Effect of "non-local" dissipation on the quantum properties of systems in a potential with degenerate minima

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We consider the effect of dissipation on the low-temperature behavior of two-dimensional and three-dimensional systems in a periodic potential. We show that as $T \to 0$ in all those systems, both in strong and in weak potentials, dissipation may lead to localization of the system while the mobility in the transition point vanishes discontinuously. We construct phase diagrams and find the low-temperature and low-frequency asymptotic behavior of the correlation functions. We also show that an external uniform field destroys the localization.

I. INTRODUCTION

Recently great attention has been paid to the problem of the effect of dissipation on quantum tunneling and such related quantum effects as: 1) the decay of a metastable state,[1–3] and 2) quantum diffusion in potentials with degenerate minima.[4–7] Of most interest in this has been the often encountered case when the dissipation in the classical region can be described in the framework of a Langevin equation with a phenomenological friction coefficient η which is independent of the frequency and depends weakly on the temperature. Such a description is widely used to take into account the effect of the interaction with the medium or with the other degrees of freedom on the motion of various coherent particle-like excitations.[8] For instance, these may be charge density waves in an external potential, fluxons in Josephson junctions, and many others. It was shown in Ref. 1 by using a simple model which describes the decay of a metastable state, when there is interaction with a thermostat consisting of a large set of oscillators with different frequencies, that when the system moves in the subbarrier region in imaginary time $\tau = it$ the dissipation is effectively described by an integral operator which is non-local in τ and which has a kernel whose decrease is determined by the low-frequency part of the spectral density $\rho(\omega)$ of the oscillators of the thermostat. The operator has a slowly decreasing kernel if the case when a Langevin equation with a frequency-independent η is applicable. A similar non-local dissipative term in the effective interaction was later derived from a microscopic theory of a Josephson junction.[2,3] It was shown in Refs. 1 to 3 that the dissipation decreases the probability of the decay of a metastable state at $T = 0$ and in Ref. 3 the dependence of the decay probability on T was studied. The effect of dissipation on class 2) of quantum properties turns out to be stronger. In Refs. 4 and 5 it is shown that a sufficiently strong dissipation may disrupt at low T the symmetry of the probability of finding the system in different degenerate minima. Finally, in the case of a system in a periodic potential, dissipation as $T \to 0$ may lead to a localization of the system while the statistical mobility vanishes jumpwise.[6,7] The phase transitions which occur in the last two cases turn out to be analogous to the phase transitions in one-dimensional ln gases.[9,10]

All results mentioned here were obtained for quantum systems with a single distinguished degree of freedom. In the present paper we consider the behavior of systems with several distinguished degrees of freedom in periodic potentials at low T. First of all we study the symmetric case which reduces to the problem of the motion of a d-dimensional particle in a crystallographic medium, and then we study the effect of an external uniform field $\mathbf{F}$ on the results. Such systems can describe the motion of particle-like formations in the modulated field of a crystalline medium or substrate, the motion of a particle injected into a crystal, and, when there is some well defined form of anisotropy present, of a sequence of Josephson junctions.

The whole study is performed using the path integral method with imaginary time τ. We shall consider the case of strong and weak potentials and find the low-temperature behavior of the main parameters of the system, the low-frequency asymptotic behavior of some correlation functions, and phase diagrams of the systems.

II. EFFECTIVE ACTION FOR SYSTEMS WITH SEVERAL DEGREES OF FREEDOM

We consider a generalization of the phenomenological Caldeira-Leggett model to the case of several degrees of freedom. Let there be a quantum-mechanical system with d degrees of freedom q_i, $i = 1, 2, \ldots, d$, which is in a bare potential $V_0(q_1, \ldots, q_d)$ and which interacts linearly with a set of oscillators x_k of a thermostat:

$$H = H_1 + H_T + H_I,$$
$$H_1 = \sum_{i=1}^{d} \frac{p_i^2}{2M_i} + V_0(q_1, \ldots, q_d),$$
$$H_T = \sum_k \left(\frac{p_k^2}{2m} + \frac{m\omega_k^2 x_k^2}{2} \right), \qquad (1)$$
$$H_I = \sum_{ik} C_{ki} q_i x_k,$$

where k runs through the number of the oscillators. In the symmetric case all parameters are independent of the index i: $M_i = M$, $C_{ki} = C_k$ and H describes a particle with coordinates $\mathbf{q} = (q_1, \ldots, q_d)$ in a d-dimensional space moving in a

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potential $V_0(\mathbf{q})$ and interacting with a thermostat. The partition function of the system (1)

$$Z=\mathrm{Sp}\,\rho=\mathrm{Sp}\,e^{-H/T} \tag{2}$$

can be written as a path integral over all closed trajectories $q_i(\tau)$, $x_k(\tau)$ with an imaginary time τ:[11]

$$Z=\int\prod_{ik} Dq_i\,Dx_k\,e^{-S},$$

$$S=S_1+S_T+S_I,\ S_1=\int_0^{1/T}d\tau\left[\sum_{i=1}^{d}\frac{M\dot q_i^2}{2}+V_0(q_1,\ldots,q_d)\right], \tag{3}$$

$$S_T=\int_0^{1/T}d\tau\sum_k\left(\frac{m\dot x_k^2}{2}+\frac{m\omega_k^2x_k^2}{2}\right),\qquad S_I=\sum_{ki}\int_0^{1/T}d\tau\,C_{ki}q_ix_k.$$

By virtue of the linearity of the interaction one can explicitly integrate over x_k. As a result we obtain for Z a path integral with an effective action S_{eff} depending only on the q_i (Ref. 11) (in what follows we drop the factor Z_T which is equal to the partition function of the thermostat):

$$Z=\int\prod_i Dq_i\,e^{-S_{\text{eff}}},\qquad S_{\text{eff}}=S_0+S_D'+S_{int}', \tag{4}$$

$$S_0=\frac{1}{2}\int_0^{1/T}d\tau\sum_{i=1}^{d}M_i\dot q_i^2,\qquad S_{int}'=\int_0^{1/T}d\tau\,V_0'(q_1,\ldots,q_d),$$

$$S_D'=-\int_0^{1/T}\!\!\int d\tau\,d\tau'\,q_i(\tau)\,q_j(\tau')D_{ij}(\tau-\tau'), \tag{5}$$

$$D_{ij}(\tau)=\sum_k\frac{C_{ki}C_{kj}}{2m\omega_k}\,\frac{\mathrm{ch}[\omega_k(1/2T-|\tau|)]}{\mathrm{sh}(\omega_k/2T)}.$$

Changing to a continuous set of oscillators and following Ref. 1 we select the low-frequency behavior of its spectral density from the condition that in the classical region for $T\gg\Omega$ (Ω is a characteristic frequency in the oscillator spectrum) the Fokker-Planck[1] or Langevin[12] equations be reproduced

$$C_{\omega i}C_{\omega j}\rho(\omega)=(2m/\pi)\,\eta_{ij}\omega^2 f(\omega/\Omega), \tag{6}$$

where η_{ij} is the phenomenological friction tensor which we assume to depend weakly on T, while $f(\omega)$ is a function which regularizes the behavior of $\rho(\omega)$ at high frequencies: $f(0)=1$, $f(\infty)=0$. Substituting (6) into (5) we get for $D_{ij}(\tau)$ at $\tau\gg\Omega^{-1}$ the following expression:

$$D_{ij}(\tau)\approx{}^1\!/_2\pi T^2\eta_{ij}\sin^{-2}(\pi\tau T), \tag{7}$$

which as $T\to0$ becomes

$$D_{ij}(\tau)\approx(\eta_{ij}/2\pi)\,\tau^{-2}. \tag{8}$$

The S_D' obtained here is not translationally invariant since the initial H is not. However, Caldeira and Leggett[1] have shown that a correct dissipative term can be obtained if one takes into account the renormalization of the potential $V_0(q_i)$. Splitting for this purpose S_D' into a translationally invariant part S_D and a local counterterm:

$$S_D'=S_D+\Delta S,$$

$$S_D=-\int_0^{1/T}d\tau\,d\tau'\,q_i(\tau)\,q_j(\tau')\,\bar D_{ij}(\tau-\tau'), \tag{9}$$

$$\Delta S=\int_0^{1/T}d\tau\,q_i(\tau)\,q_j(\tau)D_{ij}=\int_0^{1/T}d\tau\,\Delta V(q_i),\ D_{ij}=\int_0^{1/T}d\tau\,D_{ij}(\tau),$$

$$\bar D_{ij}(\tau)=D_{ij}(\tau)-TD_{ij}$$

and requiring that the renormalized potential equal the physical one:

$$V(q_1,\ldots,q_d)=V_0'(q_1,\ldots,q_d)+\Delta V(q_1,\ldots,q_d),$$

we get for the partition function of the system, in terms of the effective section, the Caldeira-Leggett expression:

$$Z=\int\prod_i Dq_i\,e^{-S_{\text{eff}}},\qquad S_{\text{eff}}=S_0+S_D+S_{int}, \tag{10}$$

$$S_{int}=\int_0^{1/T}d\tau\,V(q_1,\ldots,q_d).$$

It follows from (10), (9), and (7) that the dissipation in S_{eff} is described by the term S_D with a non-local slowly decreasing kernel $\bar D_{ij}(\tau)$. We note that now S_D remains well defined even if we assume (7) to be valid for all τ. We shall in what follows work with a kernel $D_{ij}(\tau)$ in the form (7). If the tensor $\eta_{ik}=\eta_i\delta_{ik}$ we have $D_{ij}(\tau)=\delta_{ij}D_j(\tau)$ and (10) describes a system for which each degree of freedom has its own friction coefficient η_i. One obtains a similar expression for S_D if one assumes that each distinguished degree of freedom interacts only with its own thermostat. In the isotropic case $\eta_{ik}=\eta\delta_{ik}$, $D_{ij}(\tau)=\delta_{ij}D(\tau)$ and (10) reduces to the partition function of a d-dimensional particle in a potential $V(q)$ and in a medium with a non-local interaction

$$Z=\int D\mathbf{q}\,e^{-S_{\text{eff}}},\qquad S_{\text{eff}}=S_0+S_D+S_{int}, \tag{11}$$

$$S_D=-\int_0^{1/T}d\tau\,d\tau'\,(\mathbf{q}(\tau)\mathbf{q}(\tau'))\bar D(\tau-\tau')$$

$$=\frac{1}{2}\int_0^{1/T}d\tau\,d\tau'\,(\mathbf{q}(\tau)-\mathbf{q}(\tau'))^2D(\tau-\tau').$$

It is convenient to consider jointly the quadratic part of S_0+S_D right a way:

$$S_0+S_D=\frac{1}{2}\int_0^{1/T}d\tau\,d\tau'\,(\mathbf{q}(\tau)\mathbf{q}(\tau'))\,\Box(\tau-\tau'), \tag{12}$$

$$\Box(\tau)=\Box_0(\tau)+\Box_D(\tau)=-\left[M\frac{d^2}{d\tau^2}\delta(\tau)+\eta\pi T^2\sin^{-2}(\pi\tau T)\right].$$

Its Fourier transform has the form

$$\Box(\omega_n)=\int_0^{1/T}d\tau e^{i\omega_n\tau}\Box(\tau)=M\omega_n^2+\eta|\omega_n|,\qquad \omega_n=2\pi nT, \tag{13}$$

and, hence, its dissipative part $\Box_D$ plays the main role as

$\omega \to 0$ in determining the asymptotic properties of the system (11). We note that the appearance of $|\omega_n|$ in (13) correspond to the general rule for the analytical continuation of Fourier transforms of retarded correlators when one changes from the real time t to an imaginary one:[13]

$$f_R(\omega) \to f_M(i|\omega_n|). \tag{14}$$

To within a constant, the reciprocal kernel $\Box^{-1}(\tau)$ is not uniquely determined in view of the zero eigenvalue ($n = 0$). It can be fixed by the periodicity condition $\Box^{-1}(\tau) = \Box^{-1}(\tau + T^{-1})$. As a result we get for $\Box^{-1}(\tau)$ the expression

$$\Box^{-1}(\tau) = T\sum_{n=-\infty}^{\infty} e^{-i\omega_n\tau}\Box^{-1}(\omega_n)$$

$$= T\sum_{n\neq 0} e^{-i\omega_n\tau}[M\omega_n^2 + \eta|\omega_n|)^{-1} + (\ln 2)/\pi\eta, \tag{15}$$

which has the following asymptotic behavior ($1/T \gg \tau \gg \tau_0 = M/\eta$):

$$\Box^{-1}(\tau) \approx -\frac{1}{\pi\eta}\ln|\sin\pi\tau T|, \quad \Box^{-1}(\tau) \underset{T\to 0}{\approx} -\frac{1}{\pi\eta}\ln|\pi\tau T|. \tag{16}$$

III. A PARTICLE IN A CRYSTALLINE FIELD

1. Shape of the potential

We consider the motion, described by the action (11), of a particle in a crystal with friction. For the sake of clarity we restrict ourselves to simple lattices. We denote the set of basis vectors by $\{\mathbf{a}_i\}$, $i = 1,\ldots,d$ and the set of vectors connecting lattice points with nearest neighbors by $\{\mathbf{Q}_s\}$, $s = 1,\ldots,r$, where

$$|\mathbf{Q}_s| = Q = \min|\mathbf{a}_i|, \quad \mathbf{Q}_s = \sum n_i^s\mathbf{a}_i, \quad n^s = (n_1^s,\ldots,n_d^s),$$

the n_i^s are integers. We can expand the lattice potential in the general case in a series

$$V(\mathbf{q}) = \sum_{\mathbf{n}} g_{\mathbf{k_n}} e^{i\mathbf{q}\mathbf{k_n}}, \quad \mathbf{n} = (n_1,\ldots,n_d), \quad \mathbf{k_n} = \sum n_i\mathbf{b}_i, \tag{17}$$

where the n_i are integers and the $\mathbf{b}_i$ are the reciprocal lattice basis vectors such that $(\mathbf{a}_i\cdot\mathbf{b}_k) = 2\pi\delta_{ik}$. We shall in what follows assume (we call that the isotropic case) that

$$|\mathbf{a}_i| = a_0, \quad |\mathbf{b}_i| = b, \quad g_{\mathbf{k_n}} = g_{|\mathbf{k_n}|}. \tag{18}$$

We also introduce a set of reciprocal lattice vectors with minimum length $\{\mathbf{k}_s\}$, $s = 1,\ldots,\bar{r}$, $|\mathbf{k}_s| = |\mathbf{b}_i| = b$, $\mathbf{k}_s = \Sigma n_i^s\mathbf{b}_i$. It is then convenient, if we restrict ourselves in (17) to only harmonics with minimal $|\mathbf{k}_n|$ to write (17) for the case of a weak potential in the form

$$V(\mathbf{q}) = -g\sum_{\{\mathbf{k}_s\}} e^{i\mathbf{q}\mathbf{k}_s}, \quad -g \equiv g_{|\mathbf{k}_s|}, \tag{19}$$

and in the case of a strong potential in the form

$$V(\mathbf{q}) = g\sum_{\{\mathbf{k}_s\}}(1 - \cos(\mathbf{k}_s\mathbf{q})), \quad -g \equiv g_{|\mathbf{k}_s|} = -g_0\bar{r}^{-1}. \tag{20}$$

The minima of $V(\mathbf{q})$ are situated in the points

$$\mathbf{q_m} = \sum m_i\mathbf{a}_i, \quad \mathbf{m} = (m_1,\ldots,m_d), \quad m_i \in Z. \tag{21}$$

It follows from (19), (20) that if $\{\mathbf{k}_s\}$ can be decomposed into mutually orthogonal subsets $\{\mathbf{k}_{s\alpha}\}$ all equations can be split by suitable choice of coordinates into independent equations in each of the subspaces of the corresponding sets $\{\mathbf{k}_{s\alpha}\}$. In what follows we shall therefore consider only irreducible sets $\{\mathbf{k}_s\}$. The two- and three-dimensional spaces contain from the reducible square, cubic, and hexagonal lattices the following simple Bravais lattices (**i**-, **j**-, **k**- unit vectors): $d = 2$; a triangular lattice reciprocal to it and also triagonal,

$$r = \bar{r} = 6, \quad \{\mathbf{Q}_s\}_{\rm tr} = a_0\left\{\pm\mathbf{i}, \pm\frac{1}{2}\mathbf{i} \pm \frac{3^{1/2}}{2}\mathbf{j}\right\},$$

$$\{\mathbf{k}_s\}_{\rm tr} = \left(\pm\mathbf{j}, \pm\frac{3^{1/2}}{2}\mathbf{i} \pm \frac{1}{2}\mathbf{j}\right); \tag{22}$$

$d = 3$; and the mutually reciprocal body- and face-centered cubic (BCC and FCC) lattices for which the sets $\{\mathbf{Q}_s\}$ and $\{\mathbf{k}_s\}$ can be taken in the form

$$\{\mathbf{Q}_s\}_{\rm bcc} = \frac{a_0}{3^{1/2}}(\pm\mathbf{i}\pm\mathbf{j}\pm\mathbf{k}) \quad (r = 8),$$

$$\{\mathbf{k}_s\}_{\rm bcc} = \frac{1}{2^{1/2}}(\pm\mathbf{i}\pm\mathbf{j}, \pm\mathbf{i}\pm\mathbf{k}, \pm\mathbf{j}\pm\mathbf{k}) \quad (\bar{r} = 12),$$

$$\{\mathbf{Q}_s\}_{\rm fcc} = a_0\{\mathbf{k}_s\}_{\rm bcc} \qquad \{\mathbf{k}_s\}_{\rm fcc} = \frac{1}{a_0}\{\mathbf{Q}_s\}_{\rm bcc}. \tag{23}$$

Here and henceforth to simplify the formulae we put $|\mathbf{b}_i| = 1$ so that q becomes dimensionless, and $a_0 = 2\pi\cos\gamma$ where γ is the angle between $\mathbf{a}_i$ and $\mathbf{b}_i$.

The partition function of the problem of a particle with the action (11) and $V(\mathbf{q})$ from (19), (20) is exactly the same as the partition function of a one-dimensional ln gas with generalized (isovector) charges $\{\mathbf{k}_s\}$.[10] The sets $\{\mathbf{k}_s\}$ from (22), (23) are particular cases of the charge spaces $M = \{\mathbf{k}_s\}$ considered in Ref. 10:

$$\{\mathbf{k}_s\}_{\rm tr} = M_{z_6}, \quad \{\mathbf{k}_s\}_{\rm bcc} = M_{A_3} = M_{D_3}, \quad \{\mathbf{k}_s\}_{\rm fcc} = M_{P_4} + (-M_{P_4}),$$

where M_{P_4} is a set of vectors directed towards the vertices of a tetrahedron, while the reciprocal temperature β of the ln gas is connected wi[illegible]he friction coefficient η through the relation

$$\beta = 1/\pi\eta.$$

2. Weak potential

The renormalization group (RG) method was used in Ref. 10 to study the properties of ln gases in the case of a weak potential and as $T \to 0$. The condition for the potential to be weak has the form $gM/\eta \ll 1$, where $M/\eta = \tau_0$ is that length over which the logarithmic asymptotic behavior of $\Box^{-1}(\tau)$ is cut off. The RG equations have in the general case for dimensionless charges $u = g\tau_0$, $v = M/\tau_0$,

$x = 1 - (2\pi\eta)^{-1}$ the form (up to the first non-vanishing non-linearities, $dl = d\tau_0/\tau_0$)

$$\frac{du}{dl} = xu + B_n u^n, \quad \frac{dv}{dl} = -v + C_{\{k\}} u^2, \quad \frac{dx}{dl} = 0, \qquad (24)$$

where $n = 2$ or 3 and the coefficients $C_{\{k\}}$, B_n depend on the geometry of the set $\{\mathbf{k}_s\}$. If amongst the angles between the vectors $\mathbf{k}_s \in \{\mathbf{k}_s\}$ there are some equal to $\pi/3$ and $2\pi/3$, then $B_2 > 0$; if there are no such angles, then $B_2 = 0$ and the behavior of the charges is determined by the sign of B_3. The triangular and the BCC lattices belong to the first kind and all hypercubic and FCC lattices belong to the second kind. The values of the coefficients $C_{\{k\}}$, B_2, B_3 for the lattices (22), (23) are the following (close to the critical point $x = 0$): $B_2 = \Theta(1 - x)$, where Θ is the multiplicity of the representation of each vector $\mathbf{k}_s \in \{\mathbf{k}_s\}$ in the form $\mathbf{k}_s = \mathbf{k}_{s1} + \mathbf{k}_{s2}$, $\mathbf{k}_{s1,s2} \in \{\mathbf{k}_s\}$; $C_{\{k\}} = N_{\{k\}}/3\pi\eta$, where $N_{\{k\}}$ depends on the geometry of $\{\mathbf{k}_s\}$ and is determined by the condition

$$\sum_{\{k_s\}} k_i^s k_j^s = N_{\{k\}} \delta_{ij},$$

and their values are the following: triangular lattice: $\Theta = 2$, $N_{\text{tr}} = 3$; BCC: $\Theta = 4$, $N_{\text{BCC}} = 4$; FCC: $\Theta = 0$, $N_{\text{FCC}} = \frac{8}{3}$, $\beta_3 = 16 \cdot \frac{43}{15}$ It follows from (24) that in our approximation the friction coefficient η (or x) is not renormalized and the behavior if the charge v for u, $v \ll 1$ is determined by the behavior of the charge u which depends on the quantity x. It was shown in Ref. 10 that as $T \to 0$ the system can be in two phases and the line separating the phases in the phase diagram depends on the geometry of $\{\mathbf{k}_s\}$. For $\{\mathbf{k}_s\}$ belonging to the first kind or the second kind with $B_3 > 0$ the phase diagram is as shown in Fig. 1, and for those belonging to the second kind with $B_3 < 0$ as shown in Fig. 2 (to the latter class belongs also the one-dimensional case[7]). The phases A_{II}, B_{II} in these diagrams correspond to localized states and the phases A_1, B_1 to delocalized ones, as will be shown below. The shape of the line separating the phases B_1 and B_{II} close

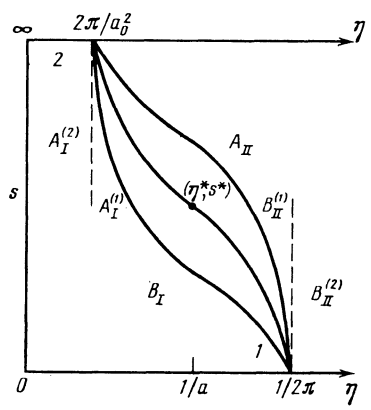

FIG. 1. Upper curve: phase-separation line in a FCC lattice, middle curve: in a self-dual triangular lattice, lower curve: in the BCC lattice dual to the FCC lattice.

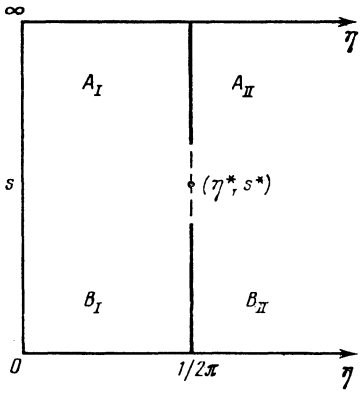

FIG. 2. Phase diagram for systems with coefficients $B_2 = 0$, and $B_3 < 0$.

to $\eta_c = \frac{1}{2}\pi$ is given by the expression

$$s = 8\cos\gamma \left[N_{\{k\}} \eta_c \left(\frac{\Delta\eta}{B_n} \right)^{1/n-1} \right]^{1/2}, \quad \Delta\eta = \frac{\eta - \eta_c}{\eta_c}. \qquad (25)$$

We note that the behavior of the phase boundary in the intermediate region $s \sim 1$ for the BCC and FCC lattices is not at all determined: as there is no self-duality, the phase boundaries continued from different regions could intersect at an angle and not go over smoothly into one another. We have drawn in Fig. 1 a smoothed variant which is the most probable one.

For finite but rather low $T > 0$, $T \ll g$ we can use Eq. (24), but now the renormalization must be discontinued at a scale l_m determined by the smallest of the two characteristic "lengths" of the problem: the correlation length l^* and the temperature length l_T;

$$l_m = \min[l_T, l^*], \quad l_T = \ln\left(\frac{1}{T\tau_0} \right), \quad l^* = \ln \xi/\tau_0.$$

Using the results of Ref. 10 we get for the low-temperature dependence of the renormalized amplitude of the potential $g_R(T)$ the following expressions:

For potentials with $\{\mathbf{k}_s\}$ of the first type or of the second type with $B_3 > 0$: a) in the region B_{I}

$$g_R(T) = \bar{g} e^{-l_T} [1 + \exp[(n-1)|\Delta\eta|l_T] \left(\frac{\bar{g}}{g_0} \right)^{n-1} (\Delta_- g^{n-1})]^{-1/(n-1)}, \qquad (26)$$

$$\Delta_\pm g^n \equiv \frac{\bar{g}^n \pm g_0^n}{\bar{g}^n}, \quad \bar{g} = \frac{\eta}{M} \left(\frac{\Delta\eta}{B_n} \right)^{1/(n-1)},$$

where g_0 is the initial value of g or, in the limiting cases

$$l_T \gg l_{\Delta_-}^{(n-1)} \text{ and } l_T \ll l_{\Delta_-}^{(n-1)},$$

where

$$l^{(n-1)} = [(n-1)|\Delta\eta|]^{-1} \ln(\Delta_- g^{(n-1)}), \qquad (26a)$$

$$g_R(T) = \begin{cases} g_0 e^{-l_T} = g_0(\tau_0 T), & l_T \ll l_{\Delta_-}^{(n-1)}, \\ g_0 (\Delta_- g^{n-1})^{-\frac{1}{n-1}} (\tau_0 T)^{1/2\pi\eta}, & l_T \gg l_{\Delta_-}^{(n-1)}; \end{cases} \qquad (26b)$$

b) in the region B_{II}

$$g_R(T)=Cg_0e^{-l_m},\quad l_m=\min[l_T,\ l^*], \tag{27}$$

where $C>1$ is a constant of order 1 determining the value of l^* from which we find ξ; in the case l^* is independent of C only near the critical separatrices 1,2 and

$$l^*=\frac{1}{n-1}\begin{cases}|\Delta\eta|^{-1}\ln\left|\dfrac{1}{\Delta^-g_0^{n-1}}\right|, & (B_{II}^{(1)}),\\ (\Delta\eta)^{-1}\ln\dfrac{1}{\Delta^+g_0^{n+1}}, & (B_{II}^{(2)}),\end{cases} \tag{28}$$

$$\Delta^{\pm}g_0{}^n\equiv\frac{\bar g^n\pm g_0{}^n}{g_0{}^n}.$$

For potentials with $\{\mathbf{k}_s\}$ of the second kind with $B_3<0$: a) in the region B_I

$$g_R(T)=\bar g e^{-l_T}[(\Delta_-g^2)e^{2|\Delta\eta|l_T}-1]^{-1/2}, \tag{29}$$

whence in the limiting cases $l_T\gg l_{\Delta_-}^{(2)}$ and $l_T\ll l_{\Delta_-}^{(2)}$

$$g_R(T)=g_0\begin{cases}(\tau_0T), & l_T\ll l_{\Delta_-}^{(2)}, \quad (29a)\\ (\Delta_-g^2)^{-1/2}(\tau_0T)^{1/2\pi\eta}, & l_T\gg l_{\Delta_-}^{(2)}; \quad (29b)\end{cases}$$

b) in the region B_{II}

$$g_R(T)=Cg_0e^{-l_m}, \tag{30}$$

$$l_m=\min[l_T,l^*],\ l^*=\begin{cases}\dfrac{1}{2\Delta\eta}\left[C'+\ln\left(1-\dfrac{\bar g^2}{g_0^2}\right)\right], & g_0>\bar g,\\ \dfrac{1}{\Delta\eta}\ln\left(\dfrac{\bar g}{g_0}\right), & g_0\ll\bar g.\end{cases} \tag{31}$$

To determine the low-temperature dependence of the correlation functions we need also the temperature dependence of the renormalized mass $M_R(T)$. From (27) we get

$$M_R(T)=M_0+C_{(k)}\tau_0\int_0^{l_m}dl'u^2(l')e^{l'},$$

where M_0 is the initial value of M, whence we have in the region B_I in the limit (26a)

$$M_R(T)\approx C_{(k)}(g_0\tau_0)^2(\Delta_-g^{n-1})^{-2/n-1}(\tau_0T)^{2|\Delta\eta|}T^{-1}. \tag{32}$$

The asymptotic forms (at low frequencies $|\omega_n|$) of the Matsubara correlator

$$\langle q_i(\tau)q_j(\tau')\rangle=\delta_{ij}T\sum_{\omega_n}e^{i\omega_n(\tau-\tau')}\langle qq\rangle(\omega_n)$$

and mobility

$$\mu(\omega_n)=|\omega_n|\langle qq\rangle(\omega_n)$$

have the form

$$\langle qq\rangle(\omega_n)\approx[g_R(T)+\eta|\omega_n|+M_R(T)\omega_n{}^2]^{-1},$$
$$\mu(\omega_n)\approx|\omega_n|[g_R(T)+\eta|\omega_n|+M_R(T)\omega_n{}^2]^{-1}. \tag{33}$$

Formulae (33) are valid for frequencies $\omega>T$. When $\omega<T$ we assume in accordance with the RG reasoning that the correlator is given by (33) with ω replaced by T. According to the rule (14) [T in (33) to (36) must be taken to mean $2\pi T$] we get

$$\mu(\omega)=\frac{\eta\omega^2-i\omega(g_R-M_R\omega^2)}{(g_R-M_R\omega^2)^2+\eta^2\omega^2},\quad \omega\gtrsim T,$$
$$\mu(\omega)=\frac{\eta T^2-iT(g_R-M_RT^2)}{(g_R-M_RT^2)^2+\eta^2T^2},\quad \omega<T. \tag{34}$$

It follows from (26) to (32) that when $T<T^*=\xi^{-1}$ the temperature dependence of g_R and of M_R become different in different regions of the phase diagram. The static mobility $\mu_{st}=\lim_{\omega\to0}\mathrm{Re}\,\mu(\omega)$ therefore begins to differ in the regions B_I and B_{II} and when $T\ll T^*$ we get

$$\mu_{B_I}\approx\frac{1}{\eta}\left(1-\left(\frac{g_R(T)}{\eta T}\right)^2\right)\approx\frac{1}{\eta}[1-O(T^{2\Delta\eta})],$$
$$\mu_{B_{II}}\approx\eta\left(\frac{g_R(T^*)}{T}\right)^{-2}\propto T^2. \tag{35}$$

In the limit as $T\to0$ we get from (35) change in μ_{st} in the form of a jump equal to

$$\Delta\mu=1/\eta(s), \tag{36}$$

where $\eta(s)$ is the function at the phase boundary. This effect is similar to the jump in the density ρ_s of the superfluid liquid in thin films of ^{4}He or of the XY model.[14] We have thus shown that as $T\to0$ dissipation may lead to particle localization in multi-dimensional spaces with a discontinuous change of the static mobility μ_{st}.

3. Strong potential (quasi-classical region)

One can also study the properties of the system (11) in the region where the quasi-classical approximation is applicable. To do this we must find all classical trajectories with period $1/Tn$ ($n\gtrsim1$) of the appropriate Euler equations:

$$\frac{\delta S_{\text{eff}}}{\delta\mathbf{q}(\tau)}=M\ddot{\mathbf{q}}-\int G(\tau-\tau')\dot{\mathbf{q}}(\tau')d\tau'-g\sum_{\{\mathbf{k}_s\}}\mathbf{k}_s\sin\mathbf{k}_s\mathbf{q}=0,$$
$$\frac{d}{d\tau}G(\tau)=D(\tau). \tag{37}$$

Such solutions exist only in the finite-motion regions in the inverse potential $-V(\mathbf{q})$. The quasi-classical method is convenient when the main contribution to the path integral comes from only a few solutions. There are two such regions. In the first, for large T, the stable periodic solutions are situated near the minima of $-V(\mathbf{q})$ and there are few of those in each minimum. When T decreases the number of periodic solutions increases and it becomes difficult to take them into account. However, for sufficiently low T the situation simplifies again and one can use only those solutions which give a contribution $\propto T^0$ to S_{eff}. In such an approach the main role is played by instantons,[15] various superpositions of which can approximate exact solutions. We shall use this method to study the system (11).

We consider first of all our system for $\eta = 0$. In that case for $T = 0$ Eq. (37) has for the potentials (22), (23) instanton solutions in the form of kinks (see Appendix)

$$q_k(\tau) = Qf(\tau - \tau_0), \quad Q = q_{m_f} - q_{m_i} \in \{Q_s\}, \tag{38}$$

where $\mathbf{q}_{m_f}$ and $\mathbf{q}_{m_i}$ are the final and initial nearest minima, Q is the topological charge of the kink, τ_0 is an arbitrary parameter which turns out to be the position of the kink, and

$$f(\tau) = \frac{2}{\pi} \operatorname{arctg}(e^{-\tau/\tau_*}), \quad \tau_*^{-1} = (Kg/K_1 M)^{1/2}, \tag{39}$$

i.e., it is formally the same as a sine-Gordon kink but with changed parameters g and M: $g \to Kg$, $M \to K_1 M$ where K_1 depends on the geometry of $\{\mathbf{k}_s\}$:

$$K = N_{\{k\}} K_1, \quad K_1 = (Q/2\pi)^2 = (a_0/2\pi)^2 = \cos^2\gamma. \tag{40}$$

The action of a kink with charge $\mathbf{Q} \in \{\mathbf{Q}_s\}$ equals

$$s_0 = 8K(N_{\{k\}}^{-1} gM)^{1/2} = 8\cos\gamma (N_{\{k\}} gM)^{1/2}. \tag{41}$$

We approximate the set of periodic trajectories for $1/T \gg \tau_*$ by all possible closed trajectories consisting of kinks (39):

$$q_{cl}(\tau) = \sum_{i=1}^{N} Q_{s_i} f(\tau - \tau_i), \quad \sum_{i=1}^{N} Q_{s_i} = 0. \tag{42}$$

Such an approximation is similar to the low-temperature expansion for the partition function of lattice spin models.[16] It is the more exact the lower T and the larger s_0. Then, neglecting the interaction between the kinks which decreases exponentially with distance we can write (11) in the form of the grand partition function of a perfect classical gas of particles with charges $\mathbf{Q}_s \in \{\mathbf{Q}_s\}$ which is neutral as a whole because of the periodicity condition:

$$Z_{(\eta=0)} = \sum_{n=0}^{N} \sum_{\{Q_{si}\}}{}' y_0^n \int_0^{1/T} d\tau_n \int_0^{\tau_n - \tau_*} d\tau_{n-1} \cdots \int_0^{\tau_2 - \tau_*} d\tau_1 \approx \sum_{n=0}^{N} \frac{(y_0/T)^n}{n!} G_n, \tag{43}$$

where $N = [1/\tau_* T]$ is the integral part of $1/T\tau_*$, the sum with the prime indicates summation over all neutral configurations of charges $\mathbf{Q}_{s_i} \in \{\mathbf{Q}_s\}$, G_n is the number of such configurations, and the chemical activity is

$$y_0 = e^{-s_0} J [\mathrm{Det}'/\mathrm{Det}_0]^{-1/2},$$

where $J = (s_0/2\pi)^{1/2}\tau_*$ is the Jacobian for the change from the zero mode to $d\tau$, $[\mathrm{Det}'/\mathrm{Det}_0]$ is the ratio of the determinants of the quadratic forms of quantum fluctuations against the background of the kink (excluding the zero mode) and the vacuum (see Appendix). If there is no interaction between the kinks and at sufficiently low T the activity y_0 determines the amplitude of the tunneling transition between nearest minima of $V(q)$. The partition function Z from (11) describes at $\eta = 0$ the motion of a quantum particle in a crystal. It is well known that in that case the particle may move freely in the allowed energy bands. We show that the approximation of a perfect gas of kinks is sufficient to describe the band structure. We introduce the Bloch wave function

$$\langle \mathbf{q}|\mathbf{p}\rangle \equiv \Psi_{\mathbf{p}}(\mathbf{q}) = \sum_{\mathbf{m}} e^{i(\mathbf{p}\mathbf{q}_m)} \Psi(\mathbf{q} - \mathbf{q}_m), \quad \Psi(\mathbf{q} - \mathbf{q}_m) \equiv \langle \mathbf{q}|\mathbf{m}\rangle, \tag{44}$$

where $\langle \mathbf{q}|\mathbf{m}\rangle$ is the wave function of the ground state of a particle near the minimum $\mathbf{q}_m$ with energy E_g neglecting the overlap of $\langle \mathbf{q}|\mathbf{m}\rangle$ with different $\mathbf{q}_m$, $\mathbf{p} = \Sigma_i l_i \mathbf{b}_i / L$ is the quasi-momentum, La_0 the size of the crystal, $-L/2 \leqslant l_i \leqslant L/2$. In the case of tunneling to only the nearest minima we get for the first band as $T \to 0$

$$\frac{\langle \mathbf{p}|e^{-H/T}|\mathbf{p}\rangle}{\langle \mathbf{p}|\mathbf{p}\rangle} = \frac{\langle 0|e^{-H/T}|0\rangle}{\langle 0|0\rangle} \left[1 + \sum_{\{Q_s\}} \frac{e^{i\mathbf{p}\mathbf{Q}_s} \langle \mathbf{Q}_s|e^{-H/T}|0\rangle}{\langle 0|e^{-H/T}|0\rangle} + \ldots\right] \approx \exp\left\{-\frac{1}{T}\left[E_g + E_0 - y_0 \sum_{\{Q_s\}} \cos \mathbf{p}\mathbf{Q}_s\right]\right\},$$
$$E_0 = -T \ln Z_{(\eta=0)}, \quad \frac{y_0}{T} = \frac{\langle \mathbf{Q}_s|e^{-H/T}|0\rangle}{\langle 0|e^{-H/T}|0\rangle}. \tag{45}$$

It is clear from (45) that the band width ΔE is determined by the chemical activity of the instanton gas $\Delta E = 2y_0 \bar{r}$.

When there is friction present, $\eta \neq 0$, there appears also a long-range interaction between the kinks which arises from the dissipative term S_D and the partition function (44) changes to the grand partition function of a one-dimensional ln gas with charges $\mathbf{e}_{s_i} \in \{\mathbf{Q}_s/a_0\}$ and an effective reciprocal temperature $\beta = a_0^2 \eta/\pi$:

$$Z = \sum_{n=0}^{N} \sum_{\{\mathbf{e}_{s_i}\}}{}' \int_0^{1/T} d\tau_n \int_0^{\tau_n - \tau_*} d\tau_{n-1} \cdots \int_0^{\tau_2 - \tau_*} d\tau_1 y_0^n \exp\left\{\frac{\beta}{2} \sum_{i \neq j} (\mathbf{e}_{s_i}\mathbf{e}_{s_j}) \cdot \tilde{\Delta}(\tau_i - \tau_j)\right\},$$
$$\tilde{\Delta}(\tau) = \Delta(\tau) - \Delta(0) = \frac{T}{\beta} \sum_{n=-\infty}^{\infty} \square(\omega_n) |q_k(\omega_n)|^2 [e^{i\omega_n \tau} - 1]_{\tau \gg \tau_*} \approx \ln \frac{\sin(\pi\tau T)}{\pi\tau_* T}. \tag{46}$$

It follows from (46) that kinks with orthogonal charges again do not interact.

In the limit as $T \to 0$, neglecting the excluded volume of the kinks since the kernel $\Delta(\tau)$ is "soft" as $\tau \to 0$ (this is equivalent to taking into account kinks with large topological charges, but this is unimportant for the renormalization), we can write (46) in a form similar to (11):

$$Z = \int D\tilde{\mathbf{q}} e^{-S_{eff}}, \quad S_{eff} = S_0 + S_D + S_i,$$
$$S_0 + S_D = \frac{1}{2\beta} \int_0^{1/T} d\tau\, d\tau' (\mathbf{q}(\tau)\tilde{\mathbf{q}}(\tau')) \Delta^{-1}(\tau - \tau'), \quad S_i = \int_0^{1/T} V(\tilde{\mathbf{q}})\, d\tau,$$
$$V(\tilde{\mathbf{q}}) = -y_0 \sum_{\{e_s\}} \cos \mathbf{e}_s \tilde{\mathbf{q}}. \tag{47}$$

It follows from (47) that in the low-frequency limit Eqs. (47) and (11) change into one another through the substitutions

$$\{\mathbf{k}_s\} \leftrightarrow \{\mathbf{e}_s\}, \quad \eta \leftrightarrow 1/a_0^2\eta, \quad g_0(M/\eta) \leftrightarrow y_0\tau_*. \tag{48}$$

This duality property of the model (11) under the transformations (48) generalizes the self-duality of the one-component case[6] as now the sets $\{\mathbf{k}_s\}$ and $\{\mathbf{e}_s\}$ can not be the same. For the triangular and hypercubic lattices the geometries of $\{\mathbf{k}_s\}$ and $\{\mathbf{e}_s\}$ are the same and, hence, they are self-dual. In that case the transformation (48) has a fixed point (η^*, s^*), where $\eta^* = 1/a_0$ while s^* is determined from the equation

$$g_0(s^*) = y_0(s^*).$$

It is also convenient to study the low-frequency properties of Z from (47) using the RG method which is applicable when $y_0\tau_* \ll 1$, $T \ll y_0$. The RG equations have the form (24) where now the coefficients $C_{\{k\}}$, B_n are determined by the geometry of $\{\mathbf{e}_s\}$. As $T \to 0$ the system may again be in two different phases.[17] The shape of the phase separation line near $\eta_c = 2\pi/a_0^2$ for potentials with $\{\mathbf{e}_s\}$ of the first kind or the second kind with $B_3 > 0$ is given by the equation

$$y_0\tau_* = \left(\frac{\Delta\eta}{B_n}\right)^{1/(n-1)}, \quad s \approx -\frac{1}{n-1}\ln\frac{\Delta\eta}{B_n} + O(\ln\ln\Delta\eta). \tag{49}$$

The low-temperature behavior of the renormalized chemical activity $y_R(T)$ and mass $\bar{m}_R(T)$ is given by Eqs. (26) to (32) with the substitutions $g \to y$, $M \to \bar{m}$, $\tau_0 \to \tau_*$.

The expression for the Matsubara correlator $\langle qq\rangle(\omega_n)$ in the quasi-classical region for the mobility $\mu(\omega_n)$ is obtained from the generating functional

$$Z[\mathbf{F}] = \int D\mathbf{q} e^{-(S_{\text{eff}} + S_{\text{ext}})}, \quad S_{\text{ext}} = \int_0^{1/T} \mathbf{F}(\tau)\mathbf{q}(\tau)\,d\tau. \tag{50}$$

($\mathbf{F}(\tau)$ is the field conjugate to the coordinate $q(\tau)$), which in the considered approximation of a rarefied gas of instantons has the form

$$Z[\mathbf{F}] = \int D\tilde{\mathbf{q}} e^{-(S_{\text{eff}} - S(F))}, \tag{51}$$

$$S(\mathbf{F}) = \frac{1}{2}\int_0^{1/T} d\tau\, d\tau' (\mathbf{F}(\tau)\mathbf{F}(\tau')) D^{-1}(\tau-\tau') + \int_0^{1/T} d\tau (\mathbf{F}(\tau)\mathbf{q}(\tau)),$$

where

$$\tilde{\mathbf{q}}(\tau) = T\sum_n e^{i\omega_n\tau} f^{-1}(\omega_n) D^{-1}(\omega_n)\tilde{\mathbf{q}}(\omega_n).$$

From (51) we get

$$\langle qq\rangle(\omega_n) = D^{-1}(\omega_n)[1 - D^{-1}(\omega_n)|f(\omega_n)^2|\langle\tilde{q}\tilde{q}\rangle(\omega_n)] \approx \frac{1}{\eta|\omega_n|}[\tilde{y}_R + \delta\omega_n^2][|\omega_n| + \tilde{y}_n + \delta\omega_n^2]^{-1}. \tag{52}$$

Here $y_R = a_b^2\eta y_R$, $\delta = a_0^2\eta\bar{m}_R$, $\bar{m}_R = \bar{m}(l_m)$ is the renormalization-induced "mass." For physical frequencies, Eq. (52) goes over into

$$\langle qq\rangle(\omega) = -\frac{(\tilde{y}_R - \delta\omega^2)(\tilde{y}_R - \delta\omega^2 - i\omega)}{\eta(i\omega)[(\tilde{y}_R - \delta\omega^2)^2 + \omega^2]}, \qquad \mu(\omega) = -i\omega\langle qq\rangle(\omega). \tag{53}$$

Repeating the reasoning of Sec. 2 we get for $T \ll T^*$ for the static mobility in the different regions of the phase diagram

$$\mu_{A\,\mathrm{I}} = \frac{1}{\eta}\left[1 - \left(\frac{a_0^2\eta y_R(T^*)}{T}\right)^{-2}\right] \approx \frac{1}{\eta}[1 - O(T^2)], \qquad \mu_{A\,\mathrm{II}} \approx \frac{1}{\eta}\left[\frac{a_0^2\eta y_R(T)}{T}\right]^2 \propto \frac{1}{\eta}T^{2\Delta\eta}, \tag{54}$$

which as $T \to 0$ leads in the transition point to a discontinuity

$$\Delta\mu = 1/\eta(s).$$

We now consider the structure of the energy band for $\eta \neq 0$. As the total action S_{eff} from (11) is translationally invariant under translators by lattice vectors and $y_R(T) > 0$ we can again introduce a Bloch function $|\mathbf{p}\rangle$ which now will correspond to the renormalized spectrum

$$\frac{\langle\mathbf{p}|e^{-H/T}|\mathbf{p}\rangle}{\langle\mathbf{p}|\mathbf{p}\rangle} \approx \exp\left\{-\frac{1}{T}\left[E_g + E - y_R\sum_{\{\mathbf{Q}_s\}}\cos\mathbf{p}\mathbf{Q}_s\right]\right\}, \tag{55}$$

where $E = -T\ln Z$. For the band width $\Delta E(T)$ and the effective mass $M^*(T)$ we have

$$\Delta E(T) = 2\bar{r}y_R(T), \quad M^*(T) = [N_{\{\mathbf{Q}\}}a_0^2 y_R(T)]^{-1}. \tag{56}$$

It follows from (56) that in the localization region as $T \to 0$ we have $\Delta E(T) \to 0$ and $M^*(T) \to \infty$. Such a behavior differs greatly from the low-temperature behavior of ΔE and M^* in polaron theory[18] where the polaron term in the effective interaction adds an exponentially small but nevertheless finite factor—the Debye-Waller factor—to the width ΔE as $T \to 0$.[18]

IV. EFFECT OF AN EXTERNAL UNIFORM FIELD

In this section we consider the effect of an external uniform field $\mathbf{F}$ on the localization caused by the dissipation. When $\mathbf{F}$ is present we add to S_{eff} a term

$$S_{\text{ext}} = \int_0^{1/T} \mathbf{F}q(\tau)\,d\tau.$$

In the RG framework this term is taken into account differently in the cases of a weak and a strong potential. In our considerations we restrict ourselves to the one-component case.

1. Weak potential

When we renormalize the partition function with the complete action (50), an odd component of the potential of the form $V_1(\mathbf{q}) = g_1 \sin q$ is generated in the potential already in first order in $g\tau_0$. We must introduce this component also into the potential. In that case, by separating the

renormalization of the free energy due to F, we get to first order in $u = g\tau_0$ and $u_1 = g_1\tau_0$ the following RG equations

$$\frac{du}{dl} = xu - \tilde{A}(l)u_1, \quad \frac{du_1}{dl} = xu_1 + \tilde{A}(l)u, \quad \frac{d\tilde{A}}{dl} = \tilde{A}, \quad (57)$$

where $\tilde{A}_0 = (2/\pi\eta)F\tau_0$. They have the solution

$$\tilde{A}(l) = \tilde{A}_0 e^l, \quad u = u_0 e^{xl}\cos[\tilde{A}_0(e^l-1)],$$
$$u_1 = u_0 e^{xl}\sin[\tilde{A}_0(e^l-1)], \quad (58)$$

from which it follows that there appears in the theory a new scale l_F:

$$l_F = \ln\left(1+\frac{C\pi}{\tilde{A}_0}\right) = \ln\left(1+\frac{C\pi^2\eta^2}{2FM}\right), \quad C\sim 1,$$

over which an essential restructuring of the potential $V(q)$ occurs. Therefore, when $F \neq 0$ the renormalization must be cut off at a scale $l_m = \min[l_T, l^*, l_F]$. As a result g_R and M_R remain finite as $T\to 0$ in both the $B_{\rm I}$ and $B_{\rm II}$ regions. The static mobility is then obtained from (35) by changing T to τ_F^{-1}. An external uniform field thus destroys the localization in the case of a weak potential.

2. Strong potential

The approximation of the partition function (50) in the quasi-classical region by a gas of instanton solutions leads, in the presence of an external uniform field, to the grand partition function of the ln gas in an external field **F**:

$$Z = \sum_{n=0}^{N}\sum_{\{e_{s_i}\}}{}' y_0^n \int_0^{1/T} d\tau_n \int_0^{\tau_n-\tau_*} d\tau_{n-1}\ldots\int_0^{\tau_2-\tau_*} d\tau_1$$
$$\times\exp\left\{\frac{\beta}{2}\sum_{i\neq j}^{n}(e_{s_i}e_{s_j})\tilde{\Delta}(\tau_i-\tau_j) + \sum_{i=1}^{n}(\mathbf{F}\mathbf{Q}_{s_i})\tau_i\right\}. \quad (59)$$

The renormalization of (59) by Anderson's method[9] leads to the same equations as without the field **F** and in that case the field itself is not renormalized and does not contribute to the renormalization of the other quantities. However, the representation (59) is applicable only when $\tau_{\rm av} \sim y_0^{-1} \ll \tau_F$, where $\tau_{\rm av}$ is the average distance between the kinks, $\tau_F = \tau_*\varphi(F/g)$ is the distance between a kink and an anti-kink in the exact solution of Eq. (37) with $\eta = 0$ and $\mathbf{F}\neq 0$ and $\varphi(x)_{x\to 0} \sim -\frac{1}{2}\ln x$. The renormalization of all quantities must now therefore stop at a scale $l_m = \min[l_T, l_F, l^*]$ where $l_F = \ln(\tau_F/\tau_*)$. The static mobility is in that case obtained from (54) by the substitution $T\to\tau_F^{-1}$ which again leads to a destruction of the localization.

V. DISCUSSION

We discuss a few results obtained in the quasi-classical region. We were not able to solve the equation of motion with $\eta\neq 0$ and therefore we substituted into S_D the kinks of the equation with $\eta = 0$. This approximation cannot change the result qualitatively, since it is easily seen that the exact solutions for $\eta\neq 0$ with non-trivial topological charges must have an action $\propto \ln(T\tau_*)^{-1}$ and lead to the same consequences. In that respect the problem considered is similar to the theorem that the determinant of the massless Dirac operator vanishes in an external topologically non-trivial field. In this case S_D plays the role of the determinant.

The expression for the renormalized tunneling amplitude can be written in the form

$$y_R(T) = y_0 e^{-\Delta s(T)},$$

where the factor $e^{-\Delta S(T)}$ reflects the effect of the dissipation and of the temperature on the tunneling amplitude. This expression is convenient for a comparison with polaron theory as the Debye-Waller factor plays the role of $\Delta s(T)$. It is clear from (26) to (31) that only in the $A_{\rm II}$ region $\Delta s(T)$ retains the simple meaning of a dissipative correction to s_0 obtained through substituting the kink into S_D as this expression for $\Delta s(T)$ corresponds to a linear approximation in the RG equation which is applicable for sufficiently small y_0 only in the $A_{\rm II}$ region. In the other region, allowance of the non-linearity changes $\Delta s(T)$ appreciably.

A final remark. The jump in the mobility when localization occurs in disordered systems is actively advocated in the papers by Mott and his coworkers.

VI. CONCLUSION

For the convenience of the readers we enumerate in conclusion the main results of the present paper (again here $T\to 2\pi T$).

1. Using the example of a particle moving in the field of a crystal we showed that a sufficiently strong dissipation may as $T\to 0$ lead to the localization of the particle in crystal fields of arbitrary dimensionality both in the case of a weak and in the case of a strong potential. We constructed the corresponding phase diagrams (Figs. 1,2).

2. We found the low-temperature behavior of the mobility $\mu(\omega)$ of the particle: a) weak potential (34)

$$\mu(\omega) = \begin{cases}[\eta\omega^2 - i\omega(g_R - M_R\omega^2)][(g_R-M_R\omega^2)^2+\eta^2\omega^2]^{-1}, & \omega > T,\\ [\eta T^2 - iT(g_R - M_R T^2)][(g_R-M_R T^2)^2+\eta^2 T^2]^{-1}, & \omega < T;\end{cases}$$

b) strong potential (53)

$$\mu(\omega) = \frac{1}{\eta}\begin{cases}(\tilde{y}_R-\delta\omega^2)(\tilde{y}_R-\delta\omega^2-i\omega)[(\tilde{y}_R-\delta\omega^2)^2+\omega^2]^{-1}, & \omega > T,\\ (\tilde{y}_R-\delta T^2)(\tilde{y}_R-\delta T^2-iT)[(\tilde{y}_R-\delta T^2)^2+T^2]^{-1}, & \omega < T,\end{cases}$$
$$\tilde{y}_R = a_0^2\eta y_R, \quad \delta = a_0^2\eta m_R,$$

where g_R, y_R are the renormalized amplitudes of the potential and of the tunneling transition, respectively, while M_R, m_R are the renormalized masses in the effective Lagrangians. They are given by Eqs. (26) to (32) and (48).

3. We showed that the low-temperature behavior of the static mobility for $T \ll T^* = \xi^{-1}$ (ξ is the correlation length) has a discontinuity: a) weak potential (35)

$$\mu_{B_{\rm I}} \simeq \frac{1}{\eta}(1-(g_R(T)/\eta T)^2) \approx \frac{1}{\eta}(1-O(T^{2\Delta\eta})),$$
$$\mu_{B_{\rm II}} \approx \eta(T/g_R(T^*))^2 \propto T^2;$$

b) strong potential (54)

$$\mu_{A_{\rm I}} \simeq \frac{1}{\eta}(1-(T/\tilde{y}_R(T^*))^2) \approx \frac{1}{\eta}(1-O(T^2)),$$
$$\mu_{A_{\rm II}} \approx \frac{1}{\eta}(\tilde{y}_R(T)/T)^2 \propto T^{2\Delta\eta}.$$

In the limit as $T\to 0$ this leads to μ_{st} vanishing discontinuously on the transition line, with a jump of magnitude $\Delta\mu = 1/\eta(s)$ where the function $\eta(s)$ that defines the transition line depends on the geometry of the crystal. It is given by Eqs. (25), (49):

a) $$s=8\cos\gamma\left[N_{\{k\}}\eta_c\left(\frac{\Delta\eta}{B_n}\right)^{1/(n-1)}\right]^{1/2}, \quad \Delta\eta=\frac{\eta-\eta_c}{\eta_c},$$

b) $$y_0\tau_*=\left(\frac{\Delta\eta}{B_n}\right)^{1/(n-1)}, \quad s\approx-\frac{1}{n-1}\ln\frac{\Delta\eta}{B_n}+O(\ln\ln\Delta\eta).$$

4. We found the low-temperature behavior of the bandwidth $\Delta E(T)$ and of the effective mass of the particle $M^*(T)$ in the case of a strong potential:

$$\Delta E(T)=2\bar{r}y_R(T), \quad M^*(T)=[N_{\{Q\}}a_0^2y_R(T)]^{-1}.$$

5. We showed that the switching on of a uniform external field F destroys the localization and we found the F-dependences of $\mu(\omega)$ and μ_{st} as $T\to 0$ which are obtained from (34), (35), (53), (54) by replacing T by τ_F^{-1}.

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APPENDIX

The renormalized amplitude of the tunneling transition from $|\mathbf{O}\rangle$ to the closest minimum $|\mathbf{Q}\rangle$ for $\eta = 0$ and $T\to 0$ is equal to

$$\langle\mathbf{Q}|e^{-H/T}|0\rangle=\int D\mathbf{q}(\tau)\,e^{-S_{\text{eff}}}, \tag{A1}$$

where the integration occurs over all trajectories with $\mathbf{q}(-\infty)=0, \mathbf{q}(\infty)=\mathbf{Q}$. In the quasi-classical approximation the main contribution to the integral comes from the solutions of the Euler equations

$$M\ddot{\mathbf{q}}-g\sum_{\{k_s\}}\mathbf{k}_s\sin\mathbf{q}\mathbf{k}_s=0. \tag{A2}$$

We look for the kinks of (A2) in the form

$$\mathbf{q}(\tau)=\mathbf{Q}f(\tau), \quad \mathbf{Q}\in\{\mathbf{Q}_s\}. \tag{A3}$$

The most important properties of the sets $\{\mathbf{Q}_s\}$ and $\{\mathbf{k}_s\}$ from (22), (23) which enable us to find kinks are the following ones:

$$\mathbf{k}_s\mathbf{Q}_{s'}=2\pi\mathbf{n}^s\mathbf{n}^{s'}=2\pi\begin{cases}0,\\ 4.\end{cases} \tag{A4}$$

Substituting (A3) into (A2) and using (A4) we get for $f(\tau)$ a sine-Gordon equation with changes parameters:

$$M(\mathbf{Q})^2\ddot{f}-gK\sin(2\pi f)=0, \tag{A5}$$

where K is the number of vectors from $\{\mathbf{k}_s\}$ for which $|\mathbf{k}_s\cdot\mathbf{Q}|=1$, and $K_1=Q^2/(2\pi)^2=K/N_{\{k\}}$. From (A5) we have for $f(\tau)$

$$f(\tau-\tau_0)=\frac{2}{\pi}\operatorname{arctg}(e^{(\tau-\tau_0)/\tau_*}), \quad \tau_*^{-1}=\left(\frac{N_{\{k\}}g}{M}\right)^{1/2}. \tag{A6}$$

The action of the kink is $s_0=8K(gM|N_{\{k\}}|)^{1/2}$. As a result we get for y_0

$$\frac{y_0}{T}=\frac{\langle\mathbf{Q}|e^{-H/T}|0\rangle}{\langle 0|e^{-H/T}|0\rangle}$$

$$=e^{-s_0}\int D\mathbf{q}\exp\left[-\frac{1}{2}\int\delta q_iS_{ij}(\mathbf{q}_{cl})\,\delta q_j\right]$$

$$\Big/\int D\mathbf{q}\exp\left[-\frac{1}{2}\int\delta q_iS_{ij}(0)\,\delta q_j\right], \tag{A7}$$

where $S_{ij}(\mathbf{q}_{ks})$ and $S_{ij}(0)$ are quadratic forms of quantum fluctuations on the background of the kink and of the vacuum, respectively,

$$S_{ij}(\mathbf{q}_{cl})=\frac{\delta^2S(\mathbf{q}_{cl})}{\delta q_i(\tau)\,\delta q_j(\tau')}. \tag{A8}$$

The eigenfunctions and spectrum of the form (A8) can be found from the equation

$$\left[\delta_{ij}\frac{d^2}{dz^2}-\frac{1}{N_{\{k\}}}\sum_{\{k_s\}}k_i^sk_j^s\cos(\mathbf{k}_s\mathbf{q}_{cl})\right]\delta q_j=-\lambda^2\delta q_i, \tag{A9}$$

in which we changed to the dimensionless $z=\tau/\tau_*$. Expanding $\delta\mathbf{q}$ in the orthonormalized base $\{\mathbf{n}_i\}$ one of whose unit vectors is directed along $\mathbf{Q}$, while the others are determined from the diagonalization condition of the equations

$$\delta\mathbf{q}(z)=\sum_{l=1}^{d}\sum_{l=0}^{\infty}\mathbf{n}_i\nu_{il}(z)C_{il},$$

we get for the eigenfunctions the equations

$$\left[-\frac{d^2}{dz^2}-\frac{2(N-M_i)}{N\operatorname{ch}^2z}\right]\nu_{il}(z)=2E_{il}\nu_{il}(z),$$

$$2E_{il}=\lambda_{il}^2-1, \tag{A10}$$

where the M_i are the diagonal values of the matrix

$$M_{ij}=N_{lk}^{(1)}n_l^in_k^j, \quad \text{a} \quad N_{lk}^{(1)}=\sum_{\{k_s\perp Q\}}k_i^sk_k^s.$$

For the lattices (22), (23) they are equal to

$$M_1=0, \quad M_{2\text{tr}}=2, \quad M_{2\text{ bcc}}=M_{3\text{ bcc}}=3,$$
$$M_{2\text{ fcc}}={}^8/_3, \quad M_{3\text{ fcc}}={}^4/_3.$$

The first of Eqs. (A10) is exactly the same as the equation for the sine-Gordon kink fluctuations. The spectrum of the bound states of (A10) is given by the formula[19]

$$-2E_{il}=\frac{1}{4}[-(1+2l)+(1+8U_{0i})^{1/2}]^2, \quad 0\leqslant l\leqslant[l_{0i}],$$

$$U_{0i}=(N_{\{k\}}-M_i)/N_{\{k\}}, \quad l_{0i}=\frac{1}{2}(-1+(1+8U_{0i})^{1/2}). \tag{A11}$$

Here $U_{01}=1$, and $U_{0i}<1$ for $i\neq 1$ so that for each i we have only a single discrete eigenvalue which for U_{01} gives $\lambda_{10}^2=0$, corresponding to the zero mode

$$\nu_{10}(z)\propto\frac{d}{d\tau}f(\tau),$$

and for $i\neq 1$ it gives $\lambda_{i0}^2=1+2E_{i0}>0$ which correspond to

the transverse excited states of the kink. The remaining $\lambda_{il}^2 > 1$ and belong to the continuous spectrum. We have thus found the eigenvalues of (A10) and shown that the kinks (A3) are stable. Using Langer's method[20] for evaluating (A7) and separating the zero mode we get for the transition amplitude the expression

$$\frac{y_0}{T} = \int_0^{1/T} d\tau e^{-s_0} J \prod_{i=1}^{d} \left[\frac{\mathrm{Det}_{0i}}{\mathrm{Det}_i'} \right]^{1/2}, \quad J = \frac{1}{\tau_*} \left(\frac{s_0}{2\pi} \right)^{1/2}, \quad \text{(A12)}$$

where J is the Jacobian for the transition from integrating over dC_{10} to integrating over $d\tau$,

$$\mathrm{Det}_{0i} = \det \left[-\frac{d^2}{dz^2} + 1 \right],$$

$$\mathrm{Det}_i' = \det \left[-\frac{d^2}{dz^2} + 1 - l_{0i}(l_{0i}+1)\, ch^{-2} z \right]$$

with the zero mode eliminated for $i = 1$

$$\prod_{i=1}^{d} \left[\frac{\mathrm{Det}_{0i}}{\mathrm{Det}_i'} \right]^{1/2} = \left(\frac{1}{2\pi} \right)^{1/2} \exp \left(\sum_{i=1}^{d} \Phi_i \right), \quad \text{(A13)}$$

where

$$\Phi_1 = \frac{1}{2} \ln 4, \quad \Phi_{i>1} = \frac{1}{2} \ln \left| \frac{l_{0i}^2 \Gamma(2-l_{0i}) \Gamma(3+l_{0i})}{2-l_{0i}(l_{0i}+1)} \right|. \quad \text{(A14)}$$

[1] A. O. Caldeira and A. J. Leggett, Phys. Rev. Lett. **46**, 2118 (1981); Ann. Phys. **149**, 374 (1983); A. O. Caldeira, PhD Thesis, Univ. of Sussex, 1980.

[2] V. Ambegaokar, U. Eckern, and G. Schon, Phys. Rev. Lett. **48**, 1745 (1982).

[3] A. I. Larkin and Yu. N. Ovchinnikov, Pis'ma Zh. Eksp. Teor. Fiz. **37**, 322 (1983); Zh. Eksp. Teor. Fiz. **85**, 1510 (1983); **86**, 719 (1984) [JETP Lett. **37**, 382 (1983); Sov. Phys. JETP **58**, 876 (1983); **59**, 420 (1984)]; V. I. Mel'nikov and S. V. Meshkov, Pis'ma Zh. Eksp. Teor. Fiz. **38**, 111 (1983) [JETP Lett. **38**, 130 (1983)].

[4] S. Chakravarty, Phys. Rev. Lett. **49**, 681 (1982).

[5] A. J. Bray and M. A. Moore, Phys. Rev. Lett. **49**, 1545 (1982).

[6] A. Schmid, Phys. Rev. Lett. **51**, 1506 (1983).

[7] S. A. Bulgadaev, Pis'ma Zh. Eksp. Teor. Fiz. **39**, 264 (1984) [JETP Lett. **39**, 315 (1984)].

[8] A. R. Bishop in Solitons in Action (eds. K. Lonngren and A. Scott) New York, 1978.

[9] P. W. Anderson, G. Yuval, and D. R. Hamann, Phys. Rev. **B1**, 4464 (1970).

[10] S. A. Bulgadaev, Teor. Mat. Fiz. **51**, 424 (1982).

[11] R. P. Feynman, *Statistical Mechanics*, Addison-Wesley, Reading, Mass., 1972.

[12] A. Schmid, J. Low Temp. Phys. **49**, 609 (1982).

[13] E. M. Lifshitz and L. P. Pitaevskiĭ, Fizicheskaya kinetika (Physical kinetics) Nauka, Moscow, 1979 [English translation published by Pergamon Press, Oxford].

[14] D. R. Nelson and J. M. Kosterlitz, Phys. Rev. Lett. **39**, 1201 (1977).

[15] S. Coleman, Lectures at the Internat. School of Subnuclear Physics, 1977.

[16] M. E. Fisher, The nature of critical points, lectures in Theoretical Physics, (Univ. of Colordo Press, Boulder), VIII c, 1, 1965.

[17] S. A. Bulgadaev, Phys. Lett. **A104**, 215 (1984).

[18] J. Appel, Solid State Phys. **21**, 2 (1968).

[19] L. D. Landau and E. M. Lifshitz, Kvantovaya mekhanika (Quantum mechanics) Nauka, Moscow, 1974 [English translation published by Pergamon Press, Oxford].

[20] J. S. Langer, Ann. Phys. (N.Y.) **41**, 108 (1967).

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