Photoelectric effect in superconductors

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The photoelectric effect in superconductors, as a result of which a stationary invariant potential and an emf (in the spatially inhomogeneous case) appear under the action of an electromagnetic field, is investigated theoretically. Only those cases in which the effect manifests itself even when the parameter $\bar{\varepsilon}/\varepsilon_F$ vanishes (where $\bar{\varepsilon}\sim(T,\Delta)$ and ε_F is the Fermi energy) are analyzed. This anomaly, which is connected with an amplitude asymmetry in the electron- and hole-excitation scattering, obtains in the presence of magnetic and (under certain conditions) nonmagnetic impurities. In the latter case the situations in which the photoelectric-effect anomaly is due to an anisotropy in the order parameter and to the presence of an external magnetic field are analyzed.

1. INTRODUCTION

In many papers devoted to the investigation of the effect of an electromagnetic field on superconductors,[1–3] the phenomena due to the change in the modulus Δ of the order parameter are studied. As is well known,[1,2] under nonequilibrium conditions, the quantity Δ depends on the form of the quasiparticle distribution function, or, more precisely, on that part of the distribution function which is the same for the electron- and hole-excitation branches. It is generally believed[1,2] that it is precisely such symmetric nonequilibrium states (with identical branch populations) that arise in superconductors under the action of an electromagnetic field.

Besides the symmetric states, there can be excited in superconductors, when subjected to other influences, asymmetric states[4] characterized by the presence of a population difference (imbalance[5]) between the electron and hole branches. The creation of a branch imbalance causes the gauge-invariant potential

$$\mu=\varphi+{}^1/_2(\partial\chi/\partial t),$$

where φ is the electric potential and χ is the phase of the order parameter (the electron charge is taken to be equal to unity), and a longitudinal electric field whose stationary value is given by the time-averaged quantity $\nabla\bar{\mu}$ to appear in the superconductor. In theoretical studies (see the review in Ref. 4), which were started by Tinkham,[5] various mechanisms have been investigated for determining the branch-imbalance relaxation time τ_b, on which depends, in particular, such an important quantity as the attenuation depth l_b for a longitudinal electric field in the superconductor.[4] It has been found that l_b in superconductors with a gap can be significantly greater than the characteristic values of the coherence length ξ and the London penetration depth λ_L.

We have already noted above the prevalence of the point of view that branch imbalance is not produced under the action of an electromagnetic field. This assertion is valid only when we neglect the effects of the magnitude of the order of the small parameter $\bar{\varepsilon}/\varepsilon_F$, where $\bar{\varepsilon}\sim T$, Δ is the characteristic quasiparticle energy and ε_F is the Fermi energy. To first order in $\bar{\varepsilon}/\varepsilon_F$ the branch imbalance and the potential μ are, under the nonequilibrium conditions, bound to manifest themselves as consequences of the difference in the excitation momenta

$$p=p_F(1+\zeta/2\varepsilon_F),\qquad \zeta=v_F(p-p_F)$$

for the electron and hole branches. Note that the potential produced as a result of irradiation and due to the allowance for the terms of first order in the parameter $\bar{\varepsilon}/\varepsilon_F$ is analyzed in Ref. 6 by Aronov.[1)]

In the present paper we shall study the photoelectric effect[2)] (PE), consisting in the appearance of a stationary potential $\bar{\mu}$ and a stationary emf (in the spatially inhomogeneous case) under the action of an electromagnetic field, the investigation being restricted only to those situations in which the effect manifests itself even when the parameter $\bar{\varepsilon}/\varepsilon_F$ vanishes. As has been demonstrated by V. V. Zaĭtsev and the present author,[8] this anomaly occurs, in particular, when the superconductor contains paramagnetic impurities. The magnitude of the PE in this case is not of the order of the small parameter $\bar{\varepsilon}/\varepsilon_F$ because of a peculiarity of the amplitude of the scattering by the paramagnetic impurities, namely, the asymmetry of this quantity for the electron and hole excitation branches. Note that this asymmetry has been known for a long time in the case of normal metals.[9] In that case it is found when allowance is made for the Kondo effect resulting from the quantum nature of the spin impurity (see, for example, the reviews in Refs. 10 and 11).

The specific character of superconductors manifests itself in the fact that the scattering amplitude is asymmetric (in the above-indicated sense) even when the Kondo effect is not taken into account, i.e., even when in the operator describing the interaction between the electron and the magnetic impurity (located at the point r_i), which is usually written in the form

$$U_m(\mathbf{r}-\mathbf{r}_i)=V_m(\mathbf{r}-\mathbf{r}_i)+J(\mathbf{r}-\mathbf{r}_i)\hat{\mathbf{S}}\hat{\mathbf{s}}\qquad(1)$$

($\hat{\mathbf{S}}$ and $\hat{\mathbf{s}}$ are respectively the impurity- and electron-spin

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operators), we replace the operator $\hat{S}$ by the classical vector $\mathbf{S}$. The scattering on the magnetic impurities in superconductors was first described exactly in this model (which can be justified when $S \gg 1$) by Shiba[12] and Rusinov.[13] Note that we can find the electron-hole scattering asymmetry, which is essential to the effect we are considering, only when we go beyond the Born approximation.[3)]

In the investigation carried out in Ref. 8, of the PE in superconductors with magnetic impurities, the scattering on these impurities is described in the model adopted in Refs. 12 and 13 (henceforth referred to, for brevity, as the SR model). The case of the dirty superconductor, in which the frequency of collision with the nonmagnetic impurities $\nu_n \gg T$, Δ, is considered there. In Sec. 2 of the present paper we find out how the magnitude of the effect depends on ν_n, ν_s, T, and Δ ($\nu_s = \tau_s^{-1}$, where τ_s is the time characterizing the spin-flip scattering of an electron on a magnetic impurity) in a broad range of values of each of these parameters. We shall take account of the scattering by the magnetic impurities largely within the framework of the SR model, but we shall, to conclude Sec. 2, briefly discuss the influence of the Kondo effect on the results.

In Sec. 3 we shall investigate the anomalous PE in the absence of magnetic impurities, which remains as a result of the presence of nonmagnetic impurities when the parameter $\bar{\varepsilon}/\varepsilon_F$ goes to zero. The cause is the same scattering asymmetry, which can arise, for example, in the presence of anisotropy or spatial inhomogeneity in the (unirradiated) superconductor. We shall consider the anomalous PE due to anisotropy of the order parameter, as well as to an external magnetic field.

Usually, in experiments the effect of the irradiation or alternating electric field on the superconductor is not uniform. Some examples of this situation are shown in Fig. 1, which also shows schematic representations of the form of the spatial distribution of the induced potential $\bar{\mu}(x)$. To determine the invariant potential $\bar{\mu}$ in that part of the superconductor where it is coordinate-independent (Figs. 1a and 1b), it is sufficient to consider the spatially homogeneous problem. Let us proceed now to solve it.

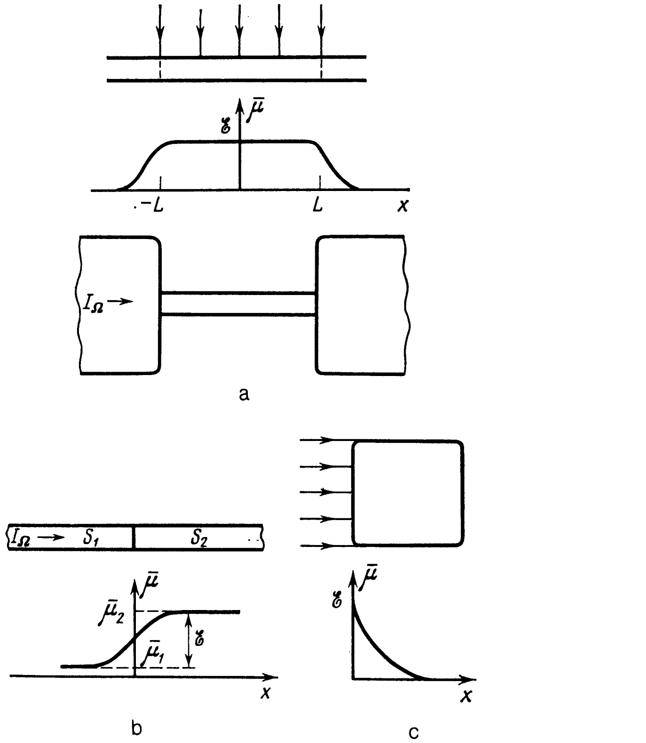

FIG. 1. Examples of structures in which under the action of radiation (depicted by arrows) or current I_Ω a nonuniform time-independent invariant potential $\bar{\mu}(x)$ is excited and, consequently an emf $\mathscr{E}$ appears across some section: a) Two methods of producing roughly the same $\bar{\mu}(x)$ distribution in a film, the difference being only that $\bar{\mu}$ decreases more rapidly in the regions $|x| > L$ in the situation depicted by the bottom diagram (of Fig. 1a), in which the film is the link member of a bridge of variable thickness; it is assumed that the length $L \gg l_b$. b) A film junction between two superconductors S_1 and S_2; an emf will develop when S_1 and S_2 differ only in their impurity concentrations. c) A bulk superconductor of thickness $d \gg \lambda_L, l_b$.

2. THE PHOTOELECTRIC EFFECT IN SUPERCONDUCTORS WITH MAGNETIC IMPURITIES

We shall, in solving the problem begin with the system of equations for the Green's functions integrated over $\zeta = v_F(p - p_F)$ (Refs. 17 and 18):

$$i\mathbf{v}_F \frac{\partial}{\partial \mathbf{R}} \check{g}_{\varepsilon,\varepsilon'} + \varepsilon \check{\tau}_z \check{g}_{\varepsilon,\varepsilon'} - \check{g}_{\varepsilon,\varepsilon'} \varepsilon' \check{\tau}_z - \int \frac{d\omega}{2\pi} (\check{\Lambda}_\omega \check{g}_{\varepsilon-\omega,\varepsilon'} - \check{g}_{\varepsilon,\varepsilon'+\omega} \check{\Lambda}_\omega)$$

$$= \check{g}\check{\Sigma} - \check{\Sigma}\check{g}, \qquad (2)$$

$$\check{\Lambda}_\omega = \mathbf{v}_F \mathbf{p}_s(\omega) \check{\tau}_z + \Delta_\omega i \check{\tau}_y + \mu_\omega \check{1},$$

where

$$\check{\tau}_j = \begin{pmatrix} \hat{\tau}_j & \hat{0} \\ \hat{0} & \hat{\tau}_j \end{pmatrix} \quad (j = z, y), \qquad \check{g}\check{\Sigma} \equiv \int \check{g}_{\varepsilon,\varepsilon_1} \check{\Sigma}_{\varepsilon_1,\varepsilon'} \frac{d\varepsilon_1}{2\pi},$$

$$\check{g} = \begin{pmatrix} \hat{g}^R & \hat{g} \\ \hat{0} & \hat{g}^A \end{pmatrix}, \quad \hat{g}^{(R,A)} = \begin{pmatrix} g & f \\ -f^+ & \bar{g} \end{pmatrix}^{(R,A)}, \quad \check{\Sigma} = \begin{pmatrix} \hat{\Sigma}^R & \hat{\Sigma} \\ \hat{0} & \hat{\Sigma}^A \end{pmatrix},$$

$$\Delta_\omega = \frac{\lambda}{4} \int_{-\omega_D}^{\omega_D} \langle f_{\varepsilon,\varepsilon-\omega} \rangle d\varepsilon, \qquad \langle (\ldots) \rangle = \int \frac{d\Omega_p}{4\pi} (\ldots),$$

$\mathbf{p}_s$ is the superfluid momentum, which is connected with the electric field $\mathbf{E}$ and μ by the relation $\partial \mathbf{p}_s / \partial t = \mathbf{E} + \nabla \mu$. Above $\check{\Sigma} = \check{\Sigma}_m + \check{\Sigma}_n + \check{\Sigma}_{ph}$, where the self-energy matrix $\check{\Sigma}_{m(n)}$ describes the interaction with the magnetic (nonmagnetic) impurities, $\check{\Sigma}_{\rm ph}$ describes the interaction with the phonons, λ is the electron-phonon interaction constant, and ω_D is the Debye frequency.

As has already been noted above, we shall describe the interaction with the magnetic impurities, using the SR model,[12,13] which allows us to relatively simply take account of that distinctive feature of the scattering which is responsible for the appearance of the anomalous PE. The question of the influence of the Kondo effect will be touched upon at the end of this section.

The expression for $\check{\Sigma}_m$ can, as in the equilibrium case,[12,13] be obtained, using the solution to the scattering problem for an isolated impurity, which leads to the following formula:

$$\check{\Sigma}_m = {}^1/_2 c_m (\check{t}^+ + \check{t}^-), \qquad (3)$$

where c_m is the magnetic impurity concentration, $\check{t}^\pm$ is the vertex matrix describing the sattering by an impurity with spin parallel (antiparallel) to an arbitrarily chosen direction, and satisfying the following equation:

$$\check{t}^{\alpha}=\check{a}^{\alpha}/\pi N_F+i\check{a}^{\alpha}\langle\check{g}\rangle\check{t}^{\alpha} \tag{4}$$

(we take the impurity potential U_m to be a δ-function: $V_m(\mathbf{r}) = V_m\delta(\mathbf{r}), J(\mathbf{r}) = J\delta(\mathbf{r})$). Here

$$\check{a}^{\alpha}=\begin{pmatrix}\hat{a}^{\alpha} & \hat{0}\\ \hat{0} & \hat{a}^{\alpha}\end{pmatrix},\quad \hat{a}^{\pm}=\begin{pmatrix}a^{\pm} & 0\\ 0 & a^{\mp}\end{pmatrix},$$

$N_F = mp_F/2\pi^2$ is the density of states at the Fermi surface, and $a^{\pm} = \pi(V_m \pm JS/2)N_F$. From (4) it follows that

$$\hat{t}^{\alpha R(A)}=(\hat{1}-i\hat{a}^{\alpha}\langle\hat{g}^{R(A)}\rangle)^{-1}\hat{a}^{\alpha}/\pi N_F,$$
$$\hat{t}^{\alpha}=(\hat{1}-i\hat{a}^{\alpha}\langle\hat{g}^{R}\rangle)^{-1}i\hat{a}^{\alpha}\langle\hat{g}\rangle(\hat{1}-i\hat{a}^{\alpha}\langle\hat{g}^{A}\rangle)^{-1}\hat{a}^{\alpha}/\pi N_F. \tag{5}$$

In the equilibrium case we have

$$\check{\Sigma}_m(\varepsilon,\varepsilon')=\check{\Sigma}_m(\varepsilon)2\pi\delta(\varepsilon-\varepsilon').$$

From (5) we obtain

$$\check{\Sigma}_m{}^R(\varepsilon)=\Big[-2\nu_s w\sin\vartheta(f_\varepsilon{}^R)^2\hat{1}+\frac{i\nu}{2}g_\varepsilon{}^R\hat{\tau}_z+i\left(\frac{\nu_m}{2}-\nu_s\right)f_\varepsilon{}^R i\hat{\tau}_y\Big][(g_\varepsilon{}^R)^2-(wf_\varepsilon{}^R)^2]^{-1}$$
$$\equiv\sigma_0{}^R(\varepsilon)\hat{1}+\sigma_1{}^R(\varepsilon)\hat{\tau}_z+\sigma_2{}^R(\varepsilon)i\hat{\tau}_y,$$
$$\nu_s=c_m(1-w^2)/2\pi N_F,\qquad \nu_m=2c_m(1-w\cos\vartheta)/\pi N_F. \tag{6}$$

Introducing the scattering phase $\delta^{\pm}$ ($\mathrm{tg}\delta^{\pm} = a^{\pm}$) in the model, we obtain[13]

$$w=\cos(\delta^+-\delta^-),\quad \vartheta=\delta^++\delta^-.$$

The expression (6) coincides with the one obtained in Refs. 12 and 13 (here the quantity $\sigma_0^R(\varepsilon)$ does not include the constant determining the renormalization of the chemical potential). As we shall see later, the occurrence in the matrix $\hat{\Sigma}_m^R(\varepsilon)$ of the component $\hat{\sigma}_0^R(\varepsilon)$, which is responsible for the breaking of the electron-hole scattering symmetry, leads to a situation in which the PE occurs even in lowest order in the parameter $\bar{\varepsilon}/\varepsilon_F$ (we take into account only the terms of this order in the quasiclassical equations we use for the Green's functions). In the presence of electron-hole scattering symmetry, the t-matrix component $t_1^R(\varepsilon)$, defined according to the relation $c_m t_1^R(\varepsilon) = (\hat{\Sigma}_m^R(\varepsilon))_{11}$, satisfies the relation $t_1^R(\varepsilon) = -(t_1^R(-\varepsilon))^*$. Since $\mathrm{Sp}\hat{\Sigma}_m^R(\varepsilon) = 2\sigma_0^R(\varepsilon) = c_m[t_1^R(\varepsilon) + (t_1^R(-\varepsilon))^*]$, it is clear that the breaking of this symmetry is due to the nonvanishing component $\sigma_0^R(\varepsilon)$.

From (2) and (6) we obtain for g_ε^R and f_ε^R the expressions[12,13]

$$g_\varepsilon{}^R=u_\varepsilon{}^R[(u_\varepsilon{}^R)^2-1]^{-1/2}=u_\varepsilon{}^R f_\varepsilon{}^R,$$
$$\frac{\varepsilon+\sigma_\varepsilon{}^{ph}}{\Delta}=\left\{1+\frac{[1-(u_\varepsilon{}^R)^2]^{1/2}}{\tau_s\Delta[(u_\varepsilon{}^R)^2-w^2]}\right\},$$
$$\sigma_\varepsilon{}^{ph}=\frac{i}{2}\mathrm{Sp}(\hat{\tau}_z+i\hat{\tau}_y u_\varepsilon{}^R)\,\mathrm{Im}\,\hat{\Sigma}_{ph}{}^R(\varepsilon), \tag{7}$$

and the expression for $\hat{\Sigma}^R_{\mathrm{ph}}$ is given in Refs. 17 and 18.

The matrix $\check{\Sigma}_n$ describing the scattering on the nonmagnetic impurities can easily be found from (3) and (4) as a particular case:

$$\check{\Sigma}_n=(c_n a_n/\pi N_F)(\check{1}-ia_n\langle\check{g}\rangle)^{-1}, \tag{8}$$

here c_n is the nonmagnetic-impurity concentration and $a_n = \pi V_n N_F = \mathrm{tg}\delta_n$.

To Eq. (2) and the relations given above must be added the normalization condition[18]

$$\check{g}^2\equiv\int\frac{d\varepsilon_1}{2\pi}\check{g}_{\varepsilon,\varepsilon_1}\check{g}_{\varepsilon_1,\varepsilon'}=2\pi\delta(\varepsilon-\varepsilon')\check{1}, \tag{9}$$

as well as the expression relating the Green's function to the potential[17,18] μ:

$$\mu_\omega=-\frac{1}{8}\int d\varepsilon\,\mathrm{Sp}\langle\hat{g}_{\varepsilon,\varepsilon-\omega}\rangle. \tag{10}$$

We shall, in determining $\hat{g}$, take into consideration the fact that this matrix can be expressed in terms of two distribution functions[18] and f_1 and f_2:

$$\hat{g}_{\varepsilon,\varepsilon'}(\mathbf{p}_F)=\int\frac{d\varepsilon_1}{2\pi}(\hat{g}^R_{\varepsilon,\varepsilon_1}(\mathbf{p}_F)\hat{f}(\varepsilon_1,\varepsilon',\mathbf{p}_F)-\hat{f}(\varepsilon,\varepsilon_1,\mathbf{p}_F)\hat{g}^A_{\varepsilon_1,\varepsilon'}(\mathbf{p}_F))\equiv\hat{g}^R\hat{f}-\hat{f}\hat{g}^A,$$
$$\hat{f}=f_1\hat{1}+f_2\hat{\tau}_z.$$

Let us now proceed to solve a specific problem. Let us consider a thin film acted upon by an electromagnetic field of frequency Ω as a result of irradiation or the passage of current. We shall assume that only the electric component $\mathbf{E} = \partial\mathbf{p}_s/\partial t$ of the microwave field is important, and that we can neglect the magnetic component $\mathbf{H} = \mathrm{curl}\,\mathbf{p}_s$ and, consequently, the variations in the p_s distribution over the cross section of the film. This is justified if the film thickness and width satisfy $d\ll\lambda(\Omega)$ and $b\ll\lambda^2(\Omega)/d$, with $\lambda(\Omega)\sim\min(\lambda_L,\lambda_{sK}(\Omega))$ where $\lambda_{sk}(\Omega)$ is the skin depth. The field-induced superfluid momentum

$$p_s(t)=A_\Omega\cos\Omega t,\quad A_\Omega=E_\Omega/\Omega$$

is assumed to be small compared to the critical unpairing momentum, which implies that the relation

$$A_\Omega\xi\ll1 \tag{12}$$

is satisfied.

In the presence of an alternating field

$$\hat{f}(\varepsilon,\varepsilon',\mathbf{p}_F)=\hat{f}(\varepsilon,\mathbf{p}_F)2\pi\delta(\varepsilon-\varepsilon')+\delta\hat{f}(\varepsilon,\varepsilon',\mathbf{p}_F),$$

where the first term determines the time-independent response to the external influence, and the second term, the time-dependent response. The distribution function $f_1(\varepsilon) = \langle f_1(\varepsilon,\mathbf{p}_F)\rangle$ characterizing the symmetric part of the branch populations has been studied in a large number of papers[1,2] for the case in which the external influence is an electromagnetic field (usually the function $n_\varepsilon = \frac{1}{2}[1 - f_1(\varepsilon)\mathrm{sign}\,\varepsilon]$ is considered). As will be shown below, allowance for the electron-hole asymmetry in the scattering on the magnetic impurities leads to a situation in which the field creates another distribution function $f_2(\varepsilon) = \langle f_2(\varepsilon,\mathbf{p}_F)\rangle$ determining the imbalance between the branch populations.

Proceeding to the derivation of the kinetic equations for $f_{1(2)}(\varepsilon)$, we consider the condition (12), which allows us to use the method of successive approximations with the small amplitude A_Ω. The ultimate aim is to separate out in the system (2) the relations which are second order in A_Ω, from which it would not be difficult to derive the required kinetic equations. Before proceeding to carry out this program, let us stipulate that the characteristic value of the energy relaxation time

$$\tau_\varepsilon \gg \tau_s,\ \Omega^{-1},\ \Delta^{-1};\qquad \tau_\varepsilon^{-1} \sim \lambda[\max(T,\Delta)]^3/\omega_D^2.$$

Because τ_ε has such a large value, the derivation of $f_1(\varepsilon)$ from the equilibrium value $\mathrm{th}(\varepsilon/2T)$ can be fairly large even for weak fields, as defined by the condition (12). This circumstance will be taken into account in the derivation.

Let us first find the first-order—in A_Ω—correction $\check{g}^{(1)}_{\varepsilon,\varepsilon'}$ to the Green's function. From (2) it follows that we can write it in the form (the x axis lies along the film in the direction parallel to $\mathbf{E}$)

$$\check{g}^{(1)}_{\varepsilon,\varepsilon'} = \pi A_\Omega v_x \check{J}_{\varepsilon,\varepsilon'}[\delta(\varepsilon-\varepsilon'+\Omega)+\delta(\varepsilon-\varepsilon'-\Omega)]. \tag{13}$$

It is convenient to represent the matrix $\hat{J}_{\varepsilon,\varepsilon'}$ as follows:

$$\hat{J}_{\varepsilon,\varepsilon'} = \hat{J}^R_{\varepsilon,\varepsilon'} f_1(\varepsilon') - \hat{J}^A_{\varepsilon,\varepsilon'} f_1(\varepsilon) + \hat{J}^a_{\varepsilon,\varepsilon'}[f_1(\varepsilon) - f_1(\varepsilon')]. \tag{13'}$$

Substituting (13) into (2), taking account of the spatial homogeneity of the problem, as well as the relation

$$\hat{g}_\varepsilon^R \hat{J}^{R(a)}_{\varepsilon,\varepsilon'} = -\hat{J}^{R(a)}_{\varepsilon,\varepsilon'} \hat{g}^{R(A)}_{\varepsilon'},$$

which follows from (9), we easily find that[4)]

$$\hat{J}^{R(a)}_{\varepsilon,\varepsilon'} = [\hat{g}_\varepsilon^R - \bar{\sigma}_0^{R(a)}(\varepsilon,\varepsilon')](\hat{g}_\varepsilon^R \hat{\tau}_z - \hat{\tau}_z \hat{g}_\varepsilon^{R(A)}) b^{R(A)}_{\varepsilon,\varepsilon'}, \tag{14}$$

where

$$\bar{\sigma}_0^{R(a)}(\varepsilon,\varepsilon') = [\sigma_0^R(\varepsilon) - \sigma_0^{R(A)}(\varepsilon')][\tilde{\xi}_\varepsilon^R + \tilde{\xi}_{\varepsilon'}^{R(A)} + i\nu_n]^{-1},$$
$$\tilde{\xi}_\varepsilon^R = [(\varepsilon+\sigma_1^R(\varepsilon))^2 - (\Delta+\sigma_2^R(\varepsilon))^2]^{1/2},$$
$$b^{R(a)}_{\varepsilon,\varepsilon'} = [1-(\bar{\sigma}_0^{R(a)}(\varepsilon,\varepsilon'))^2]^{-1}[\tilde{\xi}_\varepsilon^R + \tilde{\xi}_{\varepsilon'}^{R(A)} + i\nu_n]^{-1},$$
$$\nu_n = 4c_n \sin^2\delta_n/\pi N_F.$$

Further, we can, by substituting (14) into (2), separating out the terms proportional to $\delta(\varepsilon-\varepsilon')$, and averaging over the angles, obtain an equation for the matrix $\langle \check{g}_\varepsilon(\mathbf{p}_F)\rangle$, and from it the kinetic equations for $f_{1(2)}(\varepsilon)$. In doing this, we should take account of the fact that

$$\langle \hat{g}_\varepsilon(\mathbf{p}_F)\rangle = \langle \hat{g}_\varepsilon^R(\mathbf{p}_F) - \hat{g}_\varepsilon^A(\mathbf{p}_F)\rangle f_1(\varepsilon) + (\hat{g}_\varepsilon^R\hat{\tau}_z - \hat{\tau}_z\hat{g}_\varepsilon^A) f_2(\varepsilon)$$
$$+ \langle (\hat{g}^{R(1)}\delta\hat{f}^{(1)} - \delta\hat{f}^{(1)}\hat{g}^{A(1)})(\varepsilon)\rangle_{st}, \tag{15}$$

where the suffix attached to the last term indicates that we should take the stationary part of this expression, i.e., the coefficient of $2\pi\delta(\varepsilon-\varepsilon')$; the first-order (in A_Ω) matrix $\delta\hat{f}^{(1)}$, which can easily be determined from (14) and (13'), is equal to

$$\delta\hat{f}^{(1)}_{\varepsilon,\varepsilon'} = \pi v_x A_\Omega[\delta(\varepsilon-\varepsilon'+\Omega)+\delta(\varepsilon-\varepsilon'-\Omega)] b^a_{\varepsilon,\varepsilon'}(f_1(\varepsilon')-f_1(\varepsilon))$$
$$\times\left[\left(\frac{g_\varepsilon^R f_{\varepsilon'}^A + g_{\varepsilon'}^A f_\varepsilon^R}{f_\varepsilon^R - f_{\varepsilon'}^A}\right)\hat{1} + \bar{\sigma}_0^a(\varepsilon,\varepsilon')\hat{\tau}_z\right]. \tag{16}$$

Consequently, we can, after simple but involved calculations obtain the following kinetic equations[5)]:

$$Q_1(\varepsilon) = I_1^{ph}, \tag{17}$$

$$Q_2(\varepsilon) = \nu_b(\varepsilon) f_2(\varepsilon) + I_2^{ph}, \tag{18}$$

where the sources $Q_j(\varepsilon)$, which determine the rate of change of the function $f_j(\varepsilon)$ under the action of the field, and the integrals I_j^{ph} for the collisions with the phonons have the form

$$Q_j(\varepsilon) = Q_j(\varepsilon,\Omega) + Q_j(\varepsilon,-\Omega),$$
$$Q_1(\varepsilon,\Omega) = \alpha_\Omega \,\mathrm{Im}[b^a_{\varepsilon,\varepsilon_-}(1 - g_\varepsilon^R g_{\varepsilon_-}^A - f_\varepsilon^R f_{\varepsilon_-}^A)$$
$$- b^R_{\varepsilon,\varepsilon_-}(1 - g_\varepsilon^R g_{\varepsilon_-}^R - f_\varepsilon^R f_{\varepsilon_-}^R)][f_1(\varepsilon_-) - f_1(\varepsilon)], \tag{17'}$$

$$Q_2(\varepsilon,\Omega)$$
$$= \alpha_\Omega \,\mathrm{Im}\{\bar{\sigma}_0^a(\varepsilon,\varepsilon_-) b^a_{\varepsilon,\varepsilon_-}(g_\varepsilon^R - g_{\varepsilon_-}^A) - \bar{\sigma}_0^R(\varepsilon,\varepsilon_-) b^R_{\varepsilon,\varepsilon_-}(g_\varepsilon^R - g_{\varepsilon_-}^R)$$
$$+ 2\Delta b^R_{\varepsilon,\varepsilon_-} b^a_{\varepsilon_-,\varepsilon}$$
$$\times[(f_\varepsilon^R + f_{\varepsilon_-}^R)(f_{\varepsilon_-}^R - f_\varepsilon^A)^{-1}(g_{\varepsilon_-}^R f_\varepsilon^A + g_\varepsilon^A f_\varepsilon^R)\bar{\sigma}_0^R(\varepsilon,\varepsilon_-)$$
$$+ (g_{\varepsilon_-}^R f_\varepsilon^R + g_\varepsilon^R f_{\varepsilon_-}^R)\bar{\sigma}_0^a(\varepsilon_-,\varepsilon)]\}[f_1(\varepsilon_-) - f_1(\varepsilon)], \tag{18'}$$

$$I_1^{ph} = \frac{\lambda\pi}{8\omega_D^2}\int d\varepsilon'(\varepsilon-\varepsilon')^2 \mathrm{sign}(\varepsilon-\varepsilon')\Big\{f_1(\varepsilon)f_1(\varepsilon') - 1$$
$$+ [f_1(\varepsilon) - f_1(\varepsilon')]\,\mathrm{cth}\frac{\varepsilon-\varepsilon'}{2T}\Big\}[n(\varepsilon)n(\varepsilon') - m(\varepsilon)m(\varepsilon')], \tag{17''}$$

$$I_2^{ph} = \frac{\lambda\pi}{8\omega_D^2}\int d\varepsilon(\varepsilon-\varepsilon')^2 \mathrm{sign}(\varepsilon-\varepsilon')$$
$$\times\Big\{[f_2(\varepsilon) - f_2(\varepsilon')]\,\mathrm{cth}\frac{\varepsilon-\varepsilon'}{2T}$$
$$+ f_1(\varepsilon)f_2(\varepsilon') + f_1(\varepsilon')f_2(\varepsilon)\Big\} n(\varepsilon)n(\varepsilon'). \tag{18''}$$

Here

$$n(\varepsilon) = \mathrm{Re}\, g_\varepsilon^R,\quad m(\varepsilon) = \mathrm{Re}\, f_\varepsilon^R,\quad \varepsilon_- \equiv \varepsilon - \Omega,$$
$$\nu_b = -2\Delta\, \mathrm{Im}\, f_\varepsilon^R,\quad \alpha_\Omega = \frac{1}{12}\left(\frac{v_F E_\Omega}{\Omega}\right)^2.$$

Notice that the source $Q_2(\varepsilon)$, in contrast to $Q_1(\varepsilon)$, is not equal to zero only because of the presence of magnetic impurities. Using the solution to this system, we can find the steady-state values of Δ and $\bar{\mu}$. For the latter quantity, using (10), (15), and (16), we obtain the following expression:

$$\bar{\mu} = -\frac{1}{2}\int_{-\infty}^{\infty} d\varepsilon\, n(\varepsilon) f_2(\varepsilon)$$
$$-\alpha_\Omega \int_{-\infty}^{\infty} d\varepsilon [f_1(\varepsilon_+) - f_1(\varepsilon)]\,\mathrm{Re}\Big\{b^R_{\varepsilon,\varepsilon_+}(g^R_{\varepsilon_+} - g^R_\varepsilon)$$
$$\times\Big[\frac{f_\varepsilon^R g^R_{\varepsilon_+} + f^R_{\varepsilon_+} g_\varepsilon^R}{f_\varepsilon^R - f^R_{\varepsilon_+}} \cdot \mathrm{Re}(\bar{\sigma}_0^a(\varepsilon,\varepsilon_+) b^a_{\varepsilon,\varepsilon_+})$$
$$+ \bar{\sigma}_0^R(\varepsilon,\varepsilon_+)\frac{f_\varepsilon^R g^A_{\varepsilon_+} + f^A_{\varepsilon_+} g_\varepsilon^R}{f_\varepsilon^R - f^A_{\varepsilon_+}} b^a_{\varepsilon,\varepsilon_+}\Big]\Big\}. \tag{19}$$

The second term in (19), whose meaning is not so clear as that of the first one, is due to the allowance made for the last term in the expression (15). In the limiting cases $\nu_s \ll \Delta$ and $\nu_s \gg \Delta$, which we shall analyze later, the dominant contribution to the effect is made by the first term.

Thus, to compute $\bar{\mu}$, we need, generally speaking, to solve two equations: (17) and (18). In the present paper we limit ourselves to the analysis of the case of low-intensity pumping, for which the deviations of $f_1(\varepsilon)$ and Δ from their equilibrium values are small, and can therefore be neglected in the computation of $f_2(\varepsilon)$ and $\bar{\mu}$. In particular, as follows from (17), for $\Delta \sim T$ the perturbations of $f_1(\varepsilon)$ and Δ will be small if the following condition is fulfilled:

$$\tilde{\alpha}_\Omega = \alpha_\Omega \omega_D^2 \tau / \lambda \bar{\varepsilon}^3 [1 + (\Omega\tau)^2] \ll \bar{\varepsilon}/\Omega,$$
$$\bar{\varepsilon} = \max(\Delta, \nu_s), \quad \Omega \gtrsim \bar{\varepsilon}, \quad \tau = (\nu_n + \nu_s)^{-1}. \qquad (20)$$

Let us now consider Eq. (18) with allowance for the foregoing. We shall assume that the relaxation of $f_2(\varepsilon)$ is due largely to the interaction with the magnetic impurities. This is true provided

$$\nu_s \gg (\Omega^2 + \nu_s^2)\lambda[\max(\Omega, T)]^3/(\omega_D \Delta)^2, \quad \Omega \gtrsim \Delta, \qquad (21)$$

as a result of which I_2^{ph} is small compared to $\nu_b(\varepsilon) f_2(\varepsilon)$, and therefore the solution is easy to find:

$$f_2 = f_2^{(0)} + f_2^{(1)}, \quad f_2^{(1)} \ll f_2^{(0)}, \qquad (22)$$

$$f_2^{(0)} = Q_2(\varepsilon)\tau_b^m(\varepsilon), \quad f_2^{(1)} = \tau_b^m(\varepsilon)[I_2^{ph}(f_2^{(0)}) + \nu_b^{ph}(\varepsilon) f_2^{(0)}],$$

where we taken account of the fact that, in the region of energies $|\varepsilon| > \varepsilon_g$ (ε_g is the gap in the excitation spectrum) of interest to us, we have

$$\nu_b(\varepsilon) = \nu_b^m(\varepsilon) + \nu_b^{ph}(\varepsilon), \quad \nu_b^{ph} \ll \nu_b^m, \quad \tau_b^m = (\nu_b^m)^{-1},$$

$\nu_b^{m(ph)}$ being determined by the interaction with the magnetic impurities (phonons). Let us first analyze the dominant contribution to the potential $\bar{\mu}$, i.e., we shall set $f_2 = f_2^{(0)}$. Let us consider the limiting cases.

1. Low magnetic impurity concentration: $\tau_s \Delta \gg 1$

Relatively simple analytic expressions can be obtained under the condition that the parameter w is not close to zero or unity. A distinctive feature of the present case is the presence of impurity bands that arise as a result of the broadening of the local energy levels $\pm\varepsilon_0$ ($\varepsilon_0 = w\Delta < \Delta$), which are due to the interaction of the electrons with a single magnetic impurity. For $||\varepsilon| - \varepsilon_0| \ll \varepsilon_0$, we find from (7) that

$$u_\varepsilon^R = w \operatorname{sign}\varepsilon + {}^1/_2\gamma[y + i(1-y^2)^{1/2}],$$
$$y = (\gamma\Delta)^{-1}(|\varepsilon| - \varepsilon_0)\operatorname{sign}\varepsilon, \quad \gamma = [2(1-w^2)^{1/2}/\tau_s\Delta]^{1/2}. \qquad (23)$$

For $|y| \sim 1$, these relations are valid if $\gamma \ll w$, $1 - w$. To the impurity bands, in which the density of states $n(\varepsilon) = \operatorname{Im} u_\varepsilon^R/(1-w^2)^{1/2}$, corresponds the energy region $\varepsilon_0^- \leqslant |\varepsilon| \leqslant \varepsilon_0^+$, $\varepsilon_0^\pm = \varepsilon_0 \pm \gamma\Delta$ ($|y| \leqslant 1$).

To lowest order in the parameter $\tau_s\Delta$, we calculate $\bar{\mu}$, from the first term in (19), and find from (7), (18), (19), (22), and (23) that ($\nu_n \equiv \nu \gg \nu_s$)

$$\bar{\mu} = D(E_\Omega/\Omega)^2 F(\Omega)\sin\vartheta, \qquad (24)$$

where

$$F(\Omega) = \frac{\pi\nu\Omega}{8\Delta}\sum_{k=-1}^{1}\frac{[\nu^2 + (2k\varepsilon_0 + \Omega)\,2k\nu w(1-w^2)^{-1/2} - (2k\varepsilon_0 + \Omega)^2](2k\varepsilon_0 + \Omega)}{[\nu^2 + 2\nu\Delta(1-w^2)^{1/2} + \Omega(2k\varepsilon_0 + \Omega)]^2} q_k(\Omega)$$
$$\times\left(\operatorname{th}\frac{\varepsilon_0}{2T} - \operatorname{th}\frac{\varepsilon_0 + k\Omega}{2T}\right) + \frac{w\nu^2\Omega}{4}\int_{-\infty}^{\infty} d\varepsilon\,\theta(|\varepsilon_+| - \Delta)\,\theta(|\varepsilon| - \Delta)\left(\operatorname{th}\frac{\varepsilon}{2T} - \operatorname{th}\frac{\varepsilon_+}{2T}\right)$$
$$\times\frac{\varepsilon + \varepsilon_+}{\varepsilon^2 - \varepsilon_0^2}\sum_{k=-1}^{1}\left(\frac{\varepsilon\varepsilon_+ + \Delta^2}{\xi_\varepsilon\xi_{\varepsilon_+}} + k\right)\frac{(\xi_{\varepsilon_+} - k\xi_\varepsilon)}{[\xi_{\varepsilon_+} - k\xi_\varepsilon)^2 + \nu^2]^2},$$

$$q_k(\Omega) \equiv q\left(\frac{\Omega - \Omega_k}{\gamma\Delta}\right), \quad \Omega_k = (1 - kw)\Delta,$$

$$q(x) = \theta(1 - |x|)\left[\frac{1}{2} + \frac{1}{\pi}(\arcsin x + x(1-x^2)^{1/2})\right] + \theta(x - 1),$$
$$D = v_F^2\tau/3, \quad \xi_\varepsilon = (\varepsilon^2 - \Delta^2)^{1/2}\operatorname{sign}\varepsilon.$$

Notice that the functions $q_k(\Omega)$ govern the rapid variation of $F(\Omega)$ (in the frequency range $|\Omega - \Omega_k| < \gamma\Delta$, which is small compared to Δ), that arises as a result of the rapid growth of the amplitude for scattering on a magnetic impurity in the region of energies corresponding to the impurity bands. Let us investigate (24′) in the various limiting cases.

a) $\tau\Delta \ll 1$. For Ω satisfying the conditions $\tau\Omega \ll 1$ and $\tau\Omega^2/\Delta \ll 1$, and with only the leading terms retained, we have

$$F(\Omega) = \frac{\pi\Omega\tau}{8\Delta}\sum_{k=-1}^{1}\left(\operatorname{th}\frac{\varepsilon_0}{2T} - \operatorname{th}\frac{\varepsilon_0 + k\Omega}{2T}\right) q_k(\Omega)(\Omega + 2k\varepsilon_0)$$

$$\equiv \Omega\tau h(\Omega)$$

$$= \frac{\pi\tau\Omega^2}{4}\begin{cases} (\Omega-2\varepsilon_0)/\Delta, & \Delta\gg T, \\ w\left(\frac{\Omega}{2T}\,\mathrm{th}\frac{\Omega}{2T}-2\right)\mathrm{th}\frac{\Omega}{2T}, & \Delta\ll T, \end{cases} \quad \Omega>\Delta+\varepsilon_0^{+}. \tag{24''}$$

In the high-frequency region $\Omega\gg\Delta$, $\tau\Omega^2\gg1$ we have the following asymptotic form:

$$F(\Omega)=\frac{\pi\tau\Omega^2}{4\Delta[1+(\Omega\tau)^2]^2}\times\left\{[1-(\Omega\tau)^2]\,\mathrm{th}\frac{\varepsilon_0}{2T}-\frac{4w\Omega\tau}{(1-w^2)^{1/2}}\,\mathrm{th}\frac{\Omega}{2T}\right\}. \tag{24'''}$$

The change in sign of the function $F(\Omega)$ at certain frequencies is due to the fact that the sign of the source $Q_2(\varepsilon)$ depends on the relation between ε and Ω [notice that $\int d\varepsilon Q_2(\varepsilon)=0$].

b) $\tau\Delta\gg1$. Assuming $\Omega\gtrsim\Delta$, and taking account of only the leading terms, we obtain

$$F(\Omega)=-h(\Omega)/\Omega\tau,$$

where the function $h(\Omega)$ is defined in (24″).

In the region $\Omega<\Delta-\varepsilon_0^-$ the magnitude of the effect is markedly smaller for any value of $\tau\Delta$. Thus, we can assert that, in the case of low magnetic-impurity concentrations under consideration the frequency $\Omega_0=\Delta-\varepsilon_0^-$ (or $\tilde{\Omega}_0=\Delta+\varepsilon_0^+$ when $\Delta\gg T$) constitutes a threshold frequency for the PE. In principle, Ω_0 can be small compared to Δ; the expressions obtained are, however, valid for not too small $\Omega_0/\Delta\approx1-w\gg\gamma$. Notice also that [as can be shown, using (17)], in the case when $T\ll\Delta$ and $|\Omega-\Delta-\varepsilon_0|\sim\gamma\Delta$, the neglect of the nonequilibrium character of $f_1(\varepsilon_0-\Omega)$ is justified if the parameter $\tilde{\alpha}_\Omega\ll\gamma$.

It is worth noting that, under the condition (21), the effect in the region $\Omega>\Delta+\varepsilon_0^+$ depends weakly on ν_s and, hence, on the magnetic-impurity concentration c_m, and is determined by the parameters characterizing the scattering on a single impurity atom. This property is due to the fact that, under the condition (21), $\tau_b(\varepsilon)\sim\tau_s$ and $Q_2\sim\nu_s$, and therefore $\bar{\mu}\sim\int d\varepsilon\tau_b(\varepsilon)Q_2(\varepsilon)n(\varepsilon)$ is a slowly varying function of ν_s and c_m at all temperatures T except those in the narrow range $T_c-T\sim\nu_s\ll T_c$, where the dependence of Δ on ν_s must be taken into account. In the case of very small ν_s, when the inverse of the relation (21) is fulfilled, the magnitude of the effect will decrease as $c_m\to0$.

2. High magnetic impurity concentration: $\tau_s\Delta\ll1$

In this case, which corresponds to the gapless state, we have to leading order in the parameter $\tau_s\Delta$

$$u_\varepsilon^{R(A)}=(\varepsilon\pm i\nu_s)/\Delta,\qquad \xi_\varepsilon^{R(A)}=\pm\varepsilon+i\nu_m/2. \tag{25}$$

Taking account of (18′), (18″), (19), (22), and (25), we find $\bar{\mu}$, the expression for which can again be represented in the form (24); let us give $F(\Omega)$ in the region of frequencies (where the effect is most noticeable) satisfying the condition $(\Omega/\Delta)^2\gg1$:

$$F(\Omega)=w\left[\frac{\Omega\tau}{1+(\Omega\tau)^2}\right]^2\int_{-\infty}^{\infty}d\varepsilon\frac{(2\varepsilon-\Omega)(\varepsilon-\nu_s\nu/\Omega)(\varepsilon+\nu_s\Omega\tau)}{(\varepsilon^2+\nu_s^2)^2}\times\left(\mathrm{th}\frac{\varepsilon}{2T}-\mathrm{th}\frac{\varepsilon-\Omega}{2T}\right). \tag{26}$$

As can be seen, in the region $\Omega>\Delta$ (for Δ which are not too small; the condition (21) gives the upper bound for Ω and the lower bound for Δ) the dominant contribution to $F(\Omega)$ does not depend on Δ.[6] Let us give certain asymptotic forms that follow from (26). Taking into account the fact that at temperatures far from T_c the parameter $\tau_s T\ll1$, we obtain

$$F(\Omega)=w\begin{cases}\tau_s\tau\Omega^2[1+(1+(\Omega\tau_s)^2)^{-1}-(2/\Omega\tau_s)\,\mathrm{arctg}\,\Omega\tau_s], & T\ll\Omega,\ \nu_s\ll\nu, \\ -\nu\tau_s, & T\ll\nu_s\sim\nu\ll\Omega.\end{cases} \tag{26'}$$

In the vicinity of T_c, where the parameter $\tau_s T$ is not necessarily small, we have in the limit when Ω, $\nu\gg\nu_s$ the expression

$$F(\Omega)=\frac{2w\tau\tau_s\Omega^2[1-(\Omega\tau)^2]}{[1+(\Omega\tau)^2]^2}\int_0^\infty dy\frac{y\,\mathrm{th}(\nu_s y/2T)}{(1+y^2)^2}. \tag{26''}$$

Notice that, in the region $\nu_s\ll\Omega<\nu$, as in the case when $\nu_s\ll\Delta$ and $\Delta\ll\Omega<\nu$, the function $F(\Omega)$ increases like Ω^2.

Above we analyzed the contribution to $F(\Omega)$, found without allowance for the correction $f_2^{(1)}$ [see (22)] due to the electron-phonon interaction. By taking account of $f_2^{(1)}$, we can find the corresponding correction $\bar{\mu}^{(1)}(\Omega)$ to the potential. The contribution of this fucntion is greatest in the region of high Ω, where its asymptotic form can be represented as

$$\bar{\mu}(\Omega)^{(1)}=\frac{3\pi\lambda\tau_s\Omega^5}{80\omega_D^2\Delta^2}\bar{\mu}(\Omega)^{(0)},\qquad \Omega\gg\Delta,\ T.$$

Here $\bar{\mu}^{(0)}(\Omega)$ is a function given by the expression (24‴) and (26).

Thus far, we have carried out the computation of $\bar{\mu}$ in the SR model. To conclude this section, we shall briefly discuss the question of the influence of the Kondo effect on the results; this will give us some idea about the limits of applicability of this model. The Green's functions of a superconductor have been computed with allowance for the Kondo effect in numerous papers (see the review in Ref. 11). At present this complicated problem has been solved only by means of specific approximations. The most consistent solution, which was obtained on the basis of the Nagaoka-Suhl approximation (see Refs. 10 and 11), was given in a series of papers by Müller-Hartman and Zittartz (for references, see Refs. 11 and 19). These authors showed that, in the region of energies $|\varepsilon|\sim\Delta$, the Green's functions $\hat{g}_\varepsilon^R$ have the same form as in the SR model, i.e., the form (7), and that the influence of the Kondo effect manifests itself only in the expression for the parameter w, which, under the condition that $JSN_F \ll 1$, is given by the following formula[11,19]:

$$w=\beta[\pi^2S(S+1)+\beta^2]^{-1/2}\,\mathrm{sign}\, j,\quad \beta=j^{-1}-\ln\tilde{\varepsilon}_F/T,$$

$$\beta=\ln T/T_K \quad (j>0),$$

$$j=JN_F\cos^2(\vartheta/2),\quad e^{i\vartheta}=(1+i\pi V_mN_F)(1-i\pi V_mN_F)^{-1},$$

$$T_K=\tilde{\varepsilon}_F\exp(-j^{-1}), \tag{27}$$

where $T_K = \tilde{\varepsilon}_F \exp(-j^{-1})$ is the Kondo temperature, $\tilde{\varepsilon}_F$ being an energy of the order of ε_F (in the expression for β the difference between $\tilde{\varepsilon}_F$ and ε_F is insignificant); the introduction of T_K makes sense only in the case of antiferromagnetic coupling between the electron and impurity spins ($j>0$). Thus, allowance for the Kondo effect leads to a situation in which the parameter w depends on the temperature, as well as on the sign of the exchange interaction.

The general expressions obtained above for $\bar{\mu}$, in the derivation of which we did not use specific forms of the matrices $\hat{\Sigma}_m^{(R,A)}$, contain the function $\sigma_0^{R(A)}(\varepsilon)$ as well. From the results obtained in Ref. 11, the function $\sigma_0^R(\varepsilon)$ can, when allowance for the Kondo effect is included, be represented in the form

$$\sigma_0^R(\varepsilon)=-2\nu_s\sin\vartheta\left\{\frac{w(f_\varepsilon^R)^2}{(g_\varepsilon^R)^2-(wf_\varepsilon^R)^2}+\frac{k_\varepsilon^R\,\mathrm{sign}\,j}{[\pi^2S(S+1)+\beta^2]^{1/2}}\right\}, \tag{28}$$

where k_ε^R is some complicated function that depends on ε, Δ, ν_s, and T. It has a much simpler form in the limit $\tau_s\Delta \ll 1$, in which we can ignore the dependence of k_ε^R on Δ. In this case we have[10,11]

$$k_\varepsilon^R=\psi(1/2-i\varepsilon/2\pi T)-\psi(1/2), \tag{28'}$$

where $\psi(z)$ is the digamma function. Thus, the second term in (28), in contrast to the first term, which has the same form as in the SR model, i.e., the form (6), does not vanish as $\Delta\to 0$. It is therefore clear that the related contribution to the PE may be greatest when $\Delta \ll \nu_s$. In this case, taking account of (19), (22), and (28'), we can obtain for $\bar{\mu}$ an expression of the form (24) with F replaced by $\tilde{F} = F + F_K$, where F is given by the formula (26) and the function F_K has the following form (we assume, for simplicity, that $\Omega \ll \nu$):

$$F_K(\Omega)=-\pi\tau T\Omega^2\,\mathrm{sign}\,j/2\Delta^2[\pi^2S(S+1)+\beta^2]^{1/2}. \tag{29}$$

Notice that the expression (29) has been derived under the condition (21), which specifies, in particular, the lower bound for Δ. From (26) and (29) it follows that the relation $F_K/F \ll 1$, which allows us to use for $\bar{\mu}$ the expressions found by means of the SR model, is satisfied at sufficiently ;high values of S and β, whose magnitudes depend on Ω and the ratio T/Δ; the influence of the Kondo effect then amounts only to the renormalization of the parameter w. This is also attested by the estimate that can be obtained for arbitrary values of $\tau_s\Delta$ in the region $\Omega > \Delta, \nu_s$, the optimum region for observing the PE (we assume $\Delta \sim T$):

$$F_K/F\sim\max[1,\,(\tau_s\Delta)^{-1}][\pi^2S(S+1)+\beta^2]^{-1/2}.$$

We now proceed to investigate the anomalous PE in the absence of magnetic impurities.

3. THE PE IN SUPERCONDUCTORS WITH NONMAGNETIC IMPURITIES

It turns out that, under certain conditions, nonmagnetic impurities in superconductors can also bring about an electron-hole scattering asymmetry, and, hence, their presence can also lead to the occurrence of the anomalous PE. For example, such a situation occurs in the presence of anisotropy in the superconductor.

1. Let us first consider the case in which the superconductor possesses intrinsic anisotropy due to the dependence of the order parameter on the direction of the vector $\mathbf{n} = \mathbf{p}_F/p_F$. As is well known,[20] intrinsic anisotropy manifests itself in single-crystal samples, and is due to the direction dependence of the electron-phonon interaction, which is then described not by the constant λ, but by the function $\lambda(\mathbf{n},\mathbf{n}')$. the theoretical investigation of the effect of such an anisotropy on the properties of superconductors was begun by Pokrovskiĭ,[21] and has since been the subject of a large number of papers in which, as in the present paper, the weak-coupling model is used (see Ref. 22 and the references cited therein). We shall consider the case of the simplest anisotropy, assuming that the electron spectrum $\varepsilon(\mathbf{p})$ in the normal state can be regarded as three-dimensional. The equations for the Green's functions will then have a form similar to (2) with Δ replaced by $\Delta(\mathbf{n})$, where

$$\Delta_\omega(\mathbf{n})=\frac{1}{4}\int_{-\omega_D}^{\omega_D}d\varepsilon\langle\lambda(\mathbf{n},\mathbf{n}')f_{\varepsilon,\varepsilon-\omega}(\mathbf{n}')\rangle'.$$

Here the prime on the angle brackets denotes averaging over $\mathbf{n}'$; directional averaging should now be taken in the following sense:

$$\langle(\dots)\rangle=\int_{S_F}\frac{d^2\mathbf{p}}{v_F}(\dots)\Big/\int_{S_F}\frac{d^2\mathbf{p}}{v_F},\qquad \mathbf{v}_F=\frac{d\varepsilon(\mathbf{p})}{d\mathbf{p}}\Big|_{\mathbf{p}=\mathbf{p}_F},$$

where the integration is over the Fermi surface S_F.

Below we shall need the expression for the equilibrium matrix $\hat{\Sigma}_n^R(\varepsilon)$, which, on the basis of (8), can be represented in the form (we shall henceforth drop the index n)

$$\Sigma^R(\varepsilon)=\sigma_0^R(\varepsilon)1+{}^1/_2 i\nu_\varepsilon^R\langle\hat{g}_\varepsilon^R(\mathbf{n})\rangle, \tag{30}$$

where

$$\nu_\varepsilon^R=\nu(1+s_\varepsilon^R\sin^2\delta)^{-1},\ s_\varepsilon^R={}^1/_2\,\mathrm{Sp}\langle(\langle\hat{g}_\varepsilon^R(\mathbf{n})\rangle-\hat{g}_\varepsilon^R(\mathbf{n}))^2\rangle$$
$$=\langle g_\varepsilon^R(\mathbf{n})\rangle^2-\langle f_\varepsilon^R(\mathbf{n})\rangle^2-1,\qquad \sigma_0^R(\varepsilon)=-{}^1/_2\nu_\varepsilon^R s_\varepsilon^R\sin 2\delta.$$

In writing down the expression for $\sigma_0^R(\varepsilon)$ we left out the constant determining the renormalization of the chemical potential. Taking account of (30), we obtain for the equilibrium Green's functions $\hat{g}_\varepsilon^R(\mathbf{n})$ the expression

$$\hat{g}_\varepsilon^R(\mathbf{n})=(\varepsilon\hat{\tau}_z+\Delta_\varepsilon^R(\mathbf{n})i\hat{\tau}_y)/\xi_\varepsilon^R(\mathbf{n}),$$

$$\xi_\varepsilon^R(\mathbf{n})=[(\varepsilon+i0)^2-(\Delta_\varepsilon^R(\mathbf{n}))^2]^{1/2}, \tag{31}$$

$$\Delta_\varepsilon^R(\mathbf{n})=\Delta(\mathbf{n})+{}^1/_2 i\nu_\varepsilon^R\langle[\Delta_\varepsilon^R(\mathbf{n}')-\Delta_\varepsilon^R(\mathbf{n})]/\xi_\varepsilon^R(\mathbf{n}')\rangle'.$$

We neglect the effect of the electron-phonon interaction on $\mathrm{Im}\Delta_\varepsilon^R(\mathbf{n})$.

Thus, it follows from (30) and (31) that, because of the anisotropy in the self-energy matrix describing the scattering on the nonmagnetic impurities, the component $\sigma_0^R\hat{1}$ is nonzero. Therefore, it is to be expected that, as in the case with magnetic impurities, the PE will manifest itself in anisotropic superconductors even in zeroth order in the parameter $\bar{\varepsilon}/\varepsilon_F$.

Let us proceed to analyze this effect, assuming that a thin [characteristic cross-sectional dimension $d \ll \lambda(\Omega)$] single-crystal whisker (see the review in Ref. 23) is subjected to the homogeneous action of an electromagnetic field whose amplitude is so small that the following condition is fulfilled for typical energies:

$$f_1(\varepsilon)-\mathrm{th}(\varepsilon/2T) \ll \mathrm{th}(\varepsilon/2T).$$

In this case it is only necessary to derive the kinetic equation for the function f_2. Since its derivation is similar to the one carried out in Sec. 2, we shall omit the intermediate calculations, and write out only the result. We assume here that $\tau\nu_b(\bar{\varepsilon}) \ll 1$, which is, as a rule, guaranteed by the smallness of the quantity $(\langle\Delta^2\rangle - \langle\Delta\rangle^2)/\langle\Delta\rangle^2$. As a consequence the anisotropic part $f_2(\varepsilon,\mathbf{p}_F) - \langle f_2(\varepsilon,\mathbf{p}_F)\rangle$ is small compared to $\langle f_2(\varepsilon,\mathbf{p}_F)\rangle \equiv f_2(\varepsilon)$, and can be neglected; the resulting equation for $f_2(\varepsilon)$ is similar to (18), where now

$$Q_2(\varepsilon,\Omega)=\langle Q_2(\varepsilon,\Omega,\mathbf{n})\rangle,\quad \nu_b(\varepsilon)=-2\langle\Delta(\mathbf{n})\,\mathrm{Im}\,f_\varepsilon^R(\mathbf{n})\rangle. \quad (32)$$

Here $Q_2(\varepsilon,\Omega,\mathbf{n})$ is a function that coincides in form with (18′) with $v_F^2/3$ and Δ replaced respectively by v_{FX}^2 and $\Delta(\mathbf{n})$; furthermore, account should be taken of the fact that, in the case under consideration, the assumption that $[\sigma_0^R(\varepsilon)\tau]^2 \ll 1$ leads to the relations

$$\bar{\sigma}_0^{R(a)}(\varepsilon,\varepsilon')=[\sigma_0^R(\varepsilon)-\sigma_0^{R(A)}(\varepsilon')][\tilde{\xi}_\varepsilon^R(\mathbf{n})+\tilde{\xi}_{\varepsilon'}^{R(A)}(\mathbf{n})]^{-1},$$

$$b_{\varepsilon,\varepsilon'}^{R(a)}=[\tilde{\xi}_\varepsilon^R(\mathbf{n})+\tilde{\xi}_{\varepsilon'}^{R(A)}(\mathbf{n})]^{-1},$$

$$\tilde{\xi}_\varepsilon^R(\mathbf{n})=\xi_\varepsilon^R(\mathbf{n})[1+{}^1/_2 i\nu_\varepsilon^R\langle 1/\xi_\varepsilon^R(\mathbf{n})\rangle].$$

It is well kown that the effect of the anisotropy is strongest in fairly pure superconductors, for which the parameter $\tau\langle\Delta\rangle \gg 1$. Let us consider the limiting case

$$\nu \ll \Delta_0,\ \Delta_m-\Delta_0,\quad \Delta_0=\min\Delta(\mathbf{n}),\quad \Delta_m=\max\Delta(\mathbf{n}). \quad (33)$$

Furthermore, we shall invoke the condition $(\Omega \gtrsim \Delta \sim T)$

$$\nu_b(\Omega) \gtrsim (\langle\Delta^2\rangle-\langle\Delta\rangle^2)\nu/\Omega^2 \gg \langle\lambda\rangle\Omega^3/\omega_D^2, \quad (34)$$

which allows us to neglect I_2^{ph} in the calculation of $f_2(\varepsilon)$. Taking only the dominant contribution into account, we obtain for the stationary potential the expression

$$\bar{\mu}=-\int_{\Delta_m}^{\infty} d\varepsilon \left\langle \frac{\varepsilon}{(\varepsilon^2-\Delta^2)^{1/2}} \right\rangle \tau_b(\varepsilon) Q_2(\varepsilon)$$

$$=\left(\frac{E_\Omega}{\Omega}\right)^2 \frac{\langle v_{Fx}^2\rangle}{\Delta_0} \Phi(\Omega)\sin 2\delta. \quad (35)$$

Here $\tau_b(\varepsilon) = \tau y(\varepsilon)$, where the τ-independent function $y(\varepsilon)$ can be found from (31) and (32); below we shall need its asymptotic form for $|\varepsilon| \gg \Delta_m$: $y(\varepsilon) = \varepsilon^2(\langle\Delta^2\rangle - \langle\Delta\rangle^2)^{-1}$ (the effect of the anisotropy on the frequency ν_b was first discussed by Tinkham[5]). In principle, the function $\Phi(\Omega)$ can be computed on the basis of (35) if the function $\Delta(\mathbf{n})$ is known. For this purpose, in order to estimate $\bar{\mu}$, it makes sense to consider the simplest limiting case: $\Omega \gg \Delta_m$, in which $\Phi(\Omega)$ reduces to a constant Φ (of the same order of magnitude as Φ for the case when $\Omega \gtrsim \Delta_m$):

$$\Phi=\frac{\Delta_0}{4[\langle\Delta^2\rangle-\langle\Delta\rangle^2]}\int_{\Delta_0}^{\Delta_m} d\varepsilon\,\mathrm{th}\frac{\varepsilon}{2T}\,\mathrm{Im}\frac{s_\varepsilon^R}{(1+s_\varepsilon^R\sin^2\delta)}. \quad (35')$$

In deriving (35′) we took account of the fact that, to lowest order in the parameter $(\tau\Delta_0)^{-1}$, we can consider the function $\mathrm{Im}s_\varepsilon^R$ to be nonzero only in the region $\Delta_0 < \varepsilon < \Delta_m$, where we have, on the basis of (30) and (31),

$$\mathrm{Im}\,s_\varepsilon^R=2\langle\langle(\Delta\Delta'-\varepsilon^2)\theta(\Delta-\varepsilon)\theta(\varepsilon-\Delta')$$
$$\times(\Delta^2-\varepsilon^2)^{-1/2}(\varepsilon^2-\Delta'^2)^{-1/2}\rangle\rangle,$$

Here the double angle brackets denote averaging over $\mathbf{n}$ and $\mathbf{n}'$ and $\Delta' \equiv \Delta(\mathbf{n}')$. Let us, following Gaĭdukov,[23] further assume that the average over the Fermi surface can be written down after introducing the function $P(\Delta)$, normalized to unity in the interval (Δ_0,Δ_m), and determining the distribution density of the parameter Δ:

$$\langle(\ldots)\rangle=\int_{\Delta_0}^{\Delta_m} P(\Delta)(\ldots)d\Delta.$$

By taking account of this for the uniform distribution $P(\Delta) = (\Delta_m - \Delta_0)^{-1}$, and assuming $\Delta_0 > 2T$ and $s_\varepsilon^R \sin^2\delta \ll 1$, we can obtain the following estimate:

$$\Phi \approx 3\pi\Delta_0^3/\Delta_m^2(\Delta_m+\Delta_0),\qquad \Delta_m/\Delta_0 \leqslant 2. \quad (36)$$

Note that this estimate is correct in order of magnitude even when $\Delta_m - \Delta_0 \sim \nu$. Thus, we can conclude from (36) and the foregoing[7)] that, under the condition (34), we have $\Phi(\Omega) \gtrsim 1$ in the region $\Omega \gtrsim \Delta_m \gtrsim T$.

Let us now proceed to analyze another case, namely, the case in which the anisotropy in the superconductor is caused by an external magnetic field H.

2. We shall assume that the field H is applied parallel to the plane of a film whose thickness satisfies the condition $l = v_F\tau \ll d \ll \lambda(\Omega),\xi$. To simplify the analysis of the PE, we shall assume that the magnetic field is close to the critical field H_c for the film i.e., that $H_c \gg H_c - H$. It is well known that the gapless state, in which the equilibrium Green's functions are given by the expressions[24]

$$\langle\hat{g}_\varepsilon^R\rangle=\left[1+\frac{1}{2}\left(\frac{\Delta}{\varepsilon+i\Gamma}\right)^2\right]\hat{\tau}_z+\frac{\Delta}{(\varepsilon+i\Gamma)}i\hat{\tau}_y,\quad \langle\hat{g}_\varepsilon^R\rangle-\hat{g}_\varepsilon^R$$
$$=2i\tau v_x Hz\frac{\Delta}{(\varepsilon+i\Gamma)}i\hat{\tau}_y,\quad \Gamma=D\frac{(Hd)^2}{6}\gg\Delta, \quad (37)$$

is realized in such strong fields. To the film surfaces, which are perpendicular to the z axis, correspond the planes $z = \pm d/2$.

As we saw above, the occurrence of the anomalous PE is due to the presence of the component $\sigma_0^R(\varepsilon)$ in the self-energy matrix $\Sigma^R(\varepsilon)$. Below we shall need only the film-thickness-averaged function $\bar{\sigma}_0^R(\varepsilon)$. Computing it with allowance for (30) and (37), we obtain

$$\bar{\sigma}_0^R(\varepsilon)=-\Gamma[\Delta/(\varepsilon+i\Gamma)]^2\sin 2\delta. \quad (38)$$

Before proceeding to find the stationary invariant potential that arises in the film under the uniform action of an electromagnetic field, let us note that we shall consider the case of low-intensity pumping, in which we set $f_1(\varepsilon)=\mathrm{th}(\varepsilon/2T)$ in computing $f_2(\varepsilon)$ and $\bar{\mu}$. It is not difficult to verify that, because of the smallness of the parameter Δ/Γ, we can ignore the new terms (i.e., those that do not occur in the cases considered above), which arise because to first order in A_Ω the functions $\langle\hat{g}^{(1)}_{\varepsilon,\varepsilon-\omega}\rangle$, as well as $\Delta\omega^{(1)}$ and $\mu_\varepsilon^{(1)}$, are nonvanishing. Therefore, the derivation of the kinetic equation for $f_2(\varepsilon)$ is similar to the one carried out in Sec. 2; the result has the form of Eq. (18), in which, with account now taken of (37) and (38), we have

$$\nu_b(\varepsilon)=2\Delta^2\Gamma(\varepsilon^2+\Gamma^2)^{-1},$$

$$Q_2(\varepsilon,\Omega)=\frac{\sin 2\delta}{2}D\left(\frac{E_\Omega}{\Omega}\right)^2\Delta^2\Gamma\tau\,\mathrm{Im}\frac{1}{(1+i\Omega\tau)^2}\times\left[\frac{1}{(\varepsilon_-+i\Gamma)^2}-\frac{1}{(\varepsilon+i\Gamma)^2}\right]\left(\mathrm{th}\frac{\varepsilon_-}{2T}-\mathrm{th}\frac{\varepsilon}{2T}\right). \quad (39)$$

For

$$\nu_b(\Omega)\gg\lambda[\max(\Omega,T)]^3/\omega_D^2, \quad (40)$$

we can easily find the function $f_2(\varepsilon)$; as a result we find in the case when $\Omega\gg\Delta$ that

$$\bar{\mu}=\frac{\sin 2\delta}{2}D\left(\frac{E_\Omega}{\Omega}\right)^2\int_{-\infty}^{\infty}d\varepsilon\frac{(2\varepsilon-\Omega)(\varepsilon-\Gamma\nu/\Omega)(\varepsilon+\Gamma\tau\Omega)}{(\varepsilon^2+\Gamma^2)^2}\times\left(\mathrm{th}\frac{\varepsilon}{2T}-\mathrm{th}\frac{\varepsilon-\Omega}{2T}\right)\equiv D\left(\frac{E_\Omega}{\Omega}\right)^2F(\Omega)\sin 2\delta. \quad (41)$$

Let us note that the function $F(\Omega)$ in (41) to within the factor $2w$, after replacing $\nu_s\to\Gamma$, with the expression obtained for the case of magnetic impurities in the limit $\Delta\ll\nu_s\ll\nu$; therefore, its asymptotic forms can be obtained from (26′) and (26″). The expression (41) is not correct if the parameter Δ is so small that the inverse of the condition (40) is fulfilled. Not wanting to dwell at length on this case, we only note that, in this limit, $\bar{\mu}$ tends to zero ($\bar{\mu}\sim\Delta^2$) as Δ decreases.

We have considered only certain situations, unified by the presence of anisotropy, in which the scattering on the nonmagnetic impurities gives rise to the anomalous PE. In particular, here we have not discussed the case of nonmagnetic transition-metal impurities; it must be treated separately.

4. DISCUSSION OF THE RESULTS

Thus, the region of frequencies $\Omega\gtrsim\Delta,\nu_s,\Gamma,T$ is the optimum region for the observation of the PE. Let us estimate the magnitude of the effect for these Ω, taking account of the fact that the maximum electric-field amplitude for which these expressions can still be used to make order-of-magnitude estimates is given by the condition

$$\alpha_\Omega^{max}\sim\lambda\tilde{\varepsilon}^4[1+(\Omega\tau)^2]/\omega_D^2\Omega\tau<\alpha_\Omega^c,$$

where α_Ω^c is the critical value corresponding to the appearance of the inhomogeneous state in the superconductor[1] and $\tilde{\varepsilon}=\max(\Delta,T,\nu_s,\Gamma)$. Using, for example, the formula (24′) for the estimate,[8)] and setting $\sin\vartheta\sim 1$ and $\Delta\sim T$, we obtain

$$\bar{\mu}_{max}\sim\frac{1}{\tau_\varepsilon}\begin{cases}\tau\Omega, & \tau\Delta\ll 1 \quad (\Omega\tau\leqslant 1)\\ 1, & \tau\Delta\geqslant 1.\end{cases}$$

Here $\tau_\varepsilon^{-1}=\lambda\Delta^3/\omega_D^2$. Taking account of the fact that a typical value of $\tau_\varepsilon\sim 10^{-9}$–$10^{-10}$ s, we arrive at the conclusion that the characteristic value of $\bar{\mu}_{\max}\sim 1$-$10\,\mu$V; in the dirty limit this value is attained in the region $\Omega\gg\Delta$.

The problem we have solved in Secs. 2 and 3 clearly allows us to find the potential difference that arises in a definite region of the superconductor (or across the junction between superconductors) in the situations depicted in Figs. 1a and 1b. The voltage potential that develops in the superconductor in the case shown in Fig. 1c can also be computed from these expressions with E_Ω replaced by $E_\Omega(0)$ when the coupling between $\bar{\mu}(x)$ and $E_\Omega(x)$ is local.[9)] It is not especially difficult to generalize the results to the case of nonlocal coupling, but we shall not do this here.

Thus far we have considered the PE in superconductors. But a stationary emf $\mathscr{E}_N$ will also develop in the normal state of the metal under the inhomogeneous action of an electromagnetic field. It owes its origin to the diffusional current component j_d given by the gradient of the distribution function $f_1(\varepsilon)$: for an open circuit $j_d+\sigma E=0$; therefore, a voltage potential develops across a section in which diffusion of the excitation occurs.[10)] Thus, there is a definite analogy between the PE in the normal state (when the electromagnetic field acts nonuniformly) and the thermoelectric effect, the difference lying only in the mode of excitation (and in the form) of the distribution function $f_1(\varepsilon)$.

In the presence of magnetic impurities, both the thermoelectric[9] and photoelectric effects are anomalously large as a result of the electron-hole scattering asymmetry. Taking account of the well-known results,[10] and using the analogy noted above, we can easily estimate the magnitude $\mathscr{E}_N$ of the emf that develops, for example, in the situation depicted in Fig. 1a. Assuming that the pump is weak, i.e., that

$$E_\Omega l_\varepsilon\ll\max(T,\Omega),\quad l_\varepsilon=(D\omega_D^2/\lambda T^3)^{1/2}$$

and that $L\gg l_\varepsilon$, we obtain

$$\mathscr{E}_N(\Omega,T)\sim\sin\vartheta\frac{\nu_s\tau(E_\Omega l_\varepsilon)^2\,\mathrm{sign}\,j}{T[\pi^2S(S+1)+\beta^2]^{1/2}}\begin{cases}1, & \Omega\ll T,\\ \dfrac{T}{\Omega}, & \Omega\gg T\end{cases} \quad (42)$$

It follows from (42) and the results obtained in Sec. 2 that the ratio $\mathscr{E}_N(\Omega,T_c)/\mathscr{E}(\Omega,T)$ can be either greater or smaller than unity. Without going into details, let us note that the emf should increase (even if in a narrow temperature range $T_c-T\ll T_c$) on going from the normal into the superconducting state when magnetic-impurity concentrations are not too high, i.e., under the condition that $\tau_s T_c > 1$.

Thus, the appearance in the superconducting state, in contrast to the normal state, of an emf under the inhomogeneous action of an electromagnetic field is due to the redistribution of the excitations over the branches, and the change in the distribution function $f_1(\varepsilon)$, so long as it is small, does not play an important role. In spite of the difference in origin, the anomalies of the effect in the two cases are due to the electron-hole asymmetry in the scattering on the magnetic impurities. In the presence of nonmagnetic impurities only, an anomalously strong emf can develop only in the superconducting state.

In our opinion, the PE considered here is interesting not only from the physical point of view, but also from the standpoint of applications. Experimentally, this effect has not yet been studied.

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[1)]The effect studied in Ref. 6 is caused by the magnetic component of the microwave field (and in this sense it is similar to the effect investigated in semiconductors[7]); therefore, it disappears in very thin films. Let us note that in Ref. 6 the contribution to μ connected with the change that occurs in the quasiparticle distribution function under the action of the electric component of the microwave field is ignored.

[2)]Let us note that the effect under consideration is due to the appearance of a branch imbalance; as to the production of new quasiparticles [described by the distribution function $f_1(\varepsilon)$; see below], it does not play a significant role when pumping is weak. For this reason this effect is called light-electric, and not photoelectric.

[3)]As is well known, the first theory[14] that explained many equilibrium properties of superconductors with magnetic impurities describes the scattering on these impurities in the Born approximation. The development of a theory that went beyond this approximation led to the prediction of new states inside the energy gap of the superconductor.[12,13] These states have been investigated by means of tunneling measurements in numerous papers (see, for example, Refs. 15 and 16).

[4)]The form of the expressions (14) does not depend on the nature of the scattering from the surfaces of the film if $l = v_F \tau_n \ll d$; if on the other hand $l > d$, then these formulas are valid in the case of specular reflection from the surfaces.

[5)]Notice that Eq. (17) for $v_s = 0$ and $\Delta, \Omega \ll v_n$ goes over into the equation obtained by Éliashberg in Ref. 17, where the equation for the distribution function $n_\varepsilon = [1 - f_1(\varepsilon)\text{sign}\,\varepsilon]/2$ is given.

[6)]It can be shown that, in the Δ region defined by reversing the inequality (21) [the expressions (24') and (26) are not valid here], the potential $\bar{\mu}$ will tend to zero as $\Delta \to 0$.

[7)]It can be shown that in the case of weak anisotropy the effect tends to zero as $\Delta_m - \Delta_0 \to 0$, but that this will occur in the limits opposite to (34), when the expressions (35) and (36) are not valid.

[8)]Similar estimates are valid for the other cases in the region of the optimal frequencies.

[9)]Local coupling obtains in quite a broad range of $\Delta \sim v_s$ and $\Gamma \sim T$ in the case of dirty superconductors, for which the relation $l_b \sim [D\tau_b(\bar{\varepsilon})]^{1/2} \sim \xi \ll \lambda(\Omega)$, $\Omega \sim \Delta$ is satisfied.

[10)]In the superconducting state the diffusional component of the current is canceled out by the supercurrent, a situation which leads to the appearance of an order-parameter phase difference across the section where $\nabla f_1(\varepsilon) \neq 0$.

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