Dissipative flow of superfluid He-3 through thin channels

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Equations are obtained for the distribution of the chemical potential (pressure) of superfluid He-3 flowing through a narrow channel with allowance for the diffuse reflection of the quasiparticles from the walls. The possibility of interpreting the experimentally observed dissipation in terms of phase slippage is discussed.

1. INTRODUCTION

Experimental interest in the phenomena connected with flow of superfluid He-3 through thin channels has increased greatly in the last few years. Principal attention is being paid to measurements of the critical velocities and of the relaxation of the superfluid flow (see, e.g., Refs. 1–3). To measure the critical pair-breaking velocities it is necessary that the channel radius be comparable with the incoherence length $\xi(T)$. At such channel sizes, no vortices can be produced and also, an important factor for the A phase, the anisotropy vector **l** cannot move. These conditions were recently realized in an experiment by Manninen and Pekola.[3] They used a porous filter with hole radii $R = 0.4\ \mu$m, approximately five times larger than ξ_0. They plotted in the experiment the dependence of the pressure difference ΔP on the two sides of the filter on the helium flow I through the filter. A pressure difference was unobservable at a flow lower than a certain critical value I_c and appeared only at values above I_c. The critical flow I_c varies with temperature like $(T_c - T)^{1/2}$ and agrees well with the predictions of the Ginzburg–Landau theory for the pair-breaking flow.[4–6] At a flow $I > I_c$ the slope of the $\Delta p(I)$ curve was such that the flow of the normal component was small compared with the total liquid flow. This allows us to assume that under these conditions superconductivity is preserved at a constant pressure difference—a situation analogous in many respects to the so-called resistive state of superconductors,[7] where the superconductivity against the background of a constant electric field is maintained by phase-slippage processes. The present paper deals with the possibility of interpreting the experimental data of Ref. 3 in terms of phase slippage in superfluid He-3.

For this purpose we need first equations for the distribution of the chemical potential (pressure) along the channel. These equations cannot be obtained in the hydrodynamic approximation, for in narrow channels of radius comparable with the coherence length the particle mean free path l is always large compared with the pore radius (in Ref. 3 the ratio l/R ranged approximately from 100 to 10, depending on the pressure). Section 2 is devoted to the derivation of the corresponding equations in the basis of the microscopic theory. Also discussed in this section is sound propagation in thin channels. The experimental data are compared qualitatively with the suggested theoretical picture in Sec. 3.

2. DERIVATION OF THE KINETIC EQUATIONS

We follow the method used earlier[8] to obtain the equations of orbital dynamics in ^{3}He-A and based on an expansion of the Éliashberg equations.[9] Just as in Ref. 8, we disregard Fermi-liquid effects, which do not alter the physical picture in this case. We put in this section $\hbar = 1$.

The Éliashberg equations are expressed in terms of quasiclassical Green's functions integrated with respect to the energy variable $\zeta_p = (p^2/2m) - E_F$:

$$\hat{G} = \begin{pmatrix} \hat{g} & \hat{f} \\ -\hat{f}^+ & \hat{\bar{g}} \end{pmatrix},$$

where letters with carets denote matrices over the spin indices. The order parameter $\hat{\Delta}_p$ and the mass flux **j** are expressed with the aid of these functions in the form

$$\frac{\hat{\Delta}_{\mathbf{p}}(\omega, \mathbf{r})}{3g_0} = \nu(0) \int \frac{d\varepsilon}{4} \int \frac{d\Omega_{\mathbf{n}'}}{4\pi} \mathbf{n}\mathbf{n}' \hat{f}_{\varepsilon_+, \varepsilon_-}(\mathbf{n}', \mathbf{r}), \tag{1}$$

$$\mathbf{j}(\omega, \mathbf{r}) = -\nu(0) \int \frac{d\varepsilon}{4} \int \frac{d\Omega_{\mathbf{n}}}{4\pi} \frac{1}{2} \operatorname{Tr}\{\mathbf{p}_F [\hat{g}_{\varepsilon_+, \varepsilon_-}(\mathbf{n}, \mathbf{r}) - \hat{\bar{g}}_{\varepsilon_+, \varepsilon_-}(\mathbf{n}, \mathbf{r})]\}. \tag{2}$$

Here **n** is a unit vector in the direction of the momentum $\mathbf{p}_F$, $\nu(0) = mp_F/2\pi^2$ is the density of states on the Fermi surface, and $\varepsilon_\pm = \varepsilon \pm \omega/2$. The trace Tr is taken over the spin indices. The matrix Green's function $\hat{G}$ satisfies the equation[9]

$$-i\mathbf{v}_F \nabla \hat{G}_{\varepsilon_1, \varepsilon_2}(\mathbf{n}, \mathbf{r}) - \varepsilon_1 \tau_z \hat{G}_{\varepsilon_1, \varepsilon_2}(\mathbf{n}, \mathbf{r}) + \hat{G}_{\varepsilon_1, \varepsilon_2}(\mathbf{n}, \mathbf{r}) \tau_z \varepsilon_2 + \{\hat{H}_\omega \hat{G}_{\varepsilon_1 - \omega, \varepsilon_2}(\mathbf{n}, \mathbf{r}) - \hat{G}_{\varepsilon_1, \varepsilon_2 + \omega}(\mathbf{n}, \mathbf{r}) \hat{H}_\omega\} = \hat{I}_{\varepsilon_1, \varepsilon_2}(\mathbf{n}, \mathbf{r}), \tag{3}$$

where

$$\hat{H}_\omega = \begin{pmatrix} 0 & -\hat{\Delta}_{\mathbf{p}}(\omega, \mathbf{r}) \\ \hat{\Delta}_{\mathbf{p}}^*(\omega, \mathbf{r}) & 0 \end{pmatrix},$$

τ_z is a Pauli matrix in the space of the functions g and f, and $\hat{I}$ is the collision integral:

$$\hat{I}_{\varepsilon_1, \varepsilon_2} = \{\hat{\Sigma}^R \hat{G} - \hat{G}\hat{\Sigma}^A + \hat{\Sigma}\hat{G}^A - \hat{G}^R \hat{\Sigma}\}_{\varepsilon_1, \varepsilon_2}. \tag{4}$$

Here

$$\hat{\Sigma} = \begin{pmatrix} \hat{\Sigma}_g & \hat{\Sigma}_f \\ -\hat{\Sigma}_{f+} & \hat{\Sigma}_{\bar{g}} \end{pmatrix}$$

is the eigenvalue matrix for the pair interaction. We shall not

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write out the known cumbersome equations for $\hat{\Sigma}$, since they can be found in Refs. 8 and 9. The curly brackets denote convolution with respect to ε. The functions with superscripts R and A are respectively retarded and advanced Green's functions, which satisfy Eq. (3) with the one exception that the right-hand side contains the collision integral

$$\hat{I}^{R(A)}=\{\hat{\Sigma}^{R(A)}\hat{G}^{R(A)}-\hat{G}^{R(A)}\hat{\Sigma}^{R(A)}\}. \tag{5}$$

To derive the kinetic equation we introduce the even $f^{(2)}$ and the odd $f^{(1)}$ parts of the distribution function, in accordance with the expression[10]

$$\hat{G}_{\varepsilon_1,\varepsilon_2}=\{\hat{G}^R(f^{(1)}+\tau_z f^{(2)})-(f^{(1)}+\tau_z f^{(2)})\hat{G}^A\}. \tag{6}$$

The functions $f^{(1)}$ and $f^{(2)}$ have the following properties:

$$f_\varepsilon^{(1)}(\mathbf{p})=-f_{-\varepsilon}^{(1)}(-\mathbf{p}),\quad f_\varepsilon^{(2)}(\mathbf{p})=f_{-\varepsilon}^{(2)}(-\mathbf{p}).$$

The calculations consist of substituting expression (6) in Eq. (3) and taking into account the equation for the regular functions $\hat{G}^{R(A)}$, and of expanding the obtained expression in terms of the small time and spatial derivatives. To simplify the situation, we assume that the transverse dimensions of the pore are much larger than ξ_0, but are as before smaller than the mean free path l. In this case the stationary equation for the order parameter remains the same as in an unbounded volume of liquid, but the kinetic equation for the distribution function will differ from the hydrodynamic limit.

We begin with the case of the A-phase. In this phase the order parameter and the Green's functions can be written in the form

$$\hat{\Delta}_\mathbf{p}=i(\hat{\sigma}^{(\alpha)}\hat{\sigma}^{(2)})d_\alpha\Delta_\mathbf{p},\quad \hat{g}=\hat{\delta}_{\alpha\beta}g,\quad \hat{f}=i(\hat{\sigma}^{(\alpha)}\hat{\sigma}^{(2)})d_\alpha f_\mathbf{p}.$$

Here $\hat{\sigma}^{(\alpha)}$ are Pauli matrices in spin space, and the letters without carets denote the orbital parts of the corresponding functions. Thus,

$$\Delta_\mathbf{p}=\Delta(T)(\mathbf{\Psi n}),\quad \mathbf{\Psi}=\mathbf{\Delta}'+i\mathbf{\Delta}'',$$

where $\mathbf{\Delta}'$ and $\mathbf{\Delta}''$ are two mutually perpendicular unit vectors. Their product defines the anisotropy vector $\mathbf{l}=\mathbf{\Delta}'\times\mathbf{\Delta}''$.

We assume below for simplicity that the spin part of the order parameter remains unchanged, i.e., that the unit vector $\mathbf{d}$ in spin space is constant; we shall track only the variation of the orbital part of the order parameter. In this case the functions $f_\varepsilon^{(1)}$ and $f_\varepsilon^{(2)}$ are diagonal in the spin indices and we get[8]

$$\frac{1}{2}\mathbf{v}_F\nabla[(g^R-g^A)f_\varepsilon^{(2)}]$$
$$+\frac{1}{2}\left[(f^R-f^A)\frac{\partial\Delta_\mathbf{p}^*}{\partial t}+(f^{+R}-f^{+A})\frac{\partial\Delta_\mathbf{p}}{\partial t}\right]\frac{\partial f^{(1)}}{\partial\varepsilon}=J_1, \tag{7}$$

$$\frac{1}{2}\frac{\partial}{\partial t}[(g^R-g^A)f^{(2)}]+\frac{1}{2}(g^R-g^A)\mathbf{v}_F\nabla f^{(1)}$$
$$+\frac{i}{2}[(f^R+f^A)\Delta_\mathbf{p}^*+(f^{+R}+f^{+A})\Delta_\mathbf{p}]f^{(2)}$$
$$-\frac{1}{4}\left[(f^{+R}+f^{+A})\frac{\partial\Delta_\mathbf{p}}{\partial t}-(f^R+f^A)\frac{\partial\Delta_\mathbf{p}^*}{\partial t}\right]\frac{\partial f^{(1)}}{\partial\varepsilon}$$
$$-\frac{1}{4}\left[\left(\frac{\partial f^R}{\partial\varepsilon}-\frac{\partial f^A}{\partial\varepsilon}\right)\Delta_\mathbf{p}^*+\left(\frac{\partial f^{+R}}{\partial\varepsilon}-\frac{\partial f^{+A}}{\partial\varepsilon}\right)\Delta_\mathbf{p}\right]\frac{\partial f^{(2)}}{\partial t}$$
$$-\frac{i}{8}\left[\left(\frac{\partial f^{+R}}{\partial\varepsilon}-\frac{\partial f^{+A}}{\partial\varepsilon}\right)\frac{\partial^2\Delta_\mathbf{p}}{\partial t^2}\right.$$
$$\left.-\left(\frac{\partial f^R}{\partial\varepsilon}-\frac{\partial f^A}{\partial\varepsilon}\right)\frac{\partial^2\Delta_\mathbf{p}^*}{\partial t^2}\right]\frac{\partial f^{(1)}}{\partial\varepsilon}=J_2. \tag{8}$$

Here[1)]

$$J_1={}^1/_4 i\,\mathrm{Sp}\,[I-(I^R-I^A)f^{(1)}],$$
$$J_2={}^1/_4 i\,\mathrm{Sp}\,[\tau_z I-\tau_z(I^R-I^A)f^{(1)}]. \tag{9}$$

At this stage it is convenient to confine oneself to temperatures close to critical, $T_c-T\ll T_c$. Then $\Delta\ll T$, and furthermore at $\varepsilon\sim T$ the density of states $\frac{1}{2}(g^R-g^A)$ is constant and equal to unity, making the claculations much simpler.

To connect in Eqs. (7) and (8) the derivative of $F^{(2)}$ with the function $f^{(1)}$ and conversely, it is convenient to return for a while to the initial equations (3). The coordinate dependence of the complete functions

$$g=(g^R-g^A)f^{(1)}+(g^R-g^A)f^{(2)}, \tag{10}$$

$$\bar{g}=-(g^R-g^A)f^{(1)}+(g^R-g^A)f^{(2)} \tag{11}$$

breaks up into two parts: fast variations of g and $\bar{g}$ over distances of the order of the transverse dimension of the channel and, as will be shown below, a slow variation of the functions $f^{(1)}$ and $f^{(2)}$ over distances much larger than the transverse dimension of the channel. The time dependence of the distribution functions $f^{(1)}$ and $f^{(2)}$ is assumed to be slow compared with reciprocal time τ_ε^{-1} of the inelastic collisions. This means automatically that the corresponding frequencies are smaller than the reciprocal time $\tau_B^{-1}\sim v_F/R$ of particle transit between successive collisions with the walls. At distances on the order of the pore radius we can therefore neglect the collision integral and all but the radient term in Eq. (3) for g and $\bar{g}$. We get

$$\mathbf{v}_F\nabla g=\mathbf{v}_F\nabla\bar{g}=0. \tag{12}$$

We separate in $f^{(1)}$ the part odd in n. We write for this purpose

$$g=(g^R-g^A)f^{(0)}+g'+(g^R-g^A)f^{(2)},$$
$$\bar{g}=-(g^R-g^A)f^{(0)}+\bar{g}'+(g^R-g^A)f^{(2)},$$

where $f^{(0)}=\tanh(\varepsilon/2T)$ is the equilibrium distribution function. From (12) we obtain

$$\mathbf{v}_F\nabla g'=\mathbf{v}_F\nabla\bar{g}'=-\mathbf{v}_F\nabla[(g^R-g^A)f^{(2)}]. \tag{13}$$

We shall assume complete accommodation of the quasiparticles in collisions with pore walls. It requires that the correction g' to the Green's function vanish at that point $\mathbf{r}_B$ of the

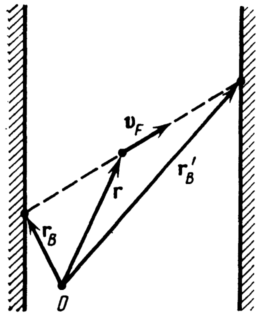

FIG. 1.

wall from which the particle departed as a result of the last collision with the wall. The solution of (13) with such a boundary condition is[11]

$$g' = -|\mathbf{r}-\mathbf{r}_B|\,(\mathbf{n}\nabla)\,[(g^R-g^A)f^{(2)}].$$

The corresponding boundary condition for the function $\bar{g}'$ should be set at the point $\mathbf{r}_B'$ at which the particle reaches the wall

$$\bar{g}' = |\mathbf{r}-\mathbf{r}_B'|\,(\mathbf{n}\nabla)\,[(g^R-g^A)f^{(2)}].$$

Here $\mathbf{r}_B$ and $\mathbf{r}_B'$ are the points where a straight line parallel to $\mathbf{v}_F$ and passing through the particle location $\mathbf{r}$ penetrates through the walls of the channel (see Fig. 1). Since $(g^R - g^A)\delta f^{(1)} = \frac{1}{2}(g' - \bar{g}')$, we have

$$(g^R-g^A)\delta f^{(1)} = {}^1/_2\{-|\mathbf{r}-\mathbf{r}_B|\,(\mathbf{n}\nabla)\,[(g^R-g^A)f^{(2)}] \\ -|\mathbf{r}-\mathbf{r}_B'|\,(\mathbf{n}\nabla)\,[(g^R-g^A)f^{(2)}]\} = -{}^1/_2 d_\mathbf{v}(\mathbf{n}\nabla)\,[(g^R-g^A)f^{(2)}],$$

where $d_\mathbf{v} = |\mathbf{r}_B - \mathbf{r}_B'|$ is the distance along the particle trajectory between the wall points. Thus,

$$f^{(1)} = f_0^{(1)} - {}^1/_2\, d_\mathbf{v}(\mathbf{n}\nabla) f_0^{(2)}. \qquad (14)$$

Analogously, separating the part of $f^{(2)}$ that is odd in $\mathbf{n}$, we get

$$f^{(2)} = f_0^{(2)} - {}^1/_2\, d_\mathbf{v}(\mathbf{n}\nabla) f_0^{(1)}, \qquad (15)$$

where $f_0^{(1,2)}$ are the parts containing even powers of $\mathbf{n}$.

To continue the derivation we need an explicit expression for the regular functions $g^{R(A)}$ and $f^{R(A)}$ with allowance for the spectrum distortion by diffusion scattering from the channel walls. The Green's functions can be calculated by the method of trajectories, as was done, e.g., in Ref. 12. It is simpler, however, to solve Eqs. (3) for regular functions with corresponding boundary conditions on the walls.

The boundary conditions on diffusely reflecting walls can be obtained by a method proposed by Ovchinnikov[13] for an analogous problem in superconductors. To this end, we replace the diffusion of the reflecting wall by a thin scattering layer of impurities, such that the mean free path in this layer is much less than its length. The impurity density in the layer is assumed so high that $\tilde{\tau}T \ll 1$, where $\tilde{\tau}$ is the path time in the layer. Since scattering by impurities disrupts the triplet pairing, we have in the interior of the layer

$$f_\mathbf{p}^{R(A)},\ f_\mathbf{p}^{+R(A)} \to 0, \qquad g_\mathbf{p}^{R(A)} = -\bar{g}_\mathbf{p}^{R(A)} \to \pm 1.$$

The equations for $f_\mathbf{p}^{R(A)}, f_\mathbf{p}^{+R(A)}$ inside the layer (the y axis is directed along the inward normal to the surface)

$$-iv_{Fy}\frac{\partial}{\partial y} f_\mathbf{p}^{R(A)} - inv_F f_\mathbf{p}^{R(A)} \int \sigma_{\mathbf{pp}'} g_{\mathbf{p}'}^{R(A)}\, d\Omega_{\mathbf{p}'} + inv_F g_\mathbf{p}^{R(A)} \\ \times \int \sigma_{\mathbf{pp}'} f_{\mathbf{p}'}^{R(A)}\, d\Omega_{\mathbf{p}'} = 0, \qquad (16)$$

$$iv_{Fy}\frac{\partial}{\partial y} f_\mathbf{p}^{+R(A)} - inv_F f_\mathbf{p}^{+R(A)} \int \sigma_{\mathbf{pp}'} g_{\mathbf{p}'}^{R(A)}\, d\Omega_{\mathbf{p}'} + inv_F g_\mathbf{p}^{R(A)} \\ \times \int \sigma_{\mathbf{pp}'} f_{\mathbf{p}'}^{+R(A)}\, d\Omega_{\mathbf{p}'} = 0. \qquad (17)$$

Here n is the impurity density in the layer $\sigma_{\mathbf{pp}'}$ is the scattering cross section, and

$$\tilde{\tau}^{-1} = nv_F \int \sigma_{\mathbf{pp}'}\, d\Omega_{\mathbf{p}'}.$$

We consider first Eq. (16) for the retarded Green's function. It can be seen that at $v_{F_y} > 0$ the function $f_\mathbf{p}^R$ increases in the interior of the layer as $y \to -\infty$, and we must therefore stipulate $f_{p_y>0}^R = 0$ on the boundary. The function $f_\mathbf{p}^R$ at $v_y < 0$ decreases the interior of the layer, except for the combination

$$\int f_\mathbf{p}^R v_{Fy}\, d\Omega_\mathbf{p},$$

which remains constant. We must therefore require that this expression also vanish on the boundary. The boundary conditions for the functions $f_\mathbf{p}^{+R}$ and $f_\mathbf{p}^A, f_\mathbf{p}^{+A}$ can be obtained similarly. Thus, we must stipulate that on diffusely reflecting walls (cf. Ref. 13)

$$f_{\mathbf{mp}>0}^R = f_{\mathbf{mp}<0}^{+R} = f_{\mathbf{mp}<0}^A = f_{\mathbf{mp}>0}^{+A} = 0, \qquad (18)$$

$$\int_{\mathbf{mp}<0} \mathbf{mv}_F f_\mathbf{p}^R\, d\Omega_\mathbf{p} = \int_{\mathbf{mp}>0} \mathbf{mv}_F f_\mathbf{p}^{+R}\, d\Omega_\mathbf{p} \\ = \int_{\mathbf{mp}>0} \mathbf{mv}_F f_\mathbf{p}^A\, d\Omega_\mathbf{p} = \int_{\mathbf{mp}<0} \mathbf{mv}_F f_\mathbf{p}^{+A}\, d\Omega_\mathbf{p} = 0, \qquad (19)$$

where $\mathbf{m}$ is the inward normal to the wall.

The stationary regular functions satisfy equations obtained from (3):

$$\begin{aligned} -iv_F\nabla f_\mathbf{p}^{R(A)} - 2\varepsilon f_\mathbf{p}^{R(A)} + 2\Delta_\mathbf{p} g^{R(A)} &= 0, \\ iv_F\nabla f_\mathbf{p}^{+R(A)} - 2\varepsilon f_\mathbf{p}^{+R(A)} + 2\Delta_\mathbf{p}^* g^{R(A)} &= 0, \end{aligned} \qquad (20)$$

as well as the normalization conditions

$$g^{R(A)} = -\bar{g}^{R(A)}, \quad [g^{R(A)}]^2 - f^{+R(A)} f^{R(A)} = 1.$$

For a narrow channel we can omit from (20) the particle-particle collision integral.

We shall see below that the important role is played by frequencies ε much higher than $\Delta(T)$. For these frequencies, Eq. (20) with boundary conditions (18) and (19) can be easily solved and yields

$$
\begin{aligned}
&g^{R(A)}=\pm 1,\\
&f_{\mathbf{p}}^{R}=\frac{\Delta_{\mathbf{p}}}{\varepsilon}\left[1-\exp\left(\frac{2i\varepsilon}{v_F}|\mathbf{r}-\mathbf{r}_B|\right)\right],\\
&f_{\mathbf{p}}^{+R}=\frac{\Delta_{\mathbf{p}}^{*}}{\varepsilon}\left[1-\exp\left(\frac{2i\varepsilon}{v_F}|\mathbf{r}-\mathbf{r}_B'|\right)\right],\\
&f_{\mathbf{p}}^{A}=-\frac{\Delta_{\mathbf{p}}}{\varepsilon}\left[1-\exp\left(-\frac{2i\varepsilon}{v_F}|\mathbf{r}-\mathbf{r}_B'|\right)\right],\\
&f_{\mathbf{p}}^{+A}=-\frac{\Delta_{\mathbf{p}}^{*}}{\varepsilon}\left[1-\exp\left(-\frac{2i\varepsilon}{v_F}|\mathbf{r}-\mathbf{r}_B|\right)\right],
\end{aligned}
\tag{21}
$$

where the vectors $\mathbf{r}_B$ and $\mathbf{r}_B'$ on the boundary are defined in Fig. 1. In the derivation of expressions (21) we took it into account that at frequencies $\varepsilon \gg \Delta$ the functions f oscillate over distances v_F/ε much shorter than the length $\xi(T)$ on which the changes of the order parameter take place. Equations (21) are valid if, furthermore, the scale of the dispersion is much smaller than the transverse dimension of the channel: $v_F/\varepsilon \ll R$. In the derivation of (21) we used only the boundary condition (18). The boundary conditions (19) are imposed on the opposite channel wall with respect to the particle motion, where

$$f_{\mathbf{p}}^{R(A)}=\pm\Delta_{\mathbf{p}}/\varepsilon,\quad f_{\mathbf{p}}^{+R(A)}=\pm\Delta_{\mathbf{p}}^{*}/\varepsilon.$$

As a result, we obtain from (1) for the normal component of the order parameter

$$\Psi\mathbf{m}=0,$$

i.e., the known condition that $\mathbf{l}$ be perpendicular to the diffusely reflecting wall.[12]

We shall need later expressions averaged over the channel cross section, of the type

$$\langle\Delta_{\mathbf{p}}^{*}(f_{\mathbf{p}}^{R}+f_{\mathbf{p}}^{A})\rangle=\frac{1}{S}\int dS\,\Delta_{\mathbf{p}}^{*}(\mathbf{r})\,[f_{\mathbf{p}}^{R}(\mathbf{r})+f_{\mathbf{p}}^{A}(\mathbf{r})].$$

With the aid of (21) we get

$$\langle\Delta_{\mathbf{p}}^{*}(f_{\mathbf{p}}^{R}+f_{\mathbf{p}}^{A})\rangle=-i\Delta_B^2(T)/\varepsilon^2\tau_B(\mathbf{v}),\tag{22}$$

where

$$\Delta_B^2(T)\,\tau_B^{-1}(\mathbf{v})=\frac{1}{2S}\int_\sigma|\mathbf{m}\mathbf{v}_F|\;|\Delta_{\mathbf{p}}|_B^2\,d\sigma.\tag{23}$$

The integration with respect to $d\sigma$ is carried out here along a line on a surface bounding the channel cross section.

The quantities $|\Delta_{\mathbf{p}}|_B^2$ and $\Delta_B(T)$ in (22) and (23) are taken on the channel walls. In the case of purely diffuse reflection the order parameter is suppressed on the walls, according to Ref. 12, to a value $\Delta_B \sim [\xi_0/\xi(T)]\Delta_0$, where Δ_0 is the value of Δ in the volume. At temperaures close to critical we have therefore $\Delta_B(T) \sim T(1-T/T_c)$. For the scattering from the walls to be more effective than inelastic scattering we must require that the temperature satisfy the condition $\tau_B/\tau_\varepsilon \ll 1-T/T_c$. The upper bound on the proximity to T_c will be established below.

If it is assumed that the order-parameter structure corresponding to the Z-phase is preserved all the way to the wall, we have

$$|\Delta_{\mathbf{p}}|_B^2=\Delta_B^2(T)\sin^2(\widehat{\mathbf{m}\mathbf{v}_F}),$$

so that

$$\tau_B^{-1}(\mathbf{v})=\frac{1}{2S}\int_\sigma|\mathbf{m}\mathbf{v}_F|\sin^2(\widehat{\mathbf{m}\mathbf{v}_F})\,d\sigma.$$

The actual expression for the characteristic time between collisions depends on the shape of the channel cross section. For a round channel, for example,

$$\tau_B^{-1}(\mathbf{v})=2|\mathbf{v}_\perp|/3\pi R,$$

where $\mathbf{v}_\perp$ is the $\mathbf{v}_F$ component perpendicular to the channel axis.

We shall need also the mean value

$$\left\langle(f_{\mathbf{p}}^{+R}+f_{\mathbf{p}}^{+A})\frac{\partial\Delta_{\mathbf{p}}}{\partial t}-(f_{\mathbf{p}}^{R}+f_{\mathbf{p}}^{A})\frac{\partial\Delta_{\mathbf{p}}^{*}}{\partial t}\right\rangle.$$

In the case of the A-phase

$$\Delta_{\mathbf{p}}^{*}\frac{\partial\Delta_{\mathbf{p}}}{\partial t}-\Delta_{\mathbf{p}}\frac{\partial\Delta_{\mathbf{p}}^{*}}{\partial t}=2i|\Delta_{\mathbf{p}}|^2\frac{\partial\Phi}{\partial t}+2i\Delta^2(T)\,(\mathbf{l}\mathbf{n})\left(\mathbf{n}\left[\mathbf{l}\frac{\partial\mathbf{l}}{\partial t}\right]\right).$$

Since $\mathbf{l}$ is secured to the boundary and cannot vary with time, we get

$$\left\langle(f_{\mathbf{p}}^{+R}+f_{\mathbf{p}}^{+A})\frac{\partial\Delta_{\mathbf{p}}}{\partial t}-(f_{\mathbf{p}}^{R}+f_{\mathbf{p}}^{A})\frac{\partial\Delta_{\mathbf{p}}^{*}}{\partial t}\right\rangle=\frac{2\Delta_B^2(T)}{\varepsilon^2\tau_B(\mathbf{v})}\frac{\partial\Phi}{\partial t}.\tag{24}$$

We return now to Eq. (8) for $f^{(2)}$. Using (14) and averaging (8) over the channel cross section taking (22) and (24) into account, we have at $\varepsilon \gg \Delta$

$$\frac{\partial f^{(2)}}{\partial t}-\frac{v_F}{2}\langle d_{\mathbf{v}}\rangle n_z^2\frac{\partial^2 f^{(2)}}{\partial z^2}+\frac{\Delta_B^2}{\varepsilon^2\tau_B}f^{(2)}-\frac{1}{2}\frac{\partial\Phi}{\partial t}\frac{\Delta_B^2}{\varepsilon^2\tau_B}\frac{\partial f^{(0)}}{\partial\varepsilon}-J_2=0.\tag{25}$$

The particle-particle collision integral at $\Delta \ll T$ is (see Refs. 8 and 9)

$$J_2=\frac{\partial f^{(0)}}{\partial\varepsilon}\left[-\frac{\chi_{\varepsilon,\mathbf{p}}^{(2)}}{\tau_\varepsilon}+\frac{\lambda^2\nu^2\pi}{2p_Fv_F}\hat{L}\{(1+f_1^{(0)}f_2^{(0)}+f_1^{(0)}f_3^{(0)}+f_2^{(0)}f_3^{(0)})\chi_{\varepsilon_3,\mathbf{p}_3}^{(2)}\}\right].\tag{26}$$

Here

$$f^{(2)}=\frac{\partial f^{(0)}}{\partial\varepsilon}\chi^{(2)},$$

$$\tau_\varepsilon^{-1}=\frac{\lambda^2\nu^2\pi}{2p_Fv_F}\hat{L}\{1+f_1^{(0)}f_2^{(0)}+f_1^{(0)}f_3^{(0)}+f_2^{(0)}f_3^{(0)}\},$$

$$\hat{L}\{\dots\}=\int\frac{d\Omega_{\mathbf{p}_1}d\Omega_{\mathbf{p}_2}}{(4\pi)^2}\left\{\delta\left(\frac{|\mathbf{p}_3|}{p_F}-1\right)\int d\varepsilon_1\,d\varepsilon_2[\dots]\right\}.$$

The expressions in the curly brackets of the operator $\hat{L}$ are taken at $\varepsilon_3=\varepsilon-\varepsilon_1-\varepsilon_2$, $\mathbf{p}_3=\mathbf{p}-\mathbf{p}_1-\mathbf{p}_2$.

Equation (25) for $f^{(2)}$ can be easily solved[14] if the particle mean free path for collisions with one another is $l=v_F\tau_\varepsilon$, while the transverse dimension R of the channel and the temperature satisfy the relation

$$\Delta_B^2/T^2 \ll \tau_B/\tau_\varepsilon \sim R/l,\tag{27}$$

i.e., in a rather narrow region near the critical temperature

$(T_c - T)^2 \ll R/l$. We shall see below that the characteristic scale of variation of $f^{(2)}$ is much larger than the "diffusion" length $l_D = (Rl)^{1/2}$. Therefore, assuming the $f^{(2)}$ varies more slowly in time than τ_ε^{-1}, the principal terms in (25) will be the third, fourth and fifth. At $\varepsilon \sim T$, the principal term is the one with the collision integral, so that at $\varepsilon \sim T$ we have

$$\chi^{(2)} = -\mu^* + \frac{1}{2}\frac{\partial\Phi}{\partial t},$$

where μ^* is independent of ε or $\mathbf{p}$. At $\varepsilon \lesssim T$ we have

$$\chi^{(2)} = \frac{1}{2}\frac{\partial\Phi}{\partial t} - \frac{\mu^*\tau_B\varepsilon^2}{\tau_\varepsilon\Delta_B{}^2+\tau_B\varepsilon^2}. \tag{28}$$

The constant μ^* can be expressed in terms of the chemical potential, by expressing the change of the mass density of the liquid in the form

$$\delta\rho = -\nu(0)m\int\frac{d\Omega_\mathbf{p}}{4\pi}\int\frac{d\varepsilon}{2}(g^R-g^A)f^{(2)}$$

$$= 2\nu(0)m\left(\mu^* - \frac{1}{2}\frac{\partial\Phi}{\partial t}\right). \tag{29}$$

Putting $\delta\rho = 2\nu(0)m\delta\mu$, we obtain

$$\mu^* = \delta\mu + \frac{1}{2}\frac{\partial\Phi}{\partial t}. \tag{30}$$

The quantity $\mu_s = -(1/2)\partial\Phi/\partial t$ has the meaning of the chemical potential of Cooper pairs, so that μ^* can be regarded as the difference between the chemical potential of the liquid, reckoned from E_F, on the one hand and the chemical potential of the condensate on the other.

Substituting now (28) in (25) and integrating with respect to $d\varepsilon$ and $d\Omega_\mathbf{p}$, we get

$$\frac{\partial\mu}{\partial t} - D\frac{\partial^2\mu}{\partial z^2} + \mu^*\int\frac{d\Omega_\mathbf{p}}{4\pi}\int_0^\infty\frac{d\varepsilon}{2T}\frac{\Delta_B{}^2}{\tau_\varepsilon\Delta_B{}^2+\tau_B\varepsilon^2} = 0, \tag{31}$$

where

$$D = \frac{v_F}{2}\int\frac{d\Omega_\mathbf{p}}{4\pi}\langle d_\mathbf{v}\rangle n_z{}^2.$$

Putting $\varepsilon = x\Delta_B(\tau_\varepsilon/\tau_B)^{1/2}$, in (31), we get

$$\frac{\partial\mu}{\partial t} - D\frac{\partial^2\mu}{\partial z^2} + \frac{\pi\Delta_B(T)\alpha}{4\pi(\tau_\varepsilon\bar\tau_B)^{1/2}}\mu^* = 0. \tag{32}$$

Here

$$\frac{\alpha}{\bar\tau_B{}^{1/2}} = \int\frac{d\Omega_\mathbf{p}}{4\pi}\tau_B{}^{-1/2}(\mathbf{v}).$$

The characteristic time between the collisions with the walls is $\bar\tau_B \sim R/v_F$. The quantities D and $\alpha/\tau_B^{1/2}$ depend on the shape of the channel. For a channel of circular cross section, for example,

$$D = 2Rv_F/3, \quad \bar\tau_B = 2R/v_F, \quad \alpha = \sqrt{2}\Gamma(1/4)/6\pi\sqrt{3}.$$

Using (14) and (38) we easily obtain the normal mass flux. The normal-flux density averaged over the cross section is

$$j_n = -2m\nu(0)D\partial\mu/\partial z. \tag{33}$$

An expression for the superfluid flux $\mathbf{j}_s$ is well known (see, e.g., the review[15]) and will not be written out here. The total flux is equal to the sum of the normal and superfluid fluxes:

$$\mathbf{j} = -2m\nu(0)D\nabla\mu + \mathbf{j}_s. \tag{34}$$

Expression (33) is the Knudsen formula for the flux density of a normal Fermi gas.[11] Equation (32) is similar to the corresponding equation of superconductivity theory,[14] except that the first term does not occur for superconductors in view of the electroneutrality, i.e., the incompressibility of the electron liquid: $\partial\rho/\partial t = 0$.

Equation (32) determines the characteristic length

$$l_\mu = [D(\tau_\varepsilon\bar\tau_B)^{1/2}\cdot 4T/\pi\alpha\Delta_B]^{1/2},$$

over which we have in superfluid He-3 relaxation of the constant component of the deviation $\delta\mu$ of the chemical potential from the chemical potential μ_s of the condensate. In order of magnitude, $l_\mu \sim l_D(R/l)^{1/4}(T/\Delta_B)^{1/2}$ and exceeds the diffusion length l_D by virtue of condition (27). This phenomenon is similar to the charge disequilibrium in superconductors, where the corresponding relaxation length is the depth of penetration of the electric field.[14]

From the continuilty equation

$$\partial\rho/\partial t + \operatorname{div}\mathbf{j} = 0, \tag{35}$$

from (34), and from (32) we easily obtain, taking (29) and (39) into account,

$$\delta\mu + \frac{1}{2}\frac{\partial\Phi}{\partial t} = \zeta_3\operatorname{div}\mathbf{j}_s. \tag{36}$$

Here

$$\zeta_3 = 2T(\tau_\varepsilon\bar\tau_B)^{1/2}/m\nu(0)\pi\alpha\Delta_B(T) \tag{37}$$

is the second-viscosity coefficient. It can be seen from (37) that the second viscosity in a narrow channel differs from its unbounded-channel value calculated earlier in Ref. 16.

We discuss now the conditions for the applicability of our results. First is satisfaction of the inequality

$$l \gg R \gg \xi_0. \tag{38}$$

Moreover, we used in the derivation of (21) the fact that at the characteristic frequencies that make the main contribution to (31), i.e., at $\varepsilon \sim \Delta_B(\tau_\varepsilon/\tau_B)^{1/2}$, the inequality $v_F/\varepsilon \ll \xi(T)$ is valid. Taking (27) into account, we find that the temperature should be in the range

$$\frac{R}{l} \ll 1 - \frac{T}{T_c} \ll \left(\frac{R}{l}\right)^{1/2}. \tag{39}$$

The condition (38) on the channel dimensions is easily met in experiment. Satisfaction of condition (39) on the temperature is a somewhat more complicated matter, but still does not raise any difficulties in principle. Experimental verification of the results is therefore perfectly feasible.

Equations (32)–(37) describe the distribution of the fluxes and of the chemical potential (pressure) in the channel. The equation for the order parameter can, generally speaking, not be obtained in closed form. The distribution function $f^{(1)}$ defined by the kinetic equation (7) depends substantially on the form of the spectrum in the energy region $\varepsilon \sim \Delta$. If we confine ourselves to the case when the channel size is much larger than $\xi(T)$, we can obtain from (3) for regular functions in the first-order approximation (without allowance for the smearing of the spectrum)

$$g^{R(A)}=\pm\varepsilon_{\mathbf p}/\xi_{\mathbf p},\quad \Delta_{\mathbf p}^{*}f_{\mathbf p}^{R(A)}=\pm|\Delta_{\mathbf p}|^{2}/\xi_{\mathbf p}, \tag{40}$$

where

$$\xi_{\mathbf p}=(\varepsilon_{\mathbf p}^{2}-|\Delta_{\mathbf p}|^{2})^{1/2},\qquad \varepsilon_{\mathbf p}=\varepsilon-{}^{1}/_{2}\mathbf v_F\nabla\Phi_{\mathbf p},$$

$$\nabla\Phi_{\mathbf p}=(\Delta_{\mathbf p}^{*}\nabla\Delta_{\mathbf p}-\Delta_{\mathbf p}\nabla\Delta_{\mathbf p}^{*})/2i|\Delta_{\mathbf p}|^{2}.$$

We put $f^{(1)}=f^{(0)}+(\partial f^{(0)}/\partial\varepsilon)\chi^{(1)}$. The collision integral J_1 in (7) can be reduced at $\Delta\ll T$ to the τ-approximation form[8]

$$J_1=-\frac{1}{\tau_\varepsilon}\frac{\varepsilon_{\mathbf p}}{\xi_{\mathbf p}}\frac{\partial f^{(0)}}{\partial\varepsilon}\chi^{(1)}.$$

As a result we get from (7)

$$-v_F\mathbf n\nabla\left(\frac{\varepsilon_p}{\xi_p}\chi^{(2)}\right)+\frac{v_Fd_v}{2}n_in_k\nabla_i\nabla_k\left(\frac{\varepsilon_{\mathbf p}}{\xi_{\mathbf p}}\chi^{(1)}\right)$$
$$-\frac{1}{2\xi_{\mathbf p}}\left(\Delta_{\mathbf p}^{*}\frac{\partial\Delta_{\mathbf p}}{\partial t}+\Delta_{\mathbf p}\frac{\partial\Delta_{\mathbf p}^{*}}{\partial t}\right)=\frac{1}{\tau_\varepsilon}\frac{\varepsilon_{\mathbf p}}{\xi_{\mathbf p}}\chi^{(1)}. \tag{41}$$

In the case of the A-phase we have

$$\Delta_{\mathbf p}\frac{\partial\Delta_{\mathbf p}^{*}}{\partial t}+\Delta_{\mathbf p}^{*}\frac{\partial\Delta_{\mathbf p}}{\partial t}=-2\Delta^{2}(T)\,(\mathbf{ln})\left(\mathbf n\frac{\partial\mathbf l}{\partial t}\right).$$

With the aid of (40) and (28) we can write down an equation for the order parameter in the A phase:

$$\frac{\nu(0)}{3}\Delta_0(T)\left\{\frac{T_c-T}{T}\Psi_i+\frac{7\zeta(3)v_F^{2}}{80\pi^{2}T^{2}}(\delta_{ik}\nabla^{2}+2\nabla_i\nabla_k)\Psi_k\right.$$
$$\left.-\frac{7\zeta(3)\Delta_0^{2}(T)}{40\pi^{2}T^{2}}[2(\Psi\Psi^{*})\Psi_i+(\Psi\Psi)\Psi^{*}]\right\}$$
$$=-\nu(0)\int\frac{d\Omega_{\mathbf p}}{4\pi}n_i\int\limits_{|\varepsilon_{\mathbf p}|>|\Delta_{\mathbf p}|}\frac{d\varepsilon}{2}\frac{\Delta_{\mathbf p}}{\xi_{\mathbf p}}\frac{\partial f^{(0)}}{\partial\varepsilon}\chi^{(1)}$$
$$+\frac{\nu(0)\pi\alpha i\Delta_B}{16T(\tau_\varepsilon\bar\tau_B)^{1/2}\Delta_0}\Psi_i\mu^{*}. \tag{42}$$

Averaging is carried out here over the channel cross section. The left-hand side of Eq. (42) is the usual Ginzburg-Landau equation,[12] while the right-hand side describes the dissipation due to motion of the anistropy vector and the deviation of the chemical potential. Equation (42) leads, in particular to Eq. (36).

The first term in the right-hand side of (42) is connected with the motion of the entropy vector **l**. If the range of variation of **l** exceeds the path length $l=v_F\tau_\varepsilon$, we obtain from (41) for the component that is even in **n**

$$\chi^{(1)}=\frac{\tau_\varepsilon}{2\varepsilon_p}\left(\Delta_{\mathbf p}\frac{\partial\Delta_{\mathbf p}^{*}}{\partial t}+\Delta_{\mathbf p}^{*}\frac{\partial\Delta_{\mathbf p}}{\partial t}\right),$$

$$\nu(0)\int\frac{d\Omega_{\mathbf p}}{4\pi}n_i\int\limits_{|\varepsilon_{\mathbf p}|>|\Delta_{\mathbf p}|}\frac{d\varepsilon}{2}\frac{\Delta_{\mathbf p}}{\xi_{\mathbf p}}\frac{\partial f^{(0)}}{\partial\varepsilon}\chi^{(1)}$$

$$=\frac{\eta}{2\Delta_0(T)}l_i\left(\Psi\frac{\partial\mathbf l}{\partial t}\right),$$

where $\eta=\nu(0)\tau_\varepsilon\Delta^{3}(T)\pi^{2}/64T$ is the Cross viscosity coefficient.[17] If, however, the changes of **l** are concentrated in a region of size smaller than l, the principal role is assumed by the diffusion term in Eq. (41), so that no closed equation can be obtained for $\chi^{(1)}$. It can be stated, however that the expression for the cross viscosity will differ from the Cross formula. In particular, the viscosity coefficient contains in place of the free-path time the diffusion time $\tau_D\sim a^{2}D$, where a is the size of the region in which **l** varies.

The equations (32) for the chemical potential and the expressions for the current (34), as well as Eqs. (36) and (37), are not restricted to the A-phase. To apply these results to the B-phase it is necessary, first, to take into account the corresponding equation for the superfluid flow in the B phase (see, e.g., Ref. 15) and second, modify the expression for the characteristic time τ_B between collisions with the walls. In the case of the B-phase the order parameter is transformed near the walls because its normal component is suppressed. The exact numerical value of the coefficient α in Eqs. (32) and (37) should therefore be found by solving the equation for the order parameter in the specified form of the channel. In order of magnitude, we can put $\alpha\sim 1$. The quantity $\partial\Phi/\partial t$ in the B-phase is simply the derivative of the order-parameter phase.

Equations (32)–(37) can be used to describe fourth sound in narrow channel. To this end they must be supplemented by an expression for the acceleration of the superfluid component of the liquid. For the A-phase we have

$$\frac{\partial v_{si}}{\partial t}=\frac{1}{2m}\nabla_i\frac{\partial\Phi}{\partial t}+\frac{1}{2m}\mathbf l\left[\frac{\partial\mathbf l}{\partial t}\times\nabla_i\mathbf l\right]$$
$$=-\frac{1}{m}\nabla_i\mu+\frac{1}{m}\nabla_i\mu^{*}+\frac{1}{2m}\mathbf l\left[\frac{\partial\mathbf l}{\partial t}\times\nabla_i\mathbf l\right]. \tag{43}$$

In the case of the B phase the last terms in the right-hand sides of (43) drop out.

If the vector **l** is fixed and cannot rotate, the equations can be written in a form that is the same for the A- and B-phases:

$$m\frac{\partial v_s}{\partial t}=-\frac{\partial}{\partial z}(\mu-\mu^{*}),\quad j_s=\rho_sv_s,$$

$$\zeta_3\frac{\partial j_s}{\partial z}=\mu^{*},\qquad \frac{\partial\mu}{\partial t}-D\frac{\partial^{2}\mu}{\partial z^{2}}+\frac{\mu^{*}}{2m\nu(0)\zeta_3}=0.$$

This yields the dispersion equation

$$\omega^{2}-[s_4^{(0)}]^{2}k^{2}-\frac{\rho_s\zeta_3}{m}Dk^{4}+i\omega\left(D+\frac{\rho_s\zeta_3}{m}\right)k^{2}=0,$$

where $s_4^{(0)}=[\rho_s/2m^{2}\nu(0)]^{1/2}$ is the fourth-sound velocity without allowance for the Fermi-liquid corrections. The fourth-sound velocity in the channel is

$$s_4=[(s_4^{(0)})^{2}-{}^{1}/_{4}(D-\rho_s\zeta_3/m)^{2}k^{2}]^{1/2}. \tag{44}$$

The sound damping is

$$\operatorname{Im}\omega=-{}^{1}/_{2}(D+\rho_s/m)k^{2}. \tag{45}$$

Estimates show that in the region (39) where results are valid we always have $\rho_s\zeta_3/m\ll D$. Therefore, notwithstanding the appreciable change of the second-viscosity coefficient by diffuse scattering from the walls, the statement made in Ref. 18, that the contribution of ξ_3 to the fourth-sound damping in a narrow channel is small, remains in force.

3. DISSIPATION IN FLOW OF He-3 THROUGH A CHANNEL

It was mentioned in the introduction that experiment[3] has demonstrated that at a flux density exceeding the critical

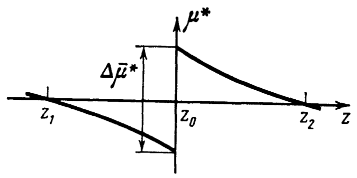

FIG. 2.

pair-breaking current density the flow is accompanied by a pressure difference and the superfluidity is not completely destroyed. Under the experimental conditions of Ref. 3, where the radius of the channel is comparable with the coherence length $\xi(T)$, vortex formation is difficult and the dissipation may be caused by the phase slippage process, which is analogous to the resistive state in superconductors.

Consider first the case of the A-phase. Since the superfluidity in the volume is not disrupted, $\mathbf{v}_s$ should be bounded. The time average of $\partial\mathbf{v}_s/\partial t$ is therefore zero. It can be seen from (43) that two mechanisms can compensate for the acceleration due to the chemical-potential gradient:

$$\nabla_i\bar{\mu}=\nabla_i\bar{\mu}^*+\frac{\hbar}{2t_0}\int_0^{t_0}\mathbf{l}\left[\frac{\partial\mathbf{l}}{\partial t}\nabla_i\mathbf{l}\right]dt \tag{46}$$

(here t_0 is the phase-slippage period). The second term in the right-hand side of (46) is specific to the A-phase; we shall discuss its role somewhat later. At high flux densities in narrow pores, the orientation of the anisotropy vector $\mathbf{l}$ is determined by the walls and by the superfluid flow of the liquid. Therefore the motion of $\mathbf{l}$ in the main volume of the channel is frozen, so that the last term of (46) is practically always zero. We have thus from (46)

$$\nabla\bar{\mu}=\nabla\bar{\mu}^* \tag{47}$$

everywhere in the channel except for a few points called the phase-slippage centers (PSC). In the vicinity of a PSC Eq. (47) no longer holds and $\bar{\mu}^*$ has a discontinuity. In a region outside the PSC, Eqs. (32) and (34) can be rewritten in the form

$$\partial^2\bar{\mu}^*/\partial z^2=l_\mu^{-2}\bar{\mu}^*, \tag{48}$$

$$j=-\sigma\partial\bar{\mu}^*/\partial z+j_s, \tag{49}$$

where $\sigma = 2mv(0)D$ is the "conductivity" of the pore.

From (48) and (49) we can determine the discontinuity $\Delta\bar{\mu}^*$ on one PSC. In the model where l_μ is assumed constant, we have (see, e.g., Refs. 7 and 19)

$$\Delta\bar{\mu}^*=\frac{l_\mu}{\sigma}\left(\operatorname{th}\frac{z_0-z_1}{l_\mu}+\operatorname{th}\frac{z_2-z_0}{l_\mu}\right)[j-j_s(z_0)].$$

Here z_1 and z_2 are the points where $\mu^* = 0$ (see Fig. 2), and z_0 the location of the PSC. If the distance between neighboring PSC is large, the differential "resistance" of one PSC is

$$\frac{\partial(\Delta p)}{\partial j}=\frac{\rho}{m}\frac{\partial(\Delta\mu)}{\partial j}=\frac{2\rho l_\mu}{m\sigma}. \tag{50}$$

The physical mechanisms of phase slippage can vary. At fixed $\mathbf{l}$ the phase slippage may be due to vanishing of the order parameter in the PSC region over the entire cross section of the channel, or due to passage of a singular vortex. The chemical-potential discontinuity $\Delta\bar{\mu}^*$ is then connected with the phase-slippage frequency:

$$\Delta\bar{\mu}^*=\frac{\hbar}{2t_0}\iint\frac{\partial}{\partial z}\frac{\partial\Phi}{\partial t}dt\,dz=\frac{N\hbar}{2}\omega_J, \tag{51}$$

where $\omega_J = 2\pi/t_0$ is the Josephson frequency. We have used here the fact that at fixed $\mathbf{l}$ the quantity Φ is the phase of the order parameter and changes by $2\pi N$ in each slippage process. The phase slippage can be produced also at nonzero order parameter through rotation of the vector $\mathbf{l}$ in the PSC region (passage of a nonsingular vortex). It must be noted, however, that the size of the nonsingular vortex must be larger than $\xi(T)$, otherwise the order parameter becomes suppressed and this vortex will be indistinguishable from a singular one. The chemical-potential discontinuity $\Delta\bar{\mu}^*$ on the PSC is in this case, as seen from (43), equal to

$$\Delta\bar{\mu}^*=-\frac{\hbar}{2t_0}\iint\mathbf{l}\left[\frac{\partial\mathbf{l}}{\partial t}\times\frac{\partial\mathbf{l}}{\partial z}\right]dt\,dz. \tag{52}$$

Such a phase slippage mechanism was proposed by Volovik.[20] The integral in the right-hand side of (52) is a topological invariant equal to $4\pi N$, where the integer N is the degree of mapping of the region (z,t) of variation of $\mathbf{l}$ on a sphere, determined by the function $\mathbf{l}(z,t)$. Therefore[20]

$$\Delta\bar{\mu}^*=N\hbar\omega_J. \tag{53}$$

This relation differs by a factor 2 from the usual Josephson relation (51). By measuring the PSC oscillation frequency it is possible in principle to distinguish between the singular and nonsingular phase slippage mechanisms.

A characteristic feature of the experimental results[3] for the A phase was observation of two dissipation regimes, as manifested by the existence of two different slopes of the lot of Δp against f. Whereas the larger slope of the $\Delta(j)$ characteristic is due to the behavior of the He-3 inside the pore and should be attributed to the phase-slippage processes discussed above, the presence of a less steep section of $\Delta p(j)$, as seen from the experiment, is due to the influence of the regions near the end poins of the pore. It was suggested in Ref. 3 that this dissipation mechanism is connected with rotation of the vector $\mathbf{l}$ near the ends of the pore, where the pinning of $\mathbf{l}$ is weaker. The order of magnitude of the dissipation due to rotation of $\mathbf{l}$ can be estimated at $V\eta(\partial\mathbf{l}/\partial t)^2$ where V is the volume in which $\mathbf{l}$ rotates and η is the viscosity coefficient. The rotation frequency, according to (53), is $\omega_J \sim \Delta\mu/\hbar$, so that the dissipation is

$$V\eta(\partial\mathbf{l}/\partial t)^2\sim V_\eta(\Delta\mu/\hbar)^2=V\eta(m\Delta p/\hbar\rho)^2.$$

Equating it to the work of the external force $Sj\Delta p/m\rho$ that maintains the flow, where S is the channel cross section, we obtain

$$j\sim(m^3\eta R/\rho\hbar^2)\Delta p. \tag{54}$$

It is assumed here that the sizes of the channel and of the region where rotation of $\mathbf{l}$ takes place are of the order of R. Since the region where $\mathbf{l}$ varies is much smaller than the mean free path l, the quantity in (54) is not the Cross viscosity (contrary to the assumption in Ref. 3); it must be obtained by solving Eq. (41) without the collision term in the right-hand part. Thus, the order of magnitude of the viscosity coefficient is

$$\eta \sim \nu(0)\Delta^3(T)R/Tv_F.$$

The slope $\partial(\Delta p)/\partial j$ has in this case a temperature dependence $(T_c - T)^{-3/2}$, which agrees with the experimental results of Ref. 3.

In the case of the *B*-phase, only one dissipation regime was observed in Ref. 3. Its behavior is similar to the dissipation regime with a large slope $\partial(\Delta p)/\partial j$ for the *A*-phase. The time derivative of the superfluid velocity in the *B*-phase is given by (43) without the last term in the right-hand side. Therefore if the superfluidity is preserved, $\overline{\partial \mathbf{v}_s/\partial t} = 0$, only one dissipation mechanism is possible and is connected with the unbalance of the chemical potentials of the liquid and of the Cooper pairs, $\bar{\mu}^* \neq 0$, described above for the *A* phase. For the slope of the characteristic of an individual PSC we obtain in this case Eq. (50). The mechanism of phase slippage in the PSC can be the usual one, i.e., due to vanishing of the order parameter or to passage of a singular vortex, or via formation of boojums on the Fermi surface in accord with the predictions of Ref. 21.

The temperature dependences (50) obtained for the slopes of the characteristic in the *B* phase and for the steeper section in the *A*-phase, $\partial(\Delta p)/\partial j \propto (T_c - T)^{-1/2}$, do not agree, however with the conclusions of Ref. 3 concerning the experimental dependence of the slope. Let us examine this situation in greater detail. The distribution of the chemical potential and the difference of the pressure on the ends of the channel depend strongly on the relation between the length of the channel and the quantity l_μ. If we use the data of Ref. 3, we see that l_μ ranges under the conditions of Ref. 3 from 1 to 10 μm, depending on the pressure, i.e., it is comparable with the approximate pore length 10 μm. If l_μ is large, only a small number of PSC can be accommodated by the length of the pore, so that the observed slope $\partial(\Delta p)/\partial j$ should actually be of the order of Eq. (50). The situation becomes more complicated, however, by the fact that on the basis of the experimental results of Ref. 3 it is impossible to separate the "resistance" introduced by PSC, primarily because the filter through which the He-3 was passed contained a very large number of channels. Thus, the experimental curves constitute an average over a large number of channels containing different numbers of PSC, so that the sections due to individual PSC are averaged out. If, however, a large number of PSC can be located along the channel, it follows from the theory of the resistive state[7] that at small Δp the characteristic will not have any linear section at all. Thus, the available data are insufficient for a detailed comparison of the theory with experiment. It would be desirable, first, to increase the range of pressure differences and fluxes so as to measure as large a part of the characteristic as possible. Second, measurements should be possible on a single channel, where the discontinuity of the pressure difference with increasing pressure would be observable. Dissipative flow in a channel should be accompanied also by generation of sound at Josephson frequency, in full analogy with the nonstationary Josephson effect that will be discussed presently. These experiments might provide an exhaustively answer to the question of the role of PSC and of the observable dissipation.

The fact that discontinuities of the chemical potential and of the pressure are produced on each PSC and are due to Josephson-frequency oscillations of the order parameter and of the superfluid component, leads to a close analogy between PSC and weak connection (narrow opening) between two reservoirs with liquid He-3. Obviously, such a system should be subject to the nonstationary Josephson effect, i.e., should generate oscillations (sound in this case) in the presence of a constant pressure difference.

Let the channel contain a weak link (narrow opening) characterized by the equation

$$j_s = j_0 \sin(\Delta\Phi), \tag{55}$$

where $\Delta\Phi = \Phi_+ - \Phi_-$ is the difference between the phases on the two sides of the narrow opening. By virtue of the condition that the superfluidity in the channel is preserved, $\overline{\partial \mathbf{v}_s/\partial t} = 0$, we have $\overline{\partial^2\Phi/\partial z\partial t} = 0$, and therefore

$$\Phi_+ = -\Delta\mu t + \bar{\Phi} + \Phi_\sim,$$

where Φ is independent of time and $\Phi_\sim$ is an oscillating increment. Thus, the superfluid flow through the narrow opening (55) will have a component that oscillates at a frequency $\omega_J = 2\Delta\mu$. where $\Delta\mu$ is the difference between the chemical potentials on the ends of a channel containing one narrow opening (one PSC). In the case of small damping of the fourth sound, the alternating part of the chemical potential is $\mu_\sim = i\omega\Phi_\sim/2$, so that the pressure difference between the ends of the channel will oscillate with an amplitude

$$\Delta p_\sim = \frac{\rho\mu_\sim}{m} = \frac{\rho}{\rho_s} j_0 s_4.$$

In conclusion, the author thanks G. E. Volovik, V. P. Mineev, Yu. N. Ovchinnikov, and J. Pekola for helpful discussions.

[1)] In contrast to Ref. 8, we have retained in the left-hand side of (8) the third and fourth terms, which will be needed later.

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