Nonlinear structures in nonequilibrium systems and the algebra of attractors

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A new method of classifying nonlinear structures in different nonequilibrium systems is proposed. Lying at the basis of the new approach is the idea of an attractor as a stable state in which a physical system can remain, in principle, for an unlimited time. Two logical operations (superposition and combination) make it possible to use algebraic methods for the analysis of complicated nonlinear structures. An algorithm for constructing complicated attractors from elementary ones is demonstrated for the example of a gas discharge.

1. INTRODUCTION

Investigations of nonlinear structures in various nonequilibrium systems[1–4] constitute one of the most rapidly developing trends of modern natural science. During the study of such structures the problem of seeking possible ways of classifying them has arisen.[1] The purpose of this paper is to describe one of these ways and to illustrate its effectiveness for the example of a gas discharge.

At the basis of the approach developed lies the idea of an attractor. By an attractor, as is well known,[5] we mean that set (in the phase space of the dynamical system of equations that describes some particular process) which is approached asymptotically by phase trajectories from a certain neighborhood of the set and which, from this point of view, is an attracting set. The concept of an attractor makes it possible to dispense with many of specific features of qualitatively different (at first glance), stable stationary states of the dynamical system, and affords the possibility of considering these states as equivalent in many aspects, in the sense that none of them is in any way unique but, depending on the conditions, can be realized on an equal footing with the others. Three varieties of attractor are known: stable static equilibrium states (or in other words, stable singular points of the dynamical system), stable limit cycles,[6] and stochastic attractors (e.g., the so-called strange attractors).[5,7]

Henceforth, by an attractor we shall mean not only an attracting set in the phase space, but also the corresponding stable state in which the particular physical system, in principle, can remain for an unlimited time. This extension of the concept of an attractor, as will be seen from the following account, makes it possible to introduce a universal classification for different nonlinear structures, without reference to the concrete mechanism of their formation. Here, we shall take into account the fact that the character of the variation of the parameters of any physical system, both in space and in time, has many general features, inasmuch as these features are a consequence of the operation of a set of particular nonlinear processes. Because of this, besides the spatially uniform structures whose temporal analogs in systems with concentrated parameters are the static states, there can exist structures with spatially periodic (the analog of cyclic oscillations) and spatially stochastic (the analog of stochastic oscillations) distributions of the parameters.

An important role in the construction of the proposed classification is played by the concept of two algebraic operations that can be applied to attractors—the operation of superposition (superposition of structures onto each other), the meaning of which is especially clear in the case of weakly interacting structures, and the operation of combination of attractors, which is physically equivalent to the simultaneous existence of different structural formations of the phases of the system. Therefore, the approach developed here to the analysis of nonlinear structures can be called an attractor algebra.

Before proceeding to the detailed account of the proposed way of classifying nonlinear structures, with the aim of giving greater clarity to the formalism developed below we shall consider a number of concrete examples from the field of gas-discharge physics, concentrating attention on the analysis of the contracted states of a glow discharge.

2. Phenomenon of dynamical contraction and superposition of nonlinear structures in a glow discharge

In the analysis of the contracted states of a gas discharge in general, and of a glow discharge in particular, it is possible to distinguish, first of all, two qualitatively different arcing regimes—static and dynamical.[8]

Whereas the static regime of contraction of a gas discharge has been studied in some detail (see, e.g., Ref. 9), the phenomenon of dynamical contraction has been established experimentally very recently[8] in a glow discharge in conditions corresponding to regular self-oscillations of the parameters characterizing this discharge.

A further dynamical-contraction regime, corresponding to stochastic self-oscillations, is observed when the discharge gap is subjected to a magnetic field (Fig. 1) which, upon increase of the power supplied to the discharge, promotes the formation of a complicated, turbulent plasma flow. Here the positive column of the glow discharge consists of a set of regularly spaced current filaments (Fig. 2), inside which the turbulent plasma flow occurs. At the same time, the discharge current varies in a pulsed-periodic manner with frequency ~1 kHz and modulation depth ~25%, and each current pulse has a wide-band high-frequency duty cycle (Fig. 3). Thus, for the dynamical-contraction regime under discussion, simultaneous manifestation of stochastic and regular variation of the discharge parameters, both in time and in space, is characteristic. In other words, what is realized in this experiment is nothing other than a superposition of several nonlinear structures; to be precise, on a spatio-

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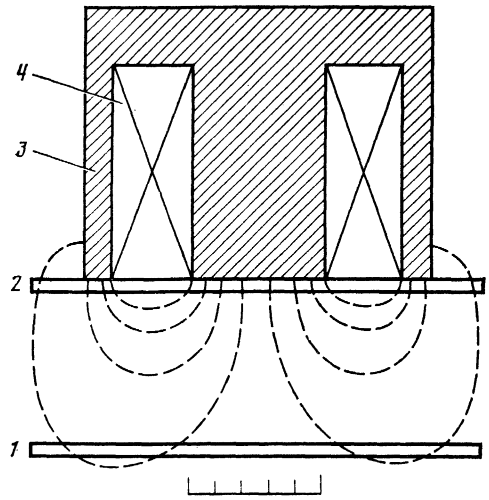

FIG. 1. Section of the discharge gap in a constant magnetic field: 1, 2) water-cooled anode and cathode; 3, 4) core and coil of a cylindrical electromagnet.

temporal stochastic structure (the turbulent state of the plasma) is superposed a structure with parameters that vary regularly in space and in time.

In both the experiments discussed above, although they demonstrate a new, general form of stable state of a discharge (the regime of dynamical contraction), essentially different structures are actually realized. In fact, in the first experiment the contracted state arose as a result of the action of an external factor, viz., the fact that the physical system under consideration has a finite size (determined by the diameter of the tube) in a direction transverse to the current. This circumstance led to the formation of a nonuniform gas-temperature profile and thereby induced a thermal mechanism of contraction.[8] Therefore, the structure that arises in this case can be called an induced structure. In the second experiment, the formation of the corresponding structure is connected primarily with features of processes occurring within the physical system itself—the gas discharge. The finiteness of this system (say, along the electric-field lines) and the nonuniformity of the magnetic field (and also, in general, the character of the variation of the external factors in

FIG. 2. Structure of the gas discharge in a magnetic field in the regime of dynamical contraction (CO_2 at a pressure of 15 Torr, and with a discharge current of 4 A).

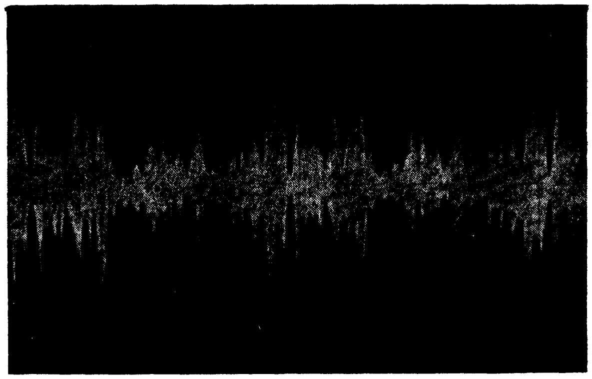

FIG. 3. Character of the variation of the discharge-current strength in the regime of dynamical contraction.

space and in time) introduce distortions that are only secondary from the standpoint of the essence of the phenomenon (see the interpretation given in Sec. 4). Therefore, it is natural to call the given structure a spontaneous structure. In the following we shall consider only such spontaneous structures.

3. THE ALGEBRA OF ATTRACTORS

3.1 The operation of superposition of attractors

We shall denote by the symbol ψ the set $\langle \psi_i \rangle$ of different internal mutually related parameters ψ_i ($i = 1, 2, \ldots$) that describe the behavior in time and in space of a certain physical system in a stationary (not necessarily static) state. We shall call ψ the state function of the system. We shall consider features of the behavior of such a system in cartesian coordinates x, y, z. Since in the following we shall be studying spontaneous structures, we shall assume, in accordance with what was said at the end of Sec. 2, that the external (controlling) parameters are constant or (as, say, in the case of structures that arise in the presence of high-frequency electromagnetic fields) averaged over the space-time continuum, and that the unperturbed system is uniform and unbounded.

Let the symbols $A_t^i, A_x^k, A_y^l, A_z^m$ or combinations of them show how the state function $\Psi = \Psi(A_t^i, A_x^k, A_y^l, A_z^m) \equiv \Psi(A^{iklm})$ varies, i.e., how each function ψ_i from the investigated set $\{\psi_i\}$ varies as a function of the time t and the spatial coordinates x, y, z. Here, taking into account what was said in the Introduction, we assume first that the indices i, k, l, m can take only three values: 0 (a static (in time) or uniform (in space) state), c (a cyclic structure),[1)] and s (a stochastic structure). Then, for example, $\Psi(A^{0000})$ will correspond to a static, spatially uniform state, $\Psi(A^{c000})$ will correspond to a time-periodic, spatially uniform structure (self-oscillations), and $\Psi(A^{0sc0})$ will correspond to a static structure with parameters that vary in a stochastic and cyclic manner in the x and y directions, respectively, but do not depend on z.

We introduce the operation of superposition of different structures. For this we consider the set of all possible states of the physical system, the elements of which are the sets of functions ψ_i, i.e., the state functions $\Psi \equiv \{\psi_i\}$. Here, we shall regard as different only those states (or, in other words, only those elements of Ψ) which cannot be obtained from each other as a result of a continuous variation of the controlling parameters without the appearance of a bifurcation. Then a superposition of structures under investigation can be described as follows in the framework of the approach being developed. Let the operator A^{iklm} act by "converting" one structure of the physical system into another, i.e., by associating one element of the set Ψ of states with another by the following rule:

$$A^{i_1k_1l_1m_1}\Psi(A^{i_0k_0l_0m_0})=\Psi(A^{i_1k_1l_1m_1}A^{i_0k_0l_0m_0})$$
$$=\Psi(A_t{}^{i_1}A_t{}^{i_0},\quad A_x{}^{k_1}A_x{}^{k_0},\quad A_y{}^{l_1}A_y{}^{l_0},\quad A_z{}^{m_1}A_z{}^{m_0}).$$

Here the combination $A_t^{i_1}A_t^{i_0}$ (or, e.g., $A_x^{k_1}A_x^{k_0}$ implies that on the structure $A_t^{i_0}(A_x^{k_0})$ a structure of the type $A_t^{i_1}(A_x^{k_1})$ is superimposed, i.e., superposition of these structures occurs. For example, if $i_0=0$ ($k_0=0$) and $i_1=c$ ($k_1=c$), we have a time-periodic (space-periodic in the direction of the x axis) modulation of the initially constant (uniform) parameters, while if $i_0=s$ ($k_0=s$) and $i_1=c$ ($k_1=c$) a stochastic structure is modulated periodically.

We shall define the product (or, in other words, superposition)

$$\prod_{\alpha=1}^{n} A^{i_\alpha k_\alpha l_\alpha m_\alpha} \equiv A^{i_n k_n l_n m_n}\ldots A^{i_1k_1l_1m_1}$$

of the operators under consideration by the rule

$$\prod_{\alpha=1}^{n} A^{i_\alpha k_\alpha l_\alpha m_\alpha}\,\Psi(A^{i_0k_0l_0m_0})=\Psi\left(\prod_{\alpha=0}^{n} A^{i_\alpha k_\alpha l_\alpha m_\alpha}\right)$$
$$=\Psi\left(\prod_{\alpha=0}^{n} A_t{}^{i_\alpha},\ \prod_{\alpha=0}^{n} A_x{}^{k_\alpha},\ \prod_{\alpha=0}^{n} A_y{}^{l_\alpha},\ \prod_{\alpha=0}^{n} A_z{}^{m_\alpha}\right).$$

Here, notation of the type

$$\prod_{\alpha=0}^{n} A_t{}^{i_\alpha} \equiv A_t{}^{i_n}\ldots A_t{}^{i_0}$$

implies the superposition of m ($m=n+1$) different structures on each other. For example, if m cyclic structures are superimposed, we can realize a situation corresponding to so-called m-dimensional tori. But if in this example m tends to infinity, then, under the condition that the motion is not periodic and that for no finite value of m can a strange attractor arise as a result of bifurcations, a stochastic structure corresponding to the Landau model of turbulence[11,12] (see also Refs. 13 and 14) is formed.

We now suppose that each of the indices i, k, l, m can take a further, fourth value, equal to unity. We shall further assume that when i, k, l, and (or) $m=1$, the identity transformation is realized, i.e., the specific features of the behavior of the nonlinear structure as functions of the corresponding coordinate of the spatio-temporal continuum are not destroyed in this case. For example, if $k=1$ the relation

$$A^{i1lm}\Psi(A^{i_0k_0l_0m_0})=\Psi(A_t{}^{i}A_t{}^{i_0},\quad A_x{}^{1}A_x{}^{k_0},\quad A_y{}^{l}A_y{}^{l_0},\quad A_z{}^{m}A_z{}^{m_0})$$
$$=\Psi(A_t{}^{i}A_t{}^{i_0},\quad A_x{}^{k_0},\quad A_y{}^{l}A_y{}^{l_0},\quad A_z{}^{m}A_z{}^{m_0})$$

is valid. It is obvious that A^{1111} will be a unit operator, i.e., will effect the identity transformation of each element of the set Ψ of states of the physical system into itself:

$$A^{1111}\Psi(A^{iklm})=\Psi(A^{1111}A^{iklm})=\Psi(A^{iklm}).$$

Now, when the formalism of the operation of superposition of attractors has been described, we shall assume that, in all situations to be considered below, the initial state of the physical system is static and spatially uniform. Therfore, to simplify the writing we shall omit the state function Ψ in the relations given below, and confine ourselves to pointing out only the operators of their superpositions (which, for brevity, will also be called attractors) corresponding to any particular nonlinear structure.

3.2 Examples of nonlinear structures in a gas discharge

To illustrate the results that can be obtained by means of the approach described above, we shall consider some of the possible nonlinear structures in an electric discharge, having assumed for definiteness that the x axis is in the direction of the electric-field intensity: A^{0c00} are the rest strata, A^{cc00} are the running regular strata, A^{0s00} are the rest irregular strata with stochastically spaced layers, A^{ccss} is the contracted state with stochastically spaced current filaments, along which regular strata propagate, and so on.

We note that in the general case several different states of the physical system can correspond to the same attractor $\Pi_\alpha A^{i_\alpha k_\alpha l_\alpha m_\alpha}$. For example, if the dependence on x and t of the discharge parameters ψ_i appears only in the form of the combination $x-vt$, where $v=\text{const}$, then A^{ssss} can be interpreted either as a set of stochastically spaced filaments along which irregular strata propagate with velocity v, or as irregular oblique strata,[15] the direction of the displacement of which does not coincide with the direction of the electric current. If, however, there is no such dependence of ψ_i on x and t, then A^{ssss} can be interpreted as a turbulent state of the plasma, and this will be assumed in the following in respect of this attractor. The turbulent regime in the plasma, as is well known, arises especially easily in the presence of an external magnetic field. For example, in the plasma of a glow discharge in crossed electric and magnetic fields there develops the so-called ionization turbulence[15] (see also Ref. 16) which, to all appearances, is also observed in the second experiment described in Sec. 2. Here, as already noted in Sec. 2, we have simultaneous superposition of regularly spaced current filaments and low-frequency modulation of the discharge parameters. Therefore, the structure under discussion corresponds to a superposition of two attractors: A^{c1cc} and A^{ssss}, i.e., to the attractor $A^{c1cc}A^{ssss}$.

We analyzed above only a small number of different variants of nonlinear structures. For example, to the attractors A^{iklm} alone, even without allowance for the possibility of associating several real physical situations with them, can correspond as many as 81 varieties of such structures. Obviously, for the attractors

$$\prod_{\alpha=1}^{n} A^{i_\alpha k_\alpha l_\alpha m_\alpha}$$

there will be many more such structures. All of this is evidence of the considerable effectiveness of the approach developed in the present paper to the systematization of different nonlinear structures.

3.3 The operation of combination of attractors

Up to now it has been assumed that the nonlinear structure corresponding to a particular attractor or superposition of attractors is formed simultaneously in the whole space. However, in practice one fairly frequently encounters situations in which nonlinear structures coexist in the same way that different phases of a substance (e.g., in a liquid-vapor system) coexist. In this case there arises a new structure that is a combination of a certain set of attractors. Below we shall confine ourselves to an analysis of the case in which the intermediate layer between one nonlinear structure and another is planar. We let the y axis be perpendicular to this layer. Then, by the combination $A^{i_+k_+l_+m_+} + A^{i_-k_-l_-m_-}$ of two attractors $A^{i_+k_+l_+m_+}$ and $A^{i_-k_-l_-m_-}$ we shall mean that attractor for which as $y\to+\infty$ a structure of the type $A^{i_+k_+l_+m_+}$ is realized and as $y\to-\infty$ a structure of the type $A^{i_-k_-l_-m_-}$ is realized.

We shall give only two examples, assuming that the operation of combination can be applied in relation to all (without exception) attractors of the type $\Pi_\alpha A^{i_\alpha k_\alpha l_\alpha m_\alpha}$ ($\alpha\geqslant1$): 1) $[A^{0000}]' + [A^{0000}]'$—in the case of a gas discharge the given layer structure corresponds to the simultaneous coexistence of a static, weak-current phase (e. g., $[A^{0000}]'$) and a static, strong-current phase ($[A^{0000}]''$); 2) $A^{0000} + A^{c0c0}A^{s0s0}$, e.g., an ionization wave, with stochastically varying parameters that are modulated in a periodic manner, propagating through a gas.

4. GLOW DISCHARGE STRUCTURE CORRESPONDING TO THE ATTRACTOR $A^{c1cc}A^{ssss}$

We shall now explain in more detail one of the possible mechanisms of formation of the nonlinear structure (discussed in Sec. 3.) of the type $A^{c1cc}A^{ssss}$, and this will enable us to give greater clarity to the operation of superposition of attractors. Here, for definiteness, we shall consider the case of a glow discharge excited in crossed electric and magnetic fields in conditions similar to those which obtained in the second of the experiments described in Sec. 2; we shall assume that the unperturbed state of this discharge is static and spatially uniform.

We suppose that at a certain time a fluctuation has appeared in the discharge under investigation. It is known (see, e.g., Ref. 17) that an arbitrary fluctuation decays in a natural way into two component parts, the development of one of which (the spatially uniform part) is accompanied by a change in the current I passing through the discharge, while the development of the other (the spatially nonuniform part) is not accompanied by such a change.

In the analysis of the distinctive features of the growth of uniform perturbations, obviously, it is necessary to take into account the influence of the external electric circuit.[17] Denoting by E the intensity of the electric field in the glow-discharge plasma, taking into account that $E\sim U$, is the voltage drop across the discharge, neglecting the role of the regions near the electrodes, and using Kirchhoff's laws, we can establish the rigorous (within the framework of the initial premises) relation

$$-\frac{\delta I}{I}=RC\left(\frac{\mathscr{E}}{IR}-1\right)\left(\frac{\delta\dot{E}}{E}+\frac{1}{RC}\frac{\delta E}{E}\right), \quad (1)$$

where $\mathscr{E}$ is the emf of the power supply, R is the ballast resitance, and C is the stray capacitance. Here and below, the symbol δ denotes fluctuations of the corresponding quantities, and a dot denotes the time derivative. The discharge parameters without the symbol δ pertain to the unperturbed (initial) state of the system.

In analyzing the development of spatially uniform perturbations we shall not take into account effects associated with the role of the magnetic field and gas heating, but draw attention to the fact that this does not affect the essence of the final conclusions. Then, in the framework of a phenomenological approach, just as was done in Ref. 8, we shall describe the kinetics of the development of the ionization by means of the equation

$$\dot{n}_e=\nu_i(n_e,E)n_e-k_rn_e^2, \quad (2)$$

where n_e is the electron concentration, ν_in_e is the ionization rate, and k_r is the rate constant of the dissociative recombination. Here, for convenience in the subsequent reasoning, we shall make use of the following approximation for ν_i: $\nu_i=\Gamma n_e^{1+\alpha}E^\beta$, where Γ, α, and β are known constants. Then, from (2),

$$\frac{\delta E}{E}=\left[1+\frac{1}{\nu_i}\left(1+\frac{\delta n_e}{n_e}\right)^{-2}\frac{\delta\dot{n}_e}{n_e}\right]^{1/\beta}\left(1+\frac{\delta n_e}{n_e}\right)^{-\alpha/\beta}-1. \quad (3)$$

Next, if we take into account the fact that the current strength I is directly proportional to the current density, equal to μ_eEn_e (μ_e is the electron mobility), and, for simplicity, neglect the dependence of μ_e on E, we can establish the relation

$$\frac{\delta I}{I}=\frac{\delta n_e}{n_e}+\frac{\delta E}{E}+\frac{\delta n_e}{n_e}\frac{\delta E}{E}. \quad (4)$$

Substituting $\delta I/I$ from (4) into (1), substituting $\delta E/E$ from (3) into the expression thus obtained, and expanding the right-hand side of (3) in a Taylor series in the fluctuations $\delta n_e/n_e$ and $\delta\dot{n}_e/n_e$, we find

$$\frac{\delta\ddot{n}_e}{n_e}+\omega_0^2\frac{\delta n_e}{n_e}=\gamma\frac{\delta\dot{n}_e}{n_e}+Q, \quad (5)$$

where

$$\omega_0^2=\frac{\nu_i}{RC}\left[\frac{IR}{\mathscr{E}}\hat{\nu}_{i,E}-(\hat{\nu}_{i,n_e}-1)\right]\left(1-\frac{IR}{\mathscr{E}}\right)^{-1},$$

$$\gamma=\nu_i(\hat{\nu}_{i,n_e}-1)-\frac{1}{RC}\left(1-\frac{IR}{\mathscr{E}}\right)^{-1},$$

and the letter Q denotes terms that are nonlinear in $\delta n_e/n_e$ and $\delta\dot{n}_e/n_e$. In the form (5) we have introduced the notation $\hat{\nu}_{i,n_e}=\partial\ln\nu_i/\partial\ln n_e$, $\hat{\nu}_{i,E}=\partial\ln\nu_i/\partial\ln E$ and have taken into account the relations $\nu_{i,n_e}=1+\alpha, \nu_{i,E}=\beta$.

Confining ourselves in the subsequent analysis to the most interesting case, when $\omega_0^2>0$, we shall seek the solution of (5), to within a phase shift that is of no fundamental importance for the gist of the problem, in the form $\delta n_e/n_e=$

$a \cos \omega_0 t$, assuming that the amplitude a can vary only very slowly with time ($|\dot{a}/a| \ll \omega_0$), so that $\delta \dot{n}_e / n_e \cong - a\omega_0 \sin \omega_0 t$. Then, multiplying both sides of the equality (5) by $\delta \dot{n}_e / n_e$, averaging the relation thus obtained over the period $T = 2\pi/\omega_0$, and retaining in the expression for Q the terms of no higher than third order of smallness in the fluctuations, we have

$$\dot{a}^2 = a^2 (\gamma - a^2 \Delta), \tag{6}$$

where

$$\Delta = \frac{1}{4} \nu_i \{ (2\bar{\omega}_0^2 + 1 - \hat{\nu}_{i,n_e}) + (RC\nu_i)^{-1} [(\bar{\omega}_0^2 + 3) - (\bar{\omega}_0^2 + 1)(1 - \mathscr{E}/IR)^{-1}] \},$$

$$\bar{\omega}_0 \equiv \omega_0 / \nu_i$$

(in writing (6) we have taken into account that usually $\hat{\nu}_{i,E} \gg 1$ while $|\hat{\nu}_{i,n_e}| \lesssim 1$, and in the right-hand side of (6) have retained only terms linear in $\hat{\nu}_{i,E}^{-1}$).

It follows from (6) that the static ($a = 0$) gas-discharge state under consideration can lose stability ($\gamma > 0$) only when the condition $\hat{\nu}_{i,n_e} > 1 + (RC\nu_i)^{-1}(1 - IR/\mathscr{E})^{-1}$ is fulfilled, i.e., in other words, only if in the neighborhood of this state the rate of development of ionization as a function of n_e grows sufficiently rapidly in comparison with the rate of recombination.[8]

If, at the same time, Δ is positive, then, as can be seen from (6), in a certain neighborhood of the unstable-equilibrium state being studied a stable limit cycle ($a^2 = \gamma/\Delta$) is formed.

We can now elucidate the physical mechanism of the onset of a nonlinear structure of the type $A^{c1cc}A^{ssss}$. For this we take into account the fact of the existence of the magnetic field, and the effects associated with the heating of the gas. Then spatially nonuniform fluctuations in crossed magnetic and electric fields, as already noted in Sec. 3.2, lead to the development of ionization turbulence. Uniform perturbations, however, because of the presence of reactive elements in the external circuit, as only just shown, promote in their turn the onset of regular self-oscillations in the system. Next, since the turbulization of the plasma is accompanied by an increase of its resistance,[15] provided that the power source is operating in the current-generator regime the power supplied to the discharge increases. It is the latter circumstance which, because of the appearance of the superheating instability,[9] promotes the formation of a periodic structure of current filaments, just as is described, e.g., in Ref. 18. As a result of the operation of the processes just considered, the complicated nonlinear structure of the attractor $A^{c1cc}A^{ssss}$ is formed.

5. CONCLUSIONS

Thus, the approach developed in this paper does indeed make it possible to construct a universal way of classifying nonlinear structures. Within the framework of this approach it is possible to exhibit the set of qualitatively different nonlinear formations that can be realized in physical systems. To a considerable extent, this is achieved because of the generality of the concept of the attractor, and because an arbitrary nonlinear structure (an arbitrary attractor) is, in the final analysis, an aggregate (superposition) of three varieties of elementary attractors: static (in space, uniform), cyclic, and stochastic.

At the same time, the given approach, by virtue of the very definition of the attractor as an attracting object (see the Introduction), rules out, generally speaking, the possibility of describing the intermediate regions in structures that are realized as a result of combining different attractors into a single whole, and assumes, in the zeroth approximation, that these regions are infinitely thin. However, if we know the distinctive features of the structure of such attractors it is usually straighforward to establish the character of the behavior of the parameters of the physical system in the intermediate zone too. Thus, in the case described in the second example of Sec. 2.3, it is obvious that damping of the cyclic and stochastic oscillations should occur in this zone.

We draw attention to one intermediate structure that arises in the formation of regular spiral self-waves, i.e., self-waves having the form of a spiral (or set of spirals)[4,10] with regularly varying values of the parameters. In the two-dimensional case, to describe such waves it is convenient to use polar coordinates r, φ (Ref. 4), taking the center of the zone around which the wave rotates to be at $r = 0$. In the neighborhood of an infinitely remote point ($r \to \infty$) it is obvious that a cyclic structure will be realized. Therefore, a spiral wave arises in the intermediate layer between, e.g., a static attractor (for $r = 0$) and a cyclic attractor (for $r \to \infty$).

In the framework of the attractor algebra specific mechanisms of formation of particular structures are not considered. The main result yielded by the approach using the attractor algebra consists, rather, in the possibility of elucidating by comparatively simple means the qualitatively different features of the construction of the nonlinear structures that can, in principle, be realized in the most diverse physical systems. It is certain that this circumstance simplifies substantially the search for such structures, both in experimental investigations and in the mathematical modeling of specific physical phenomena.

We shall give an example. It was stated above that in the intermediate zone between a static attractor (at $r = 0$) and a cyclic attractor (at $r \to \infty$) a self-wave in the form of a regular spiral can arise. But if at $r \to \infty$ a stochastic rather than a cyclic attractor is formed, then, obviously, in the intermediate layer the spiral self-wave should have a stochastic structure. The study of such self-waves, like the study of, e.g., irregular strata, is of undoubted interest.

To conclude this paper we draw attention to the fact that, using the methods of group theory, one can perform a more detailed classification of the different nonlinear structures in terms of symmetry characteristics.

[1)]We note also that isolated nonlinear structures of the self-wave type[10] will be considered below as the limiting case of cyclic structures with infinitely large period.

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