Parametric x-ray radiation with highly asymmetric diffraction

V. G. Baryshevskiĭ, A. O. Grubich, and I. D. Feranchuk

V. I. Lenin Belorussian State University

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The spectral and angular distributions for parametric x-ray radiation are found in the limit of extreme asymmetric diffraction. An expression is derived for the total number of photons recorded by a detector with a specified angular resolution.

1. INTRODUCTION

In Refs. 1–3 a novel Vavilov-Cherenkov mechanism was proposed for parametric x radiation by relativistic charged particles passing through a crystal. According to Refs. 1–3, this type of emission occurs when the Cherenkov radiation condition

$$1-vn\cos\vartheta=0 \tag{1}$$

($\hbar=c=1$) and the Bragg diffraction condition

$$(\mathbf{k}+\boldsymbol{\tau})^2-\mathbf{k}^2\leqslant 2\omega^2|n-1| \tag{2}$$

are both satisfied. Here $\mathbf{v}$ is the velocity of the particle, ϑ is the emission angle for a photon of momentum $\mathbf{k}$ and frequency ω, and τ is a reciprocal lattice vector. Condition (2) requries that the refractive index n be greater than unity even for energies above a few refractive index n be greater than unity even for energies above a few keV (hard x-ray limit). A time-dependent theory of parametric x radiation (PXR) for perfect crystals was developed in Refs. 2–5 for Laue and Bragg diffraction, while the influence of block structure on the integrated PXR intensity was considered in Refs. 6 and 7.

Because $|n-1|\approx 10^{-5}$–10^{-6} at x-ray frequencies, conditions (1) and (2) are consistent only for ultrarelativistic particles of energy[1]

$$E\gg E_{\text{thr}}=\frac{m}{(2|n-1|)^{1/2}}\approx(10^2–10^3)m, \tag{3}$$

where m is the particle mass. If inequality (3) is not satisfied, the spectral density of the PXR drops as $\propto(E/E_{\text{thr}})^4$ when the particle energy decreases.[5] Rivlin[8] made the interesting observation that Cherenkov x-ray emission should be possible even for nonrelativistic particles ($v\ll 1$), because the crystal supports stationary Bloch waves which always have harmonics whose phase velocity is much less than the speed of light. However, if (2) [and therefore also (3)] is not satisfied, the amplitude of these harmonics is very low and the intensity of the Cherenkov x-rays excited by nonrelativistic electrons is therefore nearly zero (only 10^{-15}–10^{-17} photons per electron are generated over a photon absorption length).[9] Parametric x-ray radiation was recently observed experimentally,[10] and its kinematic properties (the resonance frequencies and the photon energies angles) were found to agree with the theoretical results.[1–4] However, the explicit equations derived in Refs. 1–7 for Laue and Bragg diffraction do not yield a detailed description of the spectral and angular distributions of the XPR under the experimental conditions in Ref. 10 because the x-ray diffraction there was highly asymmetric (see, e.g., Ref. 11).

In this paper we analyze a dispersion equation for PXR in the asymmetric diffraction limit and derive explicit formulas for the spectral and angular distributions and for the total x-ray intensity.

2. SPECTRAL AND ANGULAR DISTRIBUTIONS

In the experiment in Ref. 10, electrons for energy 900 MeV bombarded the edge of a diamond single crystal, and the yield of x-ray photons emitted normal to the incident electron beam was measured (Fig. 1a). The irradiated crystal was oriented with its ⟨110⟩ axis parallel to the beam, so that the x-rays emitted parallel to the beam were diffracted by the (100) planes of the diamond crystal and were recorded by a detector (Fig. 1b). (There was also another series of measurements in Ref. 10 in which the beam was parallel to the ⟨100⟩ axis.) The Bragg angle in this case was equal to 45°, i.e., the x-ray scattering geometry in the crystal differed from both the Laue and the Bragg configurations. This situation has been referred to in the literature as transitional Laue-Bragg (or extreme asymmetric[11]) diffraction. An exhaustive theoretical analysis of asymmetric diffraction was recently given in Ref. 12 for the case of neutron scattering.

To find the PXR intensity, we note that far from the crystal the diverging electromagnetic wave emitted by the crystal may be regarded as spherical, and the number of photons (radiation intensity) of frequency ω and polarization $s=\sigma,\pi$ along the direction $\Omega=\mathbf{k}/k$ is given by the formula[13]

$$N_{\Omega\omega s}=\frac{e^2\omega}{(2\pi)^2}\left|\int dt\,\mathbf{v}\mathbf{E}_{\mathbf{k}s}^{(-)}(\mathbf{r},\omega)e^{-i\omega t}\right|^2, \tag{4}$$

which can be derived classically or by using quantum electrodynamics. Here $e^2=1/137$, and $\mathbf{v}\equiv\mathbf{v}(t)$ and $\mathbf{r}\equiv\mathbf{r}(t)$ are the velocity and position vector of the charged particle. The radiation field $\mathbf{E}_{\mathbf{k}s}^{(-)}$ is the exact solution of the homogeneous Maxwell equations for the scattering of a plane wave $\mathbf{e}_s\exp/(i\mathbf{kr})$ of unit amplitude by the crystal ($\mathbf{e}_s$ is the polarization vector of the photon); it is given asymptotically as the sum of the incidenct plane wave plus a converging spherical wave. If one knows the solution $\mathbf{E}_{\mathbf{k}s}^{(+)}$ of the homogeneous Maxwell equations, where $\mathbf{E}_{\mathbf{k}s}^{(+)}$ describes the photon scattering by the crystal and reduces to an ordinary diverging spherical wave at infinity, the field $\mathbf{E}_{\mathbf{k}s}^{(-)}$ can be found from the relation[13,14]

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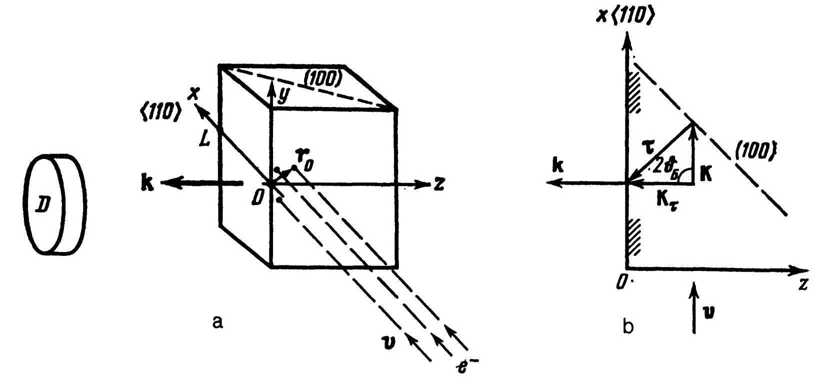

FIG. 1. Geometry of the incident electron beam (e^-), crystal, and detector (D); b) Diffraction geometry for the radiation field $\mathbf{E}_{\mathbf{k}s}^{(+)}$.

$$\mathbf{E}_{\mathbf{K}s}^{(-)}=(\mathbf{E}_{-\mathbf{K}s}^{(+)})^*. \tag{5}$$

We thus obtain the familiar system of equations[15]

$$\begin{aligned}&(\mathbf{K}^2/\omega^2-1-\chi_0^*)\mathbf{E}_{\mathbf{K}s}^{(-)}-C_s\chi_{\bar\tau}^*\mathbf{E}_{\mathbf{K}_\tau s}^{(-)}=0,\\&(\mathbf{K}_\tau^2/\omega^2-1-\chi_0^*)\mathbf{E}_{\mathbf{K}_\tau s}^{(-)}-C_s\chi_\tau^*\mathbf{E}_{\mathbf{K}s}^{(-)}=0,\end{aligned} \tag{6}$$

with complex-conjugate polarizabilities χ_0 and $\chi_{0\bar\tau}$ for the field $\mathbf{E}_{\mathbf{k}s}^{(-)}$ in the crystal for the case of two-wave diffraction. Here $\mathbf{K}_\tau=\mathbf{K}+\tau$ and $C_s=\mathbf{e}_s\mathbf{e}_{\tau s}$.

The tangential components of the wave vectors must be continuous at the crystal/vacuum interface; we allow for this in the usual way by setting

$$\mathbf{K}=\mathbf{k}-\omega g\mathbf{N}, \tag{7}$$

where $|\mathbf{k}|\equiv k=\omega$ and the vector $\mathbf{N}$ is normal to the crystal ($\mathbf{N}=-\mathbf{e}_z$). We also write

$$\gamma_0=\mathbf{kN}/\omega,\quad \gamma_\tau=(\mathbf{k}+\boldsymbol{\tau})\mathbf{N}/\omega,\quad \alpha=-(2\mathbf{k}\boldsymbol{\tau}+\boldsymbol{\tau}^2)/\omega^2. \tag{8}$$

The condition for the linear system (6) to be solvable leads to a dispersion equation that determines the possible values of g in (7):

$$g^4+bg^3+cg^2+dg+e=0, \tag{9}$$

where $b=-2\gamma_0$, $c=4\gamma_0\gamma_\tau$, $d=2\gamma_0(\alpha+\chi_0^*)$, $e=\chi_0^{*2}-C_s^2|\chi_\tau|^2+\alpha\chi_0^*$ (we set $\chi_\tau^*=\chi_{\bar\tau}$). Equation (9) is similar to the dispersion equation whose roots were found in Ref. 12. Only two roots of Eq. (9) give wave vectors (7) which describe waves that decay as a function of depth z, and this fact will be important in what follows; we denote them by $\mathbf{K}_\mu$, where $\mu=1, 2$. We will also see that for at least one of the differential waves, the wave vector $\mathbf{K}_{\tau\mu}=\mathbf{K}_\mu+\tau$ has a positive projection on the normal $\mathbf{N}$ to the crystal ($\mathbf{K}_{\tau\mu}\mathbf{N}>0$). When joining the wave solutions at the crystal/vacuum interface one must thus consider the refracted wave with wave vector $\mathbf{k}'$ propagating in the vacuum along the surface of the crystal. The amplitudes

$$\mathbf{E}_{\mathbf{K}s}^{(-)}=\sum_{\mu=1}^{2}[\mathbf{e}_sA_\mu\exp(i\mathbf{K}_\mu\mathbf{r})+\mathbf{e}_{\tau s}A_{\tau\mu}\exp(i\mathbf{K}_{\tau\mu}\mathbf{r})] \tag{10}$$

of the waves $E_{\mathbf{k}s}^{(-)}$ in the crystal can be found by matching the wave solutions on the crystal surface:

$$A_1=\frac{(2g_2\gamma_0+\chi_0^*)(\gamma_0'-\gamma_\tau+g_2)}{(g_2-g_1)[2\gamma_0(\gamma_0'-\gamma_\tau)+2\gamma_0(g_1+g_2)+\chi_0^*]},\quad A_2=1-A_1,$$

$$A_{\tau\mu}=\Delta_\mu A_\mu,\quad \Delta_\mu=-(2g_\mu\gamma_0+\chi_0^*)/\chi_\tau C_s,\quad \gamma_0'=\mathbf{k}'\mathbf{N}/\omega,$$

$$\mathbf{e}_\sigma=\frac{[\mathbf{k}\boldsymbol{\tau}]}{|[\mathbf{k}\boldsymbol{\tau}]|},\quad \mathbf{e}_\pi=\frac{[\mathbf{k}\mathbf{e}_\sigma]}{|[\mathbf{k}\mathbf{e}_\sigma]|},\quad \mathbf{e}_{\tau\sigma}=\mathbf{e}_\sigma,$$

$$\mathbf{e}_{\tau\pi}=\frac{[(\mathbf{k}+\boldsymbol{\tau})\mathbf{e}_\sigma]}{|[(\mathbf{k}+\boldsymbol{\tau})\mathbf{e}_\sigma]|}. \tag{11}$$

(To simplify the notation we have omitted the polarization subscript $s=\sigma, \pi$ from the roots g_μ of the dispersion equation.) Equations (9)–(11) completely specify the eigenstates $\mathbf{E}_{\mathbf{K}s}^{(-)}$ of the electromagnetic field in the asymmetric diffraction case and can be used to calculate the PXR intensity.

Since by definition PXR is generated by a uniformly moving charged particle, we set $\mathbf{r}(t)$ in Eq. (4) equal to $\mathbf{r}_0+\mathbf{v}t$, where $\mathbf{r}_0=(0, y, z)$ is the position vector for the point at which the particle enters the crystal (Fig. 1a). Since the particle makes a small angle with the X axis, the distance vT traveled in the crystal is $\approx L$, where L is the thickness of the crystal along the X axis. If we neglect the interference of the radiation fields emitted by the particle moving in the vacuum and inside the crystal, we get the final expression

$$N_{\Omega\omega s}=\frac{e^2\omega(\mathbf{v}\mathbf{e}_{\tau s})^2}{4\pi^2}\left|\sum_{\mu=1}^{2}A_{\tau\mu}\exp(-K_{\mu z}''z)l_\mu[\exp(-iL/l_\mu)-1]\right|^2 \tag{12}$$

for the frequency-angle probability density (4) for x-ray emission. Here $K_{\mu z}''=\operatorname{Im}K_{\mu z}\simeq\omega g_\mu''$ (g_μ'' is the imaginary part of the root $g_\mu=g_\mu'+ig_\mu''$, $g_\mu''\geqslant 0$), and the radiation coherence length

$$l_\mu=(\mathbf{v}\mathbf{K}_{\tau\mu}-\omega)^{-1} \tag{13}$$

is equal to $1/q_\mu$, where q_μ is the longitudinal momentum transferred to the crystal. [We note that only the fields with wave vectors $\mathbf{K}_{\tau\mu}$ contribute in (12), because $l_\mu\approx-\omega^{-1}$ for waves with $\mathbf{K}_\mu$ and thus remains finite, unlike (13).]

Analysis of (12) shows that $N_{\Omega\omega s}$ is greatest if

$\mathrm{Re}\, q_\mu = 0$; this condition leads to a dispersion equation (1) for PXR in a crystal,[13] where $\cos\vartheta = \mathbf{K}'_{\tau\mu}\mathbf{v}/|\mathbf{K}'_{\tau\mu}\mathbf{v}|$ is the cosine of the angle between the vectors $\mathbf{K}'_{\tau\mu}$ and $\mathbf{v}$, $\mathbf{K}_{\tau\mu} = \mathbf{K}'_{\tau\mu} + i\mathbf{K}''_{\tau\mu}$, and $n = \omega^{-1}|\mathbf{K}'_{\tau\mu}|$ is the refractive index for waves with wave vector $\mathbf{K}'_{\tau\mu} = \mathbf{k} + \boldsymbol{\tau} - \omega g'_\mu \mathbf{N}$:

$$n=(1-\alpha+g_\mu'^2-2g_\mu'\gamma_\tau)^{1/2}. \qquad (14)$$

Because of the exponentials $\exp(-K''_{\mu z}z)$ in (12) (they describe the absorption in the crystal for x-rays emitted normal to v), $N_{\Omega\omega s}$ decreases for large z. We recall that z is measured from the point at which the charge particle enters the crystal (Fig. 1a). If the radius of the incident beam is $\gtrsim l_{\text{abs}} = 1/2K''_{\mu z}$, the absorption depth for the x-rays, then the total x-ray yield normal to the beam will depend on the spatial distribution of the particles, in contrast to the radiation in the forward direction.

Since the x-rays are emitted at small angles relative to the normal $\mathbf{N}$, we can write

$$\boldsymbol{\Omega}=\mathbf{N}\cos\vartheta+\boldsymbol{\vartheta}, \qquad (15)$$

with $\boldsymbol{\vartheta} \equiv (\vartheta_x, \vartheta_y)$, $\vartheta \ll 1$, for the unit vector $\boldsymbol{\Omega} = \mathbf{k}/\omega$ along the direction of emission.

We first consider the simplest case when the velocity vector $\mathbf{v}$ of the particle is strictly parallel to the surface of the crystal: $\mathbf{v} = (v_x, 0, 0)$ (Fig. 1a). According to (7) and (13), the coherence length in this case is real-valued and independent of the root g_μ of the dispersion equation:

$$l_\mu=l\equiv[\mathbf{v}(\mathbf{k}+\boldsymbol{\tau})-\omega]^{-1}. \qquad (16)$$

Moreover, the expression $|l(e^{-L/l}-1)|^2$ in (12) tends to $2\pi\delta(1/l)$ as $L\to\infty$, where $\delta(x)$ is the Dirac delta-function. The probability density (12) for emission of an x-ray photon is thus equal to

$$N_{\Omega\omega s}=L\frac{e^2\omega(\mathbf{v}\mathbf{e}_{\tau s})^2}{2\pi}\delta\left(\frac{1}{l}\right)\left|\sum_{\mu=1}^{2}A_{\tau\mu}\exp(-\omega g_\mu''z)\right|^2. \qquad (17)$$

We stress that in contrast to the situation in Refs. 1–4, the x-ray intensity is proportional to the total thickness L of the crystal rather than to the photon absorption length.

Because of the δ-function in the right-hand side of (17), the longitudinal momentum transferred to the crystal vanishes:

$$l^{-1}\equiv\mathbf{v}(\mathbf{k}+\boldsymbol{\tau})-\omega=0. \qquad (18)$$

Thus, if the natural modes of the crystal are described by the dispersion equation (9), the mode that is radiated during PXR will be determined by the additional condition (18), which reduces by one the number of independent parameters in the coefficients of Eq. (9). For example, (18) and the condition $\mathbf{v} = v\mathbf{e}_x$ lead to

$$\omega=v\tau_x/(1-v\vartheta_x) \qquad (19)$$

for the x-ray frequency, from which we obtain

$$\alpha=-2(1-v\cos\vartheta)\approx-(m^2/E^2+\vartheta^2), \qquad (20)$$

for the parameter $\alpha = -(2\mathbf{k}\boldsymbol{\tau}+\boldsymbol{\tau})^2/\omega^2$ characterizing the mismatch from precise Bragg diffraction $(\mathbf{k}+\boldsymbol{\tau})^2 = \omega^2$; here the Lorentz factor $E/m = (1-v^2)^{-1/2}$ of the particle is $\gg 1$. Because α defined by (20) and the real part $\chi_0 = -\omega_p^2/\omega^2$ of the polarizability are both <0 far from the absorption line, the real part of the coefficient $d = 2\gamma_0(\alpha+\chi_0^*)$ in the dispersion equation (9) does not vanish (here ω_p is the electron plasma frequency of the crystal). This implies that of the various solutions of (9) considered in Ref. 12, only

$$g_1=-\frac{\chi_0^*}{2\gamma_0}+\frac{C_s^2|\chi_\tau|^2}{2\gamma_0(\chi_0^*+\alpha)}, \qquad (21)$$

$$g_2=\gamma_\tau+(\gamma_\tau^2+\chi_0^*+\alpha)^{1/2} \qquad (22)$$

satisy the condition (18) for PXR. We note that the roots g_1 and g_2 describe waves that decay with distance in the crystal (along the Z axis). Indeed, the imaginary part of (21) is $g_1'' \approx \chi_0''/2\gamma_0 > 0$. To calculate the imaginary part of g_2 in (22), we must recall that by (19) and (20), $(\gamma_\tau^2+\alpha)<0$, so that $g_2'' = [-(\gamma_\tau^2+\alpha+\chi_0')]^{1/2}$. We also observe that the refractive index (14) is greater than unity for g'_μ given by (21), (22) and α by (20):

$$n_1\approx(1-\alpha)^{1/2}>1, \quad n_2\approx(1-\gamma_\tau^2-\alpha)^{1/2}>1.$$

The threshold character of the PXR results from the fact that when the particle velocity (energy) decreases, Eq. (18) is satisfied for lower frequencies (19). If the radiation wavelength exceeds the interatomic distance ($\lambda > 2d_0$), diffraction cannot occur and the refractive index $n_0 = (1+\chi_0')^{1/2}$ for the x-rays satisfies the usual inequality $n_0 < 1$ rather than (14).

Since $g_2'' \gg g_1''$ while $|g_1| \ll |g_2|$ for $\gamma_\tau \sim |\chi_0|^{1/2}$, we may neglect the term with $\mu = 2$ in the right-hand side of (17) because of the factor $\exp(-\omega g_2'' z)$; moreover, the root g_1 is negligible compared to g_2 in the remaining expression with the amplitude $A_{\tau 1}$. We then get the result

$$N_{\Omega\omega s}=L\frac{e^2\omega(\mathbf{v}\mathbf{e}_{\tau s})^2}{2\pi}\delta\left(\frac{1}{l}\right)\frac{C_s^2|\chi_\tau|^2}{|\chi_0+\alpha|^2}\exp(-\omega\chi_0''z) \qquad (23)$$

for the spectral and angular distribution for parametric x-ray radiation. Integrating (23) with respect to ω and using the defining property of the δ-function, we obtain

$$N_{\Omega s}=L\frac{e^2\omega_0(\mathbf{v}\mathbf{e}_{\tau s})^2C_s^2|\chi_\tau|^2}{2\pi|m^2/E^2+\vartheta^2-\chi_0|^2}\exp(-\omega_0\chi_0''z) \qquad (24)$$

for the angular distribution, where the frequency ω_0 is given by (19).

The coefficient C_s is given by

$$C_s=\mathbf{e}_s\mathbf{e}_{\tau s}=\begin{cases}1, & s=\sigma\\ \cos 2\vartheta_B & s=\pi\end{cases}.$$

Since $2\vartheta_B \approx \pi/2$ for extreme asymmetric diffraction, the PXR is polarized normal to the diffraction plane:

$$\sum_s N_{\Omega s}\approx N_{\Omega\sigma}=L\frac{e^2\omega_0\vartheta_y^2|\chi_\tau|^2}{2\pi|m^2/E^2+\vartheta^2-\chi_0|^2}\exp(-\omega_0\chi_0''z). \qquad (25)$$

If the particle moves at a small angle θ to the X axis, we can write

$$\mathbf{v}=v\cos\theta\mathbf{e}_x+v\boldsymbol{\theta} \quad (26)$$

for the velocity, where $\boldsymbol{\theta} = (\theta_y,\theta_z)$. In the case considered above, $\theta = 0$; however, it is clear that all of the previous arguments remain valid for $\boldsymbol{\theta} = (\theta_y,0) \neq 0$—only expression (19) requires modification, in accordance with the equality $\omega = \mathbf{v}\boldsymbol{\tau}(1 - \mathbf{v}\mathbf{k}/\omega)$.

At first glance it may appear that the situation might change with the z-component $v_z = \theta_z$ of the particle is nonzero, because the coherence length (13) then depends on the root g_μ, so that instead of condition (18), which decreases the number of independent parameters in the coefficients of (9), we get the additional constraint

$$\mathbf{v}(\mathbf{k}+\boldsymbol{\tau})-\omega+\omega g_\mu'\theta_z=0 \quad (27)$$

on g_μ' from the requirement that $\mathrm{Re}\,(1/l_\mu) = 0$. Although (27) must now be solved simultaneously with the dispersion equation (9), a detailed analysis reveals that nothing is really changed—as before, the real part of the renormalized coefficient d in (9) can never vanish, and Eqs. (21) and (22) therefore remain valid for the roots of the dispersion equation for the radiated wave. The distributions (23)–(25) found above thus remain correct.

We also observe that the above formulas describe PXR from a charged particle in any type of periodic material, including artificial materials.

3. TOTAL NUMBER OF PXR PHOTONS

We now use (25) to calculate N_d^τ, the total number of PXR photons recorded by a detector with a specified angular resolution $\vartheta_d = a/R$, where a is the length of the detector and R is its distance from the source; N_d^τ is the quantity that was measured experimentally in Ref. 10.

Integrating Eq. (25) over the angular variables, we get the expression

$$N_d^\tau = \frac{e^2\omega_0 L|\chi_\tau|^2}{4}\exp(-\omega_0\chi_0''z)\left[\ln\frac{\vartheta_d^2+\vartheta_f^2}{\vartheta_f^2}-\frac{\vartheta_d^2}{\vartheta_d^2+\vartheta_f^2}\right], \quad (28)$$

where ω_0 is given by (19) and

$$\vartheta_f^2 = m^2/E^2-\chi_0'. \quad (29)$$

We note that according to (25), the PXR intensity for a given reflection falls off quite slowly ($\propto 1/\vartheta$) as the radiation angle ϑ increases. The result (28) thus shows that N_d^τ is sensitive to the angular dimension ϑ_d of the detector even when $\vartheta_d \gg \vartheta_f$. In this respect PXR differs from the other emission mechanisms for relativistic particles, for which there always exists an effective radiation angle ϑ_{eff} with the property that the radiation intensity saturates and is independent of ϑ_d for $\vartheta_d > \vartheta_{\mathrm{eff}}$.

Finally, we discuss how multiple electron scattering and the mosaic structure of imperfect crystals affect the properties of the PXR. Among other things, elastic electron-atom scattering alters the angle between the X axis and the particle velocity $\mathbf{v}$, while the direction of the vector τ varies from block to block in the crystal. These factors change the coherence length given by Eq. (18). If the electron trajectories and the rotational angle distribution of the individual blocks in the crystal are statistically independent, we can average over them (using, e.g., the technique considered in Ref. 13) and obtain an approximate expression for N_d^τ of the same form as (28) but with ϑ_f^2 given by

$$\vartheta_f^2 \approx m^2/E^2-\chi_0'+\overline{\theta_s^2}+\overline{\delta^2} \quad (30)$$

instead of (29); here $\overline{\delta^2}$, the mean-square rotation angle of the individual blocks, is comparable in order of magnitude to the square of the block-structure parameter; $\overline{\theta_s^2}$ is the mean-square angle for multiple scattering. We can estimate $\overline{\theta_s^2}$ from the formula $\overline{\theta_s^2} = E_s^2 L(E^2 L_R)^{-1}$ valid for a homogeneous medium, where $E_s \approx 21$ MeV and L_R is the radiation length. Under the experimental conditions in Ref. 10, $\vartheta_f^2 \approx -\chi_0' \gg \overline{\theta_s^2}$, so that multiple scattering of electrons in the crystal had little effect on the total PXR intensity. The mean-square rotation angle $\overline{\delta^2}$ of the individual blocks in Ref. 10 was also much less than $-\chi_0'$, so that the block structure of the crystal also had little influence on N_d^τ.

We used Eq. (28) with ϑ_f^2 given by (30) to quantitatively describe the experimental results in Ref. 10, where the electron beam was incident on the edge of a diamond single crystal (Fig. 1a) and the beam diameter was less tthan the photon absorption length $l_{\mathrm{abs}} = (\omega\chi_0'')^{-1}$. The factor $\exp(-\omega\chi_0''z)$ in (28) can therefore be replaced by unity. The comparison of the experimental and theoretical results carried out in Ref. 10 showed that the theory of parametric x-ray radiation in the extreme asymmetric case yields results to close agreement with measurements.

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