6}$$

The infinite sequence of singular points $N=0,1,2,...$is the result of the periodic character of the motion, with period $t_0(E)$, in the classically allowed region in Fig. 2. We obtain the location of the quartet of singular points

$$t_{sN}=\pm t_1\pm i\tau_s-t_0(N+{}^1/_2)$$

in accordance with (4), by integrating along a path drawn from the turning point φ_3 to a certain arbitrary point φ_0 on the real axis and then upward or downward to infinity, without bypassing any singular points of the integrand other than the branch points at $\varphi=\varphi_1$, φ_2, φ_3. We obtain thus integrals of the type

$$\omega t_s=(V/2)^{1/2}\left\{\int_{\varphi_3}^{\varphi_0}[E-V(\varphi)]^{-1/2}\,d\varphi + i\int_0^\infty [E-V(\varphi_0+i\varphi)]^{-1/2}\,d\varphi\right\},$$
$$t_s\equiv t_1-t_0/2+i\tau_s. \tag{7}$$

The oscillation period in the classically allowed region is determined by the relation

$$\omega t_0=(2V)^{1/2}\int_{\varphi_1}^{\varphi_2}[E-V(\varphi)]^{-1/2}\,d\varphi. \tag{8}$$

With this information on the unperturbed trajectory, we calculate the integral (3) with the aid of (6). The result is

$$D(I_1)=D_0\exp\left\{\frac{2\pi I_1}{e\Omega}\left|\frac{\sin\Omega t_1}{\sin(\Omega t_0/2)}\right|\exp(\Omega\tau_s)\right\}, \tag{9}$$

where D_0 is the transmission coefficient in the absence of alternating current. In the derivation of (9) the frequency was assumed high compared with the reciprocal time of below-barrier motion, $\Omega\tau_s\gg 1$. This imposes the frequency limits $V\gg\Omega\gg\tau_s^{-1}$ if the quasiclassical approach is valid. Equation (9) describes the time-averaged transmission coefficient in the principal exponential approximation. The averaging eliminated the dependence on the alternating-current phase at the instant of passage through the barrier.

As regards the oscillating terms in (9), the following remark is in order. Action of even a weak alternating current can alter greatly the state of the junction compared with the unperturbed one, through resonant effects, if the frequency

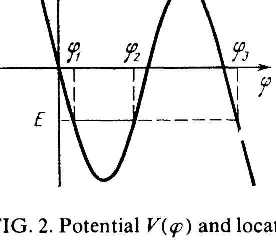

FIG. 2. Potential $V(\varphi)$ and locations of the turning points φ_1, φ_2, and φ_3.

of the alternating current coincides with one of the distances between the quantum levels. In a quasiclassical potential, the levels are locally equidistant, and since the alternating-current frequency is low compared with barrier height V, the resonance condition can be met for states having energies E that satisfy the condition

$$\Omega=2\pi N/t_0(E),\quad N=1,\ 2,\ 3\ldots,$$

where $t_0(E)$ is the period of the classical oscillations. This is just the condition under which the linear increment to the action diverges, as seen from (9). To obtain a finite answer in this case it is necessary either to take into account the nonlinearity with respect to the alternating current, in the spirit of the theory of resonance in anharmonic systems, or take a small damping into consideration.

Expression (9) is valid for low-dissipation Josephson junctions at arbitrary values of the direct current through the junction, right up to the critical one, and at any temperature right up to the value T_0 at which the macroscopic quantum tunneling goes over into the activation regime.[4] The critical temperature T_0 is obtained from relation (5) if it is assumed that the energy E tends to the top of the potential barrier,

$$2\pi T_0=\omega(1-k_0^2)^{1/4}. \qquad (10)$$

The calculation of the linear-in-I_1 increment to the tunneling exponential has thus been reduced to calculation of the zero-voltage and temperature dependences of the quantities τ_s, t_1, and t_2 that enter in (9), using the relations (5), (7), and (8). In the limiting cases when I_0 is close either to the critical current or to zero, simpler relations can be obtained.

We consider first the most vital case, when the direct current through the junction is close to the critical values, $I_c-I_0\ll I_c$, or $1-k_0\ll 1$. The potential can be regarded in this case as cubic

$$V(\varphi)=V[(1-k_0)\varphi-\varphi^3/6], \qquad (11)$$

and expression (5) reduces to an elliptic integral. For the oscillation period t_0 we obtain from (8)

$$t_0=\frac{1}{\omega}\left(\frac{2}{1-k_0}\right)^{1/4}\begin{cases}2\pi, & T\ll T_0,\\ \ln[T_0/(T_0-T)], & T_0-T\ll T_0.\end{cases}$$

Substituting the expansion (11) in Eq. (7) to find the locations of the singularities yields $\tau_s=t_1=0$; the four logarithmic singularities coalesce into one second-order pole in accordance with the results of Ref. 9. To obtain finite values of τ_s and t_1 we must substitute in (7) the exact potential $V(\varphi)$ from (2). Integrating along a contour drawn at $k_0=1$ from the turning point $\varphi_3=0$ to the point $\varphi_0=\pi$, and then vertically upward, we get

$$\omega\tau_s=\omega t_1=\frac{1}{2}\int_0^\infty\left\{\frac{[\pi^2+(z+\operatorname{sh}z)^2]^{1/2}+\pi}{\pi^2+(z+\operatorname{sh}z)^2}\right\}^{1/2}dz\approx 1.177.$$

In the considered range of currents, the values of τ_s and t_1 depend little on temperature or current. We note that τ_s and t_1 were found to be substantially smaller than $t_0\sim\omega^{-1}(1-k_0)^{-1/4}$ This is an indirect reflection of the absence of exponential amplification of the alternating current in the cubic-potential approximation.

In the other limiting case of low currents, $I_0\ll I_c$ ($k_0\ll 1$) we obtain for the oscillation period in the classically accessible region

$$t_0=\frac{1}{\omega}\begin{cases}2\pi, & T\ll T_0,\\ 4\ln[T_0/(T_0-T)], & T_0-T\ll T_0.\end{cases}$$

If the direct current is exactly zero, the potential $V(\varphi)$ becomes symmetric. For such potentials, t_1 amounts to half the time of motion, in the classically accessible region, between the turning points, while τ_s is equal to half the time of motion in the classically inaccessible region,

$$\tau_s=1/4T,\quad t_1=t_0/4. \qquad (12)$$

The time τ_s diverges at low temperature, and t_1 at temperatures close to critical. To cut off these divergences we must include a small but finite current I_0. The potential is then no longer strictly symmetric and relations (12) no longer hold. It follows in this case from (7) that

$$\tau_s(T=0)=\frac{1}{2\omega}\ln\frac{I_c}{I_0},\qquad t_1(T=T_0)=\frac{1}{2\omega}\ln\frac{I_c}{I_0}.$$

Numerical results based on relations (5), (7), and (8) are shown in Fig. 3 in the form of zero-voltage dependences of the parameters τ_s, t_1, and t_0 at different temperatures. Since the critical temperature T_0 is a function of the current [see (10)], the curves in Figs. 3a and 3b have termination points corresponding to the change from the tunneling to the activation mechanisms. The corresponding boundary curves are shown dashed. It follows from (7) that the boundary curve for τ_s coincides with the plot of $t_1(k_0)$ at $T=0$ and conversely. The solid curves in Figs. 3a and 3b intersect the dashed ones outside the borders of the figures. We note that for numerical calculations in the entire range of currents it is convenient to use in (7) $\varphi_0=3\pi/2$ when calculating τ_s and $\varphi_0=-3\pi/2$ when calculating t_1.

The sine in expression (9) appears on summation over nearby singular points. The decay probability of the zero-voltage state becomes as a result an oscillating function of the quantities Ωt_0 and Ωt_1, that depend on the direct current I_0 $k_0 I_c$, on the frequency of the alternating current, and on the temperature. At low temperature $T\ll T_0$ the position of the resonances in the denominator of (9) is determined by the frequencies of the small oscillations in the potential (2). Resonances takes place at the current values

$$I_{0n}=I_c[1-(\Omega/n\omega)^4]^{1/2},\quad n=1,\ 2\ldots.$$

We note that to obtain good resolution of the resonances in (9), the junction must have high W, and the ratio Ω/ω must not be too large, otherwise the resonances will be close to one another and the picture becomes smeared out even at relatively weak dissipation. If, on the contrary, we are interested in the exponential amplification effect, the ratio Ω/ω must be chosen as large is possible at the limit $\Omega\lesssim V$ of the validity of the classical approach

2. DECAY OF ZERO-VOLTAGE STATES IN THE CASE OF STRONG DISSIPATION

In the case of a Josephson junction with finite dissipation, Eq. (1) contains a relaxation term $\eta d\varphi/dt$, and the

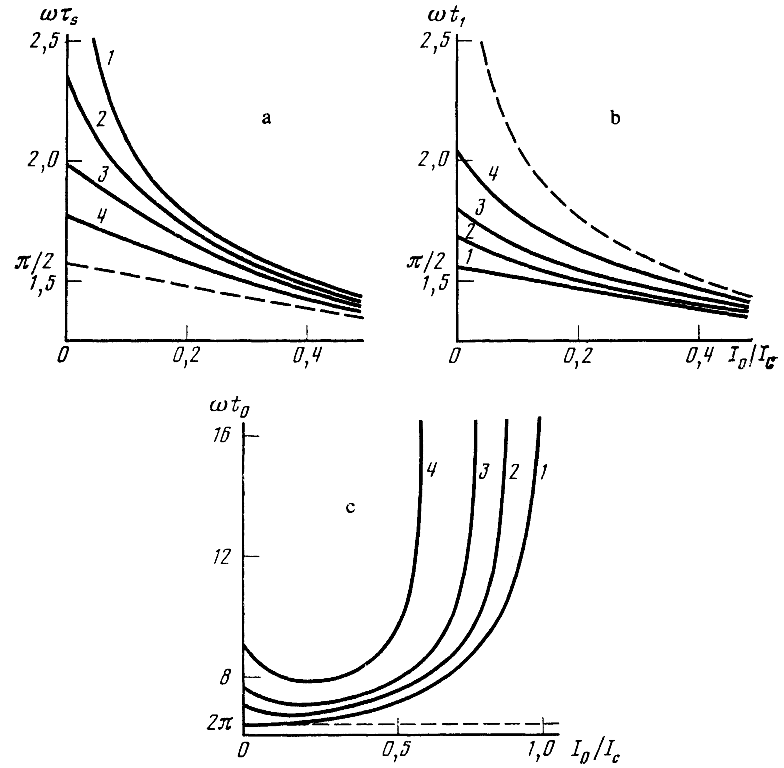

FIG. 3. Zero-voltage plots of $\omega\tau_s$, (a), ωt_1, (b), ωt_0, (c) at different values of the parameter $T^*\equiv 2\pi T/\omega$: 1—0; 2—0.7; 3—0.8; 4—0.9.

Lagrangian formalism cannot be directly applied. We use therefore the theory, developed in Refs. 2–4, of quantum tunneling in the presence of friction. It follows from these references that the dynamics of a Josephson junction is equivalent in the quasiclassical limit to a problem with effective action

$$A=-iV\int_C\left\{\frac{1}{2\omega^2}\left(\frac{d\varphi}{dt}\right)^2-(\sin\varphi-k_0\varphi-k_1\varphi\cos\Omega t)+\frac{4i\pi\eta T^2}{\omega^2}\int_C\frac{dt_1}{\operatorname{sh}^2[\pi T(t_1-t)]}\sin^2\frac{\varphi(t)-\varphi(t_1)}{4}\right\}dt,\quad(13)$$

where the notation is the same as in (1), and the contour C is shown in Fig. 1. In the absence of alternating current the interval along the horizontal sections of the contour vanishes. This relatively simple form of the effective action corresponds, as does also Eq. (1), to the adiabatic situation $\Omega\ll T_c$.

Varying the action (13), we obtain the equation of motion in complex time:

$$\frac{d^2\varphi}{dt^2}+\omega^2(\cos\varphi-k_0-k_1\cos\Omega t)-2\pi i\eta T^2\int_C\frac{dt_1}{\operatorname{sh}^2[\pi T(t_1-t)]}\sin\frac{\varphi(t)-\varphi(t_1)}{2}=0,\quad(14)$$

where the principal value of the integral is understood.

It is possible to transform from the contour C in the t_1 plane to a contour $\tilde{C}$ that differs from C only in that in the vicinity of the point $t_1=t$ it is a small semicircle located inside the contour C. It is then necessary to add a compensating half-residue to (14). The contour C has the advantage that it can be freely displaced when the position of t is fixed. As a result, Eq. (14) takes the form

$$\frac{d^2\varphi}{dt^2}+\eta\frac{d\varphi}{dt}+\omega^2(\cos\varphi-k_0-k_1\cos\Omega t)-2\pi i\eta T^2\int_{\tilde{C}}\frac{dt_1}{\operatorname{sh}^2[\pi T(t_1-t)]}\sin\frac{\varphi(t)-\varphi(t_1)}{2}=0.\quad(15)$$

At large real $t-t_1$ the integrand is small and the integral can be neglected so that at real t Eq. (15) coincides with (1).

Just as in the preceding section, we seek a correction, linear in the alternating current, to the action by using Eq. (3) in which the unperturbed trajectory $\varphi(t)$ must be substituted. We consider below the case of strong friction $\eta\gg\omega$ and of a current close to critical $I_c-I_0\ll I_c$ $(1-k_0\ll 1)$. Under these conditions the term with the second derivative can be omitted, and we can use for the potential the expansion (11), so that Eq. (15), in terms of the imaginary time it takes the form

$$\omega^2\left(\frac{\varphi^2}{2}-1+k_0\right)+\eta T\int_{-1/2T}^{1/2T}\operatorname{ctg}[\pi T(\tau_1-\tau)]\frac{d\varphi}{d\tau_1}d\tau_1=0\quad(16)$$

the solution of which was obtained by Larkin and Ovchinnikov,[4]

$$\varphi(\tau)=[2(1-k_0)]^{1/2}\left\{\frac{T^2}{T_0}\frac{2}{T_0-(T_0^2-T^2)^{1/2}\cos(2\pi T\tau)}-1\right\},\quad(17)$$

where

$$T_0\equiv\omega^2(\pi\eta)^{-1}[(1-k_0)/2]^{1/2}.$$

According to (3), exponential amplification takes place if the singular points of the unperturbed solutions do not lie on the real axis of the time t. It follows from (17), however, that for a cubic potential the singularity of the trajectory lies exactly on the real axis, as in the previously considered case of weak dissipation. This means that to find τ_s it is necessary, as before, to use the exact form of the potential.

Returning to Eq. (14), we delete its term with the second derivative, as in the transformation to (16), but do not regard φ as small. It followed from the preceding section that τ_s at $I_c - I_0 \ll I_c$ is small compared with T_0^{-1} in terms of the parameter $(1-k_0)^{1/4} \ll 1$. Assuming that the same property is preserved also at $\eta \gg \omega$, we suggest that the significant values of t are those for which $|t| \ll T_0^{-1}$. The argument of the hyperbolic sine in Eq. (14) can then be regarded as small, so that when k_0 is replaced by unity Eq. (14) takes the form

$$\eta \frac{d\varphi}{dt} + \omega^2 (\cos\varphi - 1) - \frac{2i\eta}{\pi} \int_{\tilde{C}} \frac{dt_1}{(t_1 - t)^2} \sin\frac{\varphi(t) - \varphi(t_1)}{2} = 0. \tag{18}$$

This equation is satisfied by the function

$$\varphi = i \ln \frac{t - i\tau_s}{t + i\tau_s}, \qquad \tau_s = \frac{\eta}{\omega^2} = \frac{[2(1-k_0)]^{1/4}}{2\pi T_0}. \tag{19}$$

The contour $\tilde{C}$ contracts then into a double vertical section from $-i\tau_s$ to $i\tau_s$.

We note that the reason why we can solve (18) exactly is that when the contour C is shifted in it to infinity, the integral over the contour tends to zero because of the simplified form of the integral kernel in (18) compared with (14). Bypassing the singular point $t_1 = t$ reverses then the sign of $d\varphi/dt$, so that (18) is equivalent to a relaxation equation with the time reversed:

$$-\eta \frac{d\varphi}{dt} + \omega^2 (\cos\varphi - 1) = 0.$$

At $t \gg \tau_s$ we obtain from Eq. (19) the pole part of the solution (17). It can also be seen that $\tau_s \ll T_0^{-1}$, and the parameter has the meaning of the time that the particle remains below the barrier. These results are valid so long as $T \ll T_0$.

Substitution of (19) in (3) yields

$$D(I_1) = D_0 \exp\left\{ \frac{2\pi I_1}{e\Omega} \operatorname{sh} \frac{\Omega\eta}{\omega^2} \right\}. \tag{20}$$

It can be seen that at $\eta\Omega \gg \omega^2$ we obtain exponential amplification, while at $\eta\Omega \ll \omega^2$ the argument of the exponential in (20) coincides with the variation of the static action with respect to the current, since, according to Ref. 4,

$$A_0 = \frac{4\pi V \eta (1-k_0)}{\omega^2} \left[1 - \frac{T^2}{3T_0^2} \right].$$

Evidently, in the case of strong friction the quantity $\tau_s = \eta\omega^{-2}$ greatly exceeds its value $\sim \omega^{-1}$ in the nondissipative limit. It is therefore better to study the effect of exponential amplification in low-Q junctions. It must be recognized here that the quantum regime is realized at $T < T_0$, while T_0 decreases substantially in the strong-dissipation limit.

3. CONCLUSION

A common feature of the effects considered is the strong influence of the alternating current flowing through the junction on the decay probability of a macroscopically coherent Josephson state. The physical reason for this is that during the imaginary time of motion below barrier the oscillating current acting on the system is transformed into an exponentially increasing one.

Let us discuss the order of magnitude of the effects. The formal region of validity of the derived expressions is the interval $V \gg \Omega \gg \omega$, where V is the height of the quasiclassical barrier and ω is the characteristic frequency of the motion within it. The effect is largest if the frequency of the alternating current is a maximum, $\Omega \sim V$. In the absence of alternating current the transmission coefficient $D_0 \propto \exp(-A_0)$, where $A_0 \sim V/\omega$. Since $\tau_s \sim \omega^{-1}$, the amplifying exponential of the amplitude of the alternating current can be of the order of $D_0^{-\alpha}$, where $\alpha \sim 1$. Therefore in the case of a low-dissipation Josephson junction the decay probability of a zero-voltage state, disregarding resonance effects, can be represented in the form of the following schematic expression that gives a general idea of the maximum stimulation of tunneling processes by a weak alternating current:

$$D \sim D_0 \exp\left(\frac{\beta I_1}{I_c} D_0^{-\alpha} \right), \qquad \beta \sim 1.$$

Notwithstanding the difficulty of estimating α, whose order of magnitude at $\Omega \sim V$ is unity, this expression reflects a general tendency, viz., the less probable the tunneling in the static case, the more effective the stimulating action of the alternating current.

The characteristic parameters obtained experimentally[6] for a niobium Josephson junction were of the order of

$$\omega \sim 10^{10}\ \text{s}^{-1}, \quad V \sim 10^{12}\ \text{s}^{-1}, \quad \eta/\omega \sim 10^{-1} - 10^{-2},$$

and the temperature could be chosen lower than the plasma frequency. The junctions realized under these conditions are those with weak dissipation, discussed in Sec. 1, and the effects indicated there can be observed by passing through the junction an alternating current of frequency $\Omega \sim 10^{11} - 10^{12}$ s^{-1}.

[1] Yu. M. Ivanchenko, Pis'ma Zh. Eksp. Teor. Fiz. **6**, 879 (1967) [JETP Lett. **6**, 313 (1967)].

[2] A. O. Caldeira and A. J. Leggett, Phys. Rev. Lett. **46**, 211 (1981).

[3] V. Ambegoakar, U. Eckern, and G. Schon, *ibid.* **48**, 1745 (1982).

[4] A. I. Larkin and Yu. N. Ovchinnikov, Phys. Rev. **B28**, 6281 (1983).

[5] L. D. Jackel, J. P. Gordon, E. L. Hu, *et al.* Phys. Rev. Lett. **47**, 697 (1981).

[6] R. F. Voss and R. A. Webb, *ibid.* **47**, 265 (1981).

[7] R. H. Koch, D. J. Van Harlingen, and J. Clarke *ibid.* **47**, 1216 (1981).

[8] K. K. Likharev, Usp. Fiz. Nauk **139**, 169 (1983) [Sov. Phys. Usp. **26**, 87 (1983)].

[9] B. I. Ivlev and V. I. Mel'nikov, Pis'ma Zh. Eksp. Teor. Fiz. **41**, 116 (1985) [JETP Lett. **41**, 142 (1985)]; Phys. Rev. Lett. **55**, 1614 (1985).

[10] M. H. Devoret, J. M. Martinis, D. Esteve, and J. Clarke, Phys. Rev. Lett. **53**, 1260 (1984).

[11] V. I. Mel'nikov, Zh. Eksp. Teor. Fiz. **88**, 1429 (1985) [Sov. Phys. JETP **61**, 855 (1985)].

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