Anomalous magnetoresistance of two-dimensional electrons in a lateral superlattice

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The kinetics of a degenerate electron gas in the inversion channel on the vicinal surface of a many-valley semiconductor is considered (with Si as the example) in the region of classical magnetic fields. It is shown that the possession of many valleys by the initial crystal leads to selective scattering of two-dimensional electrons to various parts of their Fermi surface (contour). In the greater part of the region of weak magnetic fields ($\omega_c\tau \ll 1$ the dependence of the resistance on the magnetic field differs substantially from quadratic. The agreement with the reported experiments, and possible new ones, are discussed.

One of the attractive features of inversion channels on vicinal surfaces of semiconductors is the possibility of easily changing the Fermi-surface topology of quasi-two-dimensional carriers [it is more correct to speak in this case of a Fermi contour (FC)]. The presence of a superlattice located on the periphery of the inversion layer (lateral superlattice) leads to a splitting of the energy spectrum of each transverse-quantization subband into a number of one-dimensional minibands separated by minigaps. By using the gate voltage to vary the density N_s of the carriers in the channel, and accordingly the degree of occupancy of such minibands, it is possible to alter substantially the shape (and topology) of the FC, thereby affecting primarily the kinetic effects.

We consider here the anomalous behavior of the magnetoresistance (MR) of a degenerate electron gas in an inversion channel on a high-index surface of a multivalley semiconductor, when the Fermi energy $\mathscr{E}_F$ lies near an intervalley minigap. This situation was investigated experimentally in Ref. 1 for n-channels on Si surfaces tilted $\sim 10^\circ$ away from the (100) face, in a magnetic field $H \sim 3$ kG and at a mobility $\mu \sim 10^4$ V · sec and at helium temperatures. The obtained blip in the plot of the MR vs N_s is attributed to the abrupt change of the shape of the FC when $\mathscr{E}_F$ passes through the first intervalley minigap (for a round FC and an isotropic relaxation time, the magnetoresistance is small in terms of the parameter $(T/\mathscr{E}_F)^2$ and is much less than the experimentally observed value). The magnetoconductance of electrons in a lateral superlattice was the subject of several theoretical papers.[2,3] They dealt, however, with the ultraquantum case of a strong magnetic field, when the magnetic length is shorter than the period of the superlattice, and the role of the latter reduces to a broadening of the Landau levels into magnetic bands. We, conversely, consider the region of classical fields that do not alter the electron dispersion law, so that the Boltzmann kinetic equation can be used. This is precisely the situation realized in the experiments.[1]

It will be shown that the MR produced is due not only to the obvious presence of a strong anisotropy of the carrier effective masses, but also to singularities in the scattering processes in the indicated system. Both factors are of equal importance for the onset of an anomalous field dependence of the MR in weak magnetic fields. From the procedural point of view, our problem is of interest because the kinetic equation with an integral collision term can be solved exactly. This circumstance is of fundamental importance, since the relaxation time leads, in view of the strong anisotropy of the scattering, to qualitatively incorrect results.

To be definite, we shall consider hereafter the system experimentally realized in Ref. 1. In this case the lower minigap is produced by hybridization of the (100) and ($\bar{1}00$) valleys, and the dispersion law in its vicinity (within the framework of the weak coupling model[4]) takes the form

$$\mathscr{E}_n(\mathbf{p}) = \frac{p_y^2}{2m} + \frac{\hbar^2}{2mb^2} + \frac{p_x^2}{2m} + (-1)^n[(\hbar p_x/mb)^2 + |V|^2]^{1/2} \tag{1}$$

where $m \equiv 0.19 m_e$ is the electron effective mass, b is the effective period of the superlattice (x is directed along its axis), $n = 1$ and 2 are the numbers of the miniband, and $2|V|$ is the width of the minigap ($|V| \ll \hbar^2/mb^2$).

If $\hbar/|V|$ is substantially smaller than the characteristic relaxation time τ of the electrons,[1)] the expression for the MR can be obtained by solving the classical Boltzmann equation with allowance for the dispersion law (1).

In the system considered, at low temperatures, the principal relaxation mechanism is scattering by charged impurities. For two-dimensional degenerate electrons the screening radius is known to be of the order of the effective Bohr radius a_B. In the important energy region we have $k_F a_B \ll 1$, so that the impurity potential is effectively short-range relative to the period of the superlattice. On the other hand, the potential is smooth relative to the period of the principal lattice. Taking these circumstances into account, we obtain for the scattering probability the expression

$$W_{nn'}(\mathbf{k},\mathbf{k}') = \frac{W_0}{2}\left[1 + (-1)^{n+n'}\frac{k_x k_x' b^2 + \alpha^2}{(k_x^2 b^2 + \alpha^2)^{1/2}(k_x'^2 b^2 + \alpha^2)^{1/2}}\right], \tag{2}$$

where $\mathbf{k} = \mathbf{p}/\hbar$, $\alpha = mb^2|V|/\hbar^2 \ll 1$, and W_0 is the probability of scattering of a plane wave by the same center. On the greater part of the FC ($k, k' \sim 1/b$), the value of $W_{nn'}(\mathbf{k},\mathbf{k}')$

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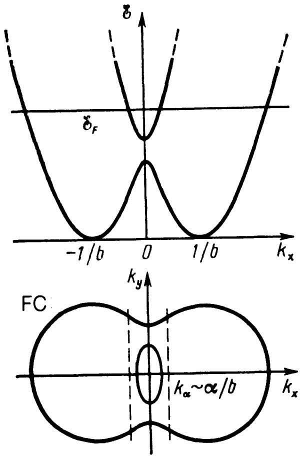

FIG. 1.

is proportional to $\theta((-1)^{n+n'}k_x k'_x)$ (see Fig. 1), where $\theta(x)$ is the step function. The reason is that the wave functions of the electron $\psi^{(+)}_{nk}(\mathbf{r})$ and $\psi^{(-)}_{nk}(\mathbf{r})$ on the right and left halves of the FC preserve (to the extent that the weak-coupling parameter α is small) the connection with their valleys. The matrix element of the $\psi^{(+)}_n \to \psi^{(-)}_n$ scattering vanishes practically even upon integration over a region of the order of the original-crystal cell dimensions (owing to the fast oscillations of the Bloch factors that pertain to different lengths). Good intermixing of $\psi^{(+)}$ and $\psi^{(-)}$ is reached only in a narrow strip (of width $\sim\alpha/b$) near $k_x = 0$, so that $W = W_0\delta_{nn'}$ inside the strip, and $W = W_0/2$ for transitions between the strip and the remainder of the FC.

The presence of the small parameter α in the expressions for $W_{nn'}(\mathbf{k},\mathbf{k}')$ and $E_n(\mathbf{k})$ leads to the appearance of an additional characteristic scale of the magnetic field ($\omega_c\tau \sim \alpha^{1/2}, \omega_c = eH/mc$). In the greater part of the weak-field region ($\alpha^{1/2} < \omega_c\tau < 1$) the dependence of the MR on the magnetic field has an anomalous character that is essentially determined by the indicated selectivity of the scattering.

The structure of the scattering probability (2) ensures separability of the kernel of the integral term in the kinetic equation, so that a solution can be obtained in closed form. Indeed, Eq. (2) can be rewritten in the form

$$W_{nn'}(\mathbf{k}, \mathbf{k}') = {}^1\!/\!_2 W_0\{1 + (-1)^{n+n'}[F_0(k_x)F_0(k_x') + F_1(k_x)F_1(k_x')]\}, \qquad (3)$$

where

$$F_0(k_x) = \frac{\alpha}{[(k_x b)^2 + \alpha^2]^{1/2}}, \quad F_1(k_x) = \frac{k_x b}{[(k_x b)^2 + \alpha^2]^{1/2}}. \qquad (4)$$

With this form of $W_{nn'}(\mathbf{k},\mathbf{k}')$, the arrival terms in the kinetic equations for the distribution function $f_i(\mathbf{k})$ ($i = 1, 2$ is the index of the miniband) depend on $f_i(\mathbf{k})$ only via constants, viz., integrals of the type $\int d\,\mathbf{k} F_{0,1}(k_x) f_i(\mathbf{k})$. Thus, the $f_i(\mathbf{k})$ are determined not by integrodifferential equations but by first-order linear partial differential equations. The constants mentioned above enter in the solutions of these equations as parameters that are self-consistently determined later.

The kinetic equations are solved by the characteristics method. We transform to variables φ and $\mathscr{E}$, where $\varphi = \omega_c t$ is the dimensionless time of electron motion along an equal-energy trajectory (with energy $\mathscr{E}$) in the magnetic field, and with periods $\Phi_1(\mathscr{E})$ and $\Phi_2(\mathscr{E})$ for the first and second minibands. With this choice of variables, the function $f_1(\varphi)$ and $f_2(\varphi)$ are periodic in φ with periods Φ_1 and Φ_2 respectively. The solutions are:

$$f_i(\varphi, \mathscr{E}) = -[2ef_0'\mathbf{E}/W_0(\Phi_1+\Phi_2)] \times\Big\{J_i\{\mathbf{v}^{(i)}(\varphi)\} + (-1)^i J_i\{F_1[k_x^{(i)}(\varphi)]\} \times\Big[\sum_{j=1}^{2}(-1)^j\int_0^{\Phi_j} d\varphi\, F_1(k_x^{(j)})J_j\{\mathbf{v}^{(j)}\}\Big] \times\Big[\sum_{j=1}^{2}\Big(\Phi_j - \int_0^{\Phi_j} d\varphi\, F_1(k_x^{(j)})J_j\{F_1(k_x^{(j)})\}\Big]^{-1}\Big\}. \qquad (5)$$

Here $\mathbf{E}$ is the electric field and f'_0 the derivative of the equilibrium Fermi distribution function with respect to the energy. The functional $J_i\{\chi(\varphi)\}$ are defined as

$$J_i\{\chi(\varphi)\} = \int_0^\infty d\theta\, \chi(\varphi - \beta\theta) \times \exp\Big\{-\theta - (-1)^i\delta\int_0^\theta d\theta'\, F_0[k_x^{(i)}(\varphi - \beta\theta')]\Big\}, \qquad (6)$$

$$\delta = \Big[\int_0^{\Phi_1} d\varphi\, F_0(k_x^{(1)}) - \int_0^{\Phi_2} d\varphi\, F_0(k_x^{(2)})\Big]\Big/(\Phi_1 + \Phi_2). \qquad (7)$$

Naturally, if the Fermi level is lower than the bottom of the second miniband, we must put $\Phi_2 \equiv 0$ everywhere. The function $k_x^{(i)}(\varphi)$ is given by the solutions of the equations of electron motion in the magnetic field, and the expressions for the velocity $v(i)[k_x(\varphi)]$ follow from the dispersion law:

$$v_y^{(i)}(k_x) = \frac{\hbar}{mb}[\tilde{\varepsilon} - k_x^2 b^2 - (-1)^i \cdot 2(k_x^2 b^2 + \alpha^2)^{1/2}]^{1/2},$$
$$v_x^{(i)}(k_x) = \frac{\hbar k_x}{m}[1 + (-1)^i/(k_x^2 b^2 + \alpha^2)^{1/2}], \qquad (8)$$
$$\tilde{\varepsilon} = 2mEb^2/\hbar^2 - 1.$$

The degree of influence of the magnetic field is characterized by the parameter

$$\beta = 2eH/[mcW_0(\Phi_1 + \Phi_2)]. \qquad (9)$$

A characteristic singularity of the distribution function (5) is that besides the usual shift $J_i\{\mathbf{v}\}$ in $\mathbf{k}$ space, which depends on the carrier velocity, there exists an additional shift proportional to $J_i\{F_i\}$ determined by the scattering singularities. This circumstance influences directly the field dependence of the resistance tensor $\rho_{ik}(H)$.

As noted above, the kinetic coefficients of the investigated system depend substantially on the location of the Fermi level. We confine ourselves to the most interesting energy region, near the minigap ($\varepsilon = \tilde{\varepsilon}(\mathscr{E}_F)$, $|\varepsilon| \sim \alpha$). The criterion of the weakness of the magnetic field (in the sense of the ability to expand ρ_{ik} in powers of H) is smallness of the parameter β in the functional (6) compared with the characteristic scales of the corresponding functions. At the same time, the functions in (5) contain the small parameter β. It can be shown that the desired criterion in the considered region will be $\beta \ll \alpha^{1/2} \ll 1$. The procedure of expanding in powers of small β yields for the MR along the superlattice axis the "normal" quadratic field dependence:

$$\Delta=(\rho_{xx}(H)-\rho_{xx}(0))/\rho_{xx}(0)$$
$$=\beta^2\sum_{i=1}^{2}\int_0^{\Phi_i} d\varphi\left[\frac{dv_x^{(i)}}{d\varphi}-(-1)^i G\frac{dF_1(k_x^{(i)})}{d\varphi}\right]^2 \Big/ \sum_{i=1}^{2}\int_0^{\Phi_i} d\varphi[v_x^{(i)}]^2\sim(\omega_c\tau)^2\alpha^{-1/2}. \quad (10)$$

Here

$$G=\left[\sum_{i=1}^{2}(-1)^{i-1}\int_0^{\Phi_i} d\varphi\, v_x^{(i)} F_1(k_x^{(i)})\right]\Big/\left[\sum_{i=1}^{2}\int_0^{\Phi_i} d\varphi\, F_0(k_x^{(i)})\right]. \quad (11)$$

At $\alpha^{1/2} \lesssim \beta \ll 1$ one can no longer expand in powers of β, although the period of motion over the equal-energy trajectory (in the first miniband) still exceeds the relaxation time. The field dependence of the MR differs thus substantially from quadratic in a wide range of intermediate weak fields.

If $\beta \gg \alpha^{1/2}$ we can use the fact that in the first miniband the dispersion law (1) deviates from quadratic, and the functions $F_{0,1}(k_x^{(1)}(\varphi))$ deviate from constants only during a small fraction of the period $(\Delta\varphi/\Phi_1\sim\alpha)$ in the vicinity of $k_x^{(1)}(\varphi)=0$, where the electron passes with practically no scattering. This circumstance allows us to put $\alpha=0$ in Eqs. (1)–(4). We then obtain in the entire region $\alpha^{1/2} \ll \beta < \infty$ (accurate to $(\alpha^{1/2}/\beta)^3$) the following expression for the MR:

$$\Delta=\frac{3}{\pi}(\varepsilon+2\alpha)^{1/2}+\frac{2}{\pi}(\varepsilon+2\alpha)\left[\frac{9}{2\pi}-\frac{1}{2\beta\,\mathrm{th}(\pi/2\beta)}-\frac{2}{\pi}\frac{(2\beta)^2}{1+(2\beta)^2}\right], \quad (12)$$

i.e., in the region of intermediate weak fields $\alpha^{1/2} \ll \beta < 1$ we have for the MR $\Delta\propto \text{const}-|H|^{-1}$, and only at $\beta \gg 1$ is the normal relation $\Delta\propto\text{const}-H^{-2}$ reached.

Note that the anomalous field dependence of the MR can be the result of abrupt singularities both in the dispersion law and in the scattering operator. (Simple models that illustrate this statement are considered in the Appendix.) The concrete form of the $\Delta(H)$ dependence is governed in our case precisely by the selectivity of the scattering processes. Neglect of this selectivity (i.e., when the τ approximation is used) leads to the qualitatively incorrect result $\Delta\propto|H|$.

It must be borne in mind that the described anomalies do not occur if the Fermi level is quite close to the saddle point: $|\varepsilon+2\alpha|\sim\alpha^2$. In this case the electron stays during the major part of the cyclotron period near $k_x=0$ (Ref. 5), so that the functions in expression (5) do not have two substantially different scales. Accordingly, the quadratic law (8) extends over the entire $\beta \ll 1$ region.

We conclude by assessing the dependence of the MR on the position of the Fermi level (i.e., on the near-surface carrier density), as determined by the parameter ε. At $|\varepsilon| \gg \alpha$ there is practically no MR. Indeed, at large negative ε/α the FC consists of two separate parts with almost-isotropic dispersion law. At large positive ε/α the fraction of carriers with mass anisotropy is negligibly small, and in the constant G [Eq. (1)] the contributions of the minibands cancel one another accurate to $\sim\alpha/\varepsilon$. The anomalous MR of the first miniband [Eq. (12)] is cancelled out, according to the general form of the function $f_i(\varphi)$ (5), by an analogous contribution from the second miniband. As a result, $\Delta\sim\alpha/\varepsilon$ in all the magnetic-field regions. In the region $|\varepsilon|\sim\alpha$, the dependence of the MR on ε is determined from Eqs. (10)–(12) through the energy dependences of the quantities $\beta, k_x^{(i)}, v_x^{(i)}$ they contain. As ε goes though the threshold value $\varepsilon=-2\alpha$ the MR resistance increases steeply with increasing ε, owing to the behavior of $\Phi_1(\varepsilon)$ near the saddle point. In the gap region $(-2\alpha<\varepsilon<2\alpha)$, the MR depends weakly on ε. At $\varepsilon=2\alpha$ the function undergoes a discontinuity followed by a change of slope. This is due to the jumplike appearance of a finite Φ_2 when the second miniband becomes populated by carriers.

This $\Delta(\mathscr{E}_F)$ dependence agrees with that obtained in experiment[1] (at the parameters $\alpha\sim\frac{1}{20}, \beta\approx0.6$). Direct observation of an anomalous field dependence of the MR under the conditions of this experiment should be expected in magnetic fields $H\sim 1$ to 5 kG.

Note also the following circumstance. We have assumed the unrenormalized mass of the two-dimensional electrons to be isotropic $(m_x=m_y\equiv m)$, whereas actually $m_y/m_x\approx\cos\theta$. This weak anisotropy does not alter substantially our present results (or, incidentally, those in other cases[4]). Indeed, it can be easily shown that m should be replaced in the quantities α, $\tilde{\varepsilon}$, and v_x and by $(m_y m_x)^{1/2}$ in v_y; in the definition of β [see (9)], m is replaced by m_y. Under these transformations, Eqs. (5)–(7) and (10)–(12) retain their form.

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APPENDIX

We consider two simple models that illustrate the influence exerted on the kinetic coefficients by the sections where the dependence on **k** is strong in the dispersion law and in the scattering probability.

1. Stepwise scattering. We assume the standard carrier dispersion law $\mathscr{E}(\mathbf{k})=\hbar^2k^2/2m$, and take the scattering probability in a form that imitates valley selectivity:

$$W(\mathbf{k},\ \mathbf{k}')=w\theta(k_x,\ k_x')+W_0, \quad (A1)$$

i.e., the electrons are scattered predominantly in their "own" half of **k** space. We write the kinetic equation for the distribution function $f(\varphi,\mathscr{E})$ in terms of angle variables:

$$\omega_c\frac{df}{d\varphi}+e\mathbf{E}f_0'\mathbf{v}+\int_0^{2\pi} d\varphi'\ W(\varphi,\varphi')\,(f(\varphi)-f(\varphi'))=0, \quad (A2)$$

where φ is the polar angle in the **k** plane and f_0' is the derivative of the equilibrium Fermi distribution function with respect to energy.

The departure term in the collision integral can be represented in the form

$$f(\varphi)\int_0^{2\pi} d\varphi'\, W(\varphi,\varphi')=f(\varphi)\pi(2W_0+w)\equiv f(\varphi)/\tau, \quad \text{(A3)}$$

and the arrival term in the form

$$\int_0^{2\pi} 2\varphi' f(\varphi')W(\varphi,\varphi')=wA\,\mathrm{sgn}(\cos\varphi), \quad \text{(A4)}$$

where the constant A is defined as

$$A=\int_{-\pi/2}^{\pi/2} d\varphi\, f(\varphi). \quad \text{(A5)}$$

Taking the introduced notation into account, we can rewrite (A2) in the form

$$\omega_c(df/d\varphi)+(f/\tau)=-e\mathbf{E}\mathbf{v}f_0'+wA\,\mathrm{sgn}\,(\cos\varphi). \quad \text{(A6)}$$

After finding the solution of (A6), we determine A in self-consistent manner and, calculating then the resistivity tensor ρ_{ik}, we find that ρ_{yy} does not depend on H, ρ_{xy} is given by the usual formula and contains no scattering characteristics, and the MR along the x axis is given by

$$\Delta=\frac{w}{2W_0}\frac{\psi(2\beta/\pi)-(2\beta/\pi)^2/(1+\beta^2)}{(\pi^2/8)[2W_0/w+\psi(2\beta/\pi)]+(1+\beta^2)^{-1}}, \quad \text{(A7)}$$

where

$$\psi(x)=x\,\mathrm{th}\,(1/x), \qquad \beta=\omega_c\tau. \quad \text{(A8)}$$

Thus, $\Delta(H)$ is an even function of the magnetic field, but has an essential singularity as $H\to 0$:

$$\Delta(H)=[2w^2\psi(2\beta/\pi)/W_0(4w+\pi^2W_0)] \propto |H|(1-2\exp\{-\pi/|\omega_c|\tau\}). \quad \text{(A9)}$$

$\Delta(H)\to\infty$ if $W_0\to 0$.

2. Singular dispersion law. Let, conversely, the scattering be isotropic and described by a relaxation time τ, and let the dispersion law be chosen in the form

$$\mathscr{E}(\mathbf{k})=\hbar^2[k_y^2+(k_x-k_0\,\mathrm{sgn}\,k_x)^2]/2m \quad \text{(A10)}$$

(this is the limiting case of the $\mathscr{E}_1(\mathbf{k})$ dependence in Eq. (1) as $\alpha\to 0$). At $\mathscr{E}_F>\hbar^2k_0^2/2m$ the FC constitutes two identical circular arcs whose angle is

$$\gamma=\arccos[-\hbar k_0/(2m\mathscr{E}_F)^{1/2}], \qquad \pi/2<\gamma<\pi, \quad \text{(A11)}$$

which are symmetrically attached to each other.

Solving in the usual manner the kinetic equation written in the relaxation-time approximation, with allowance for the dispersion law (A10), and calculating next the MR, we obtain, in particular, at $\pi-\gamma\ll 1$

$$\Delta\approx\frac{2\psi(\beta/\gamma)(2\gamma/\pi+\beta^2-1)}{(\gamma/\pi)[1-2\psi(\beta/\gamma)]+(3\gamma/\pi-2)\beta^2}. \quad \text{(A12)}$$

As $H\to 0$ we have $\Delta\propto\psi(\beta/\gamma)\propto|H|$.

[1)] For order-of-magnitude estimates we use here and below the phenomenological relaxation time τ determined from the mobility in the absence of a magnetic field. It will be shown that a correct description of the kinetics of the considered system is impossible in the τ approximation, and the kinetic equation must be solved with an integral collision term.

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