Cooling of nonequilibrium electrons and negative magnetoresistance in strong electric and magnetic fields

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The theory of the nonequilibrium conductivity of semiconductors in strong electric fields E, when the electron momentum is dissipated by scattering on ionized impurities and the electron energy is dissipated in the spontaneous emission of polar optical phonons, is generalized to include the presence of a magnetic field **B**⊥**E**. The resulting theory is used together with previously published studies to analyze the experimental results on the E dependence of the electron mobility μ for $B = 0$ and $B > 0$ at temperatures $T = 4.2$–77 K in the III–V compounds n-InSb, n-GaAs, and n-InP. It is shown that the parameter which determines the electric field dependence of the conductivity and average electron energy is the ratio ε_c/kT, where ε_c is the electron-ion interaction energy. For n-GaAs and n-InP the characteristic decrease in the electron mobility $\mu \sim E^{-(0.4-0.5)}$ at $T = 77$ K is explained by a "cooling" of the electrons; an interpretation is offered for the negative magnetoresistance when **B**⊥**E** and for its dependence on E. The features of the magnetoresistance in n-InSb at $T = 4.2$ K are discussed.

1. INTRODUCTION

The nonequilibrium conductivity in III–V semiconductors at low temperatures $kT \ll \hbar\omega_0$ (ω_0 is the limiting optical phonon frequency) in strong electric (E) and magnetic (B) fields has been studied in many papers (see, e.g., Refs. 1–11). However, certain aspects of the theory of nonequilibrium electrons in III–V compounds remain insufficiently elaborated, and a number of the experimental results have not been satisfactorily explained. For example, it is not clear why the E dependence of the electron mobility is different for $T \lesssim 20$ K (Refs. 3–8) and $T = 77$ K (Refs. 5 and 9–11) while the same energy dissipation and momentum scattering mechanisms operate. The experimentally determined[4] change in the conductivity with a magnetic field **B**⊥**E** in n-InSb at $T = 4.2$ K is at odds with the theory.[12]

The present study was undertaken with the aim of interpreting the hitherto unexplained[3–11] experimental results on the galvanomagnetic effects at large E and B (**E**⊥**B**) in III–V compounds.

For this purpose we have generalized the theory of Ref. 13, which treats conduction in the case when the momentum is scattered by ionized impurities and the energy is dissipated through the spontaneous emission of polar optical phonons, to the case in which a magnetic field is present. It is shown that the important parameter governing the electric-field dependence of the conductivity and average electron energy $\bar{\varepsilon}$ is the ratio ε_c/kT, where ω_c is the characteristic electron-ion interaction energy. The theory which we develop and the results of Refs. 12–18 are used to analyze the experimental results pertaining to III–V compounds, in n-InSb (both our results and those of Refs. 3 and 4) and in n-GaAs and n-InP (Refs. 5 and 9–11).

We note that up till now the only detailed interpretation that has been offered for $\mu(E)$ and the corresponding energy dissipation and momentum scattering mechanisms has been in reference to n-InSb and n-GaAs at $T = 4.2$ K and $B = 0$ (Refs. 6–8). According to those papers,[6–8] for a low electron density and weak electron-electron interaction there are three characteristic electric field regions. For $E_b < E < E_0$ (E_b is the dielectric-breakdown field of the impurities) the momentum is scattered by ionized impurities and the energy is dissipated on deformation acoustical and polar optical phonons; here $\mu \sim E^{0.3-0.4}$. In the second field region ($E_0 < E < E_1$) the momentum scattering occurs by the same mechanism as before but the energy dissipation now occurs as a result of the spontaneous emission of polar optical phonons (the fields E_0 and E_1 will be determined below). In the third field region $E > E_1$ the energy dissipation and momentum scattering occurs with the spontaneous emission of optical phonons, $\mu \sim E^{-1}$ (Ref. 16). We note that the theory predicts two different physical situations in the second field region: either μ is independent of E (the quasiohmic conductivity region[12]) or the electrons "cool off" as E increases, so that μ decreases with increasing E.[13] The reasons for the different dependences of μ on E are discussed in Ref. 17.

According to the theory,[12,16] the presence of a magnetic field **B**⊥**E** should not alter the character of the function $\mu(E)$ in field regions two and three. In addition, in those regions one should observe a negative magnetoresistance.[12,15] It was assumed in Ref. 15 that the presence of negative magnetoresistance in field region three is a manifestation of a population inversion of the electrons for **E**⊥**B**.

2. THEORY

1. The distribution function $F(p)$ and the conductivity σ in strong crossed fields **E**⊥**B** in the case where the energy is dissipated by polar optical phonons was calculated in Refs. 15 and 16 for momentum scattering by polar optical phonons and in Ref. 12 for various mechanisms of elastic momentum scattering. The applicability conditions and the results of these studies will be discussed later. For now let us discuss the case that has not been considered, viz., that of momentum scattering by ionized impurities for the field region

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$$E_0 \ll E \ll E_1. \quad (1)$$

Here

$$E_0 = \left[\frac{2}{3} mkT/\{e^2\tau(T)\tilde{\tau}_{DA}(T)\}\right]^{1/2}$$

is the electric field strength at which energy dissipation by polar optical phonons starts to become important, $E_1 = p_1/e\tau_1$ is the upper limit on the field region in which the momentum scattering is still elastic. Here $\tau(T)$ and $\tilde{\tau}_{DA}(T)$ are the energy and momentum relaxation times of the electrons on impurities and deformation acoustical phonons, τ_1 is the characteristic momentum relaxation time of nonequilibrium electrons with energy $\varepsilon < \hbar\omega_0$ ("passive" electrons[16]), $p_1 = (2m\varepsilon_m)^{1/2}$ is the characteristic momentum of the passive electrons, and ε_m is the energy corresponding to the peak of the electron distribution.

Under the action of the field, the electrons enter the energy region $\varepsilon > \hbar\omega_0$ to a small depth $\Delta\varepsilon \ll \hbar\omega_0$, emit polar optical phonons, and then enter the region $\varepsilon \sim \Delta\varepsilon$ near the origin of the energy axis; as a result of the stimulated absorption of deformation acoustical phonons, they move to higher energies.[12,13] The main contribution to the current and to the average quantities in field region (1) is from passive electrons.

2. Let us find the distribution function $F(\mathbf{p})$ and the average quantities in field region (1) in the presence of a strong magnetic field $\mathbf{B} \perp \mathbf{E}$. Let us assume that we can neglect the energy dependence of the logarithmic term $\Lambda(\varepsilon/\varepsilon_c)$ in the expression for the momentum relaxation time

$$\tau(\varepsilon) = \frac{(2m)^{1/2}}{\pi}\left(\frac{\varkappa}{e^2}\right)^2 \frac{\varepsilon^{3/2}}{N_i}\Lambda^{-1}(\varepsilon/\varepsilon_c) \quad (2)$$

(N_i is the concentration of ionized impurities, and $\varkappa$ is the dielectric constant). This assumption is valid[17] for $\varepsilon_c \ll kT(E_0/E)$.[2] The distribution function and the conductivity are found in Ref. 13 with $\tau(\varepsilon)$ as given in (2) and for $B = 0$.

In the electric field region given by (1) the kinetic equation for the distribution function $F(\mathbf{p})$ of the passive electrons can be solved in the diffusion approximation.

Using the well-known expressions (see, e.g., Ref. 17) for the flux densities $j_E(\varepsilon)$, $j_{ph}(\varepsilon)$, and $j_{in}(\varepsilon)$ of electrons in energy space due respectively to the field, quasielastic collisions with deformation acoustical phonons, and in inelastic scattering, we obtain the following expression for the symmetric part of the distribution function $F_0(\varepsilon)$:

$$F_0(x) = C\int_x^U (1+bx^3)A^{-1}x^{-2}dx, \quad (3)$$

where $x = \varepsilon/kT$, $U = \hbar\omega_0/kT$, $A = 1 + fx + bx^2$,

$$f = \left(\frac{E}{E_0}\right)^2, \quad b = \left[\frac{eB}{m}\tau(T)\right]^2 = [\omega_B\tau(T)]^2. \quad (4)$$

The parameter f gives the ratio of the power obtained from the field by electrons with energies $\varepsilon \sim kT$ to the power lost in the interaction with deformation acoustical phonons.[18]

Following Ref. 13, in obtaining (3) we have neglected the term in the flux density $j_{ph}(\varepsilon)$ that corresponds to the spontaneous emission of deformation acoustical phonons and assumed that for $\varepsilon = \hbar\omega_0$ we can set $F_0(\varepsilon) = 0$. The constant C is found from the normalization condition on the electron density n:

$$C = [{}^2/_3(kT)^{3/2}\rho I_n(f, b)]^{-1}n;$$

here $\rho\varepsilon^{1/2}$ is the density of states and I_n is the normalization integral

$$I_n(f, b) = \int_0^U x^{-1/2}A^{-1}dx + b\int_0^U x^{5/2}A^{-1}dx. \quad (5)$$

In the case when the drift field is directed along the X axis, the magnetic field is along the Z axis, the Hall contacts are open-circuited, and the Hall field E_y is nonzero,

$$f = f_x + f_y = (E_x/E_0)^2 + (E_y/E_0)^2.$$

In this case the dissipative current j_x is given by the expression

$$j_x = \sigma(f, b)E_x = [\sigma_{xx}(f, b) + \sigma_{xy}^2(f, b)/\sigma_{xx}(f, b)]E_x. \quad (6)$$

when $F_0(\varepsilon)$ is given by (3), the components of the conductivity tensor σ_{xx} and σ_{xy} are expressed in the form

$$\sigma_{xx}(f, b) = \sigma_T I_{xx}(f, b)I_n^{-1}(f, b);$$
$$\sigma_{xy}(f, b) = \sigma_T b^{1/2} I_{xy}(f, b)I_n^{-1}(f, b), \quad (7)$$

where

$$I_{xx}(f, b) = \int_0^U xA^{-1}dx, \quad I_{xy}(f, b) = \int_0^U x^{5/2}A^{-1}dx,$$

$$\sigma_T = e^2 n\tau(T)m^{-1}. \quad (8)$$

The average energy ε of the electrons in the magnetic field, according to (3), is

$$\bar{\varepsilon} = \frac{3}{5}kTI_\varepsilon(f, b)I_n^{-1}(f, b),$$

$$I_\varepsilon(f, b) = \int_0^U x^{1/2}A^{-1}dx + b\int_0^U x^{7/2}A^{-1}dx. \quad (9)$$

We note that according to (1) and (4), values $f \gg 1$ correspond to the predominance of energy dissipation by polar optical phonons.

3. Integrals (5) and (7)–(9) can be evaluated analytically in two limiting cases: $bU^2 \ll f$ and $bU^2 \gg f$. The first case is the most interesting for interpreting the experiments of Refs. 9–11. Since, as we shall show below, f can be substantially larger than U^2, the condition $bU^2 \ll f$ can be satisfied even for $b \gg 1$. In this case we find to zeroth order in bU^2/f

$$I_n(f, b) \approx \pi f^{-1/2} + \frac{2}{5}\frac{b}{f}U^{5/2},$$
$$I_\varepsilon(f, b) \approx \frac{2}{f}U^{1/2} + \frac{2}{7}\frac{b}{f}U^{7/2}, \quad (10)$$
$$I_{xx}(f, b) \approx U/f, \quad I_{xy}(f, b) \approx {}^2/_5 U^{5/2}/f.$$

It follows from (7)–(10) that the influence of a transverse magnetic field on the electron distribution and the average quantities can be characterized by two parameters:

$$\beta=\frac{4}{25}bU^3,\quad \gamma=\frac{2}{5\pi}bU^3(fU)^{-1/2}. \qquad (11)$$

The first of these, as can easily be seen from (10) and (7), is the square of the ratio of the Hall field E_y to the drift field E_x. The meaning of the second parameter is clear from the expression for I_n in (10): for $\gamma \ll 1$ the main contribution to the normalization integral of I_n comes, as in the case $B=0$, from the energy region $x\sim f^{-1}\ll 1$, while for $\gamma \gg 1$ it is from the energy region $x\sim U$. The explanation for this is that at $B=0$ the energy obtained from the field by an electron with energy ε is proportional to τ, while in strong magnetic fields it is proportional to τ^{-1}. We note that $\gamma \ll \beta$ in fields which satisfy (1).

The conductivity $\sigma(f, b)$ and the average electron energy $\bar{\varepsilon}(f, b)$ in a magnetic field are conveniently expressed[13] in terms of the corresponding quantities for $B=0$:

$$\sigma(f,0)=\sigma_T\frac{U}{\pi}f^{-1/2},\quad \bar{\varepsilon}(f,0)=\frac{2}{5}\frac{kTU^{1/2}}{\pi f^{1/2}},$$
$$\sigma(f,b)=\sigma(f,0)\frac{(1+\beta)^{1/2}}{1+\gamma},\quad \bar{\varepsilon}(f,b)=\bar{\varepsilon}(f,0)\frac{1+25\beta/28}{(1+\gamma)(1+\beta)^{1/2}}. \qquad (12)$$

Formulas (12) can be used to find σ and $\bar{\varepsilon}$ in strong and weak magnetic fields at various positions of the peak of the electron distribution $n_0(\varepsilon)$.

4. Let us consider the case $\gamma\ll 1$, when the majority of the electrons, as for $B=0$, are concentrated in the region $x\sim f^{-1}$. We see from (11) and (12) that σ and $\bar{\varepsilon}$ fall off with increasing E, i.e., the electrons are cooled. This cooling occurs not only for $B=0$ (Ref. 13) but also in a magnetic field **B**⊥**E**. The physical cause of the cooling in magnetic fields **B**⊥**E** for $b \ll f/U^2$ is the same as in the absence[17] of B: When $\tau(\varepsilon)$ is given by (2) and $\Lambda(\varepsilon/\varepsilon_c)=\text{const}$, the electrons cross most slowly the energy region $\varepsilon\sim\varepsilon_f=kT/f\ll kT$, where the momentum relaxation time is small. It is in this region that the peak of $n_0(\varepsilon)$ occurs. In a constant magnetic field, ε_f falls off with increasing electric field, and the peak of $n_0(\varepsilon)$ is shifted to lower energies.

In the opposite case, when $\gamma\gg 1$, the majority of the electrons are found at $x\sim U$ [see Eq. (11) and below], there is no cooling, and σ and $\bar{\varepsilon}$ increase with increasing magnetic field (for $E_x=\text{const}$). In strong magnetic fields, when $\beta\gg 1$, the conductivity and average energy are independent of the electric and magnetic fields and, in the limit, reach the values

$$\bar{\varepsilon}(f,b)\approx\frac{5\pi}{4}(fU)^{1/2}\bar{\varepsilon}(f,0)\approx\frac{3}{14}kTU,$$
$$\sigma(f,b)\approx\frac{2\pi}{5}(fU)^{1/2}\sigma(f,0)\approx\frac{2}{5}\sigma_TU^{3/2}. \qquad (13)$$

The increase of σ and $\bar{\varepsilon}$ in a magnetic field can be explained as follows: From the expression for the electron flux density caused by the electric field,

$$j_E(\varepsilon)\propto\frac{\varepsilon^{3/2}\tau(\varepsilon)(eE)^2}{1+b(\varepsilon/kT)^3}\frac{\partial F_0}{\partial\varepsilon},$$

we see that the magnetic field has almost no effect on the form of $j_E(\varepsilon)$ for $\varepsilon\ll kTb^{-1/3}$, as long as $\omega_B\tau(\varepsilon)\ll 1$. If $\varepsilon\gg kTb^{-1/3}$, the magnetic field causes a noticeable decrease in the flux density $j_E(\varepsilon)$ and alters its energy dependence. As a result, the electrons cross the region $\varepsilon>kTb^{-1/3}$ more slowly than for $B=0$, and the peak of $n_0(\varepsilon)$ is shifted to higher energies; the drift velocity thus increases. In strong magnetic fields ($\gamma\gg 1$) the peak of $n_0(\varepsilon)$ lies in the energy region $\varepsilon\gtrsim\hbar\omega_0$, and its position is independent of the electric field; the cooling ceases. Thus the dependence of σ and $\bar{\varepsilon}$ on E_x is determined by the value of γ given by (11): for $\gamma\gg 1$, σ and $\bar{\varepsilon}$ are independent of E_x, while for $\gamma\ll 1$ they fall off with increasing E_x ($\sigma\propto E_x^{-1}$, $\varepsilon\propto E_x^{-1}$). The dependence of σ on E_x and B that we have described is illustrated schematically in Fig. 1a (curves 1 and 2).

5. The above calculation of σ and ε is for the case $\Lambda(\varepsilon/\varepsilon_c)=\text{const}$ [see Eq. (2)]. To complete the discussion of the behavior of nonequilibrium electrons in strong electric and magnetic fields, let us give the results of Refs. 12 and 18, in which $\sigma(f,0)$ and $\bar{\varepsilon}(f,0)$ were calculated with allowance for the dependence of $\Lambda(\varepsilon/\varepsilon_c)$ on ε. In those papers[12,18] $\Lambda(\varepsilon/\varepsilon_c)$ was evaluated using the Conwell-Weisskopf formula $\varepsilon_c=e^2N_i^{1/3}/\varkappa$.

The computer calculation of $F(\mathbf{p})$, σ, and $\bar{\varepsilon}$ in Ref. 18 was done for various ratios ε_c/kT over a wider field range than in Ref. 12: $0\leqslant E\leqslant E_1$. According to Ref. 18, for $\varepsilon_c>kT$ in fields $E<E_0$, the peak of $n_0(\varepsilon)$ shifts with increasing E from the region $\varepsilon\sim(1/2)kT$ into the region $\varepsilon\sim\varepsilon_c>kT$; the electrons are heated. For $E_0<E<E_1$ the position of the peak of $n_0(\varepsilon)$ is independent of the field, and $\sigma(E)=\text{const}$ (the quasiohmic conductivity region), as in Ref. 12. In the intermediate case, when $kT/f\ll\varepsilon_c\ll kT$, cooling occurs[18] in fields satisfying (1), as in the case $\varepsilon_c\ll kT/f$. The difference from the case $\varepsilon_c\ll kT/f$ lies in the fact that cooling of the electrons can now occur only to an energy $\varepsilon\sim\varepsilon_c$; subsequent growth of the field has almost no effect on $\bar{\varepsilon}$, so that the decrease of σ with field is replaced by a quasiohmic region. The described dependence of σ on E_x is shown schematically in Fig. 1a for various ratios ε_c/kT.

The values of σ, $\bar{\varepsilon}$, and $F(\mathbf{p})$ for $B>0$ in fields satisfying (1) were calculated in Ref. 12 with allowance for the dependence of $\Lambda(\varepsilon/\varepsilon_c)$ on ε, i.e., under the assumption that $\varepsilon_c\gg kT/f$. The calculation showed that the quasiohmic region $\sigma(E_x)=\text{const}$ remains present in magnetic fields **B**⊥**E**, both for $\mu B\ll 1$ and $\mu B\gg 1$.

6. Information on the electron distribution in strong fields **E**⊥**B** can be obtained from the magnetoresistance.[12,15] For $\gamma\ll\beta$ (11), formula (12) implies that the average electron energy and the conductivity are larger in a magnetic field than for $B=0$, i.e., there is a negative magnetoresistance:

$$\frac{\Delta\rho}{\rho}\equiv\frac{\sigma(f,0)}{\sigma(f,b)}-1=\frac{1+\gamma}{(1+\beta)^{1/2}}-1<0. \qquad (14)$$

According to (11) and (14), the negative magnetoresistance at $B=\text{const}$ increases in absolute value with increasing drift electric field. An analogous result has also been obtained[12] with allowance for the energy dependence of $\Lambda(\varepsilon/\varepsilon_c)$.

Qualitatively, the growth of the conductivity and average electron energy with increasing B in both cases can be explained by a decrease in the relative value of $j_E(\varepsilon)$ with

TABLE I.

Sample No.	Material	Source	$N_d-N_a\cdot 10^{14}$, cm^{-3}	$N_i\cdot 10^{14}$, cm^{-3}	$\mu_{77}\cdot 10^{-5}$, cm^2/V·sec	ε_c, K
1	n-InSb	Present study	0.6	1.4	9	5.4
2	n-InSb	,,	0.72	1.1	10	5
3	n-InSb	,,	0.75	2.7	7	6.7
4	n-InSb	,,	0.87	1.1	11	5.0
5	n-InSb	,,	3	8	4.2	9.7
6	n-InSb	[3]	0.46	5	5	8.2
7	n-GaAs	[5]	11,3	15	1	16
8	n-GaAs	[9–11]	10	40	0.4	22
9	n-InP	[9–11]			0.2	

increasing B in the energy region $\varepsilon \gg \varepsilon_c$ and a shift of the peak of $n_0(\varepsilon)$ into the region $\varepsilon \sim \hbar\omega_0$; the drift velocity would thereby increase. It should be noted that in Ref. 12, as in the above, it is assumed that the drift velocity v_d is much less than v_0 even for $\mu B \gg 1$ and that the anisotropy of the distribution function is small. Here v_0 is the velocity of an electron with energy $\varepsilon = \hbar\omega_0$.

The function $\Delta\rho/\rho(E_x)$ for various ε_c/kT is shown schematically in Fig. 1b.

3. EXPERIMENTAL RESULTS AND DISCUSSION

1. Let us discuss our experimental results and the previously published results on the nonequilibrium conductivity of n-InSb, n-GaAs, and n-InP at $T \leqslant 20$ K and $T = 77$ K from a unified point of view. The theory implies (see Fig. 1) that the parameter which determines the character of the functions $\sigma(E, B)$ and $\mu(E, B)$ is the ratio ε_c/kT. The values of N_i necessary for calculating ε_c were determined according to Ref. 19 for n-InSb, while for the n-GaAs samples[3,9–11] N_i was determined from the equilibrium ability at $T = 20$ K and 77 K. Table I shows that $\varepsilon_c/kT > 1$ when $T = 4.2$ K, while $\varepsilon_c/kT < 1$ for $T = 77$ K. In view of the difference in the ratios ε_c/kT, it is more convenient to consider the experimental results for $B = 0$ first, and then those for $B > 0$.

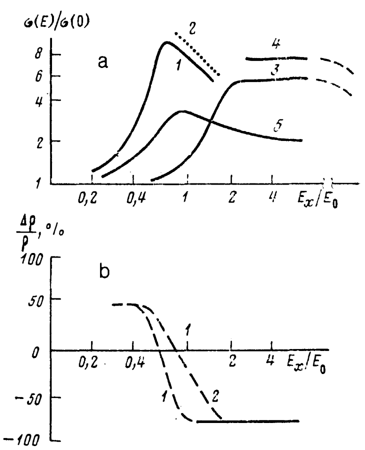

FIG. 1. a) Theoretical dependence of the conductivity on the drift field E_x for $\mathbf{B}\perp\mathbf{E}$ and for various values of ε_c/kT, according to Refs. 12, 13, and 18 and formulas (23) of this paper (schematic): 1,2) $\sigma(E_x)$ for $\Lambda(\varepsilon/\varepsilon_c) = \text{const}$ ($\varepsilon_c \to 0$) and $B = 0$ (1), $B > 0$ (2); 3,4) $\sigma(E_x, B)$ for $\varepsilon_c \gtrsim kT$ and $B = 0$ (3), $B > 0$ (4); 5) $\sigma(E_x)$ for $kT/f \ll \varepsilon_c \ll kT$ and $B = 0$. The dashed parts of curves 3 and 4 show $\sigma(E_x)$ for $E_x \gg E_1$; b) the magnetoresistance as a function of E_x in fields $E_0 \ll E \ll E_1$: 1) $\Delta\rho/\rho$ for $\varepsilon_c \gg kT/f$ (Ref. 12; see text), 2) $\Delta\rho/\rho$ for $\varepsilon_c \ll kT/f$ according to formula (14).

2. Let us briefly discuss $\mu(E)$ in n-GaAs and n-InSb for $T \leqslant 77$ K and $B = 0$. Figure 2 shows the $\mu(E)$ curves[1)] for samples 3 and 7 (see Table I). We see that for a given temperature the $\mu(E)$ curves in n-InSb and n-GaAs are similar, but they change appreciably with temperature. This, we believe, is the most important experimental result. At temperatures $T \lesssim 12$ K we have $\varepsilon_c \gg kT/f$, and so according to Ref. 12 there is an electric field region in which $\mu(E)$ for the nonequilibrium electrons is either independent of E or increases slowly with E, i.e., there is a region of quasiohmic conductivity. Sample No. 6 (n-InSb), with $\varepsilon_c \sim kT/2$ at $T = 20$ K, exhibited a quasiohmic conductivity region, since $\varepsilon_c > kT/f$ for $f \approx 10$.

3. Let us analyze the E dependence of the electron mobility at $T = 77$ K for sample No. 7 (n-GaAs), since here the characteristic regions of $\mu(E)$ are more pronounced than in n-InSb (see Fig. 2). At $T = 77$ K, $\mu(E)$ is constant in fields up to $E \approx 20$–30 V/cm and then begins to fall off, first as $\mu \propto E^{-(0.4-0.5)}$, then $\propto E^{-(0.8-0.9)}$. A similar decay of $\mu(E)$ was observed in Refs. 9–11. We note that the mobility falls off in fields in which the drift velocity v_d is considerably slower than thermal velocity v_T [$v_T = (3kT/m)^{1/2} = 1.9\cdot 10^7$ cm/s] and v_0[$v_0 = (2\hbar\omega_0/m)^{1/2} = 4.4\cdot 10^7$ cm/s].

Let us estimate field range (1) ($E_0 \leqslant E \leqslant E_1$) for samples No. 7 and 8 using the values of the equilibrium mobility $\mu(0)$ for deformation acoustical and compound polar optical scattering. According to Refs. 20 and 21, we find that the main contribution to $\mu(0)$ is from scattering by ionized impurities, and that for $E \to 0$ the energy is dissipated on deformation acoustical phonons. This agrees with the assumptions for Refs. 3 and 11–13 and let us calculate the fields E_0 and E_1 using the values of $\mu(0)$, taking p_1 to be the thermal momen-

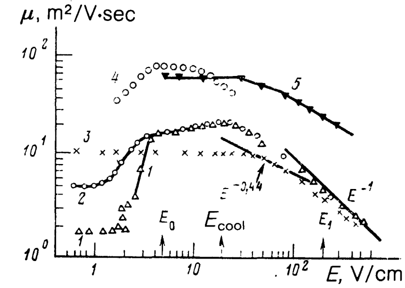

FIG. 2. Dependence of the Hall mobility of the electrons on the strength of the electric field in n-GaAs (sample No. 7, curves 1–3) and n-InSb (sample No. 3, curves 4 and 5) for $B = 0$ at temperatures in the range 4.2–77 K: 1) $T = 5.12$ K, 2) $T = 11.8$ K, 3) $T = 77$ K, 4) $T = 4.2$ K, 5) $T = 77$ K.

tum $p_T = (2mkT)^{1/2}$ and τ_1 to be $\tau(T)$. For sample No. 7 (Ref. 5) we get $E_0 \approx 4.8$ V/cm and $E_1 \approx 2 \cdot 10^2$ V/cm; for sample No. 8 (Refs. 11–13) we get $E_0 \approx 7$ V/cm and $E_1 \approx 4.5 \cdot 10^2$ V/cm. In Figs. 2–4 the values of E_0 and E_1 are indicated by vertical arrows. It follows from Fig. 2 that the energy dissipation on polar optical phonons should come into play even in fields at which $\mu(E)$ is not yet very different from $\mu(0)$, that $\mu(E)$ begins to fall off for $E \ll E_1$, where the momentum scattering by polar optical phonons should not yet be manifested, and that the decay of $\mu(E)$ corresponds to values $f > 10^2$, so that $kT/f < \varepsilon_c \ll kT$.

The decrease of μ with increasing E for $E_0 < E < E_1$ is evidently due to "cooling" of the electrons.[13,18] Since we have $kT/f < \varepsilon_c \ll kT$ for samples No. 7 and 8 at $T = 77$ K. We should expect $\mu(E)$ to resemble curve 5 in Fig. 1a. The numerical calculation of $\mu(E)$ for $kT/f < \varepsilon_c \ll kT$ in Ref. 18 was done for values of ε_c/kT which were different from the experimental values,[5,9–11] and we can therefore expect only a qualitative explanation.

Let us estimate the values of the drift velocity v_d and electric field $E_{\rm cool}$ at which the cooling of the electrons begins. For this we taken $F(\mathbf{p})$ to the equilibrium distribution function and compare $j_E(\varepsilon)$ with

$$j_{\rm in}(\varepsilon) = \rho(\hbar\omega_0) F_0(\hbar\omega_0) \frac{eE\hbar\omega_0}{m^{1/2}}.$$

Here $j_E(\varepsilon) > j_{\rm in}(\varepsilon)$ for the majority of equilibrium electrons if $v_d > 2 \cdot 10^6$ cm/s; $E_{\rm cool} \approx 20$ V/cm for sample No. 7 and $E_{\rm cool} \approx 50$ V/cm in sample No. 8. The values of $E_{\rm cool}$ are indicated by vertical arrows in the figures. We see from Fig. 2 that $\mu(E)$ begins to fall off when the electric field is close to $E_{\rm cool}$. The slower decay of $\mu(E)$ as compared to Ref. 13 apparently comes of not satisfying the condition $\varepsilon_c \ll kT/f$. We note in this regard that according to the numerical calculations of Ref. 18, at $\varepsilon_c \sim 0.1kT \sim kT/f$ one has $\mu(E) \propto E^{-0.33}$, which is rather like the experimental behavior (see Fig. 2).

Electron cooling at $B = 0$ was used in Ref. 22 to explain the electric field dependence of $\bar{\varepsilon}$ in highly compensated n-InSb at $T = 77$ K. It was shown in Ref. 23 that in this material ε_c is substantially smaller than the electron-ion interaction energy as given by the Conwell-Weisskopf formula. Evidently the conditions in Ref. 22 were such that $\varepsilon_c \lesssim kT/f$, and the observed growth [to a value of $\varepsilon \approx 1.4\varepsilon(0)$] and subsequent decrease [to $\varepsilon \approx 0.6\varepsilon(0)$] of ε with electric field are in qualitative agreement with the results presented in paragraph 2 (see also Ref. 18).

4. The influence of a magnetic field on $\mu(E)$ and on the magnetoresistance $\Delta\rho/\rho(E)$ in n-GaAs and n-InP at 77 K was studied in Refs. 9–11. Figure 3 reproduces the drift mobility curves $\mu_d(E,B)$ given in Refs. 9–11 for $E = 10^2$–10^4 V/cm and $B = 0$–1.3 T for samples No. 8 and 9. The arrows in Fig. 3 indicate the values of E_0, $E_{\rm cool}$, and E_1. We see that in a magnetic field ($\mu_d B > 1$) the $\mu_d(E)$ curve goes through a maximum, which is shifted to higher electric fields with increasing B. For $E > E_{\rm cool} \approx 50$ V/cm and $B > 0.5T$, μ_d decreases with increasing $E_{\rm cool}$ in almost the same way as for $B = 0$, i.e., the electrons cool. The function $\mu_d(E, B)$ has the feature that the decay of μ_d with increasing E corresponds to

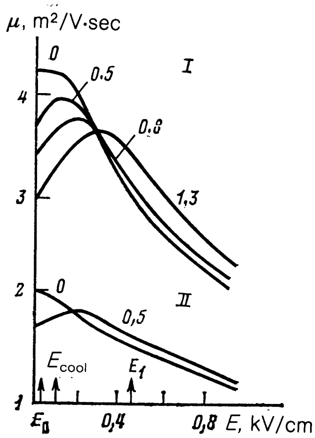

FIG. 3. Drift mobility versus the electric field for various values of the magnetic field (in teslas) in n-GaAs (I) and n-InP (II) (samples No. 8 and 9) at $T = 77$ K.

a negative magnetoresistance. Figure 4 shows the curve of $\Delta\rho/\rho(E)$ for sample No. 8 (n-GaAs) at $T = 77$ K and $B = 0.5$ T (Refs. 9–11); the fields $E_{\rm cool}$ and E_1 are also indicated. For $E \leqslant 2 \cdot 10^2$ V/cm we see the positive magnetoresistance that is characteristic for equilibrium electrons. In the range $E = 2 \cdot 10^2$–$1.5 \cdot 10^3$ V/cm we have $\Delta\rho/\rho < 0$, and the negative magnetoresistance reaches its maximum absolute value at $E \approx 6 \cdot 10^2$ V/cm. The onset of negative magnetoresistance for $E > E_{\rm cool}$ and its growth in absolute value with increasing E are in qualitative agreement with the conclusions of paragraphs 2 and 6. In fact, as E increases, so does the relative strength of the energy scattering by optical phonons in the case of a quasielastic mechanism of momentum scattering, and this gives rise to an increase in the absolute value of the negative magnetoresistance. According to the theory, this increase should occur at fields up to $E \approx E_1 = 4.5 \cdot 10^2$ V/cm. We see from Fig. 4 that $|\Delta\rho/\rho|$ continues to increase to somewhat higher values ($E \approx 6 \cdot 10^2$ V/cm). The subsequent decrease in $|\Delta\rho/\rho|$ agrees qualitatively with the calculation of Ref. 15, where it was assumed that the energy and momentum are scattered by polar optical phonons.

5. The influence of a transverse magnetic field on the mobility of nonequilibrium electrons in n-InSb has been studied, so far as we know, only at $T = 4.2$ K (Ref. 4). Since the results given in Ref. 4 are insufficient to completely char-

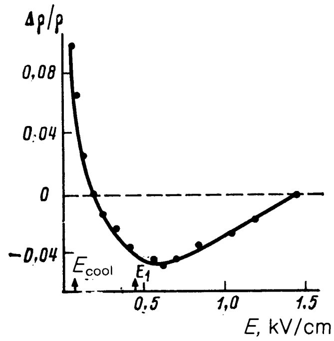

FIG. 4. Magnetoresistance as a function of the electric field for sample No. 8 (n-GaAs) at $T = 77$ K and $B = 0.5$ T.

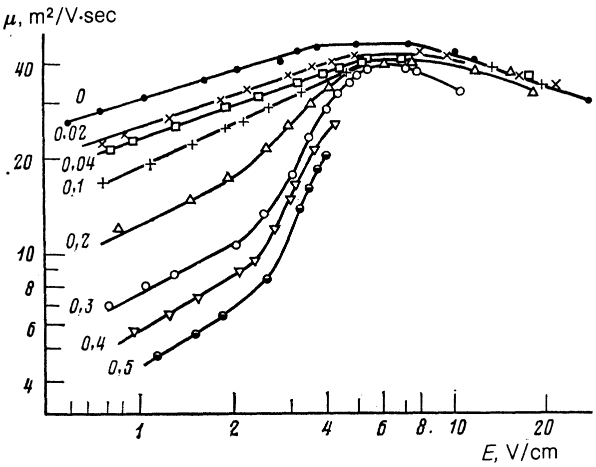

FIG. 5. Influence of the magnetic field on $\mu(E)$ in sample No. 5 for $T = 4.2$ K. The curves are labeled with the value of B (in teslas).

acterize $\mu(E, B)$, we measured the Hall constant R_H and the conductivity in five more samples of n-InSb in the range $E = 0.1$–100 V/cm, $B = 0$–$0.7T$ at $T = 4.2$ K. The parameters of the samples are given in Table I.

Figure 5 shows typical $\mu(E, B)$ curves for sample No. 5. In magnetic fields of up to 0.04 T, the $\mu(E, B)$ and $\mu(E, 0)$ curves are qualitatively similar.[2)] However, as B increases, the quasiohmic region shrinks in size and shifts to higher electric fields, and for $B \gtrsim 0.1$ T it vanishes altogether. At electric fields in the range 1–10 V/cm a positive magnetoresistance is observed. For $E = 1$–3 V/cm it does not depend appreciably on E, but, beginning at $E > 3$ V/cm, the positive magnetoresistance decreases, falling practically to zero in electric fields corresponding to the end of the quasiohmic region.

Figure 6 shows the dependence of $\Delta\rho/\rho$ on E at several values of B for the same sample. We see that for $B = 0.05$ and 0.1 T the positive magnetoresistance falls off practically to zero with increasing E. In a magnetic field $B \geqslant 0.3$ T the $\Delta\rho/\rho$ curve passes through a minimum at $E \approx 20$–30 V/cm, but the positive magnetoresistance for $E > 20$ V/cm is substantially smaller in absolute value than for $E \approx 1$–3 V/cm.

The data in Figs. 5 and 6 disagree qualitatively with the theory of Ref. 12 (curves 1 and 2 in Fig. 1a, curve 1 in Fig.

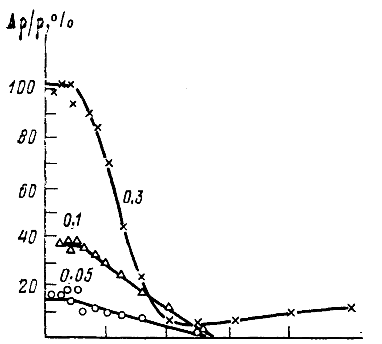

FIG. 6. The change in the magnetoresistance with electric field in sample No. 5 (n-InSb) at $T = 4.2$ K. The curves are labeled with the magnetic induction (in teslas).

1b), where, on the assumption that the anisotropy of the distribution function $F(\mathbf{p})$ is small both for $B = 0$ and for $B > 0$, it was found that the quasiohmic region in $\sigma(E)$ should remain present and that the magnetoresistance should be negative. For the particular sample under study we are convinced that the assumption that the anisotropy of $F(\mathbf{p})$ is small does not hold in the experiments. As a measure of the anisotropy we can take the ratio v_d/v_0. After determining the drift velocity v_d^0 in the absence of magnetic field from the experimental values of the mobility $v_d^0 = \mu E$, we find that $v_d^0 v_0 \approx 0.1$. We then estimate the drift velocity v_d^B in a magnetic field under the assumption of small anisotropy. Because the peak of $n_0(\varepsilon)$ occurs at $\varepsilon \sim \varepsilon_c$ for $B = 0$ and at $\varepsilon \sim \hbar\omega_0$ for $\mu B > 1$, we have $v_d^B = v_d^0(\hbar\omega_0/\varepsilon_c)^{3/2}$. Using the known values of $\hbar\omega_0$ and ε_c (see Table I), we find that $v_d^B/v_0 \approx 30$, i.e., $v_d^B \gg v_0$. Consequently, for $\mu B > 1$ the assumption that the anisotropy of $F(p)$ is small does not hold. This leads to the shrinking and subsequent vanishing of the quasiohmic region with increasing B and to the absence of negative magnetoresistance. The decrease in the positive magnetoresistance with increasing E (practically to zero for $E \approx E_1$) occurs because the peak of $n_0(\varepsilon)$ shifts to higher values of E as B increases, and for a small anisotropy of $F(\mathbf{p})$ this would result in a negative magnetoresistance.

4. CONCLUSION

This study has shown that the heating and cooling of the electrons in strong electric and magnetic fields $\mathbf{E}\perp\mathbf{B}$ are similar in all III–V semiconductors. For inelastic energy scattering due to the spontaneous emission of polar optical phonons and momentum scattering by ionized impurities, an important parameter is ε_c/kT, whose value determines which of the various physical situations will be realized. For example, for $\varepsilon_c \gtrsim kT/2 \gg kT/f$ the average electron energy is independent of the electric field in field range (1), and a region of quasiohmic conductivity is observed. For $\varepsilon_c \ll kT/f$ the average energy falls off with increasing field, i.e., there is a cooling of the electrons. In this case the magnetic field does not affect the E dependence of the conductivity and average electron energy, and electron cooling is therefore observed not only for $B = 0$ but also in strong fields $\mathbf{B}\perp\mathbf{E}$.

For $\varepsilon_c \gtrsim kT/f$ and $\hbar\omega_0/kT \gtrsim 70$, the increase of the anisotropy of $F(\mathbf{p})$ in a magnetic field $\mathbf{B}\perp\mathbf{E}$ in the present n-InSb samples cannot be neglected; it causes the region of quasiohmic conductivity to vanish and eliminates negative magnetoresistance.

We wish to express our deep gratitude to Yu. A. Gurvich for a discussion of this study.

[1)]Analogous $\mu(E)$ curves have been observed[6,7] in n-InSb and n-GaAs at $T = 4.2$ K and are interpreted in detail in Refs. 6 and 7.

[2)]For $B > 0.1$ T ($\mu B > 1$) the Hall mobility $\mu(E, B) = R_H(E, B)\sigma(E, B)$ is the same as the drift mobility.

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