Nonlinear effects in the propagation of an ion beam across a magnetic field

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Plasma turbulence excited by an ion beam, propagating across a magnetic field, is considered. It is shown that the induced scattering on plasma particles is the main nonlinear process characterizing oscillations with frequencies not too close to the lower hybrid frequency. The distribution of the oscillations in the k-space is found in the weak turbulence approximation. The induced scattering gives rise to the transfer of oscillations to the region $\omega - \omega_{LH} \ll \omega_{LH}$, where the modulational instability and collapse of the plasma waves play an important role. Due to the smallness of the phase volume of the strong turbulence region, the energy flux into the plasma is determined by the weak turbulence region and can be found exactly. The energy dissipation is mainly due to the strong turbulence effects and the absorbed energy is transferred to the accelerated ions and electrons. It is shown that the distribution of the accelerated particles has a power-type dependence and, generally, most of the energy is transferred to the electrons.

INTRODUCTION

In many astrophysical problems it is necessary to consider the interaction of plasmas with ion beams propagating across a magnetic field. As an example, we can mention the problem of the structure of a transverse shock wave,[1] in which the beam is created by the ions reflected from the wave front, or the problems of anomalous plasma ionization[2] and of the nonlinear stage of the loss-cone instability in open magnetic mirror systems.

The relative motion of ions excites intense plasma oscillations with frequencies in the vicinity of the lower hybrid frequency ω_{LH}. These oscillations, in turn, interact with both the beam and the plasma, giving rise to an efficient collisionless beam relaxation.

The quasilinear theory of the relaxation has been developed in Refs. 1 and 3. Nevertheless, the predicted oscillation level is usually so high that nonlinear effects are quite important. These effects have been considered in Ref. 4 in the framework of the weak turbulence theory.

Scattering on particles is the main nonlinear process at frequencies which are not too close to the lower hybrid frequency. The turbulence spectra resulting in this process have been found in the present work. The scattering gives rise to the transfer of energy to the frequency region $|\omega_{LH} - \omega| \ll \omega_{LH}$ (the strong turbulence region). Here the modulational instability of the lower hybrid wave develops, leading to a collapse and energy transfer to fast ions and electrons. These phenomena cannot be studied within the weak turbulence theory and require a dynamical description.

At low oscillation intensity $W/nT \ll m/M$, the instability and collapse of the lower hybrid waves reminds one qualitatively of the strong turbulence of Langmuir oscillations.[5–7] At higher turbulence levels, the situation changes dramatically. Because of the development of the modulational instability, oscillations interacting strongly with the bulk of the particles are excited in this case. Therefore, the inertial range is absent and the turbulence is superstrong.[8]

It will be shown in the present work that because of the narrowness of the strong turbulence region, the energy transfer from the beam to oscillations is usually determined by the weak turbulence effects. The absorption does not lead to the plasma heating as a whole, but rather to a formation of tails in the electron and ion velocity distribution functions with a power-law dependence, and the oscillation energy is transfered primarily to the accelerated electrons.

1. THE LINEAR THEORY

The oscillations, excited by the ion beam, belong to the frequency region $\omega_{Hi} < \omega < \omega_H$. We can also assume that the electron motion is one-dimensional and that the ions are unmagnetized. If, in addition, the condition $\omega > k_z v_{Te}, kv_{Ti}$ is satisfied, then the plasma can be described hydrodynamically. The present work is limited to the study of oscillations associated with the dispersion relation

$$\omega_k^2 = \omega_{LH}^2 (1 + (M/m)\cos^2\theta), \qquad (1.1)$$

$$\omega_{LH}^2 = \omega_H^2 \omega_{pi}^2 / (\omega_H^2 + \omega_p^2), \qquad (1.2)$$

where θ is the angle between the wave vector and the direction of the magnetic field. At frequencies in the vicinity of the lower hybrid frequency, $\omega - \omega_{LH} < \omega_{LH}$, it is necessary to include the thermal effects in the dispersion relation:

$$\omega_k = \omega_{LH}(1 + k^2R^2 + y^2)^{1/2}, \quad y = (M/m)^{1/2}\cos\theta,$$

$$R^2 = \begin{cases} 3r_d^2, & \omega_H \gg \omega_p \\ \left(\dfrac{3}{4} + \dfrac{3T_i}{T_e}\right) r_H^2, & \omega_H \ll \omega_p \end{cases}, \qquad (1.3)$$

where r_d and r_H are the Debye and gyration radii, respectively.

We will consider the frequently encountered situation in which the relaxation length greatly exceeds the ion gyration radius. The ion velocity distribution function f_b in this

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case is isotropic in the plane perpendicular to the magnetic field. By assuming that the instability is kinetic, we obtain the following expression for the growth rate[9]:

$$\gamma_b=\frac{\omega_k}{2}\frac{\omega_{LH}^2}{k^2}\frac{n'}{n}\int\limits_{\omega/k}^{\infty}\frac{\partial f_b}{\partial v_\perp}\left(\frac{(kv_\perp)^2}{\omega^2}-1\right)^{-1/2}dv_\perp$$

$$=\frac{\omega_k\omega_{LH}^2}{2k^2}\frac{n'}{n}\int\frac{1}{v_x}\frac{\partial f_b}{\partial v_x}dv_y, \tag{1.4}$$

where n and n' are the plasma and beam densities, respectively.

Let $\Delta v \ll v_b$ be the characteristic width of f_b, v_b being the velocity of the beam:

$$f_b=f\left(\frac{v-v_b}{\Delta v}\right).$$

Then the maximum growth rate is achieved at

$$\omega/k\approx v_b-\Delta v\approx v_b$$

and Eq. (1.4) yields the following estimate for the maximum value of the growth rate:

$$\gamma_{b\,max}\sim\frac{n'}{n}\frac{\omega_k\omega_{pi}^2}{k^2v_b^2}\left(\frac{v_b}{\Delta v}\right)^{3/2}\approx\frac{\omega_{pi}^2}{\omega_k}\frac{n'}{n}\left(\frac{v_b}{\Delta v}\right)^{3/2}. \tag{1.5}$$

The growth rate is a smooth function of the angle θ, since $\gamma_{b,\max}\sim 1/\omega_k$ (see Fig. 1). The dependence of the growth rate on the absolute value of the wave vector is shown in Fig. 2. We notice that $\int\gamma_b(k)dk<0$. The maximum growth rate is achieved at $\omega_k\sim\omega_{LH}$ and $k_0\sim\omega_{LH}/v_b$ and we will assume that the latter quantity is much less than r_d^{-1} and r_H^{-1}. With increasing y the graph of the maximum growth rate bends towards the region of large values of

$$k_0R\sim(\omega_{LH}/v_b)(v_{Te}/\omega_p)y.$$

At $kR\sim 1/3$, a strong Landau damping is switched on and the instability disappears. Furthermore, with the increase of y, the value of the growth rate itself is decreasing and the threshold $\gamma_b=\nu_{ei}/2$ of the instability may be inaccessible at large values of y. These effects allow us to restrict the discussion to the propagation of quasitransverse oscillations only.

2. WEAK TURBULENCE

The increase of the oscillation level is limited by nonlinear effects, the relative role of which is now well understood

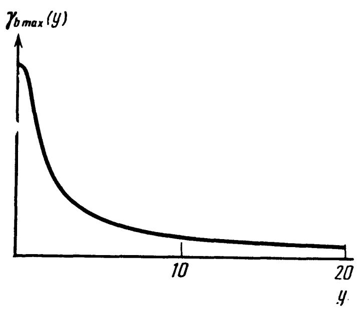

FIG. 1. The dependence of the maximum value $\gamma_{b\max}$ of the growth rate of the beam instability on the parameter $y=(M/m)^{1/2}\cos\theta$.

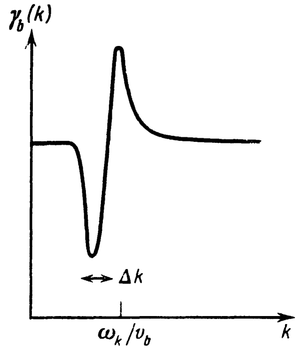

FIG. 2. The dependence of γ_b on the absolute value of the wave vector for a fixed value of y. The distribution function is chosen to have the form $f_0\propto\exp[-(v_\perp-v_b)^2/\Delta v^2]$, where $v_b=10v_{Ti}$ and $\Delta v=0.1v_b$.

(see, for example, Ref. 10). When the turbulence level is not too high, and the oscillation frequencies are not too close to the lower hybrid frequency, the turbulence can be described as a gas of quasiparticles (plasmons). The main nonlinear process in this approximation (the weak turbulence theory), in an isothermal plasma, is the induced scattering on the particles. At $y>1$, the major role is played by the scattering on ions, $\omega_k\to\omega_{k'}+|k-k'|v_{Ti}$, whereas, at $y<1$ the scattering on electrons, $\omega_k\to\omega_{k'}+|k_z-k_z'|v_{Te}$, becomes the most important. A role can be also played by decay processes, which will be considered below.

We will consider the region $y>1$ first. Scattering on ions leads to a decrease of the oscillation frequency and their transfer to the vicinity of the lower hybrid frequency.

The equation, describing the evolution of the plasma density has the form[10]

$$\partial N_{\mathbf{k}}/\partial t+\Gamma_{\mathbf{k}}N_{\mathbf{k}}=\gamma_{nl}(k)N_{\mathbf{k}}, \tag{2.1}$$

where Γ_k includes the Landau damping and the collisional damping, as well as the growth rate of the beam instability:

$$\Gamma_k=\nu_{ei}+\gamma_L(k)-\gamma_b(k). \tag{2.2}$$

The matrix element of the induced scattering is considerably simplified when $\omega_k\omega_H<\omega_p^2$. This condition is practically always satisfied in astrophysical applications.[1–4]

Due to the smoothness of the growth rate as a function of y, we can make further simplifications by going to the differential approximation. As a result γ_{nl} becomes[7,10]:

$$\gamma_{nl}=C_0k^2+C_1,\qquad C_0=\alpha\frac{d}{dy}\int n_{k'}\,dk',$$

$$C_1=\alpha\frac{d}{dy}\int n_{k'}k'^2\,dk', \tag{2.3}$$

$$n_k=2\pi k^2N_{\mathbf{k}},\qquad \alpha=\frac{\pi}{2Mn}\frac{\omega_p^2}{\omega_p^2+\omega_H^2}\left(\frac{M}{m}\right)^{1/2}.$$

The equation for the stationary solutions (2.2),

$$\Gamma_k=\gamma_{nl}\quad\text{at}\quad n_k\neq 0 \tag{2.4}$$

allows a high degree of freedom and must be completed by the stability condition

$$\Gamma_k>\gamma_{nl}\quad\text{at}\quad n_k=0. \tag{2.5}$$

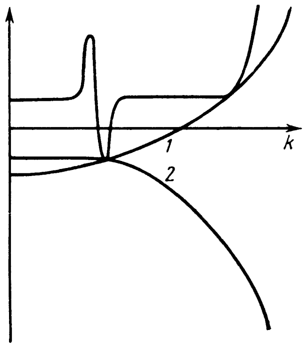

FIG. 3. The linear wave damping rate Γ_k and the damping rate γ_{nl} as function of the wave vector at a fixed value of y. Curves 1 and 2 correspond to the two stable, stationary solutions of Eq. (2.1).

Geometrically, this means that the surface Γ_k lies higher than γ_{nl} and is tangent to it at the points where the solution exists. Figure 3 shows the dependence of Γ_k on the wave vector at a fixed value of y. Since γ_{nl} is a parabola, there exist two stable stationary solutions, corresponding to the curves 1 and 2 in Fig. 3. In both cases, the distribution of the oscillations is singular and has the form of a jet in k-space.

Let us begin by considering the first case. Here there exist two jets:

$$n_k = n_b(y)\delta(k-k_0(y)) + n_1(y)\delta(k-k_1(y)).$$

The first stretches along the line of the maximum growth rate $k=k_0(y)$ and the second is located in the region of large k, where the Landau damping becomes important. Substituting this solution into (2.4) yields

$$\gamma_L(k_0)+\nu_{ei}-\gamma_b(k_0) = \alpha k_0^2 \frac{d}{dy}(n_b+n_1) + \alpha\frac{d}{dy}(k_0^2 n_b + k_1^2 n_1), \tag{2.6}$$

$$\gamma_L(k_1)+\nu_{ei} = \alpha k_1^2 \frac{d}{dy}(n_b+n_1) + \alpha\frac{d}{dy}(k_0^2 n_b + k_1^2 n_1). \tag{2.7}$$

By subtracting (2.6) from (2.7), we obtain

$$\alpha(k_1^2-k_0^2)\frac{d}{dy}(n_b+n_1) = \gamma_L(k_1) - \gamma_L(k_0) + \gamma_b > 0. \tag{2.8}$$

Thus, the total number $n_b + n_1$ of waves increases with y, and the natural boundary condition $n_k = 0$ cannot be satisfied for $y > y_0$, where $\gamma_b(y_0) = \nu_{ei} + \gamma_L$. Therefore, the distribution of the oscillations has the form of a single jet. It can be seen in Fig. 3 that for a narrow beam, when $\Delta v \ll v_b$, the jet is located at the maximum of the growth rate $k_0 \cong (\omega_k/v_b)(1+\Delta v/v)$. For $\Delta v \sim v$, the precise location of the jet in the region of positive growth rates is determined by conditions (2.4) and (2.5) (see, for example, Ref. 10). Since the obtained results are weakly dependent on k_0, we will assume, in the following, that $k_0 \cong \omega_k/v_b$. The establishment of such a distribution of jets is confirmed by our numerical solutions of Eq. (2.1). This result is related in an essential way to the axial symmetry of the growth rate.

The distribution of oscillations along the jet is described by the equation

$$\Gamma(k_0) = \alpha\left[k_0^2\frac{dn_b}{dy} + \frac{d}{dy}(k_0^2 n_b)\right] = 2\alpha k_0 \frac{d}{dy} k_0 n_b. \tag{2.9}$$

Typically in astrophysical applications the collisional damping is negligible. The Landau damping is also small on almost the whole jet. By taking this account, Eq. (2.9) yields

$$n_b(y) = \frac{1}{2k_0\alpha}\int_y^{y_0} \frac{\gamma_b}{k_0(y')}dy'$$

$$= \frac{\gamma_b(0)v_b}{2\alpha k_0(y)\omega_{LH}}[\operatorname{arctg} y_0 - \operatorname{arctg} y], \tag{2.10}$$

where y_0 is the initial point of the jet. In weakly collisional plasmas the value y_0 is defined by the Landau damping, which becomes significant at $k_z v_{Te}/\omega_k \sim 1/3$, i.e.,

$$y_0 \approx v_b/3v_{Ti}. \tag{2.11}$$

At $y < 1$, the main nonlinear process is scattering on the electrons. The equation describing the evolution of plasmons in this case has the same form as Eq. (2.1), and γ_{nl} has been found, for example, in Ref. 7:

$$\gamma_{nl} = \alpha\left(\frac{m}{M}\right)^{1/2}\int\left(k^2 y\frac{\partial}{\partial y}\frac{n_{k'}}{y} + \frac{1}{y}\frac{\partial}{\partial y} y n_{k'} k'^2\right)dk'. \tag{2.12}$$

It can be seen that γ_{nl} is a parabola as a function of k. Therefore, by repeating all the arguments used earlier, we find that the spectrum, in this case, also is built up of a single jet, located on the line of the maximum growth rate.

The damping (2.12) of the jet spectrum $n_k = n(y)\delta(k-k_0)$, coincides with Eq. (2.9) and, therefore expression (2.10) is valid up to the limit of applicability of the differential approximation $\omega_{LH}y^2 \sim k_z v_{Te}$, i.e., $y \sim k_0 R$.

The energy density of oscillations in the interval between y^* and y_0 is

$$W = \left(\frac{m}{M}\right)^{1/2}\int_y^{y_0}\omega_k n_b(y')dy'$$

$$\approx nT\frac{\gamma_b(0)(\omega_p^2+\omega_H^2)}{\pi\omega_p^2\omega_{LH}}\frac{v_b^2}{v_{Te}^2}[g(y_0)-g(y)],$$

$$g(y) = \int_0^y[\operatorname{arctg} y_0 - \operatorname{arctg} y']dy' \tag{2.13}$$

$$= \ln(1+y^2) - y\operatorname{arctg}\left(\frac{y_0-y}{1+y_0 y}\right),$$

and the energy flux entering the plasma in this interval is

$$Q = \left(\frac{m}{M}\right)^{1/2}\int_{y^*}^{y_0}\gamma_b(y')\omega_k n_b\,dy'$$

$$= nT\frac{\gamma_b^2(0)(\omega_H^2+\omega_p^2)}{\pi\omega_p^2\omega_{LH}}\frac{v_b^2}{v_{Te}^2}[f(y_0)-f(y^*)], \tag{2.14}$$

$$f(y) = \int_0^y(\operatorname{arctg} y_0 - \operatorname{arctg} y')\frac{dy'}{(1+y'^2)^{1/2}}.$$

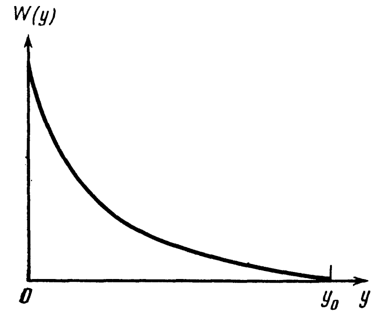

FIG. 4. The angular distribution of the oscillation energy density $W(y)$ at $y_0 = 20$ in the weak-turbulence approximation.

The plot of W is shown in Fig. 4, and $f(y_0)$ is given in Fig. 5.

Oscillation energy is accumulating in the transfer process at the vicinity of the lower hybrid frequency. Weak turbulence theory is inapplicable in this frequency region, and a dynamical description should be used. The applicability limit on the weak turbulence theory determines the lower integration limits in Eqs. (2.13) and (2.10). This question will be discussed in detail later.

We consider now the relative role of quasilinear effects. We will estimate first the turbulence level in the quasilinear regime. Initially the oscillations are absent, and at the end of the relaxation the beam energy is distributed between the oscillations and particles. Since the beam transfers a considerable part of its energy during the relaxation process, $W_{ql} = n'v_b\,\Delta v M$. By using the expression given above for W, we find

$$\frac{W_{ql}}{W}\sim\frac{Mn'\Delta v v_{Te}^2\omega_p^2\omega_{LH}}{Tnv_b\gamma_b(0)\,(\omega_p^2+\omega_H^2)}.$$

By using the expression (1.5) for the growth rate, we conclude that the quasilinear turbulence level exceeds W if

$$\left(\frac{M}{m}\right)^2\frac{\omega_{LH}^2}{\omega_p^2+\omega_H^2}\left(\frac{\Delta v}{v_b}\right)^{5/2}>1.$$

Thus, practically always, and at least in the final relaxation stage, the turbulence level is determined by nonlinear effects.

Finally, we discuss the role of the decay processes of the type $\omega_k\to\omega_{k_1}+\omega_{k_2}$. Their growth rate $\gamma_d\sim\omega(W/nT)(kv)^2$ is sufficiently large, and they may compete with the induced scattering (see Ref. 11). Nevertheless, for $\omega<2\omega_{LH}$, the decay processes are forbidden, because of the form of the spectrum. Since the energy density in a jet increases with decreasing y, all the oscillation energy is, in effect, concentrated in the vicinity of ω_{LH} and, therefore, the role of the decay processes is small and they can be neglected.

In summarizing the results of this section, we conclude that for $y>1$, the spectrum is concentrated in the region $k\sim\omega_k/v_b$, and mainly in the region of small k and large phase velocities. Therefore, quasilinear effects do not affect the bulk of the ions and electrons and do not give rise to particle acceleration.

The oscillations are transferred, preserving a considerable fraction of their energy, to the region where strongly nonlinear effects are significant. The development of the modulational instability creates contracting cavitons, filled with oscillations. In the final stage, the oscillations are damped, transferring their energy to ions and electrons and accelerating the particles. We proceed to the discussion of these effects in the following section.

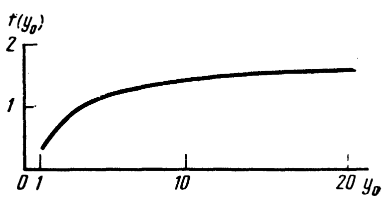

FIG. 5. The function $f(y_0)$, describing at $y^*\ll 1$ the energy flux into the plasma [see Eq. (2.14)]. At $y_0\gtrsim 10$, it is seen that the quantity Q is practically independent of y_0 and $\max f(y_0) = 1.6$.

3. STRONG LOWER-HYBRID TURBULENCE

Thus the oscillations excited by the ion beam are inevitably transferred to the frequency region in the vicinity of the lower-hybrid frequency. In order to describe the interaction of fluctuations in this region we have to use dynamical equations which preserve all the information about the phases of the waves. As with Langmuir turbulence, the proximity of the frequencies to the lower-hybrid frequency allows us to introduce a simplified, averaged description (see Ref. 12).

The equation for the high frequency potential ψ averaged over the lower hybrid frequency has the form[6,7]

$$\Delta\left(i\frac{\partial}{\partial t}+\omega_{LH}R^2\Delta_\perp\right)\psi-\omega_{LH}\psi_{zz}\frac{M}{m}=i\frac{M}{m}\frac{\omega_{LH}^2}{\omega_H n}[\nabla\delta n,\nabla\psi]_z. \tag{3.1}$$

The left hand side of Eq. (3.1) describes the propagation of oscillations characterized by the dispersion relation (1.2), and the nonlinear term in the right hand side describes their scattering on density fluctuations δn, occuring due to the drift of the particles in the magnetic field as a result of the low-frequency electric field.

The high-frequency oscillations can be described hydrodynamically, but the low-frequency motion requires a kinetic description. The low-frequency motions induced by the pondermotive forces can be described within the linear approximation, which yields an equation relating δn and ψ[7]:

$$\delta n_{k\Omega}=iG_{k\Omega}\frac{\omega_H\omega_p^2}{2nT(\omega_p^2+\omega_H^2)}\times\int[\mathbf{k}_1\mathbf{k}_2]_z\psi_{k_1}\psi_{k_2}^*\delta(\mathbf{k}-\mathbf{k}_1+\mathbf{k}_2)\,d\mathbf{k}_1\,d\mathbf{k}_2, \tag{3.2}$$

where in an isothermal plasma ($T_e = T_i$)

$$G_{k\Omega}=-\frac{L_eL_i}{L_e+L_i},\quad L_e=\int\frac{\varkappa_z v_z f_{0e}}{\varkappa_z v_z-\Omega}dv_z,\quad L_i=\int\frac{(\varkappa\mathbf{v})f_{0i}}{(\varkappa\mathbf{v})-\Omega}d\mathbf{v}.$$

In order to understand the characteristic time and spatial scales of the phenomena of interest, we consider the stability problem of a monochromatic wave. We will restrict ourselves to the wave with $k = 0$, $\delta n = 0$ and energy density W_0. The dispersion relation for perturbations $\propto\exp(-i\Omega t+i\varkappa r)$ at $y\lesssim 1$, has the form[7]

$$1+\frac{W_0}{4nT}\frac{M}{m}\omega_{LH}\frac{\omega_p^2}{\omega_p^2+\omega_H^2}G_{\varkappa\Omega}$$
$$\times\sin^2\varphi\left[\frac{1}{-\Omega+\omega(\varkappa)-\omega_{LH}}-\frac{1}{\Omega+\omega(\varkappa)-\omega_{LH}}\right]=0. \quad (3.3)$$

Here φ is the angle between $\varkappa$ and the electric field of the wave. The maximum growth rate is achieved at $\varphi=\pi/2$, and we will consider only these values of φ in the following.

We consider the static limit, $\omega<(kv_{Ti},k_z v_{Te})$, first. In this case $G_{\varkappa\Omega}\simeq-1$ and Eq. (3.3) reduces to the form

$$\Omega^2=(\omega_\varkappa-\omega_{LH})^2-(\omega_\varkappa-\omega_{LH})\frac{W_0}{2nT}\frac{M}{m}\omega_{LH}\frac{\omega_p^2}{\omega_p^2+\omega_H^2}. \quad (3.4)$$

An aperiodic instability exists when

$$\omega_\varkappa-\omega_{LH}<\Gamma,\quad \Gamma=\omega_{LH}\frac{W_0}{2nT}\frac{M}{m}\frac{\omega_p^2}{\omega_p^2+\omega_H^2}.$$

The maximum growth rate is achieved at $\omega_\varkappa-\omega_{LH}=\Gamma/2$ and equals[7]

$$\gamma_{max}=\frac{\Gamma}{2}=\omega_{LH}\frac{W_0}{4nT}\frac{M}{m}\frac{\omega_p^2}{\omega_p^2+\omega_H^2}. \quad (3.5)$$

The static condition, $\Omega<k_z v_{Te}<kv_{Ti}$, is satisfied, at $y\ll1$, up to $kR\sim1$, where a strong Landau damping is switched on and W/nT can reach the value of $W/nT\sim(m/M)((\omega_p^2+\omega_H^2)/\omega_p^2)$. Far from the diagonal $y=kR$, the static condition is rapidly violated and the instability growth rate decreases.

In the framework of the static approximation, $G=-1$ and in the **r**-representation

$$\delta n=i\frac{\omega_H\omega_p^2}{2T(\omega_p^2+\omega_H^2)}[\nabla\psi\,\nabla\psi^*]_z. \quad (3.6)$$

Structurally, equations (3.1) and (3.6) are close to those describing the collapse of Langmuir waves. Therefore, the inevitability of the collapse of localized distributions of lower hybrid waves can be confirmed, by using arguments similar to those discussed for Langmuir turbulence (see, for example, Refs. 12–15). The final evidence of the concept of the Langmuir collapse has been obtained in numerical simulations. The simulation of lower hybrid wave collapse requires solving a three-dimensional problem, because of the vector nature of the nonlinearity and the structure of the dispersion relation.

Nevertheless, as have been shown above, the following relation between the longitudinal and transverse scales $l_\parallel$ and $l_\perp$ of the collapsing caviton must be satisfied during the development of the modulational instability:

$$\frac{l_\parallel}{l_\perp}\sim\left(\frac{M}{m}\right)^{1/2}\frac{1}{k_\perp R}\sim\left(\frac{M}{m}\right)^{1/2}\frac{l_\perp}{l_\parallel}. \quad (3.7)$$

This means that the second and the third terms in Eq. (3.1) have to be of the same order. This observation allows us to use the following two-dimensional model equation in order to qualitatively analyze the three-dimensional problem

$$\Delta(i\psi_t+\Delta\psi)-\operatorname{div}([\nabla\psi\nabla\psi^*]_z[\mathbf{h}\nabla\psi])=0;\quad \mathbf{h}=\mathbf{H}/H. \quad (3.8)$$

Numerical calculations, performed in Ref. 6, have demonstrated convincingly the collapse of an initially localized distribution in a finite time interval.

Eqs. (3.1) and (3.6) admit a self-similar solution of the form[5]

$$\psi(r,t)=\psi\left(\frac{r_\perp}{(t_0-t)^{1/2}},\frac{z}{t_0-t}\right). \quad (3.9)$$

In the case of the two-dimensional equation (3.8), the self-similar solution describes the collapse, preserving the number of plasmons, trapped in the caviton (strong collapse). In the three-dimensional form of the self-similar solution (3.9), the number of plasmons in the caviton decreased during the collapse process. A similar situation exists in subsonic Langmuir collapse and in the self-focusing of a quasi-monochromatic wave (weak collapse). Solutions characterized by a loss of plasmons from the caviton during the three-dimensional collapse seem to be physically unjustified. Indeed, the role of nonlinear terms increases with the increase in the dimensionality, and it is unlikely that the strong collapse in the two-dimensional case will be replaced by weak collapse in the three-dimensional situation. At present a number of numerical simulations have shown convincingly that in the subsonic (or hydrodynamic) limit, the self-similar regime of the collapse does exist. In this case the collapse is strong. In the subsonic limit, corresponding to a low turbulence level and a large caviton size, the question of the establishment of a self-similar regime remains open. Furthermore, Ref. 16 has shown recently that the self-focusing of light, considered as a scalar model of Langmuir collapse, is not characterized by a self-similar solution (3.9) and the collapse is strong, i.e., the number of quanta is conserved in the collapse process. Naturally, we can assume the lower hybrid collapse to be strong, which will be assumed in the following.

Since a collapse is a self-accelerating process, and initially, the distance between the cavitons is of the order of their size, we can write the following estimate for the dissipated energy density:

$$Q\sim W\gamma_{mod}(W)\sim\frac{\omega_{LH}}{4}\frac{M}{m}\frac{\omega_p^2}{\omega_p^2+\omega_H^2}\left(\frac{W}{nT}\right)^2 nT. \quad (3.10)$$

If the turbulence level is high enough and the static approximation is invalid, naturally another limiting case should be considered, i.e., $\Omega>k_z v_{Te}$. Generally, the static approximation is inapplicable even at a low turbulence level, if the ratio between the longitudinal and transverse scales differs from the one given by (3.7), as, for example, for strictly transverse perturbations.

Now the Green's function $G_{\varkappa\Omega}$ is equal to

$$G_{\varkappa\Omega}=\varkappa_z^2 v_{Te}^2/\Omega^2. \quad (3.11)$$

As with isotropic plasma turbulence, this approximation can be referred to as "supersonic", or hydrodynamic, despite the fact that in the region $y<1$, the proper eigenmodes are absent because of the strong damping on electrons. It should be mentioned, that at $\omega>kv_{Ti}$, the expression (3.11) is valid in both $\omega/\varkappa_z v_{Te}<1$ and the inverse cases.

The dispersion relation now assumes the following form

$$1+\frac{\varkappa_z^2 v_{Te}^2}{\omega^2}\left[\frac{1}{\omega+\omega_\varkappa-\omega_{LH}}-\frac{1}{\omega+\omega_{LH}-\omega_\varkappa}\right]\Gamma=0,$$

$$\Gamma=\frac{\omega_{LH}}{4}\frac{W_0}{nT}\frac{M}{m}\frac{\omega_p^2}{\omega_p^2+\omega_H^2} \tag{3.12}$$

or

$$\omega^2(\omega^2-(\omega_\varkappa-\omega_{LH})^2)-(\omega_\varkappa+\omega_{LH})\varkappa_z^2 v_{Te}^2\Gamma=0.$$

It can be seen, in this case, that the maximum growth rate is achieved on the "diagonal" $y = kR$. By considering the dispersion relation on this line only, we can rewrite Eq. (3.12) in the form

$$\omega^4-4\omega_{LH}^2\omega^2(kR)^4-2(kR)^6\omega_{LH}^3\Gamma=0. \tag{3.13}$$

In the hydrodynamic approximation $\Gamma/\omega_{LH} > 1$, and therefore the second term in Eq. (3.13) can be neglected. The growth rate is then given by

$$\gamma_{mod}\sim\omega_{LH}(kR)^{3/2}\left(\frac{W}{nT}\frac{M}{m}\frac{\omega_p^2}{\omega_p^2+\omega_H^2}\right)^{1/4}; \tag{3.14}$$

and the maximum of the growth rate is achieved at $kR\sim 1$. Thus, we can see that the inertial interval is absent in this case and the oscillations are transferred immediately to the strong damping region. Such a situation is usually called a superstrong turbulence.[8]

The dynamical equations describing the nonlinear instability stage can be obtained easily by using expression (3.11) for the Green's function. Then instead of Eq. (3.2) we have

$$\frac{\partial^2\delta n}{\partial t^2}=-i\frac{\omega_p^2}{4\pi\omega_{LH}m}\frac{\partial^2}{\partial z^2}[\nabla\psi\,\nabla\psi^*]_z. \tag{3.15}$$

It should be mentioned that the growth rate of the oscillations with $kR\sim 1$ is larger than ω_{LH} and this description is no longer applicable. The division into high- and low-frequency oscillations itself loses its meaning. The stability of a monochromatic, large-amplitude wave has been investigated in Ref. 17 in the framework of the kinetic description, and it has been shown that the maximum growth rate is less than ω_{LH}. Therefore, it is natural to assume that at large kR the growth rate $\gamma_{\rm mod}(k)$ approaches ω_{LH} and the characteristic evolution time of the clump is

$$\tau\sim\gamma_{mod}^{-1}\sim\omega_{LH}^{-1}. \tag{3.16}$$

The clump breaks up into fragments of size R and the plasmons transfer their energy to the particles. The dynamics of this process has not yet been studied, but it can be assumed that a significant fraction of plasmons, trapped in the process of the development of the modulational instability, are absorbed in this case also. The energy flux into the plasma, similar to Eq. (3.10), is

$$Q\sim\gamma_{mod}W\sim\omega_{LH}W. \tag{3.17}$$

4. ENERGY RELATIONS AND PARTICLE HEATING

If the turbulence level in the vicinity of ω_{LH} is sufficiently low, i.e., $W/nT < m/M$, the width of the strong turbulence region is determined by the relation

$$(W/nT)(M/m)\sim(\varkappa r_d)^2\sim y^2. \tag{4.1}$$

The energy flux into the plasma consists of the absorption in the weak turbulence region (2.13), where y^* is given by (4.1), and of the absorption in the strong turbulence region. If y^* calculated from Eq. (4.1) is less than unity, the integration limit in Eq. (2.14) can be replaced by zero and the energy, entering the plasma, is given by

$$Q\sim nT\frac{\gamma_b^2(0)(\omega_H^2+\omega_p^2)}{\omega_p^2\omega_{LH}}\frac{v_b^2}{v_{Te}^2}f(y_0)$$

$$\approx nT\left(\frac{n'}{n}\right)^2\left(\frac{m}{M}\right)\left(\frac{v_b}{\Delta v}\right)^3\frac{v_b^2}{v_{Ti}^2}$$

$$\times\frac{(\omega_p^2+\omega_H^2)\omega_{Pi}^4}{\pi\omega_{LH}^3\omega_p^2}f(y_0). \tag{4.2}$$

The quantity $f(y_0)$ depends weakly on y_0, when the latter exceeds unity (see Fig. 5) and, therefore, as an estimate, we can use $f(y_0)\sim 1$.

The energy Q released in the plasma can be divided into two parts. Due to the conservation of the number of plasmons in the process of the induced scattering, a significant fraction of the energy is transferred to the frequency region in the vicinity of ω_{LH}. Since the frequency of the plasmons changes significantly in this transformation process, a significant fraction of the energy is transferred to the particle via the process of induced scattering, i.e., to the ions for $y > 1$ and to the electrons for $y < 1$.

By multiplying the time-independent equation (2.1) by ω_k and integrating it with respect to y, we obtain

$$Q(y)=\alpha\left(\frac{m}{M}\right)^{1/2}\int_y^{y_0}\frac{d\omega_k}{dy'}k_0^2n_b^2\,dy'+\alpha\left(\frac{m}{M}\right)^{1/2}\omega_k k_0^2 n_b^2. \tag{4.3}$$

The first term in Eq. (4.3) describes the energy absorption due to the induced scattering, and the second $\alpha(m/M)^{1/2}\omega_k k_0^2 n_b^2 = P(y)$ represents the energy flux into angles smaller than y. Therefore the fractions of the energy δ_i and δ_e absorbed by the ions and electrons, respectively, are given by

$$\delta_i=1-P(1)/Q, \tag{4.4}$$

$$\delta_e=[P(1)-P(y^*)]/Q. \tag{4.5}$$

Here, y^* is the boundary of the weak turbulence region (see below). Numerical calculations have shown that for $y_0\gg 1$ we obtain $\delta_i\sim 0.15$ while δ_e, for $y^*\ll 1$, is usually much smaller.

The energy absorbed due to the induced scattering is transferred to ions with energies close to thermal, and we can assume that the absorption results in plasma heating. Because the volume of the strong turbulence region is small, we can neglect the energy change of the plasmons due to their interaction with the beam inside this region. An accurate criterion for the validity of this assumption will be obtained below.

By comparing the energy flux into the plasma with the absorption due to the collapse, we obtain the energy level of oscillations in the strong turbulence region:

$$\frac{W_0}{nT} \sim \frac{\gamma_b(0)v_b}{\omega_{LH}v_{Ti}}\frac{m}{M}\frac{(\omega_p^2+\omega_H^2)}{\omega_p^2} \sim \frac{n'}{n}\frac{(\omega_p^2+\omega_H^2)}{\omega_{LH}^2}\frac{v_b}{v_{Ti}}\left(\frac{v_b}{\Delta v}\right)^{1/2}. \tag{4.6}$$

It can be seen that for low density beams, the static approximation is valid, despite the narrowness of this region, especially for a space plasma.

As can be seen from (4.6), we find for the ratio

$$\frac{\gamma_{mod}}{\gamma_b} \approx \frac{\omega_{LH}}{\gamma_b}\frac{M}{m}\frac{W_0}{nT}\frac{\omega_p^2}{(\omega_p^2+\omega_H^2)} = \frac{v_b}{v_{Ti}} \gg 1. \tag{4.7}$$

This means that the energy distribution in the strong turbulence region is independent of the structure of the beam, is determined only by the modulational instability, and can be assumed to be quite uniform.

As has been mentioned earlier, the growth rate averaged over the absolute value of k is negative, i.e., $\bar{\gamma}_b = -\gamma_b(\Delta v/v_b)$. Therefore, the interaction of the plasmons with the beam in the strong turbulence region results in reemission of the energy with the rate

$$Q_1 \sim -\gamma_b(\Delta v/v)W. \tag{4.8}$$

The ratio

$$Q_1/Q \sim (\Delta v/v_b)(v_{Ti}/v_b)^2 \ll 1,$$

meaning that the interaction of the beam with the plasmons in the strong-turbulence region introduces, as have been assumed earlier, a negligible contribution to the energy balance of the system.

As can be seen from Eq. (4.6), the static approximation is violated for $\gamma_b \sim \omega_{LH}(v_{Ti}/v_b)$. At larger values of the growth rate the energy absorption is given by the estimate (3.17). By equating it to Q, we obtain the level of the turbulence $W = Q/\omega_{LH}$. Since the maximum growth rate of the modulational instability $(\gamma_{\rm mod})_{\rm max} \sim \omega_{LH}$, the distribution of oscillations in the strong turbulence region can be assumed to be uniform up to

$$\gamma_b \sim \omega_{LH}. \tag{4.9}$$

For larger growth rates we cannot expect to have a uniform distribution in the turbulence region, and on the "ridge" of the growth rate the density of oscillations should increase. We have mentioned earlier that, even if the condition (4.9) is violated, the growth of the oscillations on the "ridge" of the growth rate reduces the amount of energy reemitted into the beam, and only then gives rise to absorption. As a result, when the condition (4.9) is violated, apparently, there also exists a significant region of the plasma parameter space where the absorbed energy is determined by expression (4.2).

Thus, in the problem just considered, we can estimate the rate of the energy release in the plasma because it is determined by the weak turbulence region. This in turn allows us to find the turbulence level in the vicinity of ω_{LH}.

5. PARTICLE ACCELERATION

A considerable fraction of the plasmon energy is transferred to the particles, due to the development of the modulational instability, thus giving rise to the formation of tails in the ion and electron distribution functions.

Because cavitons are located randomly and the phase of the field in the cavitons is random, the acceleration of the particles takes the form of diffusion in velocity space and is described by the conventional quasilinear equations. If the turbulent region is sufficiently large, and the velocities of the particles are small, we can neglect the particle loss and consider a one-dimensional problem. Particle acceleration obviously takes place in the velocity space region where Landau damping becomes significant, changing the energy density of the oscillations considerably. Therefore, an iterative-type solution of the problem of determining the particle distribution functions self-consistently seems to be unrealistic.

Nevertheless, in studying strong Langmuir turbulence we can, by using quite reasonable estimates,[18,19] obtain stationary particle distribution functions. Similar arguments will be employed below in our case.

The "tail-stretching" process evidently continues until $\gamma_{\rm mod}$ is balanced by the Landau damping associated with the new distribution function:

$$\gamma_{mod} = \gamma_L. \tag{5.1}$$

At lower values of v, the distribution function in this process has to become stationary, since the plasmons do not reach this region of phase velocities and the diffusion coefficient vanishes. Thus the time-independence is established due to the motion of the boundary towards the region of larger values of v. At large values of v, however, Landau damping can be neglected and we can, therefore, use the results of Section 3 in calculating $\gamma_{\rm mod}$.

We consider the static case first. In order to find the change in

$$\gamma_{mod} = \omega_{LH}\frac{W}{nT}\frac{M}{m}\frac{\omega_p^2}{\omega_H^2+\omega_p^2}$$

in the caviton contraction process, we assume that the number of plasmons is conserved:

$$W_0 l_{0\perp}^2 l_{0\parallel} = W l_\perp^2 l_\parallel; \tag{5.2}$$

where $l_\perp$ and $l_\parallel$ are related via condition $y = kR$, or

$$(l_\perp/l_\parallel)(M/m)^{1/2} \sim R/l_\perp. \tag{5.3}$$

We consider the acceleration of ions first. Only the transverse component of the velocity is increasing in this case and, therefore,

$$\gamma_L \sim \frac{\omega_{LH}}{n}v^2 f(v)\,|_{v=\omega_{LH}/k}. \tag{5.4}$$

By using Eqs. (5.3) and (5.4), and since initially

$$\frac{W_0}{nT}\frac{M}{m}\frac{\omega_p^2}{\omega_p^2+\omega_H^2} \sim \frac{R^2}{l_\perp^2}, \tag{5.5}$$

we obtain

$$\gamma_{mod} \sim \omega_{LH}\left(\frac{nT}{W_0}\frac{m}{M}\right)\frac{\omega_p^2+\omega_H^2}{\omega_p^2}\left(\frac{R}{l_\perp}\right)^4. \quad (5.6)$$

Equating (5.5) and (5.6), we have

$$f \sim \left(\frac{nT}{W_0}\frac{m}{M}\right)\frac{\omega_p^2+\omega_H^2}{\omega_p^2}\left(\frac{v_{Ti}}{v}\right)^4\frac{n}{v^2}\propto v^{-6}. \quad (5.7)$$

It can be seen that the distribution function is decreasing so rapidly that the accelerated ions do not contribute to the particle or energy balance in the plasma. The power-law tail (5.7) "stretches" until v becomes of the order of v_b, or until the ion loss from the turbulent region becomes significant. In the latter case, at large velocities an exponentially decreasing particle distribution is formed.

We consider now the electron acceleration. Here,

$$\gamma_L \sim \omega_{LH} v f(v)\,|_{v=\omega_{LH}/k_z}. \quad (5.8)$$

By expressing $\gamma_{\rm mod}$ via $k_z = l_\parallel^{-1}$, we obtain

$$\gamma_{mod} \sim \omega_{LH}\left(\frac{nT}{W_0}\frac{m}{M}\right)\frac{\omega_p^2+\omega_H^2}{\omega_p^2}\frac{M}{m}\frac{R^2}{l_\parallel^2}, \quad (5.9)$$

which yields the following expression for the electron distribution function

$$f = \left(\frac{nT}{W_0}\frac{m}{M}\right)\frac{\omega_p^2+\omega_H^2}{\omega_p^2}\frac{v_{Te}^2}{v^2}\frac{n}{v}. \quad (5.10)$$

It can be seen that the number of particles in the tail is finite; however, the total energy in the tail

$$\varepsilon = \int_{v_{Te}}^{v_{max}} f\frac{mv^2}{2}dv = nT\left(\frac{nT}{W_0}\frac{m}{M}\right)\frac{\omega_p^2+\omega_H^2}{\omega_p^2}\ln\frac{v_{max}}{v_{Te}} \quad (5.11)$$

diverges logarithmically at the upper limit, i.e., all the energy is contained in fast particles. The thermal transport is determined by these particles. As in the ion case, Eq. (5.10) is valid only for velocities $v < (Da)^{1/3}$ (where a and D are the size of the turbulence region and the quasilinear diffusion coefficients, respectively). If the opposite inequality holds, the particle distribution decreases exponentially.

We consider next the hydrodynamical limit, i.e., the strong turbulence limit. Here the inertial interval is absent. Consider the acceleration of ions first. By using expression (3.14) for the growth rate of the modulational instability, we obtain

$$f_i = \frac{n}{v^2}\left(\frac{W_0}{nT}\frac{M}{m}\frac{\omega_p^2}{\omega_p^2+\omega_H^2}\right)^{1/4}\left(\frac{v_{Ti}}{v}\right)^{3/2}\propto\frac{1}{v^{7/2}}. \quad (5.12)$$

Similarly, the electron distribution function is

$$f_e = \frac{n}{v}\left(\frac{W_0}{nT}\frac{M}{m}\frac{\omega_p^2}{\omega_p^2+\omega_H^2}\right)^{1/4}\left(\frac{v_{Te}}{v}\right)^{3/4}. \quad (5.13)$$

It can be seen that again the number of particles in the tail is finite. The integral defining the total energy of the particles diverges, however, at the upper limit. If we assume that the maximum ion and electron velocites are related as $v_{e\max}(v_{i\max}^2/v_{Ti})(M/m)^{1/2}$, which follows from (5.4), then the ratio between the energies of the ion and electron tails is

$$\varepsilon_i/\varepsilon_e = {}^5/_2(v_{Ti}/v_{i\,max})^2,$$

i.e., in this case also, most of the energy goes to the electrons.

6. DISCUSSION

We have seen that ion flow across a magnetic field is accompanied by a large number of nonlinear phenomena. The ion flow excites intense lower-hybrid oscillations, propagating almost perpendicularly to the magnetic field. The phase velocities of these oscillations are large and, practically speaking, they do not interact with the bulk of the electrons and ions. Therefore the induced wave scattering becomes the main nonlinear process. The induced scattering transfers the oscillations to the frequency region in the vicinity of the lower hybrid frequency. About 15% of the energy, in this process, is spent on the heating of the ion component of the plasma.

The modulational instability, which develops in the lower-hybrid frequency region, leads to a decrease of the phase velocities of the oscillations and of their interaction efficiency with ions and electrons. In this process most of the absorbed energy goes into accelerating the ions. The acceleration of the particles results in a formation of power-law tails in the particle distribution functions, and the maximum energy of the accelerated electrons exceeds the ion energy in the beam. The turbulence level drops when the tails in the distribution functions reach the velocity of light and then the quasilinear effects become important.

The results of the present work, applied to the anomalous ionization problem, mean that energy is transferred efficiently to the electrons and that the critical velocity is close to the Alfven limit.[3]

The excitation of electrostatic oscillations with $k_0c/\omega_p \sim (\omega_{LH}/\omega_p)\times(c/v_b)\gg 1$ has been considered in the present work. This condition is apparently well satisfied in the anomalous ionization problem.[2,20] In a near-terrestrial shock wave, where the ion stream velocities are considerably larger than those of the oscillations excited in the vicinity of ω_{LH}, the electromagnetic corrections to the dispersion relation are important. The corrections change the estimate of the energy flux Q into the plasma, but the overall picture of strong turblence discussed here remains unchanged.

Experimental data on shock waves[2–5] are in good agreement with the predictions of the present work. We have confirmed the excitation of the lower hybrid waves, electron acceleration to high energies, etc. It is still difficult, however, to test the details of the picture of the phenomena just developed. One should mention also the existence of purely physical problems, preventing full understanding. These problems are related primarily to a class of cases associated with lower hybrid collapse in the strong turbulence regime, which requires the use of detailed numerical simulations.

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