Influence of three-electron collisions on the galvanomagnetic properties of an electron gas in longitudinal electric and ultraquantum magnetic fields

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A study is made of the behavior of an electron gas in longitudinal electric and quantizing magnetic fields when the electrons occupy a single lowest Landau level. Only the interactions of the electrons with charged impurities, with optical phonons, and with one another in the field of randomly distributed impurities are taken into account. The inelastic interaction of the electrons and optical phonons is assumed to be strong enough to prevent the electrons from penetrating into the active region. In this case a closed nonlinear integro-differential kinetic equation is obtained for the distribution function in the passive region. This kinetic equation is used to find the distribution function in the limits of weak and strong electric fields. The nonequilibrium properties of the electron gas are studied for the case of the electrical conductivity in longitudinal and transverse electric fields.

1. INTRODUCTION

In ultraquantum magnetic fields, when practically all of the electrons are found in the lowest Landau subband, the nonequilibrium thermogalvanomagnetic effects exhibit a number of features due to the quasi-one-dimensional character of the motion of the charge carriers.[1] In particular, in a system of electrons with a substantially nonmonotonic energy distribution, effects such as absolute negative conductivity[2] and the generation of plasma oscillations[3,4] or electromagnetic radiation[5,6] can arise.

An applied electric field can cause appreciable heating of the charge carriers due to the decrease in the electron scattering efficiency with increasing energy $\mathscr{E}$, primarily on account of the decrease in the density of electron states in the lowest Landau subband. In this case, Kogan[7] has predicted a possible thermal instability of the electron gas, assuming the existence of an electron temperature. Such an assumption is justified, for example, if the density of charge carriers is large enough[8] or if the heating electric field is oriented perpendicular to the magnetic field and the electron energy dissipation mechanism is quasielastic.[9] The behavior of an electron gas having a low carrier density in parallel electric and ultraquantum magnetic fields was first considered in Ref. 19, where it was shown that for quasielastic relaxation mechanisms so-called "electron runaway" takes place.

Substantial heating of the electron gas or the development of thermal instability either causes the next Landau subbands to be filled and thereby violates the ultraquantum condition or "switches on" some deeply inelastic scattering mechanism at high energy such as the spontaneous emission of optical phonons.[11,12] The kinetic properties of the electron gas in the first case have been analyzed in a number of papers (see, e.g., Ref. 1), and it was found that the distribution function can retain its two-scale nature.[13–15]

In the second case, if the energy $\hbar\omega_0$ of the optical phonons is less than the energy separation of the lowest and next-lowest Landau subbands, if the lattice temperature T_0 is low ($T_0 \ll \hbar\omega_0$), and if the interaction between the electrons and the optical phonons is sufficiently strong, then the electrons are practically all found in the lowest Landau subband even in the presence of appreciable heating of the electron gas. If the density of charge carriers is small, then collisions between electrons are unimportant, and one can find the distribution function with the aid of a linearized (with respect to the distribution function) version of the quantum-mechanical kinetic equation.[1] This limit has been studied in several papers (see, e.g., Ref. 1); we might mention one of the more detailed later studies,[16] in which the current-voltage characteristic in crossed electric and magnetic fields was found for semiconductors having a complex band structure, and Refs. 17 and 18, which give an analysis of kinetic phenomena in longitudinal electric and magnetic fields.

Because binary collisions of electrons are suppressed in ultraquantum magnetic fields,[1] one can also use a linearized kinetic equation in a strong electric field parallel to the magnetic field when the penetration of the charge carriers into the active region ($\mathscr{E} > \hbar\omega_0$) is the order of the characteristic energy for changes in the distribution function in the passive region ($\mathscr{E} < \hbar\omega_0$). This limiting case was considered in its most general form in Ref. 19.

If, on the other hand, the carrier density n and charged-impurity density N_i are not too high and not too low (10^{14} cm$^{-3} \lesssim n, N_i \lesssim 10^{16}$ cm^{-3}) and the electrical field is not too strong ($E \lesssim 100$ V/cm),[19] the behavior of the electrons in the active region is, as before, governed by their interaction with optical phonons, whereas in the passive region for $T_0 \ll \hbar\omega_0$ the charge-carrier energy is redistributed in three-particle electron-electron-electron or electron-electron-impurity collisions.[8] In fact, for polar semiconductors (e.g., InSb or GaAs), in which the interaction between the electrons and optical phonons is fairly intense, the relaxation of the charge-carrier momentum at the given values of the densities n and N_i occurs through elastic collisions with charged

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impurities or with optical phonons; at an energy $\mathscr{E} \sim \hbar\omega_0 \sim 0.02$ eV the characteristic elastic collision frequency is $\nu_i \sim 10^{10}$–10^{11} sec^{-1}, much smaller than the characteristic rate of spontaneous emission of optical phonons ($\nu_0 \sim 10^{12}$–10^{13} sec^{-1}), while the three-particle collision frequency $\nu_{ee} \sim 10^9$–10^{10} sec^{-1} is considerably higher than both the characteristic frequency for energy relaxation on acoustic phonons (10^7–10^8 sec^{-1}) and the frequency of compound scattering by optical phonons[20] as a consequence of their weak dispersion. In this case one cannot use the effective-temperature approximation but must solve the nonliner quantum-mechanical kinetic equation directly in order to find the distribution function. The case when the heating of the electron gas is due to the optical generation of "hot" electrons or to an applied electric field transverse to the magnetic field was considered by the present authors in Refs. 21 and 22. In the present paper we study the kinetic effects due to the heating of the electrons by an electric field directed parallel to the magnetic field, in the given ranges of carrier density and applied fields.

2. KINETIC EQUATION; BOUNDARY CONDITION

Let us consider a homogeneous n-type semiconductor in parallel magnetic (H) and electric (E) fields, assuming that the following condition is satisfied:

$$\hbar\Omega > \hbar\omega_0 \gg T_0, \tag{1}$$

where Ω is the cyclotron frequency of the charge carriers and T_0 is the lattice temperature. We assume that the electrons interact with optical phonons sufficiently intensely that only an insignificant number of the electrons penetrate into the active region under the influence of the electric field. Therefore, in accordance with inequality (1), practically all the electrons are found in the lowest Landau subband. Then the electron gas can be described by a distribution function f (the diagonal elecment of the electron density matrix) which in the steady and spatially uniform case depends only on the longitudinal (with respect to the magnetic field) quasimomentum P of the electrons and satisfies a one-dimensional quantum-mechanical kinetic equation[1]

$$\nu_E \partial f/\partial p = I\{f,\ p\}. \tag{2}$$

Here $I\{f, p\}$ is the collision integral, which generally takes into account all the electron interaction mechanisms, $\nu_E = eE/P_0$, and $p = P/P_0$, where $P_0 = (2m_e\hbar\omega_0)^{1/2}$, with m_e being the effective mass of the electrons. However, we shall take into account only the interaction of the electrons with one another, with charged impurities, and with optical phonons.

For the sake of simplicity, we shall assume in the following analysis that the lifetime of the nonequilibrium optical phonons is short enough that we can neglect the departure from equilibrium in the phonon distribution function N_q, i.e., by virtue of condition (1) we have $N_q = \exp(-\hbar\omega_0/T_0)$. Then, under the assumptions we have adopted, the collision integral for electrons with optical phonons can be written (cf. Ref. 16, for example).

$$I_{ef}\{f,p\} = -\nu_0 \frac{1}{(\varepsilon-1)^{1/2}}\theta(\varepsilon-1) \times\left\{f(p) - \frac{1}{2}\exp\left(-\frac{\hbar\omega_0}{T_0}\right)[f((p^2-1)^{1/2}) + f(-(p^2-1)^{1/2})]\right\} + \nu_0\frac{1}{(\varepsilon+1)^{1/2}}\theta(1-\varepsilon)\left\{\frac{1}{2}[f((p^2+1)^{1/2}) + f(-(p^2+1)^{1/2})] - f(p)\exp\left(-\frac{\hbar\omega_0}{T_0}\right)\right\}. \tag{3}$$

Here $\theta(x)$ is the Heaviside step function, and $\varepsilon = P^2/2m_e\hbar\omega_0 = p^2$ is the kinetic energy of the electron motion along the magnetic field scaled by the optical phonon energy $\hbar\omega_0$.

The interaction of the electrons with charge impurities is elastic, and the collision integral for these collisions is of the form[3,19]

$$I_{ei}\{f,\ p\} = -\nu_i\varepsilon^{-3/2}[f(p) - f(-p)], \tag{4}$$

where

$$\nu_i = \frac{\pi}{2\sqrt{2}}\frac{e^4N_i}{\varkappa^2 m_e^{1/2}(\hbar\omega_0)^{3/2}}[ue^u|\mathrm{Ei}(-u)|+1], \tag{5}$$

$\varkappa$ is the low-frequency dielectric constant, N_i is the density of charged impurities, $u = 8\varepsilon\omega_0/\Omega$, and Ei is the exponential integral function. In this expression we have neglected the screening of the Coulomb potential, since at the densities under consideration the Debye radius is quite large. Furthermore, we note that $ue^u|\mathrm{Ei}(-u)| \lesssim 1$, so that in our subsequent analysis we can for simplicity treat the quantity in square brackets in (5) as a constant.

In the first order of perturbation theory the collision integral for binary collisions of electrons vanished identically as a consequence of the quasi-one-dimensionality of the electron motion in an ultraquantum magnetic field. Therefore, we must take into account the interaction of the electrons with a third body. Let us for simplicity study a compensated semiconductor in which the density of charged impurities is much higher than the electron density: $N_i \gg n$ (obviously, for $n \sim N_i$ the results obtained below will remain qualitively the same). In this limit we may ignore all but the scattering of the electrons in the field of the randomly distributed impurities. Furthermore, at the electron densities under consideration the characteristic value of the quasimomentum transfer is much lower than the average quasimomentum. Consequently, the diffusion approximation is valid for the electron-electron-impurity collision integral, which in the second order of perturbation theory is given by the expression (see Appendix 1)

$$I_{ee}\{f,p\} = \frac{e^8N_i}{4\hbar q_D\lambda^2\varkappa^4(\hbar\omega_0)^3}\operatorname{sign} p\frac{\partial}{\partial p} \times\left\{\frac{1}{\varepsilon}\int_\Delta^\infty\frac{d\varepsilon'}{\varepsilon'}\frac{1}{\sqrt{\varepsilon}+\sqrt{\varepsilon'}}\left[\frac{1}{2p}\frac{\partial}{\partial p}f(p)f_s(\varepsilon') - f(p)\frac{\partial}{\partial\varepsilon'}f_s(\varepsilon')\right]\right\}. \tag{6}$$

Here $f_s(\varepsilon) = 1/2[f(p) + f(-p)]$ is the symmetric part of the distribution function, Δ is the cutoff parameter for the logarithmic divergence at low energies, $\lambda^2 = \hbar c/eH$, and q_D^{-1} is the static Debye screening radius (cf. Refs. 1 and 23, for example):

$$q_D^2=\frac{4\pi e^2}{\varkappa}\rho(\hbar\omega_0)\int_0^\infty\frac{d\varepsilon}{\varepsilon^{1/2}}\left(-\frac{\partial f_c}{\partial\varepsilon}\right), \tag{7}$$

where $\rho(\mathscr{E})=m_e^{1/2}/2\sqrt{2}\pi^2\hbar\lambda^2\mathscr{E}^{1/2}$ is the density-of-states function of the lowest Landau level.

Thus, in the approximation under consideration we have

$$I\{f,p\}=I_{ef}\{f,p\}+I_{ei}\{f,p\}+I_{ee}\{f,p\}, \tag{8}$$

where the symmetric part of the electron–optical–phonon collision integral $I^s_{ef}\{f,p\}$ is nonzero only in narrow energy regions near the bottom of the condition band and near the point of the optical phonon energy $\hbar\omega_0$. Then kinetic equation (2) with collision integral (8) can be written for $p>0$ as a system of two equations for the symmetric part $f_s(\varepsilon)$ and antisymmetric part $f_a(p)=[f(p)-f(-p)]/2$ of the distribution function:

$$\nu_E\partial f_a/\partial p=I_{ee}\{f_s,p\}+I_{ef}{}^s\{f,p\}, \tag{9}$$

$$\nu_E\,\partial f_s/\partial p=-2\nu_i\varepsilon^{-1/2}f_a+I_{ef}{}^a\{f,p\}. \tag{10}$$

Here $I^a_{ef}\{f,p\}$ is the antisymmetric part of the electron–optical–phonon collision integral. In Eq. (10) we have neglected the three electron collision integral, since in the gas approximation it is small compared to the collision integral for elastic collisions of electrons with charges impurities.

In what follows we shall assume that the electric field is not too strong, so that the energy "penetration depth" of the nonequilibrium electrons into the active region ($\sim\nu_E^2/\nu_0^2$; Ref. 19) is much smaller than the characteristic energy for changes in the distribution function near the bottom of the conduction band; we can then write $I^s_{ef}\{f,p\}$ in the form of a δ function. In addition, we shall assume for simplicity that

$$\nu_i\gg\nu_0\exp(-\hbar\omega_0/T_0), \tag{11}$$

as this is usually the case at the actual values of the impurity density for $T_0\ll\hbar\omega_0$. Then, by virtue of expression (3) and inequality (11), equation (10) directly yields a relation between the symmetric and antisymmetric parts of the distribution function in the passive region ($\varepsilon<1$):

$$f_a(\varepsilon)=-\frac{\nu_E}{\nu_i}\varepsilon^2\frac{\partial f_s(\varepsilon)}{\partial\varepsilon}. \tag{12}$$

Integrating kinetic equation (9) over the quasimomentum from $0<p<1$ to infinity and using expressions (6), (7), and (12), we get

$$\frac{\nu_E^2}{\nu_i}\varepsilon^2\frac{\partial f_s(\varepsilon)}{\partial\varepsilon}+\nu_{ee}\frac{1}{\eta^{1/2}}\left[\int_0^1\frac{d\varepsilon'}{\sqrt{\varepsilon'}}\left(-\frac{\partial f_s(\varepsilon')}{\partial\varepsilon'}\right)\right]^{-1/2}$$

$$\times\frac{1}{\mathscr{L}_q}\frac{1}{\varepsilon}\int_\Delta^1\frac{d\varepsilon'}{\varepsilon'}\frac{1}{\sqrt{\varepsilon}+\sqrt{\varepsilon'}}\left[\frac{\partial f_s(\varepsilon)}{\partial\varepsilon}f_s(\varepsilon')-f_s(\varepsilon)\frac{\partial f_s(\varepsilon')}{\partial\varepsilon'}\right]=i. \tag{13}$$

Here the value of the dimensionless electron density $\eta=n/\hbar\omega_0\rho(\hbar\omega_0)$ determines the normalization condition for the distribution function:

$$\eta=\int_0^1\frac{d\varepsilon}{\varepsilon^{1/2}}f_c(\varepsilon), \tag{14}$$

ν_{ee} is the characteristic frequency of three-electron collisions:

$$\nu_{ee}=\left(\frac{\pi}{2}\right)^{3/2}\frac{e^4N_i\mathscr{L}_q}{\varkappa^2m_e^{1/2}(\hbar\omega_0)^{3/2}}\left[\frac{e^2n^{1/2}}{\varkappa\hbar\omega_0}\right]^{3/2} \tag{15}$$

(clearly, in the gas approximation $\nu_{ee}\ll\nu_i$), $\mathscr{L}_q$ is the Coulomb logarithm, which in the present analysis is, for simplicity, assumed constant, and

$$i=\int_0^1dp'I_{ef}{}^s\{f,p'\}=-\int_1^\infty dp'I_{ef}{}^s\{f,p'\}.$$

This last identity is a consequence of conservation of electron number and so is valid independently of the applicability conditions or expression (6).

In addition, after integrating (13) over ε from zero to one, we find

$$i=\int_0^1d\varepsilon\frac{\nu_E^2}{\nu_i}\varepsilon^2\left(-\frac{\partial f_c(\varepsilon)}{\partial\varepsilon}\right). \tag{16}$$

This relation directly determines the energy balance of the electron gas, since collisions between electrons in the field of randomly distributed impurities cannot, in any order of perturbation theory, effect a dissipation of the energy acquired by the charge carriers in the electric field. Relation (16) can also be written in the equivalent form

$$JE=\frac{(m_e/2)^{1/2}}{\pi^2\lambda^2\hbar}(\hbar\omega_0)^{3/2}i, \tag{16'}$$

where J is the electrical current density.[1)]

If the solution of the kinetic equation (2) in the active region is known, then under the assumptions made above, the set of equations consisting of (13) together with (12), (14), and (16) yields the distribution function in the passive region; the usual boundary condition of the distribution function at the point $\varepsilon=1$ is the requirement that the solutions of the corresponding kinetic equations be continuous there. However, for a number of reasons it is difficult to obtain the solution of the kinetic equation in the active region. First, the quasimomentum P is a good quantum number only in strong electric fields, when the energy "penetration depth" of the electrons into the active region is much larger than the energy uncertainty $\hbar\nu_0$ due to emission of optical phonons.[19] Second, the electron energy spectrum in a quantizing magnetic field is restructured in the presence of strong coupling with optical phonons, and an energy gap arises[24] near the energy $\mathscr{E}=\hbar\omega_0$. Third, in the active region the three-electron collision integral can be comparable to the field term of the kinetic equation, while at the same time the diffusion approximation for the electron–electron-impurity collision integral (6) ceases to be valid, and the distribution functions suffers a discontinuity.[25] However, in the weak electric field limit the distribution function is practically symmetric, and the flux of electrons in quasimomentum (p) space is governed by three-electron collisions and by the interaction of the electrons with optical phonons. This makes the electron motion in p space diffusive, and from the requirement that the flux of particles from the active region into the passive region be bounded in the presence of opticalphoton emission, the boundary condition on the distribution function at high values of ν_0 is (see Appendix 2)

$$f(-1) \approx f_s(1) \approx f_s(0)\exp(-\hbar\omega_0/T_0). \tag{17}$$

In strong electric fields the three-particle collision integral $I_{ee}\{f, p\}$ can be neglected in the solution of kinetic equation (2) in the active region. Then for $\nu_i \ll \nu_0$ we find immediately from the solution of this equation that

$$f(-1) = f_s(0)\exp(-\hbar\omega_0/T_0). \tag{18}$$

Comparing relations (17) and (18), which were obtained in opposite limits, we see that the latter expression can be taken as a boundary condition on the distribution function in the passive region; by virtue of (12), this expression is equivalent to

$$\frac{\nu_E}{\nu_i}\frac{\partial f_c(\varepsilon)}{\partial \varepsilon}\bigg|_{\varepsilon=1} + f_s(1) = f_s(0)\exp\left(-\frac{\hbar\omega_0}{T_0}\right). \tag{19}$$

Equation (13) in conjuction with expressions (12) and (16), normalization condition (14), and boundary condition (19) determines the distribution function in the passive region in closed form. We point out that near the boundary of the active and passive regions, these relations are practically insensitive to the conditions that these approximations be applicable and thus permit one to study the kinetic properties of the electron gas over a wider range of system parameters.

The methods of solving kinetic equation (13) differ substantially depending on the relationship of the frequencies ν_i, ν_{ee}, and ν_E, and we shall therefore consider the weak-electric-field and strong-electric-field cases separately (however, we assume as before that $\nu_E \ll \nu_0$).

3. DISTRIBUTION FUNCTION AND CURRENT-VOLTAGE CHARACTERISTIC IN WEAK ELECTRIC FIELDS

The weak-electric-field limit in the model under study corresponds to the condition that the redistribution of energy between the electrons of the passive region be dominated by three-electron collisions. In this limit the left-hand side of (13) is dominated by the second term. Consequently, to a first approximation the symmetric part of the distribution function $f_s(\varepsilon)$ is Maxwellian, with an effective (dimensionless) temperature t, where t should be much less than unity ($t \ll 1$), since the distribution function in the active region is substantially non-Maxwellian. In this case it follows from the results obtained below that the value of $\nu_E = eE/P_0$ satisfies the inequality

$$\nu_E \ll (\nu_{ee}\nu_i)^{1/2}. \tag{20}$$

At such values of ν_E the deviation of the distribution function from Maxwellian in the energy region $\varepsilon \sim t$ is insignificant, in accordance with the exponentially small parameter $e^{-1/t}$, and so in relations (14) and (16) and in the integral factors in (13) we may use a first approximation to the distribution function. Then, from (14) and (16) we find

$$i = \frac{2}{\pi^{1/2}}\eta\frac{\nu_E^2}{\nu_i}t^{1/2}, \tag{21}$$

and in the next order of perturbation theory we obtain from Eq. (13) an expression for the symmetric part of the distribution function:

$$f_s(\varepsilon) \approx \frac{\eta}{(\pi t)^{1/2}}e^{-\varepsilon/t} - \pi^{1/2}\frac{i}{\nu_{ee}}t\varepsilon^{3/2}. \tag{22}$$

By virtue of expression (22), boundary condition (19) becomes

$$e^{-1/t} - e^{-1/t_0} \approx \pi\frac{i}{\eta\nu_{ee}}t^{3/2} + \frac{\nu_E}{\nu_i}\frac{1}{t}e^{-1/t}, \tag{23}$$

where $t_0 = T_0/\hbar\omega_0$.

Relations (21) and (23) determine in closed form the dependence of i and t on the applied electric field E; the second term on the right-hand side of boundary condition (23) can be neglected unless the frequency ν_E is so small that it lies in the region where the electron gas is not heated significantly, $(t - t_0)/t_0 \ll 1$; in that region one can use a linear theory of the kinetic effects.[1] With this simplification in mind, we obtain from (21) and (23) a relation between the effective temperature t and the strength of the electric field (in units of ν_E), viz.,

$$\frac{\nu_E}{(\nu_{ee}\nu_i)^{1/2}} = \left\{\frac{1}{2\pi^{1/2}}\frac{1}{t^3}\left(e^{-1/t} - e^{-1/t_0}\right)\right\}^{1/2}, \tag{24}$$

and a monotonically increasing current-voltage characteristic, which is given by the expression

$$\frac{\nu_E}{(\nu_{ee}\nu_i)^{1/2}} = \frac{\pi^{1/4}}{4}j\left(\frac{\nu_i}{\nu_{ee}}\right)^{1/4}\left|\ln\left[e^{-1/t_0} + \frac{\pi^{1/2}}{8}j^2\frac{\nu_i}{\nu_{ee}}\right]\right|^{3/4}. \tag{25}$$

Here $j = 2i/\eta\nu_E$ is the dimensionless electrical current density [in fact, by virtue of Eq. (16), $J = eP_0nj/2m$]. The functions $t(\nu_E)$ and $j(\nu_E)$ are represented by curves 1 and 2 in Fig. 1.

In concluding this part of the paper, we note the validity of one of the basic assumptions, viz., that the number of electrons in the active region is small in the electric-field range under study. Actually, the energy "penetration

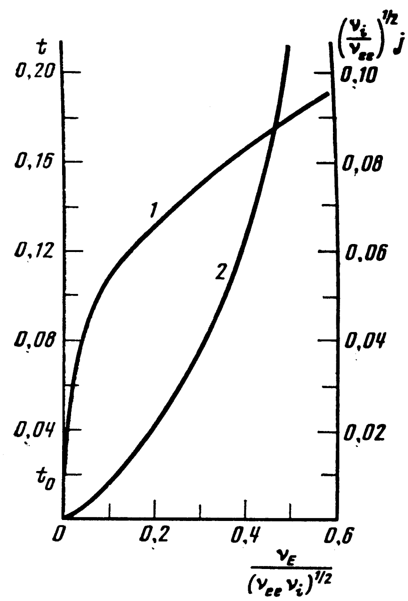

FIG. 1. Effective temperature t as a function of the electric field (curve 2) and the current-voltage characteristic (curve 2) in weak electric fields.

depth" of nonequilibrium electrons into the active region ($\sim\nu_E^2/\nu_0^2$) is much smaller than unity, since, by assuming $t\sim 1$ in (24), we obtain an estimate of the maximum possible value here, $\nu_{E\max}\sim(\nu_{ee}\nu_i)^{1/2}$, from which we find that

$$(\nu_E/\nu_0)_{max}\sim(\nu_{ee}\nu_i/\nu_0^2)^{1/2}\ll 1.$$

4. DISTRIBUTION FUNCTION AND CURRENT-VOLTAGE CHARACTERISTIC IN STRONG ELECTRIC FIELDS

Let us now consider the limit of strong electric fields, when three-electron collisions have an important effect on the form of the distribution function only in a rather narrow energy region near the bottom of the conduction band, $\varepsilon\lesssim\Theta\ll 1$. This limit corresponds to having a small second term on the left-hand side of (13) for $\Theta\ll\varepsilon\leqslant 1$, while ν_E satisfies

$$\nu_E\gg(\nu_{ee}\nu_i)^{1/2}. \tag{26}$$

Thus, in the energy region $\Theta\ll\varepsilon\leqslant 1$ the second term on the left-hand side of (13) can be neglected, whereupon, by virtue of boundary condition (19) and expression (12), we find ($\varepsilon\gg\Theta$)

$$f_s(\varepsilon)\approx f_s(0)e^{-1/t_0}+{}^1/_2\eta j+{}^1/_2\eta j\left(\frac{\nu_i}{\nu_{ee}}\right)^{1/2}\frac{(\nu_{ee}\nu_i)^{1/2}}{\nu_E}\left(\frac{1}{\varepsilon}-1\right) \tag{27}$$

and

$$f_a(\varepsilon)\approx{}^1/_2\eta j. \tag{28}$$

At energies $\varepsilon\lesssim\Theta$, three-electron collisions "smooth out" the distribution function, eliminating the nonintegrable singularities in the expressions for the kinetic coefficients that follow formally from relation (27) when $\varepsilon\to 0$. In addition, in the leading approximation in the energy region $\varepsilon\sim\Theta$, elastic collisions of electrons with charged impurities ensure that the antisymmetric part of the distribution function will be small compared to the symmetric part. Therefore, an estimate of the value of the distribution function $f_s(0)$ near the bottom of the conduction band can be obtained by setting $\varepsilon\sim\Theta$ in (27). One thereby demonstrates that the first two terms on the right-hand side of (27) are small in the energy region $\varepsilon\sim\Theta$, i.e.,

$$\left(\frac{\nu_i}{\nu_{ee}}\right)^{1/2}\frac{(\nu_{ee}\nu_i)^{1/2}}{\nu_E}\frac{1}{\Theta}\gg 1 \tag{29}$$

and for $\varepsilon\gtrsim\Theta$

$$f_s(\varepsilon)\approx\frac{1}{2}\eta j\left(\frac{\nu_i}{\nu_{ee}}\right)^{1/2}\frac{(\nu_{ee}\nu_i)^{1/2}}{\nu_E}\frac{1}{\varepsilon}. \tag{30}$$

No general analytical method of solving the kinetic equation (13) has been found, but the method of scale transformations of the distribution function $f_s(\varepsilon)$ and energy ε can yield fairly complete information on the distribution function and the kinetic characteristics of the electron gas if the asymptotic behavior of the distribution function in the energy region $\Theta\ll\varepsilon\ll 1$ has a power-law character.

The behavior of the distribution function $f_s(\varepsilon)$ in the low-energy region, where three-particle collisions have an appreciable influence, is rather insensitive to the actual values of the quantities appearing in boundary condition (19) and so is practically the same as the solution of the analytical continuation of equation (13) to the energy interval $0<\varepsilon<\infty$:

$$\frac{\nu_E^2}{\nu_{ee}\nu_i}\varepsilon^2\frac{\partial f_s(\varepsilon)}{\partial\varepsilon}+\frac{1}{\eta^{1/2}}\left[\int_0^\infty\frac{d\varepsilon'}{\sqrt{\varepsilon'}}\left(-\frac{\partial f_s(\varepsilon')}{\partial\varepsilon'}\right)\right]^{-1/2}\frac{1}{\mathscr{L}_q}\frac{1}{\varepsilon}\times\int_\Delta^\infty\frac{d\varepsilon'}{\varepsilon'}\frac{1}{\sqrt{\varepsilon}+\sqrt{\varepsilon'}}\left[\frac{\partial f_s(\varepsilon)}{\partial\varepsilon}f_s(\varepsilon')-f_s(\varepsilon)\frac{\partial f_s(\varepsilon')}{\partial\varepsilon'}\right]=-\frac{1}{2}\eta j\left(\frac{\nu_i}{\nu_{ee}}\right)^{1/2}\frac{\nu_E}{(\nu_{ee}\nu_i)^{1/2}} \tag{31}$$

with the auxiliary boundary condition

$$f_s(\infty)=0. \tag{32}$$

We define the scale transformations of the distribution function and the energy as follows:

$$f_s(\varepsilon)=\xi\varphi(x),\quad\varepsilon=\Theta x, \tag{33}$$

where the coefficients ξ and Θ of the scale transformations satisfy the relations

$$\Theta\left(\frac{\xi}{\eta}\right)=j\left(\frac{\nu_i}{\nu_{ee}}\right)^{1/2}\frac{(\nu_{ee}\nu_i)^{1/2}}{\nu_E}, \tag{34}$$

$$\Theta^{-5/4}\left(\frac{\xi}{\eta}\right)^{1/2}=j\left(\frac{\nu_i}{\nu_{ee}}\right)^{1/2}\frac{\nu_E}{(\nu_{ee}\nu_i)^{1/2}}. \tag{35}$$

Then Eq. (31) becomes

$$x^2\frac{\partial\varphi(x)}{\partial x}+\left[\int_0^\infty\frac{dx'}{\sqrt{x'}}\left(-\frac{\partial\varphi(x')}{\partial x'}\right)\right]^{-1/2}\frac{1}{\mathscr{L}_q}\frac{1}{x}\times\int_{\Delta_x}^\infty\frac{dx'}{x'}\frac{1}{\sqrt{x}+\sqrt{x'}}\left[\frac{\partial\varphi(x)}{\partial x}\varphi(x')-\varphi(x)\frac{\partial\varphi(x')}{\partial x'}\right]=-\frac{1}{2}. \tag{36}$$

Equation (36) does not have any large or small parameters, and the function $\varphi(x)$ should therefore vary smoothly in the interval $0<x\lesssim 1$, with the asymptotic behavior $\varphi(x)\approx 1/2x$ for $x\gg 1$, and the value $\varphi(0)\sim 1$. (A distribution function consistent with this behavior of $\varphi(x)$ is shown in Fig. 2.) Hence we also see that it is the parameter Θ that sets the

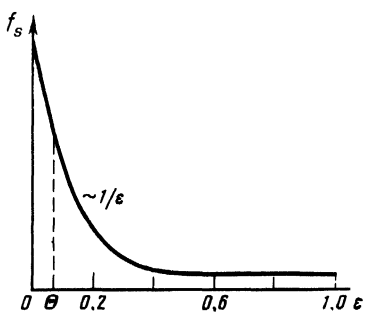

FIG. 2. Qualitiative shape of the distribution function in strong electric fields.

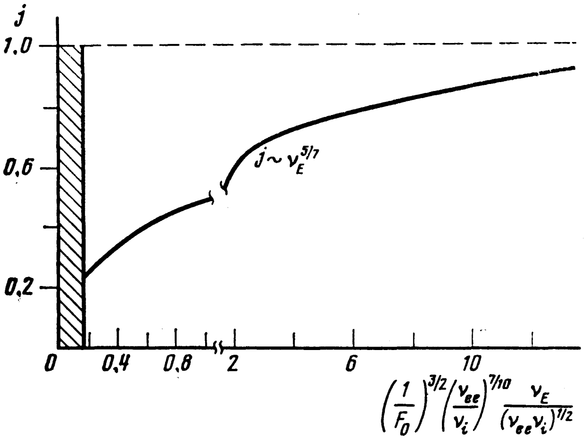

FIG. 3. Current-voltage characteristic in strong electric fields (the shaded area is the weak-field region).

characteristic energy scale near the bottom of the conduction band.

Making use of the fact that Θ is small, we find from the normalization condition (14) and expressions (34) and (35) that

$$\Theta=\frac{1-j}{F_0}\left(j\left(\frac{\nu_i}{\nu_{ee}}\right)^{1/2}\right)^{-4/5}, \tag{37}$$

$$\xi=\eta\left(\frac{1-j}{F_0}\right)^{1/2}\left(j\left(\frac{\nu_i}{\nu_{ee}}\right)^{1/2}\right)^{2/5}, \tag{38}$$

$$\frac{\nu_E}{(\nu_{ee}\nu_i)^{1/2}}=\left[\frac{F_0}{1-j}\right]^{3/2}\left(j\left(\frac{\nu_i}{\nu_{ee}}\right)^{1/2}\right)^{7/5}, \tag{39}$$

where the number F_0 is

$$F_0=\int_0^\infty \frac{dx}{x}\varphi(x)\sim 1.$$

Relation (39) gives the current-voltage characteristic in Fig. 3, while the set of relations (37)–(39) gives, in parametric form, the dependence on the applied electric field of the characteristic scales Θ and ξ of the distribution function near the bottom of the conduction band (see curves 1 and 2, respectively in Fig. 4). The maximum strength of the electric field (in units of ν_E) at which the basic approximations of the model remain in force in this case is determined by inequality (29), which serves as a measure of the validity of the gas approximation:

$$\nu_{E\,max}\sim(\nu_{ee}\nu_i)^{1/2}(\nu_i/\nu_{ee})^{13/10}. \tag{40}$$

Let us analyze these results in greater detail. If

$$(\nu_{ee}\nu_i)^{1/2}\ll\nu_E\ll(\nu_{ee}\nu_i)^{1/2}(\nu_i/\nu_{ee})^{7/10},$$

then $j\ll 1$, and we find from relations (37)–(39) that $j\sim\nu_E^{5/7}$, $\Theta\sim\nu_E^{-4/7}$, and $\xi\sim\nu_E^{2/7}$ ($\xi\Theta^{1/2}=\text{const}$). When

$$(\nu_{ee}\nu_i)^{1/2}(\nu_i/\nu_{ee})^{7/10}\ll\nu_E\lessapprox\nu_{E\,max},$$

the electrical current density is practically independent of the impressed electric current ($j\approx 1$), since the number of electrons located near the bottom of the conduction band is rather small, and all the electrons of the passive region actually move without collisions under the influence of the electric field (the streaming condition), and we have $\Theta\sim\nu_E^{-2/3}$ and $\xi\sim\nu_E^{-1/3}$ ($\xi/\Theta^{1/2}=\text{const}$).

In concluding this section of our paper, we call attention to the fact that the condition that the penetration depth of the nonequilibrium electrons into the active region ($\sim\nu_E^2/\nu_0^2$) be small with respect to the energy scales of the distribution function, in particular Θ, does not impose any fundamental limitations on the existence of the limit under study. In fact, the characteristic value of Θ corresponding to the maximum electric field at which the gas approximation still holds [see Eq. (40)] is found to be of the order of $(\nu_{ee}/\nu_i)^{4/5}$. We thus have $\Theta\gg\nu_{E_{\max}}^2/\nu_0^2$ if the frequency ν_0 is high enough.

5. CONCLUSION

Let us give a numerical estimate of the characteristic electric field at which $\nu_E^*\sim(\nu_{ee}\nu_i)^{1/2}$. If $\nu_i\sim10^{10}$–10^{11} sec^{-1}, $\nu_{ee}\sim10^9$–10^{10} sec^{-1}, $\hbar\omega_0\sim0.02$ eV, and $m_e\sim0.1m_0$ (m_0 is the free-electron mass), then $E^*\sim1$–10 V/cm; this value, as expected, is considerably smaller than the electric field necessary for the kinetic equation to hold in the active region, $E_a\gtrsim100$ V/cm (Ref. 19).

We note that in ultraquantum magnetic fields one can experimentally obtain fairly detailed information on the properties of the distribution function near the bottom of the conduction band by studying transport phenomena in the direction perpendicular to the magnetic field. For example, the diagonal component of the transverse conductivity tensor (perpendicular to the magnetic field) is given by[1]

$$\sigma_\perp\approx\frac{\nu_i}{\Omega^2}\frac{e^2}{m_e}\rho(\hbar\omega_0)(\hbar\omega_0)\int_\Delta^1\frac{d\varepsilon}{\varepsilon}\left(-\frac{\partial f_s(\varepsilon)}{\partial\varepsilon}\right) \tag{41}$$

and the curve of $\sigma_\perp(E)$ as a function of the applied longitudinal electric field is plotted in Fig. 5.

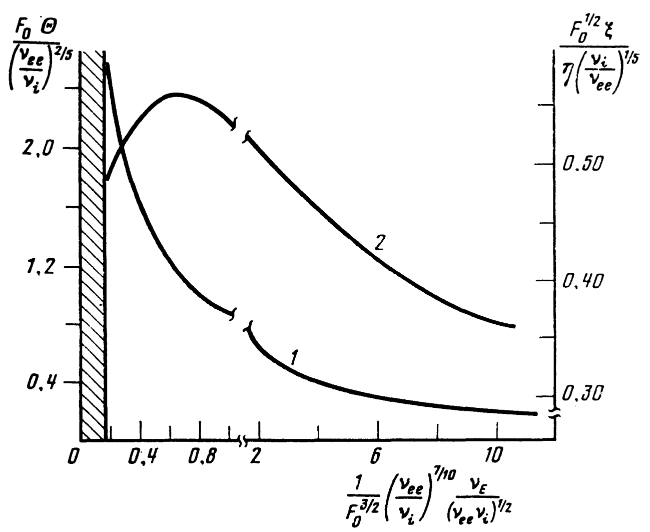

FIG. 4. Electric-field dependence of the characteristic energy scale Θ (curve 1) and amplitude scale ξ (curve 2) of the distribution function near the bottom of the conduction band in strong electric fields, $\Theta=[d\ln\varphi(x)/dx|_{x=0}][d\ln f(\varepsilon)/d\varepsilon|_{\varepsilon=0}]^{-1}$, $\xi=f(0)/\varphi(0)$ (the weak-field region is shaded).

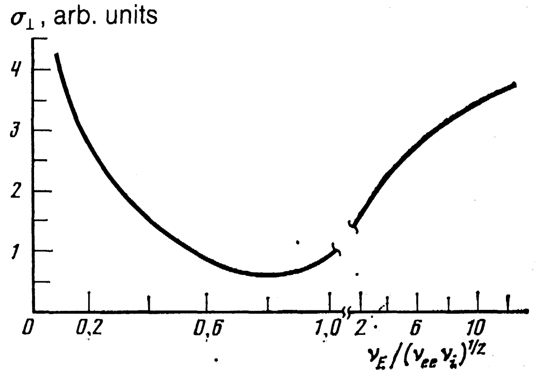

FIG. 5. Transverse conductivity of the electron gas as a function of the longitudinal electric field.

We also note that the results obtained in the strong-electric-field limit are actually independent of the specific form of the collision integral for the three-particle electron–electron-impurity collisions, being determined by its group properties with respect to the scale transformations of the distribution function and energy.

We wish to thank I. B. Levinson for his interest in this study and for a detailed discussion of the results.

APPENDIX 1

The electron-electron-impurity collision integral in an ultraquantum magnetic field was first obtained in general form in Ref. 8. However, in the cases studied in the present paper the characteristic value of the electron quasimomentum transfer as a result of such collisions ($\sim \hbar q_D$, where q_D^{-1} is the Debye screening radius) is much smaller than the average electron quasimomentum $\langle P \rangle$, and so one can use the diffusion approximation for the three-particle collision integral. Furthermore, in accordance with the gas approximation, the energy transfer $\langle \partial \mathscr{E}_P/\partial P \rangle \hbar q_D$ in such collisions is substantially larger than the energy smearing $\hbar \nu_i$ of the electron state with quasimomentum $\langle P \rangle$ in elastic collisions with impurities. Therefore, one does not have to go beyond second order in the interactions of the electrons with one another and with impurities.

One can easily obtain an expression for the three-particle electron–electron-impurity collision integral in the diffusion approximation by using the obvious relation

$$I_{ee}\{f,P\} = -\frac{(2\pi)^2\lambda^2}{V}\hbar\frac{\partial}{\partial P}J_{ee}\{f,P\}, \qquad (\text{A.1.1})$$

where $\lambda^2 = \hbar c/eH$, V is the volume of the semiconductor, P is the electron quasimomentum parallel to the magnetic field, and $J_{ee}\{f,P\}$ is the flux of particles in quasimomentum space due to three-particles collisions.

The value of the flux $J_{ee}\{f,P\}$ in the second order of perturbation theory is determined by the scattering processes whose diagrams are shown in Fig. 6 (the exchange diagrams are unimportant, since they correspond to a large quasimomentum transfer). Using the standard procedure of calculating the transition matrix elements in the second order of perturbation theory,[26] we get for a nondegenerate electrons gas

$$J_{ee}\{f,P\} = \frac{V}{(2\pi)^2\lambda^2}\frac{N_i}{(2\pi)^7\hbar^3\lambda^2}\int\limits_{P'<P} dP' \int\limits_{P'+\hbar q_z'>P} dq_z' \times \int dP'' \int dq_z'' \int d^2q_\perp' \int d^2q_\perp'' M_3^2 [f(P')f(P'') - f(P'+\hbar q_z')f(P''+\hbar q_z'')] \times\delta[\mathscr{E}_{P'+\hbar q_z'} + \mathscr{E}_{P''+\hbar q_z''} - \mathscr{E}_{P'} - \mathscr{E}_{P''}]. \qquad (\text{A.1.2})$$

Here N_i is the density of charged impurities, $f(P)$ is the distribution function, $\mathscr{E}_P = P^2/2m_e$ is the kinetic energy, q_z and $q_\perp$ are the longitudinal and transverse components (with respect to the magnetic field) of the vector $\mathbf{q}$, and M_3 is the interaction matrix element

$$M_3 = v(\mathbf{q}+\mathbf{q}')\{v(\mathbf{q}')[(\mathscr{E}_{P''+\hbar q_z''+\hbar q_z'} - \mathscr{E}_{P''})^{-1} + (\mathscr{E}_{P''-\hbar q_z'} - \mathscr{E}_{P''+\hbar q_z''})^{-1}] + v(\mathbf{q}'')[(\mathscr{E}_{P'+\hbar q_z'+\hbar q_z''} - \mathscr{E}_{P'})^{-1} + (\mathscr{E}_{P'-\hbar q_z''} - \mathscr{E}_{P'+\hbar q_z'})^{-1}]\}, \qquad (\text{A.1.3})$$

where $v(\mathbf{q}) = 4\pi e^2/\varkappa(q^2+q_D^2)$ is the scattering potential of a screened Coulomb center.

Because the electron momentum transfer in three-particle collisions is small ($\hbar q_D \ll P$), one can neglect in the expansion of the factors in expression (A.1.2) all but the terms which are nonzero in q_z' and q_z''. Furthermore, the diffusion coefficients and the dynamic friction in quasimomentum space are determined by the integral of the distribution function over all values of the energy. Therefore, if the distribution function is not too highly localized about some quasimomentum $P_l \neq 0$, the terms with $|P'-P''| \ll P$ make an insignificant contribution to the integral in (A.1.2). Because

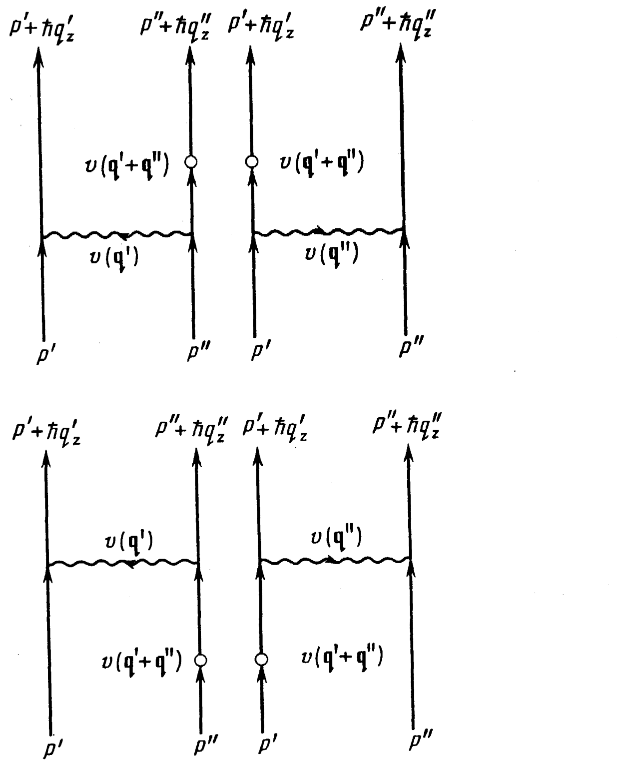

FIG. 6. Diagrams of the scattering processes which determine the value of the electron–electron-impurity collision integral.

of this, in expression (A.1.3) we can to good accuracy take into consideration only the terms in the first or in the second set of square brackets. Then, doing the integration over $\mathbf{q}'$ and $\mathbf{q}''$ in expression (A.1.2), we get

$$J_{ee}\{f,P\}=-\frac{V}{(2\pi)^2\lambda^2\hbar}\frac{N_i m_e^{1/2}e^8}{2\cdot 2^{1/2}\hbar q_D\lambda^2\varkappa^4}\operatorname{sign}P$$

$$\times\frac{1}{\mathscr{E}}\int_\Delta^\infty\frac{d\mathscr{E}'}{\mathscr{E}'[\mathscr{E}^{1/2}+(\mathscr{E}')^{1/2}]}\left[\frac{m_e}{P}\frac{\partial f_s(P)}{\partial P}f_s(\mathscr{E}')-f(P)\frac{\partial f_s(\mathscr{E}')}{\partial\mathscr{E}'}\right],\qquad\text{(A.1.4)}$$

where $f_s(\mathscr{E}) = 1/2[f(P) + f(-P)]$ is the symmeric part of the distribution function, and Δ is the cutoff parameter of the logarithmic divergence at low energies. Relations (A.1.2) and (A.1.4) imply expression (6).

APPENDIX 2

Let us consider the limit of weak electric fields, when the energy "penetration depth" of electrons into the active region is due mainly to three particle collisions and is small compared to the characteristic energy for changes in the distribution function near the bottom of the conduction band. Let us first assume that the diffusion approximation of the collision integral $I_{ee}\{f,p\}$ is valid even for $p>1$. In the active region the distribution function decays rapidly, and therefore in expression (6) we need keep only the term of the form $D\partial^2 f_s/\partial p^2$, where D is the electron diffusion coefficient in quasimomentum space; here it can be assumed constant. Then the kinetic equation (2) for $p>1$ becomes

$$D\frac{\partial^2 f_s}{\partial p^2}-\nu_0\frac{1}{(p^2-1)^{1/2}}\left[f(p)-f_s(0)\exp\left(-\frac{\hbar\omega_0}{T_0}\right)\right].\qquad\text{(A.2.1)}$$

Introducing the variable $\zeta=(p-1)(2^{1/2}D/\nu_0)^{-2/3}$ and the quantity $u(\zeta)=f(p)-f_s(0)\exp(-\hbar\omega_0/T_0)$, we find from (A.2.1) that

$$\partial^2 u/\partial\zeta^2\approx\zeta^{-1/2}u.\qquad\text{(A.2.2)}$$

The desired solution of equation (A.2.2) is clearly an expression of the form $Au^*(\zeta)$, where $u^*(\zeta)$ is decreasing function of ζ with a characteristic scale for changes with respect to the coordinate ζ and an amplitude both of order unity, and A is a constant factor. In this case the flux Λ of electrons from the active region to the passive region due to their interaction with optical phonons is

$$\Lambda\sim\nu_0\int_1^\infty\frac{dp}{(p^2-1)^{1/2}}u\approx\frac{DA\nu_0^{2/3}}{(2D^2)^{1/3}}\int_0^\infty\frac{d\zeta}{\zeta^{1/2}}u^*(\zeta)\sim D^{1/3}u(0)\nu_0^{2/3}.\qquad\text{(A.2.3)}$$

The electron flux Λ is a bounded quantity at arbitrarily large values of ν_0; consequently, $u(0)\sim\nu_0^{-2/3}$ and so, to within the small parameter $(D/\nu_0)^{2/3}$, we have $u(0)=0$. However, when the frequency ν_0 is so high that the diffusion approximation is no longer valid, the value of the flux Λ can be estimated by assuming that $\partial^2 f_s/\partial p^2|_{p=1}\sim u(0)q_D^{-2}$, and the distribution function $u(p)$ for $q_D\ll 1-p\ll 1$ is therefore considerably larger than $u(0)$ and practically agrees with the solution of the kinetic equation (2) with the boundary condition $u(0)=0$.

If, on the other hand, the energy spectrum is appreciably restructured and an energy gap arises[25] at $\mathscr{E}=\hbar\omega_0$, then in some neighborhood (of size Q_p) of the gap the interaction between the electrons and optical phonons is important; here Q_p increases with increasing ν_0. Then either $u(0)Q_p\nu_0\sim\Lambda$ and thus $u(0)=-\nu_0^{-\gamma}$, where $\gamma>0$, or the situation is analogous to the case in which the diffusion approximation breaks down, and we can therefore assume $u(0)=0$ in this case.

Thus, in the three possible limiting cases the solution of the kinetic equation (2) in the passive region is practically independent of $u(0)$, and one can therefore use relation (17) as the desired boundary condition.

[1)] Expression (16) enables one to find the current-voltage characteristic in explicit form if the dependence of i on the applied electric field E is known.

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