Coherent effects in fermion emission

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The coherent effects that arise when fermions are emitted by two sources are considered. It is shown that these effects are similar to the effects exhibited in photon emission by excited atoms. The feasibility of experimental observation of these effects is discussed.

1. The purpose of the present paper is to present a quantum-mechanical description of fermion (neutrino, neutron, etc.) emission by a system of identical nuclei. As will be seen below, a rigorous analysis of this problem yields the prediction that coherent effects exist similar to those first investigated by Dicke[1] and Feynman *et al.*[2] for bosons (photons) emitted by a system of excited atoms. The evolution in time of an atomic ensemble has been studied by Ernst and Stehle[3] by means of a quantum-mechanical description obtained with the aid of the Weisskopf-Wigner method.[4] It is shown in these papers that the phase coherence of the wave functions of atoms spontaneously and simultaneously emitting photons results in a significant change in the temporal and angular characteristics of the radiation (superradiant radiation, photon echo, etc.). Below we shall show that coherent transition effects occur also in the case when fermions are emitted by a system of identical nuclei.

2. Let us, for the sake of simplicity, consider a system of two identical atoms that at time $t = 0$ are in the same excited state. The term "excited state" needs in the present case a special explanation. An excited state is that state of the system of electrons and nuclei from which the system can emit a fermion and go into another state. For example, an atom in the process of K capture emits a neutrino: the excited state in this case is the state preceding the capture and the emission of the neutrino; the ground state, the state of the atom after the emission of the neutrino. Similarly, we can define these states for the process of neutron emission from some specific level of the atomic nucleus.

We can assume that the atoms are simultaneously in the excited state if this state is produced in accordance with the uncertainty principle over a period of time $\Delta t \sim \hbar/2\Gamma$, where Γ is the width of the initial state of the isolated atom.

In order to avoid unwieldy expressions below, we shall ignore the delay in the emission of the particles. This is justified when the condition $R/v \lesssim \hbar/\Gamma$, where R is the distance between the atoms and v is the velocity of the emitted particles, is fulfilled.

3. Let us, for definiteness, consider the process of emission of K-capture neutrinos by two identical atoms located at points with coordinates $\mathbf{r}_1$ and $\mathbf{r}_2$ [note at once that similar results and expressions can be obtained for the processes of emission of neutrons by excited nuclei (see Sec. 4)]. Let us write the wave function of the system of two atoms in the initial excited state in the form

$$\psi(t) = A\varphi_1^*\varphi_2^* + \sum_{\mathbf{q}\sigma} C^{(1)}_{\mathbf{q},\sigma}(t)\,\varphi_1\varphi_2^*|\mathbf{q},\sigma\rangle + \sum_{\mathbf{q}\sigma} c^{(2)}_{\mathbf{q},\sigma}(t)\,\varphi_1^*\varphi_2|\mathbf{q},\sigma\rangle + \sum_{\mathbf{q}\sigma\mathbf{q}'\sigma'} C_{\mathbf{q}\sigma\mathbf{q}'\sigma'}(t)\,\varphi_1\varphi_2|\mathbf{q},\sigma;\mathbf{q}',\sigma'\rangle, \tag{1}$$

Here φ_1^* is the wave function of the atom located at the point $\mathbf{r}_1$ before the K capture; φ_2^* is the wave function of the second atom at the point $\mathbf{r}_2$ in the same state; φ_1 and φ_2 are the wave functions of the atoms after the K capture and neutrino emission;

$$|\mathbf{q},\sigma\rangle = a_{\mathbf{q}\sigma}^+|0\rangle, \quad |\mathbf{q},\sigma;\mathbf{q}',\sigma'\rangle = 2^{-1/2}(a_{\mathbf{q}\sigma}^+ a_{\mathbf{q}'\sigma'}^+)|0\rangle,$$

where $|0\rangle$ is the neutrino vacuum and $a^+_{\mathbf{q}\sigma}$ and $a_{\mathbf{q}\sigma}$ are the creation and annihilation operators for a neutrino with wave vector $\mathbf{q}$ and spin component σ ($\hbar = c = 1$). Let us write the interaction describing the K capture in the two-atom system in the form

$$V = \sum_{j=1,2}\sum_{\mathbf{q}\sigma}(H_{\mathbf{q}\sigma}^* e^{-i\mathbf{q}\mathbf{r}_j} a_{\mathbf{q}\sigma}^+ + \text{c.c.}), \tag{2}$$

where $H^*_{\mathbf{q}\sigma}$ is the matrix element corresponding to the K capture and the first summation is over the two atoms. Using Heitler's method,[5] we can obtain for the Fourier transforms of the coefficients in Eq. (1) the system of equations

$$(\omega - 2E_1 + i\varepsilon)A(\omega) = 1 + \sum_{\mathbf{q}\sigma} H_{\mathbf{q}\sigma} e^{i\mathbf{q}\mathbf{r}_1} C^{(1)}_{\mathbf{q}\sigma}(\omega) + \sum_{\mathbf{q}'\sigma} H_{\mathbf{q}'\sigma} e^{i\mathbf{q}'\mathbf{r}_2} C^{(2)}_{\mathbf{q}'\sigma}(\omega),$$

$$(\omega - E_1 - E_{\mathbf{q}\sigma} + i\gamma/2) C^{(1)}_{\mathbf{q}\sigma}(\omega) = H_{\mathbf{q}\sigma}^* e^{-i\mathbf{q}\mathbf{r}_1} A(\omega) + \sum_{\mathbf{q}'\sigma'} H_{\mathbf{q}'\sigma'} e^{i\mathbf{q}\mathbf{r}_2} C_{\mathbf{q}\sigma,\mathbf{q}'\sigma'}(\omega),$$

$$(\omega - E_1 - E_{\mathbf{q}\sigma} + i\gamma/2) C^{(2)}_{\mathbf{q}\sigma}(\omega) = H_{\mathbf{q}\sigma}^* e^{-i\mathbf{q}\mathbf{r}_2} A(\omega) + \sum_{\mathbf{q}'\sigma'} H_{\mathbf{q}'\sigma'} e^{i\mathbf{q}\mathbf{r}_1} C_{\mathbf{q}\sigma,\mathbf{q}'\sigma'}(\omega), \tag{3}$$

$$(\omega - E_{\mathbf{q}\sigma} - E_{\mathbf{q}'\sigma'} - i\gamma) C_{\mathbf{q}\sigma,\mathbf{q}'\sigma'}(\omega) = H^*_{\mathbf{q}'\sigma'} e^{-i\mathbf{q}'\mathbf{r}_1} C^{(2)}_{\mathbf{q}\sigma}(\omega) - H_{\mathbf{q}\sigma}^* e^{-i\mathbf{q}\mathbf{r}_1} C^{(2)}_{\mathbf{q}'\sigma'}(\omega) + H^*_{\mathbf{q}'\sigma'} e^{-i\mathbf{q}'\mathbf{r}_2} C^{(1)}_{\mathbf{q}\sigma}(\omega) - H_{\mathbf{q}\sigma}^* e^{-i\mathbf{q}\mathbf{r}_2} C^{(1)}_{\mathbf{q}'\sigma'}(\omega),$$

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where E_1 is the total energy released as a result of the electron capture, $E_{\mathbf{q}\sigma}$ is the neutrino energy, and γ is the energy width of the state of the atom after the K-electron capture. The system (3) is entirely similar to the system of equations obtained in Ref. 4, where photon emission by a system of two atoms is considered, with one very important exception: the last equation for the amplitude $C_{\mathbf{q}'\sigma',\mathbf{q}\sigma}(\omega)$ describing the state with two fermions $\mathbf{q}'\sigma'$ and $\mathbf{q}\sigma$ differs significantly from the corresponding equation with two photons. For the system with two photons all four terms in the analogous equation have the same sign, whereas for the system with two fermions the terms in the equation that correspond to the interchange of $\mathbf{q}\sigma$ and $\mathbf{q}'\sigma'$ have opposite signs. This difference is a consequence of the fact that the statistics for bosons and fermions are different. The solution to the system of equations (3) can be written in the form

$$A(\omega)=(\omega-2E_1+i\Gamma_0)^{-1},$$

$$C_{\mathbf{q}\sigma}^{(1)}(\omega)=\frac{H_{\mathbf{q}\sigma}{}^{*}e^{-i\mathbf{q}\mathbf{r}_1}}{\omega-E_1-E_{\mathbf{q}\sigma}+i\gamma/2}\left[\alpha_{\mathbf{q}\sigma}^{+}(\omega)+e^{i\mathbf{q}(\mathbf{r}_1-\mathbf{r}_2)}\alpha_{\mathbf{q}\sigma}^{-}(\omega)\right],$$

$$C_{\mathbf{q}\sigma}^{(2)}(\omega)=\frac{H_{\mathbf{q}\sigma}{}^{*}e^{-i\mathbf{q}\mathbf{r}_2}}{\omega-E_1-E_{\mathbf{q}\sigma}+i\gamma/2}\left[\alpha_{\mathbf{q}\sigma}^{+}(\omega)+e^{-i\mathbf{q}(\mathbf{r}_1-\mathbf{r}_2)}\alpha_{\mathbf{q}\sigma}^{-}(\omega)\right],$$

$$C_{\mathbf{q}'\sigma',\mathbf{q}\sigma}(\omega)$$

$$=\frac{H_{\mathbf{q}\sigma}{}^{*}H_{\mathbf{q}'\sigma'}}{\omega-E_{\mathbf{q}\sigma}-E_{\mathbf{q}'\sigma'}+i\gamma}\left[\beta_{\mathbf{q}'\sigma',\mathbf{q}\sigma}^{+}(\omega)\left(e^{-i\mathbf{q}\mathbf{r}_2-i\mathbf{q}'\mathbf{r}_1}+e^{-i\mathbf{q}\mathbf{r}_1-i\mathbf{q}'\mathbf{r}_2}\right)\right.$$

$$\left.+\beta_{\mathbf{q}'\sigma',\mathbf{q}\sigma}^{-}(\omega)\left(e^{-i\mathbf{q}\mathbf{r}_1-i\mathbf{q}'\mathbf{r}_1}+e^{-i\mathbf{q}\mathbf{r}_2-i\mathbf{q}'\mathbf{r}_2}\right)\right], \tag{4}$$

where

$$\alpha_{\mathbf{q}\sigma}^{+}(\omega)$$

$$=A(\omega)\frac{(\omega-E_{\mathbf{q}\sigma}-E_1+i\gamma/2)\left[\omega-E_{\mathbf{q}\sigma}-E_1+i(\Gamma_0+\gamma)/2\right]}{\left[\omega-E_1-E_{\mathbf{q}\sigma}+i(\Gamma_0+\gamma)/2\right]^2+\Gamma_{12}{}^2/4},$$

$$\alpha_{\mathbf{q}\sigma}^{-}(\omega)$$

$$=A(\omega)\left(-\frac{i\Gamma_{12}}{2}\right)\frac{(\omega-E_{\mathbf{q}\sigma}-E_1+i\gamma/2)}{\left[\omega-E_1-E_{\mathbf{q}\sigma}+i(\Gamma_0+\gamma)/2\right]^2+\Gamma_{12}{}^2/4},$$

$$\beta_{\mathbf{q}\sigma,\mathbf{q}'\sigma'}^{+}=\frac{\alpha_{\mathbf{q}\sigma}^{+}(\omega)}{\omega-E_1-E_{\mathbf{q}\sigma}+i\gamma/2}-\frac{\alpha_{\mathbf{q}'\sigma'}^{+}(\omega)}{\omega-E_1-E_{\mathbf{q}'\sigma'}+i\gamma/2}, \tag{5}$$

$$\beta_{\mathbf{q}\sigma,\mathbf{q}'\sigma'}^{-}=\frac{\alpha_{\mathbf{q}\sigma}^{-}(\omega)}{\omega-E_1-E_{\mathbf{q}\sigma}+i\gamma/2}-\frac{\alpha_{\mathbf{q}'\sigma'}^{-}(\omega)}{\omega-E_1-E_{\mathbf{q}'\sigma'}+i\gamma/2}.$$

Here

$$\Gamma_0=2\sum_{\mathbf{q}\sigma}|H_{\mathbf{q}\sigma}|^2\xi(\omega-E_{\mathbf{q}\sigma}),$$

$$\Gamma_{12}=2\sum_{\mathbf{q}\sigma}|H_{\mathbf{q}\sigma}|^2e^{i\mathbf{q}(\mathbf{r}_1-\mathbf{r}_2)}\xi(\omega-E_{\mathbf{q}\sigma}).$$

By definition,[5]

$$\xi(x)=\mathscr{P}/x-i\pi\delta(x),\qquad x\xi(x)=1.$$

The probability for simultaneous emission of two neutrinos following the capture of an electron

$$W\sim|C_{\mathbf{q}'\sigma',\mathbf{q}\sigma}(\omega=E_{\mathbf{q}\sigma}+E_{\mathbf{q}'\sigma'})|^2.$$

Let us consider, without loss of generality, the two limiting cases: $qR\ll1$ and $qR\gg1$, where $R=|\mathbf{r}_1-\mathbf{r}_2|$. In the first case $\Gamma_{12}\approx\Gamma_0$, and, using (5), we obtain from the system (4) the following expression for the probability:

$$W\sim\frac{|H_{\mathbf{q}\sigma}|^2|H_{\mathbf{q}'\sigma'}|^2}{(x+y)^2+(\Gamma_0+\gamma)^2}\cdot\frac{x^2y^2+(\Gamma_0+\gamma)^2[x^2+y^2+(\Gamma_0+\gamma)^2]}{[y^2+(\Gamma_0+\gamma)^2]^2[x^2+(\Gamma_0+\gamma)^2]^2}$$

$$\times(x-y)^2, \tag{6}$$

where $x=E_{\mathbf{q}\sigma}-E_1$ and $y=E_{\mathbf{q}'\sigma'}-E_1$. As was to be expected, in this case, when the wavelength of the emitted particles is much greater than the distance between the emitters, the probability for simultaneous emission of two neutrinos does not depend on the angle between their momenta $\mathbf{q}$ and $\mathbf{q}'$ and their orientation relative to the vector $\mathbf{R}=\mathbf{r}_1-\mathbf{r}_2$. An exception is the case when $\mathbf{q}=\mathbf{q}'$, for which, as follows from the expression (6), $W=0$, in accord with the Pauli principle.

In the second limiting case $qR\gg1$, we can show that $\Gamma_{12}\sim\Gamma_0(qR)^{-2}$, and therefore we can, to within terms of order $(qR)^{-2}$, neglect Γ_{12} in comparison with Γ_0. Then for the probability of emission of two neutrinos in the state with $\sigma=\sigma'$ we obtain the expression

$$W\sim\frac{|H_{\mathbf{q}\sigma}|^2|H_{\mathbf{q}'\sigma}|^2}{(x+y)^2+(\Gamma_0+\gamma)^2}\left\{\left[xy-\left(\frac{\Gamma_0+\gamma}{2}\right)^2\right]^2\right.$$

$$\left.+\left(\frac{\Gamma_0+\gamma}{2}\right)^2\left[y^2-\left(\frac{\Gamma_0+\gamma}{2}\right)^2\right]\right\}$$

$$\times\left\{\left[x^2+\left(\frac{\Gamma_0+\gamma}{2}\right)^2\right]^2\left[y^2+\left(\frac{\Gamma_0+\gamma}{2}\right)^2\right]^2\right\}^{-1}$$

$$\times(x-y)^2[1+\cos(\mathbf{q}-\mathbf{q}')(\mathbf{r}_1-\mathbf{r}_2)]. \tag{7}$$

It follows from the expression (7) that in this case the value of the probability for emission of two fermions depends essentially on the direction of their emission and the mutual orientation of the emitters. The probability is greatest when

$$(\mathbf{q}-\mathbf{q}')\mathbf{R}=2\pi n,\quad n=1,2,3,\ldots, \tag{8}$$

and zero for

$$(\mathbf{q}-\mathbf{q}')\mathbf{R}=\pi(2n+1),\quad n=0,1,2,\ldots. \tag{9}$$

Notice that the condition (8) starts from $n=1$, since for $n=0$ we have $\mathbf{q}=\mathbf{q}'$, and the probability (7) vanishes because the factor $(x-y)^2=0$ (the Pauli principle). As shown in Ref. 4, a very similar situation obtains in the $qR\gg1$ limit in the case of a system that simultaneously emits two photons. The probability for emission of two photons has the form[4]

$$R_{\mathbf{k}_1\mathbf{k}_2}\sim[1+\cos(\mathbf{k}_2-\mathbf{k}_1)(\mathbf{r}_1-\mathbf{r}_2)]f(x,y), \tag{10}$$

where $\mathbf{k}_1$ and $\mathbf{k}_2$ are the wave vectors of the photons and $f(x,y)$ is a function that depends on the emitted-photon energy, and satisfies $f(x = y) \neq 0$, in contrast to the analogous function in (7); therefore, the condition, analogous to (8), for the probability to have its maximum value in the case of photons includes the $n = 0$ case.

Thus, in the case of a system of two atoms that simultaneously emit two neutrinos following an electron capture coherent effects occur similar to those that are exhibited in the case of two-photon emission by a system of two excited atoms. Here, as in the case of photon emission by atoms, the nuclei can be considered to be identical if at the initial moment of time they are in a definite spin state. The angular distribution of the particles emitted by each nucleus is determined by the multipole order of the transition from the excited to the ground state, and determines, together with the conditions (8) and (9), the angular directivity of the coherent emission of two particles. For example, for the K-capture reaction

$$_4^7Be \xrightarrow{K} {}_3^7Li$$

(the energy release $E_1 \approx 0.864$ MeV) the position of the first peak in the angular distribution in the case of two-neutrino emission ($R \approx 4$ Å) corresponds to the coherence angle $\vartheta \approx 9°$ (ϑ is the angle between $\mathbf{q}$ and $\mathbf{q}'$). Notice that the angle ϑ depends on the neutrino mass, it is not difficult to show that the coherence angle, given by the condition (8), between the momenta of the two neutrinos for $m_\nu \neq 0$ differs by the amount $\Delta\vartheta = m_\nu^2/E^2$ (if $m_\nu < E$) from the corresponding angle for the $m_\nu = 0$ case. Furthermore, the two-neutrino emission probabilities for the $m_\nu = 0$ and $m_\nu \neq 0$ cases are also different. Thus, for the first ($n = 1$) peak, setting $x \sim y \sim \Gamma_0 + \gamma$, we obtain the estimate

$$W(m_\nu=0)/W(m_\nu\neq 0) \sim m_\nu/40(\Gamma_0+\gamma). \qquad (11)$$

4. Let us consider another example of fermion decay, namely, the emission of neutrons from a compound-nucleus state, in the course of which coherent effects similar to those discussed above should also be observed. We note that the detection of the correlated-emission effect is more attractive in the case of neutrons than in the case of neutrinos, since the cross section for interaction of neutrons with nuclei is many orders of magnitude greater than the cross section for the corresponding processes involving the neutrino. But in the case of neutron decay it is quite difficult to simultaneously produce two nuclei in an excited state at points so close to each other that the effect in question is not affected by retardation. This is due to the fact that the lifetimes of the excited states of a compound nucleus are very short ($\sim 10^{-14}$ sec) and the velocities v_n of neutrons with resonance energies are very low, $v_n \ll c$.

Let us recall once more the conditions necessary for the observation of correlated neutron emission: a) the time Δt required to produce a pair of excited nuclei should be shorter than, or of the order of, the lifetime $\hbar/\Gamma$ of the excited state:

$$\Delta t \leqslant \hbar/\Gamma = 0{,}66 \cdot 10^{-15}/\Gamma, \qquad (12)$$

where Γ is the width of the excited state in eV and Δt is the time in seconds; b) the retardation effects will be insignificant if a neutron emitted from one excited nucleus with velocity v_n can get to the other excited nucleus located at a distance Δx from the first nucleus in a time $\Delta t \approx \hbar/\Gamma$, i.e., if

$$\Delta x \leqslant v_n \Delta t \approx 9 \cdot 10^{-10} E^{1/2}/\Gamma, \qquad (13)$$

where E is the neutron energy in eV.

We can, when these two conditions are fulfilled, expect that the correlated-emission effects will, as in the case of the neutrino, be described by formulas similar to (6)–(9), and that the probability for simultaneous emission of two neutrons in the direction given by the condition (8) will be twice that for emission in any other direction. Thus, the simultaneous detection of two neutrons in the solid angle $\theta \approx 4\pi/2k\Delta x$, where k is the modulus of the neutron wave vector, will constitute the detection of the correlated-emission effect.

Let us proceed to consider some possible ways of setting up experiments on correlated neutron emission and to estimate the orders of magnitude of the quantities required for the detection of the effects of the neutron fluxes.

Resonance neutron-beam scattering, in which two neutrons occasionally get captured by two close nuclei within a time interval on the order of the lifetime of the resonance level, is a natural method for observing correlated neutron emission. In such an experimental setup the effect will intensify as the volume of the target is increased at fixed density. But then there is also a rapid increase in the background noise incident to the detection within a time interval equal to the apparatus resolving time T of two neutrons that have undergone scattering in which at least one of the conditions (12) and (13) is violated. If we choose the target volume so that the effect and the background noise have equal intensities, then we obtain the following expression for the number of useful events per second:

$$N = 5{,}8 \cdot 10^{-76} \frac{E}{\Gamma^6} \frac{N_0^2(E)}{T}, \qquad (14)$$

where $N_0(E)$ is the resonance-neutron flux density in the beam. In deriving the expression (14), we used the following specific values for the resonance-scattering cross section σ_s, the scattering-nucleus mass number A, and the target density ρ:

$$\sigma_s \approx 2 \cdot 10^6/E, \quad A \approx 100, \quad \rho \approx 10 \text{ g/cm}^2. \qquad (15)$$

For $E \sim 100$ eV, $\Gamma \sim 0.1$ eV, and $T \sim 10^{-9}$ sec, we must have a flux density $N_0 \sim 2 \times 10^{27}$ cm^{-2} – sec^{-1} – eV^{-1} in order to obtain one useful event per hour. This is many orders of magnitude higher than the flux densities actually available at present.

The high density of the required flux is in many respects due to the fact that the distribution of the neutrons over the volume of the beam is random. The required flux density would be significantly lower if, for example, neutrons separated by distances $\lesssim \Delta x$, where Δx is defined by (13), moved in pairs. A flux of paired neutrons can be obtained with the aid of threshold reactions involving the emission of two neutrons, e.g., $(n, 2n)$ reactions. Such neutrons are initially separated by distances much smaller than Δx, and, moreover, they have, in the vicinity of the reaction threshold, nearly equal energies and emission angles in the laboratory system. Let us estimate the effect for such a neutron flux. The probability for correlated scattering of two neutrons into the solid angle $2\pi/k\Delta x$ in the case of a scatterer of thickness d is

$$W=(n\sigma_s)^2\frac{1}{2k\Delta x},\qquad(16)$$

where n is the number of nuclei in a volume ($1\ \mathrm{cm}^2$) Δx of the scatterer. The probability for a background event is

$$\Phi=\left(n\sigma_s\frac{d}{\Delta x}\frac{1}{2k\Delta x}\right)^2.\qquad(17)$$

From (16) and (17) we find that the ratio W/Φ attains a value equal to unity when $d=\Delta x(2k\Delta x)^{1/2}$.

Let us specifically consider the $(n, 2n)$ reaction. To estimate the order of magnitude of the effect, we use the cross-section data given in Ref. 6 for this reaction. Then, taking into account the effective coincidence of the neutron energy with the resonance energy, we find the number of useful events per second:

$$N=4\cdot10^{-25}N_0S/E\Gamma^{1/2},\qquad(18)$$

where N_0 is the fast-neutron flux density and S is the area of the target. For $S\sim10^2\ \mathrm{cm}^2$ and a characteristic $E\cdot\Gamma^{1/2}$ value $\sim10^2$, a fast-neutron flux density of $7\times10^{20}\ \mathrm{cm}^{-2}$–$\mathrm{sec}^{-1}$–$\mathrm{eV}^{-1}$ is needed to produce one useful event per hour. This flux density is seven orders of magnitude smaller than the resonance-neutron flux density required in the preceding method. But it is clear that the required flux densities are still high compared to the fluxes obtainable from stationary neutron sources.

Finally, let us consider a third method. In this method a sequence of two reactions, e.g.,

$$n+(Z, N)\rightarrow(Z, N)^*+n',\quad n'+(Z, N-1)\rightarrow(Z, N)^*.\qquad(19)$$

are used to produce two "long-lived" nuclei with excitation energy higher than the neutron binding energy. It is assumed that the target consists of a mixture of (Z, N) and $(Z, N-1)$ isotopes. The incoming neutron (n) is inelastically scattered by the (Z, N) isotope, leaving it in the excited state correspoinding to the neutron resonance state. The scattered neutron (n') is captured by the $(Z, N-1)$ nucleus, and produces the same excited state. In those cases when the distance between these nuclei is smaller than Δx, correlated neutron emission can occur. The correspondence of the energy of the inelastically scattered neutron with the resonance energy is ensured by choosing the fast-neutron energy to correspond to the resonance energy. The effect can be expected to have a greater magnitude in this case than in the preceding procedure because of the smaller number of successive reactions (two instead of three) and the fact that the effect can be simultaneously observed at several resonances.

Let us estimate the order of magnitude of the effect. For this purpose let us find the target for which the effect and the background noise have the same magnitude. Let there be produced as a result of the occurrence of the first reaction in excited nucleus and a neutron with the required energies. The probability for correlated scattering is given by

$$W=n\sigma_s/2k\Delta x.\qquad(20)$$

The probability for a background event is

$$\Phi\approx n\sigma_s\frac{d}{\Delta x}\left(\frac{1}{2k\Delta x}\right)^2.\qquad(21)$$

From the requirement that $W=\Phi$, we obtain

$$d=2k\Delta x^2.\qquad(22)$$

Estimates show that the quantity d is comparable to the mean free path of the resonance neutron. Therefore, we can use greater thicknesses without significantly worsening the signal-to-background noise ratio. Let us consider the case of the (n, n') reaction. Since we are, in the final analysis, interested in inelastic scattering with subsequent emission of a second neutron, we can in fact take the cross section for the corresponding $(n, 2n)$ reaction. Therefore, for a rough estimate, we can use the same data used in the analysis of the preceding method. It at the same time allows us to compare the two methods under essentially identical conditions. Using (20) and (15), and setting $n(d)\sim0.1\ \mathrm{b}^{-1}$, we find the number of useful events per second:

$$N\approx5\cdot10^{-11}N_0S\Gamma\nu/E^{3/4},\qquad(23)$$

where ν is the number of observed resonances. In order to produce one event per hour in the case when $E\sim10^3$ eV, $\Gamma\sim10^{-1}$ eV, $\nu\sim20$, and $S\sim10^2\ \mathrm{cm}^2$, we need a flux of density $N_0\approx8\times10^6\ \mathrm{cm}^{-2}$–$\mathrm{sec}^{-1}$–$\mathrm{eV}^{-1}$, and this value is now comparable to the presently available flux densities. Let us note that we do not exclude the possibility that the use of another reaction involving the emission of a neutron and the production of an excited residual nucleus will cause the effect to be enhanced.

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[1]R. H. Dicke, Phys. Rev. **93**, 99 (1954).
[2]R. P. Feynman, F. L. Vernon, and R. W. Hellwarth, J. Appl. Phys. **28**, 49 (1957).
[3]V. Ernst and P. Stehle, Phys. Rev. **176**, 1456 (1968).
[4]V. Weisskopf and E. Wigner, Z. Phys. **63**, 54 (1930).
[5]W. Heitler, The Quantum Theory of Radiation, Clarendon Press, Oxford, 1954 (Russ. Transl., Izd-vo Inostr. Lit., Moscow, 1956).
[6]V. M. Bychkov, A. B. Pashchenko, and V. I. Plyatskin, Veprosy atomnoĭ nauki i tekhniki: Ser. Yadernye konstanty (Problems of Atomic Science & Technology: Nuclear Constants Series), Vol. 2, No. 29, 1978, p. 7.

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