Change, due to scattering-act correlation, of electron diffusion in a classically strong magnetic field

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It is shown that the electron-diffusion rate in a strong magnetic field **H** is determined by multiple returns to one and the same scattering center. The scattering-act correlations connected with the returns accelerate the transverse diffusion and slow down the longitudinal one. In scattering by charged centers, the effect turns out to be strong when the Larmor radius becomes smaller than the screening radius. The transverse-diffusion coefficient $D'_\perp$ is found in this case to be proportional to $1/H^{4/3}$. With increasing magnetic field, the cross section for Coulomb scattering becomes dependent on H. As a result, $D'_\perp$ decreases like $1/H^{16/9}$. If negatively charged (repelling) centers are present, slow electrons become "trapped" between pairs of such centers. The coefficients of both transverse and longitudinal diffusion become proportional to $1/H$. Electron diffusion is considered in a film placed in a perpendicular magnetic field. The size effect due to the scattering-act correlation is described. The effect consists of acceleration of the diffusion across **H** when the film thickness L is decreased, such that $D'_\perp \propto 1/L^{1/2}$. It comes into play in a strong magnetic field if the film thickness exceeds greatly the mean free path. In this case $D'_\perp$ is proportional to $1/H$. The manner in which the correlations of the scattering acts are "destroyed" by motion of centers (e.g., ions in a gas plasma) is considered.

1. This paper is devoted to a qualitative description of the kinetics of electrons scattered by randomly placed potential centers. The medium is assumed to be macroscopically homogeneous. It is shown that the Boltzmann kinetic equation cannot be used to describe such a system in a sufficiently strong magnetic field. We emphasize that we are dealing with fields that are only classically strong. The quantum properties of the electrons are immaterial. It is found nonetheless that the electron motion cannot be regarded as a sequence of independent scattering acts. The point is that a magnetic field makes the electron trajectory one-dimensional, and in a sufficiently strong field to such an extent that multiple returns of the electron to one and the same center become significant. As a result, many scattering acts become correlated, whereas the usual theory presupposes the absence of any correlation whatever.

The change produced in the character of transverse diffusion by the fact that the electron leaves repeatedly identical regions of a random electric field was considered earlier by Dreĭzin and Dykhne.[1,2] In Ref. 1 was investigated diffusion in a medium with macroscopic inhomogeneities (whose dimension is much larger than the mean free path l). Reference 2 dealt with the kinetics of electrons in a hot excited plasma. The assumed excitation wavelength, while smaller than l, was larger than the screening radius. In the present paper principal attention is paid to the description of a homogeneous equilibrium plasma. It is shown that effects similar to those investigated by Dreĭzin and Dykhne occur also in the case. It is also shown that the correlations of the scattering acts can alter substantially the character of the longitudinal diffusion.

The plan of the exposition is the following. In Sec. 2 the general laws are illustrated by considering a simple model. The correlations investigated throughout are for scattering by charged centers. The transverse component of the diffusion-coefficient tensor is calculated in Secs. 3–7, and the longitudinal in Sec. 8. The influence of collisions with short-range centers is treated in Sec. 6. Motion of charged centers (e.g., ions in a gas plasma), is taken into account in Sec. 7. The size effect due to scattering-act correlations is considered in Sec. 9.

2. It is convenient to elucidate the gist of the topic by using the following model. Assume that the entire space occupied by the medium can be subdivided into regions of two types, which we call reflecting and displacing. An electron incident on a region of the first type undergoes a reversal of the sign of its longitudinal velocity. This is the only function of a reflecting region. The deflecting region, on the contrary, does not influence the longitudinal motion. However, passage through it is accompanied by a displacement of the Larmor-orbit center in a direction transverse to the magnetic field. We designate the typical dimensions of reflecting and displacing regions by ξ_r and ξ_d, respectively. The characteristic length of the transverse displacement during the free path time τ will be designated $R(\tau)$. (More accurately speaking, τ is here the time between reflections.) Assume that the Larmor radius $r_L \lesssim R(\tau)$. The mean free path l along the magnetic field will be assumed larger than either of the lengths $\xi_r, \xi_d, R(\tau)$. Thus the distances between reflecting regions are much larger than the sizes of the regions. On the basis of the solution of Boltzmann kinetic equation, the expressions for the respective coefficients $D_\perp$ and $D_\parallel$ of the transverse and longitudinal diffusion are given by

$$D_\perp \sim R^2(\tau)/\tau, \quad D_\parallel \sim l^2/\tau.$$

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These (Boltzmann) diffusion coefficients do not depend explicitly on ξ_r and ξ_d. In fact, however the true coefficients $D'_\perp$ and $D'_\parallel$ differ considerably, depending on the relation between the lengths ξ_r, ξ_d, and $R(\tau)$. Namely, $D'_\perp = D_\perp$ only if $R(\tau) \gg \xi$, and $D'_\parallel = D_\parallel$ only if $R(\tau) \gg \xi_r$. Actually, at $R(\tau) \lesssim \xi_d$ it is highly probable that the electron will return upon reflection to the same displacing regions through which it passed prior to the reflection. In the case of the strong inequality $R(\tau) \ll \xi_d$ the electron will return to them many times. If, however, the condition $R(\tau) \gg \xi_r$ is not met, the successive reflections will not be independent. At $R(\tau) \ll \xi_r$ the electron is "trapped" between a pair of reflecting regions.

Let us estimate the diffusion coefficients for the case when the correlations are important. To this end we must know the time dependence of $R(t)$, the transverse displacement after a time t. We introduce also the symbol $t(R)$ for the time of displacement by a distance R.

A. $\xi_r \gg R(t) \gg \xi_d$. By virtue of the last inequality the transverse diffusion obeys Boltzmann's law (the correlations in the transverse displacements are small). The longitudinal diffusion is much slower than than according to Boltzmann. The value of $D'_\parallel$ can be estimated to equal the ratio $l^2 t(\xi_r)$. Indeed, to traverse a length l along H the electron must be displaced by a distance $\sim \xi_r$ in the transverse direction. If the electron is tracked from the instant $t = 0$, it is seen to be at $t \lesssim T(\xi_r)$ in a trapped state between two reflecting regions. Its transverse displacement in this case is $R(t) \sim R(\tau)(t/\tau)^{1/2}$, and accordingly $t(\xi_r \sim \tau(\xi_r/R(\tau))^2$. We have thus

$$D_\perp' = D_\perp, \quad D_\parallel' \sim D_\parallel (R(\tau)/\xi_r)^2.$$

B. $\xi_d \gg R(\tau) \gg \xi_r$. In this case the it is the longitudinal diffusion that obeys the Boltzmann law. The transverse diffusion is faster. This is due to the strong correlations in the transverse displacements. The decisive factor here is the recurrence property of one-dimensional diffusion. In one-dimensional motion the electron passes repeatedly through the starting point of its trajectory. In fact, the path traversed by the electron with a time t is vt (where v is the longitudinal velocity), and the diffusion volume corresponding to this time is $(D_\parallel t)^{1/2}$. The ratio $vt/(D_\parallel t)^{1/2} \sim (t/\tau)^{1/2}$ yields the number of returns within a time $t \gtrsim \tau$. Since $\xi_d \gg R(\tau)$ by assumption, the electron returns at $t \lesssim t(\xi_d)$ to the same displacing regions. The contribution made to the transverse displacement by each region is "enhanced" by a factor $(t/\tau)^{1/2}$. After a time t the electron negotiates $(t/\tau)^{1/2}$ mean free paths along **H**. Since the directions of the displacements in different regions are random, $R(t)$ can be calculated by multiplying $R(\tau)(t/\tau)^{1/2}$ only by the square root of this number of free paths, i.e., by $(t/\tau)^{1/4}$. We get thus

$$R(t) \sim R(\tau)(t/\tau)^{3/4}, \quad t \lesssim t(\xi_d). \tag{1}$$

In times of the order of $t \gg t(\xi_d)$ the transverse motion is diffusive. Therefore

$$D_\perp' \sim \xi_d^2/t(\xi_d).$$

The diffusion coefficients take therefore the form[1)]

$$D_\perp' \sim D_\perp (\xi_d/R(\tau))^{2/3}, \quad D_\parallel' = D_\parallel. \tag{2}$$

Their interpretation of the macroscopic equations permits a ready generalization of their result to include also microscopic defects. Such a generalization, which we in fact duplicate in the present subsection, was carried out later in Refs. 2 and 3.

C. $\xi_r \gg \xi_d R(\tau)$. At $t \lesssim t(\xi_r)$ the electron is in a trapped state. In contrast to case A, the sizes of the displacing regions are assumed larger than $R(\tau)$. Therefore, so long as $t \lesssim t(\xi_d)$, the electron is displaced in the same direction in each passage between reflections. In other words, in a time on the order of $\tau \ll t \ll (\xi_d)$ the transverse motion takes place with constant velocity:

$$R(t) \sim R(\tau)(t/\tau), \quad t \lesssim t(\xi_d). \tag{3}$$

At still longer times $t \gg t(\xi_d)$ the transverse motion becomes diffusive. Accordingly, $D'_\perp$ must be estimated as the ratio $\xi_d{}^2/t(\xi_d)$, by finding $t(\xi_d)$ from Eq. (3). We obtain thus $D'_\perp \sim D_\perp(\xi_d/R(\tau))$. The longitudinal-diffusion coefficient is estimated as in case A at $l^2/t(\xi_r)$. Knowing $D'_\perp$, we get $t(\xi_r) \sim \xi_r{}^2/D'_\perp \sim \tau(\xi_r{}^2/R(\tau)\xi_d)$, and ultimately

$$D_\perp' \sim D_\perp \frac{\xi_d}{R(\tau)}, \quad D_\parallel' \sim D_\parallel \frac{\xi_d R(\tau)}{\xi_r^2}. \tag{4}$$

D. $\xi_d \gg \xi_r R(\tau)$. This case differs from the preceding one in that (3) is valid only if $t \lesssim t(\xi_r)$ [now $t(\xi_r)$ is shorter than $t(\xi_d)$]. Over times of the order of $t(\xi_r) \lesssim t \lesssim t(\xi_d)$, however, the variation of $R(t)$ is given by Eq. (1). The diffusion regime $R(t) \propto t^{1/2}$ is reached at $t \sim (\xi_d)$. The value of $t(\xi_d)$ is obtained by successively using Eqs. (3) and (1):

$$t(\xi_d) \sim \tau (\xi_d/R(\tau))(\xi_d/\xi_r)^{1/3}.$$

To obtain $D'_\perp$ we must substitute this time in the definition $D'_\perp \sim \xi_d{}^2/t(\xi_d)$. The value of $D'_\perp$ is estimated to be the ratio $l^2/t(\xi_r)$, where $t(\xi_r) \sim \tau(\xi_r/R(\tau))$. As a result we have

$$D_\perp' \sim D_\perp \left(\frac{\xi_d}{R(\tau)}\right)^{2/3} \left(\frac{\xi_r}{R(\tau)}\right)^{1/3}, \quad D_\parallel' \sim D_\parallel \frac{R(\tau)}{\xi_r}. \tag{5}$$

We have thus discussed the cases of all possible relations between the lengths ξ_r, ξ_d, and $R(\tau)$. We consider one more situation, in which a new characteristic length comes into play. We consider a film placed in a perpendicular magnetic field. Of greatest interest here is the size effect that manifests itself when the film thickness L is much larger than the mean free path [and accordingly ξ_r, ξ_d, and $R(\tau)$]. An effect that is similar in principle was considered for macroscopic inhomogeneities of a medium by Kvyatkovskiĭ (Ref. 4).[2)] We denote by t_L the time during which the electron diffuses from surface to surface: $t_L \sim L^2/D'_\parallel$. If t_L turns out to be shorter than the time within which the transverse motion becomes diffusive, the presence of the surfaces greatly influences the value of $D'_\perp$. Since $L \gg l$ by assumption, this can happen only in cases B and D. The size effect is strong when $_L \lesssim t(\xi_d)$. Assume that this is the case. Then, obviously, in case B Eq. (1) is valid only at $t \lesssim t_L$. For time scales $t_L \lesssim t \lesssim t(\xi_d)$, however, the transverse motion appears to be uniform: $R(t) \sim R(t_L)(t/t_L)$. As a result, the time $t(\xi_d)$ decreases and becomes of the order of

$\tau(\xi_d/R(\tau))^{4/3}(L/L_{c1})^{1/2}, \quad L_{c1}\sim l(\xi_d/R(\tau))^{2/3}.$

The length L_{c1} determines the characteristic film thickness at which the size effect becomes strong. It is important that this thickness is much larger than the mean free path. Substituting the obtained value of $t(\xi_d)$ in the relation $D_\perp' \sim \xi_d{}^2/t(\xi_d)$ we get

$$D_\perp' \sim D_\perp\left(\frac{\xi_d}{R(\tau)}\right)^{2/3}\left(\frac{L_{c1}}{L}\right)^{1/2}, \quad L\leqslant L_{c1}. \tag{6}$$

This expression is valid up to values $L\sim l$, at which it goes over into Eq. (4). For the case D we obtain similarly

$$D_\perp' \sim D_\perp\left(\frac{\xi_d}{R(\tau)}\right)^{4/3}\left(\frac{\xi_r}{R(\tau)}\right)^{1/3}\left(\frac{L_{c2}}{L}\right)^{1/2},$$

$$L\leqslant L_{c2}, \quad L_{c2}\sim l(\xi_d/\xi_r)^{2/3}.$$

The characteristic film thickness differs here from that in case B. It can be seen, however, that in the region of the strong size effect the expressions for $D_\perp'$ are the same in both cases. In the derivation above it was assumed that $L\gtrsim l$. We rewrite (6) in the form $D_\perp' \sim D_\perp(\xi_d/\delta)(\xi_d/L)^{1/2}$, where δ is the characteristic value of the transverse displacement on passage through one displacing region. It is easy to verify then that this expression is valid at $L\ll l$ (up to $L\sim\xi_d$). The same asymptotic form as assumed by the dependence of $D_\perp'$ on L in cases A and C. In the former case the size effect manifests itself strongly at $L\lesssim\xi_d(\xi_d/\delta)^2$, and in the latter at $L\lesssim l$.

3. We consider now effects due to correlations of the scattering acts in a homogeneous equilibrium rarefied plasma. We use the term "plasma" for brevity to denote in general a gas of electrons scattered by charged centers. In Secs. 3–6 these centers are assumed to be immobile. Therefore the results can be used directly to describe electrons in doped semiconductors. The differences caused by the presence of moving centers (ions in a gas plasma) are considered in Sec. 7. By plasma homogeneity is meant the absence of fluctuations with a scale larger than the screening radius.

The hierarchy of the characteristic lengths in the absence of a magnetic field is the following. The largest is the mean free path $l\sim 1/nr_0{}^2\ln(r_s/r_0)$, where n is the density of the centers, $r_0\sim e^2/\varkappa\varepsilon$ ($\varkappa$ differs from unity in a semiconductor, where it constitutes the dielectric constant of the lattice), ε is the average kinetic energy of the electron, and r_s is the screening radius. The length r_0 is the shortest ($r_0\ll n^{-1/2}$). The quantity r_s satisfies the inequalities $n^{-1/3}\ll r_s\ll l$. To be specific, we assume also that the electron wavelength $(\hbar^2/m\varepsilon)^{1/2}$ is small compared with r_0 (i.e., the average electron energy is lower than the Bohr energy). If, however, this condition is not met, the wavelength must replace r_0 in the argument of the Coulomb logarithm.

The magnetic field influences strongly the transverse diffusion if the Larmor radius $r_L\lesssim l$. According to the Boltzmann equation, the diffusion coefficient $D_\perp'$ is proportional to $1/H^2$ at $r_L\ll l$. It usually assume that the same relation holds also at $r_L\lesssim r_s$ (except that a dependence on H appears in the argument of the Coulomb logarithm). It will be shown below that in fact the "$1/H^2$ law" holds only for $r_L\gg r_s/\ln(r_s/r_0)$. For smaller values of r_L it is necessary to take correlation effects into account.

We obtain the function $D_\perp'(H)$ first at $r_0\lesssim r_L\lesssim r_s$. The case $rL\ll r_0$ will be considered later.

The scattering in the plasma is collective. The number of simultaneously interacting particles is restricted by the screening. According to the usual theory, self-consistent electric-field fluctuations of all scales are of equal importance, starting with r_0 and ending with r_s (this is the cause of the Coulomb logarithm). It is natural to consider, in a strong magnetic field, separately the scattering by fluctuations with $\xi\lesssim r_L$ and $\xi\gtrsim r_L$. In the former case the influence of the magnetic field on the scattering process is insignificant. The contribution to the Boltzmann diffusion coefficient from such collisions is of the form[6]

$$D_\perp|_{\xi\lesssim r_L}\sim r_L{}^2nvr_0{}^2\ln(r_L/r_0), \quad v\sim(\varepsilon/m)^{1/2}.$$

In the opposite limit $\xi\gg r_L$ the electric field changes little along the radius of the cyclotron orbit. The scattering reduces therefore to electric drift of the orbit as a whole. A solution of the Boltzmann equation in this (drift) approximation is given in §60 of Ref. 6. The corresponding contribution to the diffusion coefficient is

$$D_\perp|_{\xi\gtrsim r_L}\sim r_L{}^2nvr_0{}^2\mathscr{L}\ln(r_s/r_L).$$

Here $\mathscr{L}$ is the large logarithm that appears following the thermodynamic averaging (for details see Ref. 7).

A decisive factor when correlations are taken into account is scattering by fluctuations having the largest scale $\xi\sim r_s$. To verify this, we turn to the results of Sec. 2 (case B). Equation (2) contains the correlation radius ξ_d, the Boltzmann diffusion coefficient $D_\perp$, and the displacement $R(\tau)$ during the mean-free path time. A phase differs from the medium considered in Sec. 2 mainly in that the fluctuations are of large scale. We consider, however, fluctuations of only one scale ξ, so that $\xi\gg r_L$. The role of ξ_d is then assumed by ξ and expression (2) is directly applicable. It remains to find $D_\perp$ and $R(\tau)$. It is convenient to introduce the drift-velocity distribution

$$\mathbf{v}_d(\mathbf{r})=c[\mathbf{E}(\mathbf{r}),\ \mathbf{H}]/H^2.$$

Here $\mathbf{E}(\mathbf{r})$ is the self-consistent electric field.[3)] For fluctuations of scale ξ the characteristic field intensity is obviously of the order of $(e/\varkappa\xi^2)(n\xi^3)^{1/2}$. We have then $D_\perp\sim v_d^2(\xi/v)$, where v_d is the drift velocity corresponding to this field, i.e., $D_\perp\sim r_L^2nvr_0^2$ (as expected, this quantity is independent of ξ). The free path time is

$$\tau\sim 1/nvr_0{}^2\ln(r_L/r_0). \tag{7}$$

(The length ξ_r introduced in Sec. 2 corresponds to $r_0\ln^{1/2}(r_L/r_0)$. Its square is of the order of the transport cross section of the Coulomb scattering at $r_0\lesssim r_L\lesssim r_s$.) The displacement is

$$R(\tau)\sim(D_\perp\tau)^{1/2}\sim r_L/\ln^{1/2}(r_L/r_0)$$

(recall that this is the drift displacement in a field of fluctuations having only one scale). Substituting the expressions obtained in Eq. (2), we obtain for the diffusion coefficient $D_\perp'$

$$D_\perp' \sim r_L^2 n v r_0^2 (\xi/r_L)^{2/3} \ln^{1/3}(r_L/r_0).$$

It can be seen that $D_\perp'$ increases with increasing ξ. This indicates that the diffusion velocity in the plasma is determined by the drift in the field of the fluctuations with the largest ξ. These are $\xi \sim r_s$. Ultimately we have

$$D_\perp' \sim r_L^2 n v r_0^2 \left(\frac{r_s}{r_L}\right)^{2/3} \ln^{1/3}\left(\frac{r_L}{r_0}\right). \qquad (8)$$

From a comparison of this expression with the Boltzmann relation we find that the correlations in the scattering become significant at $r_L \sim r_s/\ln(r_s/r_0)$. With increasing magnetic field, $D_\perp'$ decreases mainly in proportion to $1/H^{4/3}$.

The result (8) can be derived somewhat more rigorously by considering the time "scan" of the diffusion process. In accord with the drift approximation, the transverse displacement in a time t is

$$\mathbf{R}(t) = \int_0^t dt' \, \mathbf{v}_d(\mathbf{r}(t')).$$

The mean value is

$$\langle R^2(t) \rangle$$
$$= \int_0^t dt' \int_0^t dt'' \int d^3\mathbf{r}' \int d^3\mathbf{r}'' \langle G(\mathbf{r}', t'; \mathbf{r}'', t'') \mathbf{v}_d(\mathbf{r}') \mathbf{v}_d(\mathbf{r}'') \rangle,$$

where G is the probability that, given the velocity field $\mathbf{v}_d(\mathbf{r})$, the electron trajectory passes through the points $\mathbf{r}'$ and $\mathbf{r}''$ at the respective instants of time t' and t''. The angle brackets in the right-hand scale mean averaging over the distribution of $\mathbf{v}_d(\mathbf{r})$. The integral is estimated in the following fashion:

$$\langle R^2(t) \rangle \sim \int_0^t dt' \int_0^t dt'' \int_{-\infty}^{\infty} dz \, g(z, |t'-t''|; 0, 0)$$
$$\times \langle \mathbf{v}_d(0,0,0) \mathbf{v}_d(\langle R^2(|t'-t''|) \rangle^{1/2}, \quad \langle R^2(|t'-t''|) \rangle^{1/2}, z) \rangle.$$

Here $g = (4\pi D_\parallel |t'-t''|)^{-1/2} \exp(-z^2/4D_\parallel |t'-t''|)$ is the probability of landing from the point $z=0$ on the point z after a time $|t'-t''|$. This estimate is in fact an equation for $\langle R^2(t) \rangle$. At $t \gg r_s^2/D_\parallel$ the exponential in g can be set equal to unity. Using the correlator of the drift velocities

$$\langle \mathbf{v}_d(0) \mathbf{v}_d(\mathbf{r}) \rangle = \frac{2}{3} \left(\frac{c}{H}\right)^2 \langle \mathbf{E}(0) \mathbf{E}(\mathbf{r}) \rangle$$
$$= \frac{2}{3} \left(\frac{c}{H}\right)^2 \frac{4\pi n e^2}{\varkappa^2 r} e^{-r/r_s},$$

we obtain at $\tau \lesssim t \ll t(r_s)$

$$\langle R^2(t) \rangle \sim \frac{r_L^2}{\ln(r_L/r_0)} \left(\frac{t}{\tau}\right)^{3/4} \ln\left(\frac{t(r_s)}{t}\right). \qquad (9)$$

The time of transverse displacement by a distance r_s is

$$t(r_s) \sim \tau (r_s/r_L)^{4/3} \ln^{2/3}(r_L/r_0). \qquad (10)$$

Note that, compared with expression (1), the time dependence of (9) is weaker. Equation (9) contains a logarithm that decreases with increasing t. This is a manifestation of the fact that there are fluctuations with many scales in the plasma. It can be seen, however, that when the time $t \sim t(r_s)$ is reached the logarithm becomes of the order of unity. The presence of many scales plays therefore no role in the case of longer times.

At $t \gg t(r_s)$ we have $\langle R^2(t) \rangle \propto t$, and we return to expression (8) for the diffusion coefficient $D_\perp' \sim r_s^2/t(r_s)$.

Note the following important circumstances. The distribution of the electric field $\mathbf{E}(\mathbf{r})$ [and hence of $\mathbf{v}_d(\mathbf{r})$] was assumed independent of time. It is useful to bear in mind, however, that actually $\mathbf{E}(\mathbf{r})$ oscillates rapidly, owing to the temporal fluctuations of the number of electrons in a given volume. The amplitude of the oscillations is not small — it is of the order of the characteristic time-averaged value. We must track spatial $\mathbf{E}(\mathbf{r})$ fluctuations of scale r_s. They have a characteristic frequency v/r_s equal to the plasma frequency. The period of the oscillations is much shorter than the free path time. The scattering acts are thus correlated only "in the mean." When referring above to $\mathbf{E}(\mathbf{r})$, we meant namely the field averaged over the fast plasma oscillations. The justification for this averaging is the recurrence of the passage of the electrons through the same sections of the field.[4)]

4. Equation (8) was obtained under the assumption that $r_L \gtrsim r_0$. We consider now the opposite case.[5)] The picture of the diffusion remains essentially the same. All that changes is expression (7) for the free path time. The cross section for reflection from an attracting ion now turns out to be much smaller than r_0^2, on the order of $r_0^2 (r_L/r_0)^{4/3}$. This estimate can be obtained simply by making the equations of motion nondimensional. We explain this in the following manner. We denote by ρ the impact parameter of the Larmor-orbit leading center. Electrons incident on an ion with sufficiently large ρ do not contribute to the cross section. They pass through the Coulomb field without reflection. Scattering for them is a slow drift of the orbit center along a circle around the ion. The question is: how long does such a description remain valid when ρ is decreased? The required condition is that the electric field acting on the electron change little during the period of the cyclotron revolution. It takes the form $(eH/mc)\rho/v(\rho) \gg 1$ ($v(\rho) \sim (e^2/\varkappa m \rho)^{1/2}$ is the velocity acquired by the electron in the field of the center) and is met when $\rho \gg (mc^2/\varkappa H^2)^{1/3}$. This inequality can be identified also as the condition that the drift velocity $c(e/\varkappa\rho^2 H)$ be small compared with $v(\rho)$. If, however, $\rho \ll (mc^2/\varkappa H^2)^{1/3}$, then the electric force $e^2/\varkappa\rho^2$ becomes much larger than the force $eH(v(\rho)/c)$ exerted by the magnetic field. Electrons emitted with such low ρ are scattered in the Coulomb field and reflected (just as at $H=0$). Writing the reflection cross section in the form $(mc^2/\varkappa H^2)^{2/3}$, we obtain the expression given above.

Following the same reasoning that led to Eq. (8) and replacing (7) by $\tau \sim 1/nvr_0^2(r_L/r_0)^{4/3}$, we obtain at $r_L \ll r_0$ the diffusion coefficient[6)]

$$D_\perp' \sim r_L^2 n v r_0^2 (r_s/r_0)^{2/3} (r_0/r_L)^{2/9}. \qquad (11)$$

The diffusion coefficient decreases with increasing magnetic field in proportion to $(1/H)^{16/9}$. This is close to the usual quadratic dependence. We emphasize, however, that in this case $D_\perp'$ is much larger than the Boltzmann value.

5. We have considered above only scattering by positively charged ions. We turn now to scattering by negative

ones. We assume their density n_- to be lower than n_+ of the positive ions. It turns out that even rare negative ions are important in a strong magnetic field. Their diffusion coefficient is much larger than given by Eq. (11), and varies with the magnetic field like $1/H$. The cause of this effect is that the cross section for reflection by an ion at $r_L \ll r_0$ depends strongly on the sign of its charge. For a repelling ion it equals approximately r_0^2, much more than the $r_0^2(r_L/r_0)^{4/3}$ for an attracting one. It follows hence rightaway that at $r_L \ll r_0(n_-/n_+)^{3/4}$ the free path time τ is determined by scattering from the relatively rare negative ions. (We emphasize that within the framework of the Boltzmann theory this circumstance is of no importance whatever, since $D_\perp$ continues to be proportional to $1/H^2$.) The positive ions merely displace the Larmor orbit in the transverse direction. It might seem at first glance that negative ions can be present only $r_L \lesssim r_0(n_-/n_+)^{3/4}$, in which case it suffices to change the expression for τ. We would then obtain at $r_L \lesssim r_0(n_-/n_+)^{3/4}$

$$D_\perp' \sim r_L^2 n_+ v r_0^2 (r_s/r_L)^{1/3} (n_-/n_+)^{1/3}. \tag{12}$$

We shall show that this relation can hold in fact only in a relatively narrow interval of r_L:

$$r_0(n_-/n_+)^{1/2}/\ln^{1/2}(r_s/r_0) \lesssim r_L \ll r_0(n_-/n_+)^{3/4}.$$

For such an interval to exist the ratio n_-/n_+ must be larger than $1/\ln^2(r_s/r_0)$, i.e., the Coulomb logarithm must be large enough. If, however, $n_-/n_+ \ll 1/\ln(r_s/r_0)$, as H increases the negative ions become significant at $r_L \sim r_0(n_-/n_+)^{9/14}/\ln^{3/14}(r_s/r_0)$, when the time is still not bounded by them.

We denote by t_- the time of motion from one negative ion to another. So long as $r_L \gg r_0(n_-/n_+)^{3/4}$, the time is longer than $\tau - t_- \sim (1/n_- v r_0^2)^2/\tau$. (Accordingly, $R(t_-) \sim R(\tau)(t_-/\tau)^{3/4}$.) In a stronger field, the reflections occur mainly upon collision with negative ions, so that $t_- \sim \tau \sim 1/n_- v r_0^2$. Expression (12) presupposes that $R(t_-) \gg r_0$. Indeed, only then are the successive acts of reflection by negative ions independent. In the opposite case, the time t_- is too short for the electron to move across **H** to a distance larger than their reflection radius. As a result, the electron is "trapped" between a pair of negative ions.

Clearly, the condition $R(t_-) \gg r_0$ ceases to hold on further increase of H. The value of r_L at which $R(t_-)$ comes close to r_0 depends on the parameter $(n_-/n_+)\ln^2(r_s/r_0)$. It is of the order of $r_0(n_-/n_+)^{1/2}\ln^{1/2}(r_s r_0)$ (and lands in the region $r_L \ll r_0(n_-/n_+)^{3/4}$). If this parameter is large. This corresponds to the existence of the r_L interval, indicated above, in which $D_\perp'$ is described by Eq. (12). To calculate $D_\perp'$ at smaller r_L we turn to the results of Sec. 2 (case D). The multiplicity of fluctuation scales in the plasma, which distinguishes it from the model considered in Sec. 2, manifests itself in the possibility of using Eq. (5) directly for $D_\perp'$. It is nonetheless easy to generalize this equation. It was shown in Sec. 3 that the main contribution to the diffusion acceleration is made by drift in the field of the fluctuations having the largest scale $\xi_d \sim r_s$. The Boltzmann diffusion coefficient in the field of such fluctuations is $D_\perp \sim r_L^2 n_- v r_0^2$. The corresponding displacement $R(\tau) \sim (D_\perp \tau)^{1/2}$ within the free-path time $\tau \sim 1/n_- v r_0^2$ is estimated at $r_L(n_+/n_-)^{1/2}$. This is precisely the quantity that should enter in the ratio $\xi_d/R(\tau)$ in Eq. (5). It is readily understood, however, that the number of passes that an electron must make between a pair of negative ions before it leaves the "trapped" state is determined by the displacement $R(\tau)$ due to the drift in the field of the fluctuations of all scales. It exceeds the indicated value by a factor $\ln^{1/2}(r_s/r_0)$, and it is this number which must be substituted in the factor $(\xi_r/R(\tau))^{1/3}$ of expression (5) (the role of ξ_r is assumed here by r_0). We obtain thus[7]

$$D_\perp' \sim r_L^2 n_+ v r_0^2 \left(\frac{r_s}{r_L}\right)^{1/3} \left(\frac{r_0}{r_L}\right)^{1/3} \left(\frac{n_-}{n_+}\right)^{1/2} \frac{1}{\ln^{1/6}(r_s/r_0)}. \tag{13}$$

When the magnetic field is strengthened the diffusion coefficient decreases in proportion to $1/H$.

We consider now the case $(n_-/n_+)\ln^2(r_s/r_0) \ll 1$. $R(t_-)$ becomes comparable then with r_0 at $r_L \sim (n_-/n_+)^{9/14}/\ln^{3/14}(r_s/r_0)$, hence $r_L \gg r_0(n_-/n_+)^{3/4}$. Therefore the relation (12) does not appear at all. It is easily verified that Eq. (13) is valid for all $r_L \lesssim r_0(n_-/n_+)^{9/14}\ln^{3/14}(r_s/r_0)$. It was assumed in its derivation that the positive ions do not limit the free path time. In fact, however, it is valid for all cases only if $R(t_-) \lesssim r_0$ (even when the electron motion between the collisions with the negative ions is by diffusion). The point is that at $t \gtrsim t_-$ the value of $R(t)$ depends on $R(t_-)$ and t_- via the ratio $R(t_-)/t_-$. The latter, however, is expressed by identical formulas regardless of the number of times that the electron reverses its longitudinal velocity during the time t_-.

6. Assume now that the density of the charged centers is low. Let for the sake of argument the free-path time by bounded by scattering from neutral centers. We shall show that scattering by ions is nonetheless a decisive factor in the calculation of the transverse-diffusion velocity for large H. Indeed, at $r_L \lesssim r_s$ the collisions with the ions are correlated. The electron is repeatedly scattered by one and the same ion. At large H this leads again to the relation $D_\perp' \propto 1/H^{4/3}$ [see Eq. (2)]. The diffusion coefficient, which is governed by scattering from neutral centers, is on the other hand $\sim r_L^2/\tau$ (the collisions with neutral centers are assumed to be uncorrelated). It decreases with increasing H more rapidly — in proportion to $1/H^2$. In a sufficiently strong field we have therefore $D_\perp' \gg r_L^2/\tau$. The difference from the picture described in Sec. 3 is then negligible, only that the electron is now reflected by collision with neutral centers. Accordingly,

$$D_\perp' \sim r_L^2 n v r_0^2 \left(\frac{r_s}{r_L}\right)^{4/3} \frac{1}{(n v r_0^2 \tau)^{1/3}}.$$

By comparing this expression with r_l^2/τ we find the region of its validity: $r_L \lesssim r_s(n v r_0^2 \tau)$. Recall that we have assumed that $n v r_0^2 \tau \ll 1$.

7. In the preceding reasoning the charged centers were assumed at rest. This is the case, for example, for ionized impurities in semiconductors. The ions in a gas plasma are in motion. Obviously, the correlation of the scattering acts is "characterized" by this motion. The question is: How rapidly must the ions move for the Boltzmann theory to hold? In a gas plasma, the large-scale ($\xi \sim r_s$) structure of the self-consistent electric field oscillates with two frequencies, viz., the

electron plasma frequency $\Omega_e \sim v/r_s$ and the frequency $\Omega_i \sim v_i/r_s$ connected with the motion of the ions (whose thermal velocity is v_i). In a single-temperature plasma Ω_i coincides with the plasma frequency of the ions. An electron cannot return to a given ion in a time shorter than that of the mean free path. This time is comparable with the period of the field oscillations. The oscillations due to the motion of the electrons are faster, the corresponding parameter $\Omega_e \tau \gg 1$ and the averaging was therefore carried out with respect to them. The variation of a field averaged in this manner is already relatively slow, with a characteristic frequency Ω_i. It is clear therefore that the scattering correlations can be neglected (even at $r_L \lesssim r_s/\ln(r_s/r_0)$) if $\Omega_i \tau \gg 1$. Putting $\tau \sim 1/nvr_0^2 \ln(r_s/r_0)$, we rewrite this condition in the form $nr_s^3 \gg (v/v_i)\ln(v/v_i)$. It holds only for a sufficiently rarefied plasma. It must be noted, however, that under typical experimental conditions the plasma is not very strongly rarefied, so that $\Omega_i \tau$ is large enough.

Let us describe the situation at $\Omega_i \tau \ll 1$. The correlations are then important if the field is strong enough. This, however, still does not mean that the equations derived above can be used. Their applicability calls for the more stringent condition $\Omega_i t(r_s) \lesssim 1$ which becomes less valid the stronger the magnetic field. The field must therefore be strong enough to permit correlations to manifest themselves, but still weak enough to be describable by the theory expounded above. Consider the case $\Omega_i t(r_s) \gg 1$. Clearly, the correlations vanish over time scales $t \sim \Omega_i^{-1}$. The diffusion should therefore be estimated at $D'_\perp \sim R^2(\Omega_i^{-1})\Omega_i$. To calculate $R^2(\Omega_i^{-1})$ at $r_L \gtrsim r_0$ we shall use Eq. (9). As a result we get

$$D_\perp' \sim r_L^2 nvr_0^2 \frac{1}{(\Omega_i\tau)^{1/2}} \ln\left[\frac{r_s^2(\Omega_i\tau)^{3/2}}{r_L^2 nvr_0^2\tau}\right], \tag{14}$$

where $\tau \sim 1/nvr_0^2\, ln(r_L/r_0)$. The condition $\Omega_i t(r_s) \sim 1$ corresponds to the value $r_L \sim r_s(\Omega_i\tau)^{3/4}\ln^{1/2}(r_L/r_0)$. At this values, Eqs. (8) and (14) become "joined." Thus, as H is increased, the relation $D'_\perp \propto 1/H^{4/3}$ goes over into the relation $D'_\perp \propto 1/H^2$ that follows from (14). The generalization to the case $r_L \ll r_0$ is obvious.

Equation (14) describes electron diffusion in a weakly ionized plasma. The time τ is then determined by the scattering from neutral centers, so that a situation with $\Omega_i\tau \ll 1$ is easily realized. It must be borne in mind, however, that the condition for (14) to be valid in this case is satisfaction of an inequality that is very stringent in practice, viz., the quantity (14) must be larger than r_L^2/τ.

8. We discuss now the influence of the correlation effects on the longitudinal diffusion. The transverse-displacements correlations considered in Secs. 3 and 4 are immaterial. The deviation of the prolonged-diffusion velocity from that is due exclusively to "trapping" of the electrons. Obviously, the prolonged diffusion slows down in this case. To calculate $D'_\parallel$ we turn to the results of Sec. 2. The length ξ_d in the plasma corresponds to screening radius, and the length ξ_r to the radius of reflection by the ion (ξ_r^2 is of the order of the cross section for reflection). In the case of scattering by positive ions only, $R(\tau)$ is always larger than ξ_r. The "trapping" effects are then weak. The situation changes if negative ions are present. Their reflection cross section at $r_L \ll r_0$ is $\sim r_0^2$ (and is independent of H, in contrast to the cross section for reflection by a positive ion). In a sufficiently strong field the transverse displacement $R(\tau)$ becomes smaller than $\xi_r \sim r_0$. The electrons are thus "trapped."

The situation is simplest when the positive ions do nothing but displace transversely the cyclotron orbit, and the free-path time is governed by scattering from negative ions. Equation (5) for $D'_\parallel$ can then be applied directly. On this case $\xi_r \sim r_0$, $\tau \sim 1/n_- vr_0^2$ $D_\parallel \sim /vn_- r_0^2$, and $R(\tau) \sim r_L(n_+/n_-)^{1/2}\ln^{1/2}(r_s/r_0)$. Accordingly

$$D_\parallel' \sim \frac{v}{n_- r_0^2}\left(\frac{n_+ r_L^2}{n_- r_0^2}\ln\frac{r_s}{r_0}\right)^{1/2}. \tag{15}$$

The value of $D'_\parallel$ decreases with increasing field in proportion to $1/H$ (note that under these conditions all the components of the diffusion tensor have this dependence on H). Without repeating the arguments advanced at the end of Sec. 5, we indicate that Eq. (15) is valid also for diffuse longitudinal motion between collisions with negative ions. It is necessary only to have $R(t_-) \lesssim r_0$. The $D'_\parallel(H)$ dependence is thus found to be nonmonotonic. With increasing H, the value of $D_\parallel$ increases, first in proportion to $1/\ln(r_L/r_0)$ at $r_0 \lesssim r_L \lesssim r_s$, and next in proportion to $(r_0/r_L)^{4/3}$. The acceleration of the diffusion is due simply to the decrease of the cross section for scattering by the positive ion. If, however, $R(t_-)$ becomes smaller than r_0, the rate of diffusion decreases with increasing field: $D'_\parallel \propto 1/H$. This is due to the "trapping" of the electrons by the negative ions.[8] (Note that in the case $(n_-/n_+)\ln^2(r_s/r_0) \gg 1$ there exists an interval of r_L in which there is no "trapping" but the free-path time is determined by scattering from negative ions—see Sec. 5. In this case $D_\parallel$ is independent of H.)

9. At the end of Sec. 2 we considered the size effect in a film placed in a perpendicular magnetic field. To estimate its vlaue for scattering by charged centers, the time $t_L \sim L^2/D'_\parallel$ (the time of diffusion from surface to surface at a film thickness L) must be compared with the time $t(r_s)$ (which assumes the role of the time $t(\xi_d)$ of Sec. 2). The size effect is strong when $t_L \lesssim (L/t)^2$. Assume that $r_0 \lesssim r_L \lesssim r_s$. Then $t(r_s)$ is expressed by formula (10), $t_L \sim \tau(L/l)^2$, and the condition $t_L \lesssim t(r_s)$ is satisfied at $L \lesssim L_c$, where

$$L_c \sim l(r_s/r_L)^{2/3}\ln^{1/3}(r_L/r_0), \quad l \sim 1/nr_0^2\ln(r_L/r_0).$$

The dependence of $D'_\perp$ on L and on H can be obtained by resorting to Eq. (6), which is best rewritten here in the form

$$D_\perp' \sim D_\perp^{1/2}(vr_s^2/L)^{1/2} \tag{16}$$

(in accordance with the results of Sec. 3 it is necessary to substitute here the Boltzmann diffusion coefficient D in the case of scattering by fluctuations of only one scale $\sim r_s$: $D_\perp \sim r_L^2\, nvr_0^2$). With decreasing L, the value of $D'_\perp$ increases in proportion to $1/L^{1/2}$ and is inversely proportional to the magnetic field. Since (16) does not contain the cross sections for reflection by the ion, the same field dependence remains in force also if $r_L \lesssim r_0$. All that changes is the equation for the characteristic thickness of the film, viz., we get $L_c \sim l(r_s/r_L)^{2/3}$, where $l \sim 1/nr_0^2(r_L/r_0)^{4/3}$. We emphasize the unusual character of this effect. It manifests itself when the film thickness exceeds greatly the mean free path (notwithstanding the assumption that this film is macroscopically homogeneous).

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[1] In fact, the type of transverse motion described in this subsection is similar to that first observed in Ref. 1. There Dreĭzin and Dykhne investigated the magnetoresistance of a medium with macroscopic inhomogeneities having dimensions much larger than l. Their interpretation of the macroscopic equations permits a ready generalization of their result to include also microscopic defects. Such a generalization, which we in fact duplicate in the present subsection, was carried out later in Refs. 2 and 3.

[2] We note that Dreĭzin and Dykhne[1,5] also described a size effect that manifests itself at large L (they investigated the magnetoresistance of polycrystals with open Fermi surfaces). While it is not due to scattering-event correlations, it is caused, as in our case, by the long time required for the diffusion across **H** to set in.

[3] Of course, the concept "drift velocity at a point" is meaningful if the characteristic correlation radius of the field is $\xi \gg r_L$.

[4] In the expressions derived, the diffusion coefficient was determined by electrons of approximately average energy. Yet it follows from the results of Dreĭzin and Dykhne[2] that in a strong magnetic field the main contribution to $D_\perp'$ should be made by electrons that move very slowly along **H**. These are just electrons whose energy of motion along **H** is of the order of the characteristic amplitude of the fluctuation potential γ (but higher than the percolation threshold). These electrons, however, constitute a small fraction of the total, and according to Ref. 2 they diffuse rapidly across **H**.
The derivation of Ref. 2 is based on the premise that a slow electron moving along **H** with energy $\sim\gamma$ turns out to be trapped in the fluctuation well and its transverse motion becomes drift-like. Our picture differs in principle from that described by Dreĭzin and Dykhne. The fluctuation potential oscillates rapidly with time (owing to the electron motion). In other words, the electrons rapidly exchange energy with one another. It can be *easily* verified that the time in which an electron acquires an energy $\gamma \sim (e^2/\varkappa r_s)(nr_s^3)^{1/2}$ is of the order of the plasma-oscillation period. This time is so short that no trapped state is realized.

[5] We emphasize once more that we confine ourselves to classical (nonquantizing) magnetic fields. The Larmor radius can then be comparable with r_0 only if ε is lower than the Bohr energy. We note also that for our analysis to be valid the characteristic amplitude of the self-consistent potential must be small compared with ε. These two conditions can be satisfied simultaneously if the parameter na^3 is small enough (a is the Bohr radius).

[6] In so strong a magnetic field the electrons collide with one another much more frequently than with ions. In Eq. (11), however we took τ to mean the time between ion-ion collisions. The reason is that the longitudinal-diffusion collisions of electrons that interact only with one another and not with ions becomes in fact infinite. Conversely, the coefficient of diffusion of such electrons across **H** is strictly equal to zero. Thus, electron-electron scattering does not lead by itself to diffusion. Its influence manifests itself only in the form of the electron distribution function, and it does not alter the estimates of $D_\perp'$.

[7] Expression (13) can be derived also in another manner, by considering the "time scan" of the diffusion procedure, in analogy with the procedure used to derive (9). The only difference from the derivation of (9) is that it is now necessary to use the value (15) for the diffusion coefficient (see below).

[8] In the derivation of (15) it was assumed that the electrons incident on a negative ion with impact parameters smaller than $e^2/\varkappa E$ (E is the energy of longitudinal motion) are totally reflected. The possibility of tunneling was disregarded. Yet tunneling enables electrons to leave the "trapped" state even without transverse displacements. In a sufficiently strong magnetic field this circumstance is decisive. A simple calculation of the longitudinal-diffusion coefficient yields in this case $D_\parallel' \sim D_\parallel \times \langle \exp(-16(E_B/E)^{1/2})\rangle$ ($D_\parallel$ is the Boltzmann diffusion coefficient, E_B is the Bohr energy, and the angle brackets denote thermodynamic averaging). Recall that in a nonquantizing magnetic field the Larmor radius may turn out to be smaller than r_0, but only when the electron energy is less than E_B.

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