Orbital angular momentum in superfluid ^{3}He-B in a magnetic field

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We obtain an expression for the superfluid current density in ^{3}He-B in a magnetic field, taking into account terms stemming from the internal angular momentum of the fluid. By direct calculation of the contribution, linear in the magnetic field and in the angular velocity of the container, to the energy of the fluid we find the magnitude of the internal angular momentum of ^{3}He-B in a magnetic field at arbitrary temperatures; it turns out to be small to the extent that the particle and hole distributions are asymmetric near the Fermi surface. We also discuss the role played in the calculation of the angular momentum in ^{3}He-B by non-unitary corrections to the order parameter in the B phase, which are linear in the angular velocity of the container and in the magnetic field.

1. INTRODUCTION

It is well known[1] that the superfluid A-phase of ^{3}He in which there is a Cooper pairing of particles in a quantum state with an angular momentum component $m = 1$ along a preferred direction **l** has an orbital angular momentum. In a vessel with ^{3}He-A there flows an undamped superfluid current which produces an orbital angular momentum

$$\vec{\mathscr{L}} \sim (\hbar/m_3)\rho_s V\mathbf{l};$$

here V is the volume occupied by the fluid, and ρ_s the density of the superfluid component. At the same time due to the strong overlap of the Cooper pairs the internal orbital angular momentum per unit volume of the fluid, due to the rotation of the particles in the Cooper pairs, turns out to be a negligibly small quantity:

$$\mathbf{L}^{int} \sim (T_c/\varepsilon_F)^2 \ln(\varepsilon_F/T_c)\,(\vec{\mathscr{L}}/V)$$

(see Ref. 2 and the references in that paper).

In connection with the discovery[3] of a gyromagnetic effect in rotating ^{3}He-B the problem of the angular momentum has recently become urgent also for the B-phase of ^{3}He in a magnetic field. The order parameter of the B-phase (see Ref. 4)

$$A_{\alpha i} = \Delta_0 R_{\alpha i} e^{i\Phi} \tag{1.1}$$

is given by the three-dimensional matrix $R_{\alpha i}$ of rotation of the spin space (index α) relative to the orbital space (index i), Δ_0 is the modulus of the order parameter, and $e^{i\Phi}$ is a phase factor. The wave function (1.1) corresponds to a state with total angular momentum $J = 0$. Here J is an eigenvalue of the operator

$$\hat{J}_i = \hat{L}_i + R_{\alpha i}\hat{S}_\alpha, \tag{1.2}$$

and $\hat{L}_i$ and $\hat{S}_\alpha$ are the orbital and spin angular momentum operators:

$$\hat{L}_i A_{\mu j} = -i\hbar e_{ijk} A_{\mu k}, \tag{1.3}$$

$$\hat{S}_\alpha A_{\mu i} = -i\hbar e_{\alpha\mu\nu} A_{\nu i}. \tag{1.4}$$

The average values $\langle\hat{\mathbf{L}}\rangle$ and $\langle\hat{\mathbf{S}}\rangle$ vanish in the state (1.1), i.e., there are no orbital and spin angular momenta in the B-phase. However, when there is a magnetic field present the Cooper pairs in the B-phase acquire an average magnetization, i.e., $\langle\mathbf{S}\rangle \neq 0$, and thanks to the rigidity of the state with $J = 0$ there must also appear an average orbital angular momentum

$$\langle\hat{L}_i\rangle = -R_{\alpha i}\langle\hat{S}_\alpha\rangle. \tag{1.5}$$

The magnitude of the orbital angular momentum of a container with ^{3}He-B in a magnetic field turns out to equal (see Ref. 5)

$$\mathscr{L}_i \sim -(\chi_n/\gamma) H_\alpha R_{\alpha i} V. \tag{1.6}$$

Here χ_n is the magnetic susceptibility of the normal Fermi-liquid, γ the gyromagnetic ratio, and V the volume of the container. This orbital angular momentum is produced by a boundary current arising in ^{3}He-B when there is a magnetic field present:

$$\vec{\mathscr{L}} = \int [\mathbf{r}\mathbf{j}]\,dV.$$

As in the A-phase, inside the volume occupied by ^{3}He-B in a magnetic field there occurs almost complete compensation of the orbital motion of the pairs, and the internal orbital angular momentum per unit volume turns out to be of the order of

$$\mathbf{L}^{int} \sim (T_c/\varepsilon_F)^2 \ln(\varepsilon_F/T_c)\,(\vec{\mathscr{L}}/V). \tag{1.7}$$

The results (1.6) and (1.7) obtained in Ref. 5 (see also Refs. 6 and 7) contradict the results of Ref. 8 where a generalized gauge transformation was used to find that the internal orbital angular momentum is of order

$$\mathbf{L}^{int} \sim (\rho_s/\rho)\,(\vec{\mathscr{L}}/V).$$

One can find a discussion of this contradiction in Ref. 5.

Equations (1.6), (1.7) were derived in Ref. 5 in the Ginzburg-Landau region as $T \to T_c$ (although they are independent of the temperature!). In that paper it was shown that the orbital angular momentum in the B-phase is due to the non-unitary corrections to the B-phase order parameter in a magnetic field. In the same paper it was noted (for more details see Ref. 6) that there is a local angular momentum in the B-phase itself in a magnetic field (i.e., neglecting the non-unitary corrections to the order parameter), due to the appearance of a correction, linear in the field H, to the gradient energy. The magnitude of that angular momentum is in the region $T \to T_c$ a fraction $1 - T/T_c$ of $\mathbf{L}^{int}$ given in (1.7).

We show in the present paper that the results (1.6),

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obtained in Ref. 5 in the region as $T \to T_c$, are valid at any temperature. Thus, we summarize the discussion raised by the appearance of the papers by Dombre and Combescot[9] and by Yip[10], in the latter of which it was stated that the internal angular momentum density in ^{3}He-B at $T=0$ is $(-2\chi_n/3\gamma)H_\alpha R_{\alpha i}$. We show that this statement is incorrect notwithstanding the fact that the expression for the current density obtained in Refs. 9 and 10, on which it is based, is correct. Through a direct evaluation of the internal angular momentum as the response of the energy of the fluid to the angular rotational velocity of the container $-\delta\mathcal{F}/\delta\mathbf{\Omega}$ we show in the present paper that at any temperature, neglecting the non-unitary corrections to the order parameter, we have

$$L_i^{int} = -\alpha(2\rho_s\chi_n/3\rho\gamma)H_\alpha R_{\alpha i},$$
$$\alpha \sim (T_c/\varepsilon_F)^2 \ln(\varepsilon_F/T_c),$$

where α is a coefficient which is non-zero only due to a small asymmetry in the particle-hole distribution near the Fermi surface. The discussion of the results of Refs. 9 and 10 and a correct determination of the magnitude of the internal angular momentum in the B-phase, neglecting non-unitary corrections to the order parameter at arbitrary temperatures, comprise the topic of the second section of the present paper. The corresponding very cumbersome calculations are given in Appendices A, B, and C.

The third section is devoted to the evaluation of the orbital angular momentum of ^{3}He-B in a magnetic field, taking into account non-unitary corrections to the order parameter. In it we generalize the corresponding results of Ref. 5 to the case of non-unitary order-parameter corrections arising in a container rotating with an angular velocity $\mathbf{\Omega}$.

2. CURRENT, GRADIENT ENERGY, AND ANGULAR MOMENTUM

The superfluid current density in the B-phase in the approximation which is linear in the magnetic field as $T \to T_c$ has the form (Ref. 6)[1)] (here and henceforth $\hbar = m_3 = 1$)

$$j_k = \frac{1}{5} e_{\lambda\mu\nu} \frac{\rho_s}{\rho} \frac{\chi_n H_\nu}{\gamma} (R_{\mu i}\nabla_k R_{\lambda i} + R_{\mu k}\nabla_i R_{\lambda i} + R_{\mu i}\nabla_i R_{\lambda k}) - \frac{1}{3} e_{kij} \frac{\rho_s}{\rho} \left[\left(\nabla_i \frac{\chi_n H_\nu}{\gamma} \right) R_{\nu j} + \alpha \frac{\chi_n H_\nu}{\gamma} \nabla_i R_{\nu j} \right]. \quad (2.1)$$

In a constant magnetic field and neglecting density gradients, the last term in (2.1) can be written as $\frac{1}{2}$ curl $\mathbf{L}$, where

$$L_j = -\frac{2}{3} \alpha \frac{\rho_s}{\rho} \frac{\chi_n H_\nu}{\gamma} R_{\nu j} = -\alpha M_\nu^P R_{\nu j} \quad (2.2)$$

has the meaning of the internal angular momentum density of the Cooper pairs, and M_ν^P is the spin density of the Cooper pairs in ^{3}He-B. The coefficient α is of the order of $(T_c/\varepsilon_F)^2 \ln(\varepsilon_F/T_c)$ and therefore, as we have already noted, if the non-unitary corrections to the order parameter of the B-phase are neglected the internal angular momentum density of the B-phase in a magnetic field as $T \to T_c$ is a fraction $1 - T/T_c$ of $\mathbf{L}^{int}$ of (1.7).

The statement that the internal angular momentum in the B-phase in a magnetic field is small was criticized in the papers by Dombre and Combescot[9] and Yip,[10] who used a gradient expansion of the equation for the density matrix[9] and the Gor'kov equations[10] to obtain an expression for the superfluid current density in the B-phase in a magnetic field at $T=0$:

$$j_k = M_\nu^P v_{\nu k} - \tfrac{1}{2} e_{kij} \nabla_i (M_\nu^P R_{\nu j}). \quad (2.3)$$

Here

$$v_{\nu k} = \tfrac{1}{2} e_{\alpha\beta\nu} (\nabla_k R_{\alpha j}) R_{\beta j} \quad (2.4)$$

is the spin superfluid velocity and

$$M_\nu^P = 2\chi_n H_\nu / 3\gamma \quad (2.5)$$

is the spin density of ^{3}He-B at $T=0$.

The last term in (2.3) has the form $\frac{1}{2}$ curl $\mathbf{L}$, where

$$L_j = -M_\nu^P R_{\nu j}. \quad (2.6)$$

The expression for the current in ^{3}He-B in a magnetic field at $T=0$ is thus the same as the expression for the current of a superfluid Bose-liquid consisting of molecules in the same quantum state as the Cooper pairs in ^{3}He-B (see Ref. 10). The quantity $\mathbf{L}$ in (2.6) was interpreted in Ref. 10 as the internal angular momentum density of the B-phase in a magnetic field.

There arises thus a contradiction, wherein the internal angular momentum density of ^{3}He-B in a magnetic field can be both the negligibly small quantity (2.2) as $T \to T_c$ and of the order of the spin polarization of the whole liquid at $T=0$. We see that this contradiction has its origin in the incorrect interpretation assumed in Ref. 10, of the quantity (2.6) as the internal angular momentum of ^{3}He-B in a magnetic field.

Equation (2.3) for the current density is valid under the assumption that the particle-hole distribution is symmetric near the Fermi surface. A simple calculation (see Appendix B) shows that, if this assumption is not made the following correction is added to the current density (2.3):

$$\mathbf{j}^L = \tfrac{1}{2} \operatorname{rot} \mathbf{L}^{int}, \quad (2.7)$$

where

$$\mathbf{L}^{int} = \alpha \mathbf{L}, \quad \alpha \sim (T_c/\varepsilon_F)^2 \ln(\varepsilon_F/T_c), \quad (2.8)$$

with $\mathbf{L}$ given by Eq. (2.6). Such a correction to the current density should alert one, as it reminds one of the situation for the A-phase of ^{3}He, in which[11,12] the superfluid current density has not only a term $\frac{1}{2}C_{ij}$ (curl $\mathbf{l}$)$_j$ term, but also a term $\frac{1}{2}$ curl $\mathbf{L}^{int}$ corresponding to a contribution from the internal angular momentum which is small as the asymmetry in the particle-hole distribution is small.

It is impossible to find out directly from the expression for the superfluid current density which of the quantities (2.6) or (2.8) has the meaning of the internal angular momentum of ^{3}He-B in a magnetic field. To do this it is necessary to have an expression for the free energy $\mathcal{F}$ of the fluid in a container rotating in a magnetic field with an angular velocity Ω. After that the internal angular momentum is sought for as

$$\mathbf{L}^{int} = -\delta\mathcal{F}/\delta\mathbf{\Omega}. \quad (2.9)$$

It was shown in Refs. 5 and 6 that the quantity (2.2) has, indeed, the meaning of the internal angular momentum in

^{3}He-B in a magnetic field in the Ginzburg-Landau region as $T\to T_c$, since it corresponds to $\mathbf{L}^{\rm int}$ evaluated using Eq. (2.9). At the same time it follows from the expression for the free energy of ^{3}He-B in a rotating container in a magnetic field that at $T=0$ the orbital angular momentum (2.6) has the meaning of the orbital angular momentum generated by the superfluid current, while the magnitude of the internal angular momentum and its contribution to the current are given by Eqs. (2.8) and (2.7).

We formulate the results more precisely. The free energy of ^{3}He-B, which is quadratic in the gradients and linear in the magnetic field, assuming symmetry of the particle-hole distribution, is equal to

$$\mathcal{F}_s=\int(\mathbf{v}_s-\mathbf{v}_n)\,\mathbf{j}_s\,dV+\mathcal{F}_{nc}, \tag{2.10}$$

where $\mathcal{F}_{nc}$ is that part of the energy which is independent of $\mathbf{v}_s-\mathbf{v}_n$ and therefore does not contribute to the current density, and $\mathbf{j}_s$ is the superfluid current density obtained assuming symmetry of the particle-hole distribution. At $T=0$ the quantity $\mathbf{j}_s$ is the same as (2.3), and as $T\to T_c$ it is the same as expression (2.1) without the last term. It is immediately clear from (2.10) that the internal angular momentum determined from (2.9) vanishes. The orbital angular momentum (2.6) is thus the orbital angular momentum of the superfluid current. Since the vortex term in the current (2.3) is concentrated near the surface of the vessel, it leads to the existence of an orbital angular momentum of the fluid in the integral sense (see Ref. 5), i.e., for the vessel as a whole. It is shown in Ref. 5 that the energy (2.10) turns out to affect the orientation of the order parameter in ^{3}He-B only near the surface of a rotating vessel (in a layer of thickness of the order of the magnetic length ξ_H^B), and only under non-equilibrium conditions, i.e., when there is a countercurrent $\mathbf{v}_s-\mathbf{v}_n$. At equilibrium there is in a rotating container a vortex lattice such that the surface countercurrent is greatly weakened by the vortices: $\mathbf{v}_s\approx\mathbf{v}_n$.

Taking the asymmetry of the particle and hole distribution into account adds additional $\mathcal{F}_s$ terms to the free energy:

$$\mathcal{F}=\mathcal{F}_s+\mathcal{F}_a, \tag{2.11}$$

$$\mathcal{F}_a=\int dV[\alpha' M_\nu{}^P R_{\nu i}\Omega_i+\alpha'' M_\nu{}^P R_{\nu i}e_{ijk}\nabla_j(v_s-v_n)_k]. \tag{2.12}$$

Here $\mathbf{\Omega}=\frac{1}{2}\,\mathrm{curl}\,\mathbf{v}_n$ is the angular velocity of the vessel. Both coefficients α' and α'' are of order $(T_c/\varepsilon_F)^2\ln(\varepsilon_F/T_c)$. In accordance with (2.9) the internal angular momentum of ^{3}He-B in a magnetic field equals

$$L_i^{int}=-(\alpha'+\alpha'')M_\nu{}^P R_{\nu i}, \tag{2.13}$$

where M_ν^P is given by Eq. (2.2). The coefficient α in (1.8) and (1.9) thus equals $\alpha'+\alpha''$. We note that under equilibrium conditions, i.e., when $\mathbf{\Omega}=\frac{1}{2}\langle\mathrm{curl}\,\mathbf{v}_s\rangle$, the internal angular momentum is determined solely by the coefficient α'.

$$\mathbf{j}^L=-\delta\mathcal{F}_a/\delta\mathbf{v}_n. \tag{2.14}$$

In the Ginzburg-Landau region this is the last term in (2.1). If $T=0$ this is the current (2.7) with the internal angular momentum (2.8), or, what is the same, (2.13).

The calculations leading to the results formulated here can be found in Appendices A, B, and C.

3. NON-UNITARY CORRECTIONS TO THE ORDER PARAMETER

We describe in this section a method of taking into account non-unitary corrections to the order parameter of the B-phase in order to find an internal orbital angular momentum which generalizes the results of Ref. 5. For clarity we consider only the region $T\to T_c$, although the conclusions formulated here remain valid in the whole temperature range. The free energy, linear in the external field H and in the angular rotational velocity $\mathbf{\Omega}$ of the vessel, of a spatially homogeneous superfluid Fermi liquid has in the case of p-pairing the form

$$\mathcal{F}=\mathcal{F}_0+\mathcal{F}_H+\mathcal{F}_\Omega. \tag{3.1}$$

Here

$$\mathcal{F}_0=\int dV\{-\alpha A_{\mu i}^*A_{\mu i}+\beta_1A_{\mu i}^*A_{\mu i}^*A_{\nu j}A_{\nu j}+\beta_2A_{\mu i}^*A_{\mu i}A_{\nu j}^*A_{\nu j}+\beta_3A_{\mu i}^*A_{\nu i}^*A_{\nu j}A_{\mu j}+\beta_4A_{\mu i}^*A_{\nu i}A_{\nu j}^*A_{\mu j}+\beta_5A_{\mu i}^*A_{\nu i}A_{\nu j}A_{\mu j}^*\} \tag{3.2}$$

is the condensation energy; the complex 3×3 matrix $A_{\alpha i}$ is the order parameter of an arbitrary state of the superfluid Fermi-liquid in the case of p-pairing with Roman orbital and Greek spin indices. In the weak coupling approximation the coefficients α and β equal

$$\alpha={}^1/_3N_0(1-T/T_c),\qquad -2\beta_1=\beta_2=\beta_3=\beta_4=-\beta_5=7\zeta(3)N_0/120\pi^2T_c^2, \tag{3.3}$$

where $N_0=m^*p_F/2\pi^2$ is the density of states on the Fermi surface for a single spin component. The energy linear in the field H is

$$\mathcal{F}_H=\int dV\,g_H{}^1ie_{\alpha\mu\nu}H_\alpha A_{\mu i}^*A_{\nu i}, \tag{3.4}$$

with (see Ref. 13)

$$g_H{}^1\sim\gamma N_0'\ln(\varepsilon_F/T_c), \tag{3.5}$$

N_0' is the derivative of the density of states with respect to energy on the Fermi surface. The energy linear in $\mathbf{\Omega}$ is

$$\mathcal{F}_\Omega=\int dV\,g_\Omega{}^1ie_{ijk}\Omega_iA_{\mu j}^*A_{\mu k}, \tag{3.6}$$

with (see Ref. 2)

$$g_\Omega{}^1\sim N_0'\ln(\varepsilon_F/T_c). \tag{3.7}$$

The order parameter of the B-phase of ^{3}He

$$A_{\mu i}{}^0=[\alpha/2(3\beta_{12}+\beta_{345})]^{1/2}R_{\mu i}e^{i\Phi},\qquad \beta_{12}=\beta_1+\beta_2,\quad \beta_{345}=\beta_3+\beta_4+\beta_5 \tag{3.8}$$

minimizes expression (3.2). Minimizing (3.1) we find the equilibrium order parameter with account taken of the non-unitary corrections:

$$A_{\mu i}=A_{\mu i}{}^0+i\frac{3\beta_{12}+\beta_{345}}{2\alpha(3\beta_1-\beta_4+\beta_{35})}(g_H{}^1e_{\mu\nu\alpha}A_{\nu i}{}^0H_\alpha+g_\Omega{}^1e_{ijk}A_{\mu j}{}^0\Omega_k). \tag{3.9}$$

Substituting (3.9) into (3.1) we get the increment, quadratic in H and Ω, to the equilibrium energy of ^{3}He-B on account of non-unitary corrections to the order parameter:

$$\mathcal{F}_{H\Omega}=\int dV\frac{(g_H{}^1H_\alpha-g_\Omega{}^1R_{\alpha k}\Omega_k)^2}{2(3\beta_1-\beta_4+\beta_{35})}. \tag{3.10}$$

Equation (3.10) adds to the gyromagnetic energy of ^{3}He-B a contribution which is $(1-T/T_c)^{-1}$ times larger than expression (2.12) which holds when one neglects the non-unitary corrections to the order parameter of the B-phase. Far from T_c both expressions (3.10) and (2.12) are of the same order of magnitude.

The magnitude and internal orbital angular momenta are given by the equations

$$M_\alpha=-\frac{\delta\mathcal{F}_{\Omega H}}{\delta H_\alpha}=-\frac{g_H{}^1(g_H{}^1H_\alpha-g_\Omega{}^1R_{\alpha k}\Omega_k)}{3\beta_1-\beta_4+\beta_{35}}, \tag{3.11}$$

$$L_i^{int}=-\frac{\delta\mathcal{F}_{\Omega H}}{\delta\Omega_i}=\frac{g_\Omega{}^1R_{\alpha i}(g_H{}^1H_\alpha-g_\Omega{}^1R_{\alpha k}\Omega_k)}{3\beta_1-\beta_4+\beta_{35}}, \tag{3.12}$$

which generalize the corresponding expressions (3.14) and (3.15) of Ref. 5[2)] to the case of non-unitary corrections that depend on the rotational angular velocity $\mathbf{\Omega}$.

The integral orbital angular momentum of a rotating vessel with ^{3}He-B in a magnetic field produced by the surface current has, in accordance with Eq. (2.12) from Ref. 5 the form

$$\vec{\mathcal{L}}=2(K_2+K_3)|A_{\mu i}{}^0|^2V\langle \mathbf{L}\rangle, \tag{3.13}$$

where

$$\langle L_i\rangle=-ie_{ijk}A_{\mu j}{}^*A_{\mu k}/|A_{\nu l}{}^0|^2,\quad K_2+K_3=7\zeta(3)N_0v_F{}^2/120\pi^2T_c{}^2, \tag{3.14}$$

V is the volume occupied by the fluid. Substituting expression (3.9) into (3.14) we get

$$\langle L_i\rangle=\frac{2}{3}\frac{3\beta_{12}+\beta_{345}}{\alpha(3\beta_1-\beta_4+\beta_{35})}(g_H{}^1R_{\alpha i}H_\alpha-g_\Omega{}^1\Omega_i), \tag{3.15}$$

whence

$$\mathcal{L}_i/V\sim v_F{}^2(g_H{}^1R_{\alpha i}H_\alpha-g_\Omega{}^1\Omega_i). \tag{3.16}$$

It follows from Eqs. (3.11), (3.12), (3.16) that the magnetic and orbital angular momenta of the fluid vanish when the following relation holds:

$$H_\alpha=(g_\Omega{}^1/g_H{}^1)R_{\alpha k}\Omega_k. \tag{3.17}$$

4. CONCLUSIONS

The theory of gyromagnetic phenomena in superfluid ^{3}He-B when there is no vortex lattice in the rotating fluid was given in Ref. 5. The conclusions reached in Ref. 5 on the basis of calculations in the Ginzburg-Landau region $T\to T_c$ remain valid for any temperature. The orbital angular momentum in ^{3}He-B in a magnetic field, as in ^{3}He-A without a field, is practically completely concentrated in the current flowing along the surface of the vessel and only an insignificant part $\sim(T_c/\varepsilon_F)^2\ln(\varepsilon_F/T_c)$, caused by a small asymmetry in the particle and hole distributions near the Fermi surface, is connected with the local rotational motion of the fluid. This is a manifestation of the specific nature of a superfluid Fermi liquid and distinguishes its behavior from that of a superfluid Bose liquid consisting of molecules with the same wave function as the Cooper pairs in the Fermi liquid.

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APPENDIX A

Gradient expansion of the Gor'kov equations

To find the free energy of ^{3}He-B in a rotating vessel in a magnetic field, and also for calculating the internal angular momentum of the A-phase of ^{3}He (see Ref. 2) we shall formally assume that superfluid ^{3}He-B is a charged Fermi liquid with Cooper p-pairing in an external magnetic field acting both on the spins and on the particle charges. The field of the vector potential $\mathbf{A}$ is then equivalent to the field of the normal velocity $\mathbf{v}_n$:

$$\mathbf{A}=(mc/|e|)\mathbf{v}_n,$$

and the rotational angular velocity of the thermostat is determined to be $\mathbf{\Omega}=\frac{1}{2}\,\mathrm{curl}\,\mathbf{v}_n$. In the present Appendix we obtain solutions for the Gor'kov equations for a charged Fermi liquid with p-pairing up to second order in the spatial gradients; the solutions are linear in the external magnetic field acting on the particle spins.

We write down the Gor'kov equations for the charged B-phase of ^{3}He (see Ref. 2):

$$\begin{gathered}\{i\omega+\mu(\mathbf{r})-{}^1/_2(\mathbf{k}-i\nabla-i[\mathbf{\Omega}\partial/\partial\mathbf{k}])^2\\ -\mathbf{B}\boldsymbol{\sigma}\}\bar{G}(\mathbf{k},\,\mathbf{r},\,\omega)-\bar{\Delta}(\mathbf{k}+\mathbf{O},\,\mathbf{r})\bar{F}^+(\mathbf{k},\,\mathbf{r},\,\omega)=1,\\ \{-i\omega+\mu(\mathbf{r})-{}^1/_2(\mathbf{k}+\mathbf{O}-i[\mathbf{\Omega}\partial/\partial\mathbf{k}])^2-\mathbf{B}\boldsymbol{\sigma}\}\bar{F}^+(\mathbf{k},\,\mathbf{r},\,\omega)\\ +\bar{\Delta}^+(\mathbf{k}-i\nabla,\,\mathbf{r})\bar{G}(\mathbf{k},\,\mathbf{r},\,\omega)=0.\end{gathered} \tag{A.1}$$

Here

$$\mathbf{O}=-i\nabla-2\mathbf{A}(\mathbf{r}),\quad \omega=(2n+1)\pi T,\quad \mathbf{B}\boldsymbol{\sigma}=-{}^1/_2|\gamma|H_z\sigma_z,$$

we use the system of units in which $|e|=\hbar=m=c=1$, H is the magnetic field which acts on the particle spins which we shall assume to differ from $\mathbf{\Omega}=\frac{1}{2}\,\mathrm{curl}\,\mathbf{A}=\frac{1}{2}\,\mathrm{curl}\,\mathbf{v}_n$, the magnetic field acting on the particle charges, i.e., the angular velocity of the heat bath. The Green functions with a superior bar

$$\begin{aligned}\bar{G}(\mathbf{k},\mathbf{r})&=\int\exp[-i\mathbf{k}(\mathbf{r}-\mathbf{r}')]\bar{G}(\mathbf{r},\mathbf{r}')d^3(\mathbf{r}-\mathbf{r}'),\\ \bar{F}^+(\mathbf{k},\mathbf{r})&=\int\exp[-i\mathbf{k}(\mathbf{r}-\mathbf{r}')]\bar{F}^+(\mathbf{r},\mathbf{r}')d^3(\mathbf{r}-\mathbf{r}')\end{aligned} \tag{A.2}$$

are connected with the usual Green functions as follows:

$$\begin{aligned}\bar{G}(\mathbf{r},\mathbf{r}')&=G(\mathbf{r},\mathbf{r}')\exp\Big[i\int_{\mathbf{r}'}^{\mathbf{r}}\mathbf{A}(\mathbf{x})d\mathbf{x}\Big],\\ \bar{F}^+(\mathbf{r},\mathbf{r}')&=F^+(\mathbf{r},\mathbf{r}')\exp\Big[i\int_{\mathbf{r}'}^{\mathbf{r}}\mathbf{A}(\mathbf{x})d\mathbf{x}\Big].\end{aligned} \tag{A.3}$$

Here and henceforth we shall omit the frequency dependence of F^+ and G to simplify the formulae. Under the gauge transformation $\mathbf{A}\to\mathbf{A}+\nabla\chi$ the functions $\bar{G}$ and $\bar{F}^+$ transform according to

$$\bar{G}\to\bar{G},\quad \bar{F}^+\to\bar{F}^+\exp[2i\chi(\mathbf{r})]. \tag{A.4}$$

The same relations hold also for the solutions of (A.1) obtained to any order of the expansion in the slow gradients $-i\nabla$, $\mathbf{O}$. The matrix of the order parameter of the B-phase

$$\Delta(\mathbf{k},\,\mathbf{r})=\Delta_0(T)i\sigma_\alpha d_\alpha(\mathbf{k},\,\mathbf{r})\sigma_y=\Delta_0(T)i\sigma_\alpha R_{\alpha i}(\mathbf{r})(k_i/k_F)\sigma_y e^{i\Phi}, \tag{A.5}$$

where $\sigma_\alpha=(\sigma_x,\sigma_y,\sigma_z)$ are Pauli matrices, satisfies the equation

$$\bar{\Delta}(\mathbf{k},\mathbf{r})=-3g\int\frac{d\Omega_{\mathbf{k}'}}{4\pi}\frac{\mathbf{k}\mathbf{k}'}{k_F{}^2}T\sum_\omega\bar{F}(\mathbf{k}',\mathbf{r}). \tag{A.6}$$

One can expand the solutions of the set (A.1) in power series in ∇, $\mathbf{O}$, and $\mathbf{A}$:

$$\bar{G}=G_0+G_1+G_2+\ldots,\quad \bar{F}^+=F_0^++F_1^++F_2^++\ldots, \tag{A.7}$$

or, in matrix notation,

$$\mathscr{G}=\begin{pmatrix}\bar{G}\\ \bar{F}^+\end{pmatrix}=\mathscr{G}_0+\mathscr{G}_1+\mathscr{G}_2+\ldots. \tag{A.8}$$

As we are interested in effects which are linear in the field H, we write each of the functions $\mathscr{G}_i$ as a sum:

$$\mathscr{G}_i=\mathscr{G}_i^0+\mathscr{G}_i^H, \tag{A.9}$$

where $\mathscr{G}_i^0$ is the function $\mathscr{G}_i$ in the field $H=0$, and $\mathscr{G}_i^H$ is a correction linear in the field. For the functions $\mathscr{G}_i^0$ and $\mathscr{G}_i^H$ we have the following set of equations:

$$E\mathscr{G}_0=\begin{pmatrix}1\\0\end{pmatrix}, \tag{A.10a}$$

$$E\mathscr{G}_0^H-\boldsymbol{\sigma}\mathbf{B}\mathscr{G}_0=0, \tag{A.10b}$$

$$E\mathscr{G}_1^0+R_1\mathscr{G}_0^0=0, \tag{A.10c}$$

$$E\mathscr{G}_1^H-\boldsymbol{\sigma}\mathbf{B}\mathscr{G}_1^0+R_1\mathscr{G}_0^H=0, \tag{A.10d}$$

$$E\mathscr{G}_2^0+R_1\mathscr{G}_1^0+R_2\mathscr{G}_0^0=0, \tag{A.10e}$$

$$E\mathscr{G}_2^H-\boldsymbol{\sigma}\mathbf{B}\mathscr{G}_2^0+R_1\mathscr{G}_1^H+R_2\mathscr{G}_0^H=0. \tag{A.10f}$$

Here

$$E=\begin{pmatrix}\varepsilon & -\Delta\\ \Delta^+ & \varepsilon^\cdot\end{pmatrix},\quad \varepsilon=i\omega+\mu-k^2/2=i\omega-\xi, \tag{A.11}$$

$$R_1=\begin{pmatrix}-\dfrac{\mathbf{k}\nabla}{i} & -\dfrac{\partial\Delta}{\partial\mathbf{k}}\mathbf{O}\\ \dfrac{\partial\Delta^+}{\partial\mathbf{k}}\dfrac{\nabla}{i} & -\mathbf{k}\mathbf{O}\end{pmatrix}, \tag{A.12}$$

$$R_2=\begin{pmatrix}-\dfrac{1}{2}\left(\dfrac{\nabla}{i}\right)^2-\dfrac{k}{i}\left[\boldsymbol{\Omega}\dfrac{\partial}{\partial\mathbf{k}}\right] & -\dfrac{1}{2}\dfrac{\partial^2\Delta}{\partial k_i\,\partial k_j}O_iO_j\\ -\dfrac{1}{2}\dfrac{\partial^2\Delta^+}{\partial k_i\,\partial k_j}\nabla_i\nabla_j & -\dfrac{\mathbf{O}^2}{2}-\dfrac{\mathbf{k}}{i}\left[\boldsymbol{\Omega}\dfrac{\partial}{\partial\mathbf{k}}\right]\end{pmatrix}. \tag{A.13}$$

The solutions of the set (A.10) have the form

$$\mathscr{G}_0^0=D^{-1}E^+\begin{pmatrix}1\\0\end{pmatrix}, \tag{A.14a}$$

$$\mathscr{G}_0^H=D^{-1}E^+\boldsymbol{\sigma}\mathbf{B}\mathscr{G}_0^0, \tag{A.14b}$$

$$\mathscr{G}_1^0=-D^{-1}E^+R_1\mathscr{G}_0^0, \tag{A.14c}$$

$$\mathscr{G}_1^H=D^{-1}E^+\boldsymbol{\sigma}\mathbf{B}\mathscr{G}_1^0-D^{-1}E^+R_1\mathscr{G}_0^H, \tag{A.14d}$$

$$\mathscr{G}_2^0=-D^{-1}E^+R_1\mathscr{G}_1^0-D^{-1}E^+R_2\mathscr{G}_0^0, \tag{A.14e}$$

$$\mathscr{G}_2^H=D^{-1}E^+\bar{\boldsymbol{\sigma}}\mathbf{B}\mathscr{G}_2^0-D^{-1}E^+R_1\mathscr{G}_1^H-D^{-1}E^+R_2\mathscr{G}_0^H, \tag{A.14f}$$

where

$$D=|\varepsilon|^2+\Delta\Delta^+. \tag{A.15}$$

APPENDIX B

Superfluid current in the B-phase in a magnetic field

The density of a current linear in the field H can be expressed in terms of the Green function G_1^H:

$$\mathbf{j}=\int\frac{d^3\mathbf{k}}{(2\pi)^3}\mathbf{k}T\sum_\omega\frac{1}{2}\,\mathrm{Sp}\,G_1^H(\mathbf{k},\mathbf{r})+\text{h.c.} \tag{B.1}$$

Using the results of Appendix A and dropping the terms proportional to $\nabla\mu$ and ∇H, we get

$$G_1^H=-\frac{1}{D^3}(\varepsilon^\cdot)^2\boldsymbol{\sigma}\mathbf{B}\frac{\partial\Delta}{\partial\mathbf{k}}\mathbf{O}\Delta^++\frac{1}{D^3}\Delta\boldsymbol{\sigma}\mathbf{B}\Delta^+\frac{\partial\Delta}{\partial\mathbf{k}}\mathbf{O}\Delta^+$$
$$+\frac{1}{D^3}\Delta\frac{\partial\Delta^+}{\partial\mathbf{k}}\frac{\nabla}{i}(\Delta\boldsymbol{\sigma}\mathbf{B}\Delta^+)-\frac{1}{D^3}\varepsilon^\cdot\frac{\partial\Delta}{\partial\mathbf{k}}\mathbf{O}(\Delta^+\boldsymbol{\sigma}\mathbf{B}\varepsilon^\cdot+\varepsilon\boldsymbol{\sigma}\mathbf{B}\Delta^+)$$
$$-\frac{2}{D^3}(\varepsilon+\varepsilon^\cdot)\boldsymbol{\sigma}\mathbf{B}\Delta\mathbf{k}\mathbf{O}\Delta^+-\frac{\varepsilon^\cdot}{D^3}\frac{\mathbf{k}\nabla}{i}(\Delta\boldsymbol{\sigma}\mathbf{B}\Delta^+). \tag{B.2}$$

Evaluating $\frac{1}{2}\,\mathrm{Sp}\,G_1^H$ and dropping terms with the vector potential $\mathbf{A}$, which cancel out when we take the Hermitian conjugate in (B.1), we get

$$\frac{1}{2}\,\mathrm{Sp}\,G_1^H$$
$$=-\frac{\Delta_0^2}{D^3}e_{\alpha\beta\gamma}B_\delta\frac{\partial d_\beta}{\partial\mathbf{k}}\frac{\partial d_\gamma}{\partial\mathbf{r}}\{[2\varepsilon^{\cdot2}+\varepsilon\varepsilon^\cdot+\Delta_0^2]\delta_{\alpha\delta}+2d_\alpha d_\delta\Delta_0^2\}$$
$$-\frac{2}{D^3}(\varepsilon+\varepsilon^\cdot)\Delta_0^2e_{\alpha\beta\gamma}B_\alpha d_\beta\mathbf{k}\nabla d_\gamma. \tag{B.3}$$

Using the relation (see Ref. 10)

$$e_{\alpha\beta\gamma}B_\delta d_\delta d_\alpha\frac{\partial d_\beta}{\partial\mathbf{k}}\frac{\partial d_\gamma}{\partial\mathbf{r}}=e_{\alpha\beta\gamma}B_\alpha\frac{\partial d_\beta}{\partial\mathbf{k}}\frac{\partial d_\gamma}{\partial\mathbf{r}}, \tag{B.4}$$

we have

$$\frac{1}{2}\,\mathrm{Sp}\,G_1^H$$
$$=-\frac{2\Delta_0^2}{D^3}(\varepsilon^{\cdot2}+\Delta_0^2)e_{\alpha\beta\gamma}B_\alpha\frac{\partial d_\beta}{\partial\mathbf{k}}\frac{\partial d_\gamma}{\partial\mathbf{r}}-\frac{\Delta_0^2}{D^2}e_{\alpha\beta\gamma}B_\alpha\frac{\partial d_\beta}{\partial\mathbf{k}}\frac{\partial d_\gamma}{\partial\mathbf{r}}$$
$$-\frac{2\Delta_0^2}{D^3}(\varepsilon+\varepsilon^\cdot)e_{\alpha\beta\gamma}B_\alpha d_\beta\mathbf{k}\nabla d_\gamma. \tag{B.5}$$

Substituting (B.5) into (B.1) we find

$$j_i=2\int\frac{d^3k}{(2\pi)^3}T\sum_\omega k_i\Big[-\frac{2\Delta_0^2}{D^3}(\varepsilon^{\cdot2}+\Delta_0^2)e_{\alpha\beta\gamma}B_\alpha\frac{\partial d_\beta}{\partial k_m}\frac{\partial d_\gamma}{\partial r_m}$$
$$-\frac{\Delta_0^2}{D^2}e_{\alpha\beta\gamma}B_\alpha\frac{\partial d_\beta}{\partial k_m}\frac{\partial d_\gamma}{\partial r_m}+\frac{4\xi\Delta_0^2}{D^3}e_{\alpha\beta\gamma}B_\alpha d_\beta\mathbf{k}\nabla d_\gamma\Big]. \tag{B.6}$$

We now integrate the second term in (B.6) by parts:

$$j_i=2\int\frac{d^3\mathbf{k}}{(2\pi)^3}T\sum_\omega\Big\{-\frac{\Delta_0^2}{D^2}e_{\alpha\beta\gamma}B_\alpha d_\gamma\frac{\partial d_\beta}{\partial r_i}$$
$$+k_i\Big[-\frac{2\Delta_0^2}{D^3}(\varepsilon^{\cdot2}+\Delta_0^2)e_{\alpha\beta\gamma}B_\alpha\frac{\partial d_\beta}{\partial k_m}\frac{\partial d_\gamma}{\partial r_m}$$
$$-\frac{\Delta_0^2}{D^2}e_{\alpha\beta\gamma}B_\alpha d_\gamma\frac{\partial^2 d_\beta}{\partial r_m\,\partial k_m}\Big]\Big\}. \tag{B.7}$$

We consider the case $T=0$. Using the relations ($T=0$)

$$T\sum_\omega\frac{\varepsilon^{\cdot2}}{D^3}=\frac{1}{8E^3}-\frac{3}{16}\frac{\Delta_0^2}{E^5},$$
$$T\sum_\omega\frac{1}{D^2}=\frac{1}{4E^3},\quad T\sum_\omega\frac{\Delta_0^2}{D^3}=\frac{3}{16}\frac{\Delta_0^2}{E^5}, \tag{B.8}$$

$$\int d\xi\frac{1}{E^3}=\frac{2}{\Delta_0^2},\quad \int d\xi\frac{\Delta_0^2}{E^5}=\frac{4}{3\Delta_0^2}, \tag{B.9}$$

we get from (B.7)

$$j_i=\frac{1}{3\gamma}\chi_ne_{\alpha\beta\gamma}H_\alpha\frac{\partial R_{\beta m}}{\partial r_i}R_{\gamma m}-\frac{\chi_nH_\alpha}{3\gamma}e_{ijk}\frac{\partial}{\partial r_j}R_{\alpha k}. \tag{B.10}$$

In the absence of terms proportional to ∇H and $\nabla\mu$, Eq. (B.10) is the same as (2.3). One proves easily (see Refs. 7, 10, and 14) that when those terms are taken into account Eq. (2.3) is exact.

We now note that the integration over $d\xi$ in Eq. (B.7) was performed for constant density of states, i.e., in the approximation of symmetric particle and hole distributions near the Fermi surface. Bearing in mind that the density of states near the Fermi surface can be written as

$$N_0(\xi)=N_0+N_1\xi/\varepsilon_F+N_2\xi^2/\varepsilon_F^2, \qquad \text{(B.11)}$$

we get with logarithmic accuracy from the second and third terms of (B.7) a correction to the second term of (B.10) or (2.3):

$$-\frac{1}{3}\frac{N_2}{N_0}\frac{\Delta_0^2}{\varepsilon_F^2}\frac{\chi_n H_\alpha}{\gamma}\ln\left(\frac{\varepsilon_F}{T_c}\right)e_{ijk}\frac{\partial}{\partial r_j}R_{\alpha k}, \qquad \text{(B.12)}$$

see Eqs. (2.7) and (2.8).

We go over to the case $T\to T_c$. In this case it is more convenient to start directly from Eq. (B.6). Retaining in it only the terms proportional to Δ_0^2 we have

$$j_i=2\int\frac{d^3\mathbf{k}}{(2\pi)^3}T\sum_\omega\hat{k}_i\Big\{-\Big[\frac{2}{\varepsilon^3\varepsilon^*}+\frac{1}{\varepsilon^2\varepsilon^{*2}}\Big]\times\Delta_0^2 e_{\alpha\beta\gamma}B_\alpha R_{\beta j}(\delta_{mj}-\hat{k}_m\hat{k}_j)\frac{\partial R_{\gamma l}}{\partial r_m}\hat{k}_l+\frac{4\xi k^2\Delta_0^2}{(\varepsilon\varepsilon^*)^3}e_{\alpha\beta\gamma}B_\alpha R_{\beta j}\hat{k}_j\Big(\hat{k}_m\frac{\partial}{\partial r_m}\Big)R_{\gamma l}\hat{k}_l\Big\}. \qquad \text{(B.13)}$$

Here $\hat{\mathbf{k}}=\mathbf{k}/k$ and $k^2=2(\xi+\mu)$. It is convenient to integrate the term containing $-2/\varepsilon^3\varepsilon^*$ in Eq. (B.13) with respect to $d\xi$ by parts taking into account the asymmetry in the distribution of the particles and holes distribution (B.11). We integrate the last term in Eq. (B.13) by parts using the exact expression for the density of states in the weak coupling approximation:

$$N_0(\xi)=A(\xi+\mu)^{1/2}. \qquad \text{(B.14)}$$

After regrouping the terms we get thus

$$j_i=2\int\frac{d^3\mathbf{k}}{(2\pi)^3}T\sum_\omega\Big\{\frac{6\Delta_0^2}{(\varepsilon\varepsilon^*)^2}\hat{k}_i\hat{k}_j\hat{k}_m\hat{k}_l e_{\alpha\beta\gamma}B_\alpha R_{\beta j}\frac{\partial R_{\gamma l}}{\partial r_m}+\frac{\Delta_0^2}{\varepsilon_F^2}\frac{N_2}{N_0}\frac{2\xi^2}{(\varepsilon\varepsilon^*)^2}\times\hat{k}_i\hat{k}_l(\delta_{mj}-\hat{k}_m\hat{k}_j)e_{\alpha\beta\gamma}B_\alpha R_{\beta j}\frac{\partial R_{\gamma l}}{\partial r_m}\Big\}. \qquad \text{(B.15)}$$

One sees easily that after integration and summation over the frequencies the first term in (B.15) goes over into the first term of (2.1), while from the second term of (B.15), which contains an additional small factor $(T_c/\varepsilon_F)^2\ln(\varepsilon_F/T_c)$, we can separate the last term of (2.1).

One should note that if in the limit as $T\to T_c$ one retains also terms $\propto(\Delta_0)^4$ there appears in the expression for the current density (2.1), apart from the term

$$-{}^1/_3\alpha(\rho_s/\rho)(\chi_n H_\nu/\gamma)e_{kim}\nabla_i R_{\nu m}, \qquad \text{(B.16)}$$

corresponding to the internal angular momentum with $\alpha\sim(T_c/\varepsilon_F)^2\ln(\varepsilon_F/T_c)$, also a term

$$-{}^1/_3\beta(\rho_s/\rho)(\chi_n H_\nu/\gamma)e_{kim}\nabla_i R_{\nu m}, \qquad \text{(B.17)}$$

where $\beta\sim(\Delta_0/T_c)^2$. The contribution (B.17) to the current is larger than the contribution (B.16) up to a negligibly narrow, $\Delta_0\lesssim(T_c/\varepsilon_F)T_c$, vicinity of the superfluid-transition temperature.

APPENDIX C

The gradient energy

The energy which is quadratic in the gradients of the order parameters and is linear in the magnetic field acting on the particle spins can be expressed in terms of the Green function $F_2^{+H}(\mathbf{k},\mathbf{r})$ (see Refs. 2, 15):

$$\mathcal{F}-\mathcal{F}(\lambda=0)=-\int d^3\mathbf{r}\Big\{\int\frac{d^3\mathbf{k}}{(2\pi)^3}T\sum_\omega\int_0^1 d\lambda\,\frac{1}{2}\,\mathrm{Sp}\,\Delta(\mathbf{k},\mathbf{r})\times F_2^{+H}(\mathbf{k},\mathbf{r},\lambda)+\text{h.c.}\Big\}. \qquad \text{(C.1)}$$

Here $F_2^{+H}(\mathbf{k},\mathbf{r},\lambda)$ is the function F_2^{+H} from (A.14f) in which we have made the substitution $\Delta\to\lambda\Delta$. From (A.14) we have

$$\mathcal{G}_2^H=\begin{pmatrix}G_2^H\\ F_2^{+H}\end{pmatrix}=-D^{-3}E^+[\boldsymbol{\sigma}\mathbf{B}E^+R_2+R_2E^+\boldsymbol{\sigma}\mathbf{B}]E^+\begin{pmatrix}1\\0\end{pmatrix}+D^{-4}E^+[\boldsymbol{\sigma}\mathbf{B}E^+R_1E^+R_1+R_1E^+\boldsymbol{\sigma}\mathbf{B}E^+R_1+R_1E^+R_1E^+\boldsymbol{\sigma}\mathbf{B}]E^+\begin{pmatrix}1\\0\end{pmatrix}. \qquad \text{(C.2)}$$

The evaluation of the gradient energy using Eqs. (C.1) and (C.2) belongs to that category of calculations which are more easily done independently than by following their development. We give here the least cumbersome part of the calculations, namely we obtain the contribution to the free energy, which is proportional to the angular velocity of the heat bath, i.e., the first term in Eq. (2.12).

From the first two terms in (C.2), separating in R_2 the contributions proportional to $\boldsymbol{\Omega}$, we have

$$F_{2\Omega}^{+H}=-\frac{\boldsymbol{\sigma}\mathbf{B}(2\varepsilon^2+\varepsilon^*\varepsilon)}{D^3}\frac{\mathbf{k}}{i}\Big[\boldsymbol{\Omega}\frac{\partial}{\partial\mathbf{k}}\Big]\Delta^++\frac{1}{D^3}\Delta^+\boldsymbol{\sigma}\mathbf{B}\Delta\frac{\mathbf{k}}{i}\Big[\boldsymbol{\Omega}\frac{\partial}{\partial\mathbf{k}}\Big]\Delta^++\frac{1}{D^3}\Delta^+\frac{\mathbf{k}}{i}\Big[\boldsymbol{\Omega}\frac{\partial}{\partial\mathbf{k}}\Big](\Delta\boldsymbol{\sigma}\mathbf{B}\Delta^+). \qquad \text{(C.3)}$$

Moreover,

$$\frac{1}{2}\mathrm{Sp}\,\Delta F_{2\Omega}^{+H}(\lambda)=\Big\{-\frac{\lambda\Delta_0^2}{D^3(\lambda)}(2\varepsilon^2+\varepsilon^*\varepsilon)+\frac{\lambda^3\Delta_0^4}{D^3(\lambda)}\Big\}e_{\alpha\beta\gamma}B_\alpha d_\beta\mathbf{k}\Big[\boldsymbol{\Omega}\frac{\partial}{\partial\mathbf{k}}\Big]d_\gamma,$$

$$D(\lambda)=|\varepsilon|^2+\lambda^2\Delta_0^2. \qquad \text{(C.4)}$$

Integrating over λ we get

$$\int_0^1\frac{1}{2}\mathrm{Sp}\,\Delta F_{2\Omega}^{+H}(\lambda)\,d\lambda=\Big\{(2\varepsilon^2+|\varepsilon|^2)\Big(\frac{1}{D^2}-\frac{1}{|\varepsilon|^4}\Big)$$

$$-\left(\frac{\Delta_0^2}{D^2}+\frac{1}{D}-\frac{1}{|\varepsilon|^2}\right)\Big\}\frac{1}{4}e_{\alpha\beta\gamma}B_\alpha d_\beta\mathbf{k}\left[\boldsymbol{\Omega}\frac{\partial}{\partial\mathbf{k}}\right]d_\gamma. \quad (C.5)$$

Dropping terms which are odd in the frequency we rewrite the expression in the braces as

$$4\xi^2/D^2-4\xi^2/|\varepsilon|^4+2/|\varepsilon|^2-2/D. \quad (C.6)$$

Summing (C.6) over the frequency we get

$$\xi^2\left(\frac{1}{E^2}\,\mathrm{th}\frac{E}{2T}-\frac{1}{2E^2T}\,\mathrm{ch}^{-2}\frac{E}{2T}\right)$$
$$-\left(\frac{1}{|\xi|}\,\mathrm{th}\frac{|\xi|}{2T}-\frac{1}{2T}\,\mathrm{ch}^{-2}\frac{|\xi|}{2T}\right)$$
$$+\frac{1}{|\xi|}\,\mathrm{th}\frac{\xi}{2T}-\frac{1}{E}\,\mathrm{th}\frac{E}{2T}. \quad (C.7)$$

We must note here that it is essential to sum just over the frequencies, since the substitution

$$T\sum_{\omega}\rightarrow\int d\omega$$

as $T\to 0$ is incorrect as $\xi\to 0$. We integrate next (C.7) with respect to ξ:

$$\int N_0(\xi)\,d\xi\left\{-\frac{\Delta_0}{E^3}\,\mathrm{th}\frac{E}{2T}-\frac{\xi^2}{2E^2T}\,\mathrm{ch}^{-2}\frac{E}{2T}+\frac{1}{2T}\,\mathrm{ch}^{-2}\frac{|\xi|}{2T}\right\}$$
$$\sim -2N_2\left(\frac{\Delta_0}{\varepsilon_F}\right)^2\ln\frac{\varepsilon_F}{T_c}. \quad (C.8)$$

We have thus for the contribution proportional to $\boldsymbol{\Omega}$ to the gradient energy

$$\frac{2}{3}\frac{N_2}{N_0}\frac{\chi_n H_\alpha}{\gamma}\left(\frac{\Delta_0}{\varepsilon_F}\right)^2\ln\frac{\varepsilon_F}{T_c}H_\alpha R_{\alpha q}\Omega_q. \quad (C.9)$$

We have thus shown that the contribution linear both in **H** and in $\boldsymbol{\Omega}$ to the free energy [the first term in (2.12)] is non-vanishing only when we take the particle and hole distribution asymmetry into account, and has therefore an extra small factor $(T_c/\varepsilon_F)^2\ln(\varepsilon_F/T_c)$.

Similar although considerably more complicated calculations lead to the conclusion that the second term in (2.12), which is proportional to **H** and to curl $(\mathbf{v}_s-\mathbf{v}_n)$, is also small because the particle and hole distribution asymmetry is small.

1) The values of the coefficients in Eq. (2.1) are corrected as compared to Ref. 6 where all coefficients must be multiplied by $\frac{1}{2}$.

2) On the right-hand side of Eq. (3.13) of Ref. 5 a factor $-2/3$ was omitted and hence all signs of the right-hand sides of Eqs. (3.14) to (3.16) of that paper must be reversed.

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