Quantization of the chromoelectric flux and action in the nonperturbative phase of quantum chromodynamics

G. S. Iroshnikov

Physicotechnical Institute, Moscow

(Submitted 12 December 1985)
Zh. Eksp. Teor. Fiz. **90**, 1922–1930 (June 1986)

The quantization of the chromoelectric flux and action of a relativistic string is derived from the condition of uniqueness of the solution of the effective field equations for the hadron phase of quantum chromodynamics.

1. INTRODUCTION

In a recent paper[1] an approximate method of calculating hadron field correlators, based on the $1/N$ expansion and the quasiclassical approximation, was considered. In the framework of this approach the effective two-dimensional action S_{eff} for the correlation functions of gauge-invariant operators was obtained. The Euclidean field equations associated with this action possess a topologically nontrivial solution. From a physical point of view, this solution described the chromoelectric field of an open, infinitely thin relativistic string with quarks at its ends and with a bare tension coefficient. In the present paper it is demonstrated that, because of the topological properties of this solution, quantization of the chromelectric flux and of the action of the string arises.

In Sec. 2 we give the basic stages of the derivation of S_{eff} and its explicit form. In Sec. 3 we give a description of the solution of the Euclidean field equations, with the accent on topological arguments. Section 4 is devoted strictly to the derivation of the quantization of the flux and action in the nonperturbative phase of quantum chromodynamics (QCD). In the Conclusion it is noted that when we take the limit of the Abelian theory all the pheomena considered disappear.

2. THE EFFECTIVE ACTION

In Ref. 1, in the framework of the $1/N$ expansion (N is the number of colors), the effective action S_{eff} for the Euclidean correlation functions was obtained:

$$K(1,\ldots,n)=B^{-1}\int d\mu(A)\,D\psi\, D\bar{\psi}\{\exp[-S_{Y-M}(A,\psi)]\times[M(\Gamma_n)\ldots M(\Gamma_1)]\},\quad (1)$$

where

$$B=\int d\mu(A)\,D\psi\,D\bar{\psi}\exp(-S_{Y-M}),$$
$$S_{Y-M}(A,\psi)=\int d^4x\{{}^1/_4 G_{\mu\nu}{}^a G_{\mu\nu}{}^a+\bar{\psi}(i\gamma_\mu D_\mu-m)\psi\},\quad (2)$$
$$G_{\mu\nu}{}^a=\partial_\mu A_\nu{}^a-\partial_\nu A_\mu{}^a-ef^{abc}A_\mu{}^b A_\nu{}^c,$$
$$D_\mu=\partial_\mu+ie\frac{\lambda^a}{2}A_\mu{}^a\equiv\partial_\mu+ieA_\mu.$$

The gauge-invariant quantities appearing in formula (1),

$$M(\Gamma)\equiv M_\alpha{}^\beta(\Gamma)=\bar{\psi}_c{}^\beta(y')\left[P\exp\left(-ie\frac{\lambda^a}{2}\int_y^{y'}dz_\mu A_\mu{}^a(z)\right)\right]_{cc'}\psi_{c',\alpha}(y),\quad (3)$$

are regarded as the field operators of composite extended mesons. Here α, β are combined indices of the Lorentz group $O(4)$ and the internal-symmetry group; c and c' are indices of the gauge group $SU(N)$. The integration over z_μ in formula (3) is performed along the contour Γ joining the points y and y' in the Euclidean space R^4.

We shall list briefly the main stages of the derivation of S_{eff}. Neglecting the quark loops (the $1/N$ expansion) and their spin, one can represent[2,3] the connected part of the correlator (1) as

$$K(1,\ldots,n)\approx\sum_{perm}\delta_P\prod_{q=1}^{n}\int \tilde{D}x_q\left\{\exp\left[-\frac{1}{2}\int_{\gamma_1}^{\gamma_2}d\gamma\left(\frac{\dot{x}^2}{\lambda}+\lambda m^2\right)\right]_q\times\langle O(\Gamma)\rangle_A\right\},\quad (4)$$

where

$$\tilde{D}x_q=\left[\frac{d\lambda\Delta\gamma}{4\pi i}Dx_\mu(\gamma)\right]_q,\quad \dot{x}_\mu=\frac{dx_\mu}{d\gamma},$$
$$O(\Gamma)=\mathrm{Tr}\left[P\exp\left(-ie\oint_\Gamma dz_\mu A_\mu\right)\right],\quad (5)$$

and δ_P is the parity of the permutation of the Fermi fields. The integration contour Γ in the expression (5) is shown in the figure. It consists of the contours Γ_i ($i=1,\ldots,n$) that are contained in the operators $M(\Gamma_i)$ and whose ends are linked by the quark trajectories $x_\mu(\gamma)$. In leading order in $1/N$ it is necessary to take into account only planar gluon diagrams. This means that in the integral

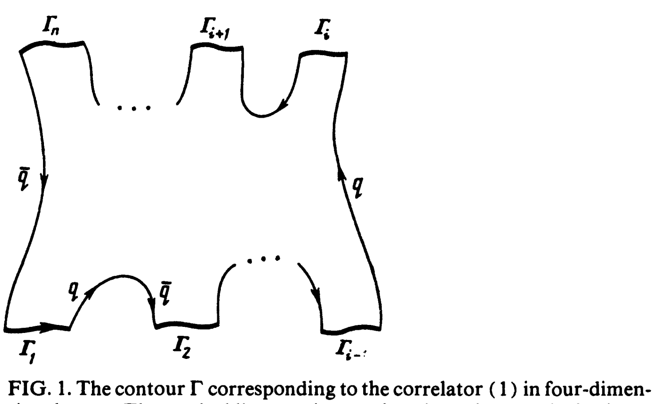

FIG. 1. The contour Γ corresponding to the correlator (1) in four-dimensional space. The vertical lines depict quark trajectories and the horizontal lines depict the paths of integration in formulas (3) and (1).

 0038-5646/86/061124-06$04.00

$$\langle O(\Gamma)\rangle_A = B^{-1}\int d\mu(A)\exp[-S_{Y-M}(A)] \times \mathrm{Tr}\left[P\exp\left(-ie\oint_\Gamma dz_\mu A_\mu\right)\right] \quad (6)$$

it is necessary to sum only over a certain subclass of the fields A_μ^a whose contribution is the most important in the indicated approximation. In Ref. 1 this subclass was defined as follows.[1)] We consider in R^4 an arbitrary two-dimensional surface Σ, without holes or handles and resting on the contour Γ. The surface is specified by the equation $x_\mu = z_\mu(\eta^i)$, where $\mu = 0, 1, 2, 3$, and $i = 0, 1$. We shall determine the tangent components $A_i^a(\eta) = (\partial z_\mu/\partial\eta^i)A_\mu^a(z(\eta))$ of the fields.

The approximate calculation of the average (6), according to Ref. 1, reduces to integration over $A_i^a(\eta) \equiv A_i^a(\Sigma)$ for a fixed surface Σ and subsequent summation over all surface Σ with the boundary $\partial\Sigma = \Gamma$, i.e.,

$$\langle O(\Gamma)\rangle_A \approx \int d\mu(\Sigma)\int d\mu[A_i^a(\Sigma)]\exp\{-S[A_i^a(\Sigma)]\}O(\Gamma). \quad (7)$$

Here,

$$S[A_i^a(\Sigma)] = \frac{1}{4}\int_\Sigma d^2\eta\, g^{1/2}g^{il}g^{kn}G_{ik}^a G_{ln}^a, \quad (8)$$

$$G_{ik}^a = \frac{\partial \bar{A}_k}{\partial\eta^i} - \frac{\partial \bar{A}_i^a}{\partial\eta^k} - \varepsilon f^{abc}\bar{A}_i^b\bar{A}_k^c, \quad (9)$$

where $g = \det g_{ik}$, $g_{ik}(\eta) = (\partial z_\mu/\partial\eta^i)(\partial z_\mu/\partial\eta^k)$ being the metric tensor on Σ. In order that the action (8) be dimensionless, we have changed the normalization of the fields in accordance with

$$eA_i^a = (e/d)(A_i^a d) \equiv \varepsilon\bar{A}_i^a, \quad \varepsilon = e/d, \quad (10)$$

where d is an arbitrary constant with the dimensions of length, ε is the bare dimensional charge (the symbol $\sim$ is henceforth omitted). Next, we represent $O(\Gamma)$ in the form of an integral over the Grassmann fields $\xi_c(\gamma)$ $(c = 1,...,N)$ (Refs. 2, 4):

$$O(\Gamma) = \int\prod_\gamma [iD\xi(\gamma)D\xi^*(\gamma)]i\xi_c(1)\xi_c^*(0)e^{-S(\xi)}, \quad (11)$$

$$S(\xi) = \oint_\Gamma d\gamma\,\xi_c^*(\gamma)\left[\frac{d}{d\gamma} + i\varepsilon A_i\frac{d\eta^i}{d\gamma}\right]_{cd}\xi_d(\gamma). \quad (12)$$

Here it has been taken into account that $A_\mu \dfrac{dz_\mu[\eta(\gamma)]}{d\gamma} = A_\mu \dfrac{\partial z_\mu}{\partial\eta^i}\dfrac{d\eta^i}{d\gamma} = A_i\dfrac{d\eta^i}{d\gamma}$.

The variable γ parametrizes the closed contour $\Gamma = \partial\Sigma$, by $x_\mu = x_\mu(\gamma)$, and varies monotonically from zero to unity, with $x_\mu(0) = x_\mu(1)$. After this, each term in the sum over permutations in (4) is written in the form

$$\int\prod_{q=1}^n \mathcal{D}x_q \int d\mu(\Sigma)\int d\mu[A(\Sigma)] \times\int [iD\xi(\gamma)D\xi^*(\gamma)]i\xi_c(1)\xi_{c*}(0)\exp(-S_{eff}), \quad (13)$$

where

$$S_{eff} = S_0(x) + S_{Y-M}(A) + S(\xi) = \sum_{q=1}^n\left[-\frac{1}{2}\int_{\gamma_1}^{\gamma_2} d\gamma\left(\frac{\dot{x}^2}{\lambda} + \lambda m^2\right)\right] + \frac{1}{4}\int_\Sigma d^2\eta\, g^{1/2}g^{il}g^{kn}G_{ik}^a G_{ln}^a + \oint_\Gamma d\gamma\,\xi_c^*(\gamma)\left[\frac{d}{d\gamma} + i\varepsilon A_i\frac{d\eta^i}{d\gamma}\right]_{cd}\xi_d(\gamma). \quad (14)$$

The term $S(\xi)$ has arisen from the total contribution of the ordered exponentials from the operators (3) and the quark propagotars. The action (14) takes into account the above-the-vacuum excitations generated by the operators (3). It is invariant under arbitrary transformations of the coordinates η^i and γ. The kinetic quark term $S_0(x)$ is invariant under the reparametrization[5] $\gamma\to\gamma' = f(\gamma)$, $\lambda\to\lambda' = \lambda/\dot{f}(\gamma)$. The Euclidean involution operation

$$(\xi_1\xi_2)^* = \xi_1^*\xi_2^* \quad (15)$$

ensures that $S(\xi)$ is real.

3. EXACT SOLUTION OF THE EUCLIDEAN FIELD EQUATIONS

The leading quasiclassical approximation is determined by the contribution of the classical fields[2)] to the correlator (4):

$$K(1,\ldots,n) \sim i\xi_c^{cl}(1)\xi_c^{cl*}(0)\exp[-S_{eff}(A^{cl},\xi^{cl},x^{cl},\lambda^{cl})]. \quad (16)$$

The equations of motion for these fields are obtained by variations of S_{eff} (14). For the field A_i^a we have the Yang-Mills equation on the surface Σ:

$$D_i^{ab}G^{b,ik}(\eta) = 0, \quad D_i^{ab} = \delta^{ab}\nabla_i + \varepsilon f^{abc}A_i^c, \quad (17)$$

$$\nabla_i G^{ik} = \frac{1}{g^{1/2}}\frac{\partial(g^{1/2}G^{ik})}{\partial\eta^i}$$

or

$$\partial_i(g^{1/2}G^{a,ik}) + \varepsilon f^{abc}A_i^c g^{1/2}G^{b,ik} = 0. \quad (18)$$

The surface term from the variation of $S_{\text{Y-M}}$ is combined with the result of varying $\delta S(\xi)/\delta A_i^a$ and leads to a boundary condition for the stress tensor on $\partial\Sigma = \Gamma$:

$$\frac{d\eta^t}{d\gamma}(g^{1/2}G^{a,ik}e_{it} + \varepsilon T^a(\gamma)\delta_t^k) = 0; \quad (19)$$

here e_{it} is the two-dimensional antisymmetric unit tensor. The quark color-spin vector

$$T^a(\gamma) = i\xi_c^*(\gamma)(\lambda^a/2)_{cd}\xi_d(\gamma) \quad (20)$$

is real by virtue of the definition (15). The classical Grassman field $\xi^{cl}(\gamma)$ appearing in it obeys the equation of motion

$$\frac{d\xi_c}{d\gamma} + i\varepsilon\left(\frac{\lambda^a}{2}\right)_{cd}\xi_d A_i^a\frac{d\eta^i}{d\gamma} = 0. \quad (21)$$

From formulas (20) and (21) follows the equation

$$\frac{dT^a}{d\gamma}-\varepsilon f^{abc}A_k{}^bT^c\frac{d\eta^k}{d\gamma}=0,\qquad (T^a)^2-\text{const.} \tag{22}$$

The formal solution of Eq. (21) is expressed in terms of an ordered exponential:

$$\xi_c{}^{cl}(\gamma)=\left[P\exp\left(-i\varepsilon\frac{\lambda^a}{2}\int_0^\gamma d\gamma'\frac{d\eta^i}{d\gamma'}A_i{}^a\right)\right]_{cd}\xi_d{}^{cl}(0), \tag{23}$$

which is an element of the group $SU(N)$. The field $\xi^{cl}(\gamma)$ is defined only on the boundary $\partial\Sigma=\Gamma$ and realizes a mapping of the closed contour Γ into the group $SU(N)$. Since $SU(N)$ for $N\geqslant 2$ is simply connected, this mapping is topologically trivial, i.e., $\pi_1(SU(N))=0$. An exception is provided by the case of the subgroup $U(1)$, for which $\pi_1(U(1))=\pi_1(S^1)=Z$, where Z is the group of integers. Here, π_1 is the homotopy group of the mappings.

It is known that the stable field configurations are topologically nontrivial solutions of the classical equations. Therefore, we shall consider the exact solution of Eqs. (18), (19), (21), which spontaneously breaks the gauge group $SU(N)$ to the local subgroup $U(1)$.

We shall start from the ansatz

$$G^{a,\,ik}(\eta)=g^{im}g^{kl}G_{ml}{}^a=\lambda e^{ik}I^a(\eta)/[g(\eta)]^{1/2}, \tag{24}$$

where λ is a constant factor, and $I^a(\eta)$ is a function of η^i that belongs to the adjoint representation of $SU(N)$. Substitution of (24) into Eq. (18) gives

$$\partial I^a/\partial\eta^i-\varepsilon f^{abc}A_i{}^bI^c=0,\qquad (I^a)^2-\text{const}, \tag{25}$$

i.e., the quantity (24) will be the solution of Eq. (18) if the vector $I^a(\eta)$ is covariantly constant. From the boundary condition (19) it follows that $\lambda=\varepsilon$ and

$$I^a(\eta(\gamma))\equiv I^a(\gamma)=T^a(\gamma). \tag{26}$$

We note that in two dimensions the Bianchi relation

$$D_l{}^{ab}G_{ik}{}^b(\eta)+D_i{}^{ab}G_{kl}{}^b(\eta)+D_k{}^{ab}G_{li}{}^b(\eta)=0$$

is fulfilled identically for the antisymmetric tensor G_{ik}^a. Thus, the field intensity has the form

$$G^{a,\,ik}(\eta)=\varepsilon e^{ik}I^a(\eta)/[g(\eta)]^{1/2},$$
$$G_{ik}{}^a(\eta)=\varepsilon e_{ik}[g(\eta)]^{1/2}I^a(\eta). \tag{27}$$

We now determine the two-dimensional potential $A_i^a(\eta)$ corresponding to (27). To begin we go over to a gauge in which the vector $I^a(\eta)$ is constant. Since $I^a(\eta)$ is defined on the open surface Σ and satisfies Eq. (25), this can always be done. The expression

$$A_i{}^a(\eta)=I^aa_i(\eta)/\varepsilon, \tag{28}$$

where $a_i(\eta)$ is an Abelian field, satisfies Eq. (25).

Substituing (28) into (9), we have

$$G_{ik}{}^a(\eta)=I^aF_{ik}(\eta)/\varepsilon,\quad F_{ik}(\eta)=\partial_ia_k(\eta)-\partial_ka_i(\eta). \tag{29}$$

This expression will satisfy Eq. (18), i.e., will coincide with formula (27), under the following condition: The quantity dual to F_{ik} should be constant:

$${}^*F\equiv\frac{1}{2}\frac{e^{ik}}{[g(\eta)]^{1/2}}F_{ik}(\eta)=\varkappa,\qquad \varkappa=\varepsilon^2. \tag{30}$$

This relation is not changed under arbitrary transformation of the coordinates η_i. Substituting the ansatz

$$a_i(\eta)=[g(\eta)]^{1/2}e_{ik}\partial^k\Lambda(\eta) \tag{31}$$

(which corresponds to the Lorentz condition $a^i_{;i}=0$) into the relation (30), we obtain an equation for the function $\Lambda(\eta)$:

$$g^{-1/2}\partial_i(g^{1/2}g^{ik}\partial_k\Lambda)=\varkappa. \tag{32}$$

The nontrivial solution of Eq. (32) for $\Lambda(\eta)$ is found by going over to conformal coordinates. Then the equation will be satisfied if we set

$$\Lambda(\eta)=\mp{}^1/_2\ln[g_R(\eta)]^{1/2}+f(\eta),\qquad R=\pm2\varkappa, \tag{33}$$

where g_R is the metric of a surface of constant scalar curvature R, f is an arbitrary function satisfying the condition $\Delta f=0$, and Δ is the Laplacian in conformal coordinates. Thus, the equation of motion (18) has a solution of the form (27), (28) only on a surface of constant scalar curvature[3] $|R|=2\varepsilon^2$. Surfaces of other types do not provide an extremum of the action S_{eff} (14).

With the world tensor (27) we can associate a Lorentz $O(4)$ tensor in accordance with

$$G_{\mu\nu}{}^a(z(\eta))=\frac{\partial z_\mu}{\partial\eta^i}\frac{\partial z_\nu}{\partial\eta^k}G^{a,ik}(\eta).$$

We then obtain the expression

$$G_{\mu\nu}{}^a(z(\eta))=\varepsilon I^a(\eta)\sigma_{\mu\nu}(\eta)/[g(\eta)]^{1/2}, \tag{34}$$

where

$$\sigma_{\mu\nu}=\frac{\partial z_\mu}{\partial\eta^0}\frac{\partial z_\nu}{\partial\eta^1}-\frac{\partial z_\mu}{\partial\eta^1}\frac{\partial z_\nu}{\partial\eta^0},$$

which coincides with the field intensity of a chromoelectric hadron string. Such a field was studied earlier in Refs. 7, 3, and 8, but its derivation from first principles of quantum theory was less systematic from a logical point of view than here.

We shall determine the contribution of the classical fields to S_{eff} (14). Substitution of the expression for G_{ik}^a (27) into $S_{\text{Y-M}}(A)$ gives

$$S_{Y\text{-}M}(A^{cl})=\frac{\varepsilon^2I^2}{2}\int_\Sigma d^2\eta[g(\eta)]^{1/2}=\frac{1}{2\pi\alpha_0'}A(\Sigma), \tag{35}$$

where $A(\Sigma)$ is the area of the surface Σ swept out by the string. The quantity

$$k_0=\frac{1}{2\pi\alpha_0'}=\frac{\varepsilon^2I^2}{2}=\frac{e^2I^2}{2d^2}=\frac{e^2C_2(F)}{2d^2}, \tag{36}$$

where $C_2(F)$ is the Casimir operator of the fundamental representation of $SU(N)$, plays the role of the bare tension coefficient of the string. It has the same structure as in lattice gauge theories.[9] The term $S(\xi^{cl})$ does not make a contribution to $S_{\rm eff}$, since it vanishes by virtue of the equation of motion (21). In the calculation of the kinetic quark term $S_0(x)$ it is necessary to take into account that the quarks situated at the ends of a string form parts of the boundary $\partial\Sigma$. By virtue of this, we write

$$\dot{x}_2=\frac{\partial x_\mu}{\partial\eta^i}\frac{\partial x_\mu}{\partial\eta^k}\frac{\partial\eta^i}{d\gamma}\frac{d\eta^k}{d\gamma}=g_{ik}(\eta(\gamma))\dot{\eta}^i\dot{\eta}^k.$$

The variation of the quark action

$$S_q=\int_{\gamma_1}^{\gamma_2}d\gamma\left\{\frac{1}{2}\left(g_{ik}\frac{\dot{\eta}^i\dot{\eta}^k}{\lambda}+\lambda m^2\right)+\varepsilon T^a(\gamma)A_i{}^a\frac{d\eta^i}{d\gamma}\right\} \tag{37}$$

with respect to $\eta^i(\gamma)$ leads to the equation of motion of a quark (in the natural parameter $\gamma = s$)

$$m\left(\frac{d^2\eta^i}{ds^2}+\Gamma_{kl}{}^i\frac{d\eta^k}{ds}\frac{d\eta^l}{ds}\right)=\varepsilon T^a G^{a,ik}\frac{d\eta^k}{ds}. \tag{38}$$

Variation with respect to λ gives $\lambda^{cl}=(\dot{\eta}^2)^{1/2}/m$. Taking this into account, we can express the kinetic term in (14) in terms of the length l of the quark trajectory:

$$S_0(x)=\sum_{q=1}^{n}\left(m\int_{\gamma_1}^{\gamma_2}d\gamma(\dot{\eta}^2)^{1/2}\right)_q=\sum_{q=1}^{n}m_q l_q, \tag{39}$$

the form of which is determined by Eq. (38).

As a result the correlator $K(1,...,n)$ takes the form

$$K(1,\dots,n)$$

$$\approx i\xi_c{}^{cl}(1)\xi_c{}^{cl^*}(0)\left\{\exp\left[-k_0A(\Sigma)-\sum_{q=1}^{n}m_q l_q\right]\right\}\Psi. \tag{40}$$

Here the factor Ψ takes into account all the nonleading (in $1/N$) contributions that arise from allowance for Gaussian fluctuations about the classical fields. Amongst these contributions, in particular, are sums over surfaces and quark trajectories (see formula (13)), over zero modes, etc.

To conclude this section we shall give a short explanation. The final answer for the correlator (1) should be a function of the contours Γ_i $(i=1,...,n)$, while the area of a surface of constant curvature is a function of the entire boundary $\partial\Sigma=\Gamma$, which includes not only the Γ_i but also the quark trajectories. However, after these trajectories have been found (by solving Eqs. (38)), they become functions of the ends of the contours Γ_i. This classical problem, like the problem of the determination of the coordinates of the surface Σ (the position of which is also determined by the contours Γ_i), can be fully solved, if this is desired, but this is not the purpose of the present paper.

4. QUANTIZATION OF THE CHROMOELECTRIC FLUX AND ACTION

According to Eq. (21) the classical color spinor (the fundamental representation of $SU(N)$), defined on the boundary $\partial\Sigma=\Gamma$, is covariantly constant. As follows from the preceding section, into Eq. (21) it is necessary to substitute the gauge field $A_i^a(\eta)$ (28), which is the exact solution of the Yang-Mills equation (18) with spontaneous breaking of the gauge symmetry $SU(N)$ to $U(1)$. According to the boundary condition (19), (26), the color vector (the adjoint representation of $SU(N)$) on the boundary is replaced by the kernel (20) of the integral operator of the quark color spin, i.e.,

$$I^a(\eta(\gamma))\equiv I^a(\gamma)=i\xi_c{}^*(\gamma)(\lambda^a/2)_{cd}\xi_d(\gamma)=T^a(\gamma). \tag{41}$$

The action of such operators in the space of holomorphic functions $f(\xi^*)$ and the action of a product of operators are specified by the formulas[10]

$$Tf(\xi^*)=\int\prod(id\zeta_c{}^*d\zeta_c)T(\xi^*,\zeta)f(\zeta^*)\exp\left\{-i\sum\zeta_c{}^*\zeta_c\right\}, \tag{42}$$

$$(T_1T_2)(\xi^*,\xi)$$

$$=\int\prod_c^c(id\zeta_c{}^*d\zeta_c)T_1(\xi^*,\zeta)T_2(\zeta^*,\xi)\exp\left\{-i\sum_c^c\zeta_c{}^*\zeta_c\right\}. \tag{43}$$

(The extra factor i has arisen because of the Euclidean conjugation rule (15) and ensures that the integration measure is real.) By means of the expression (43) one can check that the quantities (20) satisfy the algebra of $SU(N)$:

$$[T^a(\gamma),T^b(\gamma)]=if^{abc}T^c(\gamma)$$

and that

$$\sum_a T^a(\gamma)T^a(\gamma)=C_2(F)\hat{E}, \tag{44}$$

where $C_2(F)=(N^2-1)/2N$ is the Casimir operator, and

$$\hat{E}=i\sum_{c=1}^{N}\xi_c{}^*\xi_c \tag{45}$$

is the kernel of the unit operator in the fundamental representation.

Application of formula (42) to the right-hand side of the equation of motion (21) leads to the following result:

$$d\xi_c(\gamma)/d\gamma=-iC_2(F)a_i(\eta(\gamma))\frac{d\eta^i}{d\gamma}\xi_c(\gamma). \tag{46}$$

Solving this equation, we obtain

$$\xi_c(\gamma)=e^{-i\varphi(\gamma)}\xi_c(0), \tag{47}$$

where the phase

$$\varphi(\gamma)=C_2(F)\int_0^\gamma ds\frac{d\eta^i}{ds}a_i(\eta)=I^2\int_0^\gamma ds a_s \tag{48}$$

does not depend on the index $c=1,...,N$, this being a consequence of the spontaneous breaking of the $SU(N)$ symmetry to $U(1)$. The phase $\varphi(\gamma)$ forms mappings $S^1\to S^1$, which can be decomposed into homotopy classes characterized by a winding number Q, since for the mappings the homotopy group $\pi_1(S^1)=Z$. In other words, for the spinor (47) to be single-valued as a function of the parameter γ it is necessary to require that the result of going round a closed contour lead to a phase that is a multiple of 2π, i.e.,

$$\varphi(1)=I^2\int_0^1 d\gamma a_\gamma=C_2(F)\oint_\Gamma d\eta^i a_i=2\pi Q,$$

$$Q=0,\ \pm1,\quad \pm2\ldots. \qquad (49)$$

Mappings of a given class cannot be continuously deformed into mappings of another class without destroying the single-valuedness of $\xi_c(\gamma)$.

The relation (49) leads to quantization of the chromoelectric-field flux

$$\Phi=\oint_{\Gamma=\partial\Sigma} d\eta^i a_i=\frac{1}{2}\int_\Sigma d^2\eta g^{1/2}\frac{e^{ik}}{g^{1/2}}F_{ik}=\int_\Sigma d^2\eta g^{1/2}{}^*F=\frac{2\pi Q}{C_2(F)}. \qquad (50)$$

Using the expressions (30) and (36) we find that the action $S_{\text{Y-M}}(A^{cl})$ is also quantized:

$$S_{\text{Y-M}}=\pi|Q|. \qquad (51)$$

Another quantity that takes discrete values is the area $A(\Sigma)$ swept out by the string:

$$A(\Sigma)=2\pi|Q|/\varepsilon^2C_2(F). \qquad (52)$$

We note that, despite the relation (36), the action $S_{\text{Y-M}}(A^{cl})$ does not in fact depend on the magnitude of the arbitrarily introduced scale parameter d. This is a consequence of the fact that, according to (33), the surface Σ has a constant scalar curvature $R\sim1/d^2$.

The topological number Q can be written in an explicitly gauge-invariant form, using the last equality of (50) and (27):

$$Q=\frac{I^2}{2\pi}\int_\Sigma d^2\eta g^{1/2}{}^*F=\frac{\varepsilon}{4\pi}\int_\Sigma d^2\eta e^{ik}I^a(\eta)G_{ik}{}^a(\eta). \qquad (53)$$

At the end of Sec. 3 we used a gauge in which the vector I^a was constant. By means of gauge $SU(N)$ transformations it is possible to rotate the vector locally from point to point. The dual quantity ${}^*F$ is an invariant under such transformations (see (53)), despite the fact that its concrete relationship with the gauge-noninvariant quantities $a_i(\eta)$ and $I^a(\eta)$ depends on the choice of gauge. For example, if to the quantity (28) we apply a transformation from an $SU(2)$ subgroup, then ${}^*F$ is written in terms of new quantities a_i' and $I^{a'}$ as

$${}^*F=\frac{e^{ik}}{2g^{1/2}}\left\{e^{abc}I^{a'}\partial_iI^{b'}\partial_k\frac{I^{c'}}{(I^2)^2}+\partial_ia_k'-\partial_ka_i'\right\},$$

$$a,\ b,\ c=1,\ 2,\ 3.$$

The relation (50) gives the gauge-invariant flux of the field F_{ik}. One quantum of this flux is equal to

$$\Phi_0=2\pi\hbar c/C_2(F). \qquad (54)$$

(If we had followed analogies with the theory of superconductivity and considered the flux of the (total) field G^a_{ik} (29), we should have obtained a gauge-noninvariant expression for Φ_0, which, moreover, would have contained the bare charge ε, which is unreasonable.) It can be seen from the expression (54) that with increase of the number N of colors the flux quantum Φ_0 decreases, since

$$\Phi_0\approx4\pi\hbar c/N,\quad N\gg1.$$

By means of global $SU(N)$ rotations it is possible to go over in a continuous manner from one value of $\xi_c(0)$ in formula (47) to another. This freedom of choice of $\xi_c(0)$ does not correspond to yet another homotopic classification. The situation somewhat resembles that in the CP_N model, but there is an important difference. In the CP_N model the spinor field is specified on the entire surface, whereas in our case $\xi_c(\gamma)$ is defined only on the boundary $\partial\Sigma$. Henceforth, in calculating quantum fluctuations, one must regard the quantity $\xi_c^{cl}(0)$ as the zero Grassmann mode and integrate over it.

5. CONCLUSION

In the preceding sections we have described a topologically nontrivial solution of the Euclidean field equations that spontaneously breaks the gauge group $SU(N)$ to the local Abelian subgroup $U(1)$. The non-Abelian structure of the theory was used at the following points:

1) the $1/N$ expansion in the number of colors in the derivation of S_{eff} (14);

2) the intensity $G^a_{ik}(\eta)$ (27) is expressed in terms of the vector $I^a(\eta)$, the magnitude of which is determined, with allowance for the boundary condition (26), by the color spin of the quark. This spin is nonzero for $N\geqslant2$ and thereby ensures a nonzero value of the intensity (27); the requirement of covariant constancy of the vector $I^a(\eta)$ follows from the non-Abelian equation (18);

3) the quantization of the action and of the flux Φ is due to the single-valuedness of the color spinors $\xi_c(\gamma)$ defined on the closed contour Γ. We note that the quantities $\xi_c(\gamma)$ themselves appear only in the non-Abelian theory (formula (11)). Thus, in the Abelian case we would have neither S_{eff} (14), nor flux quantization, nor the very field configuration that describes the bare string.

The solution considered is nonperturbative in the charge ε. This follows from the fact that the metric $g_R(\eta)$ in terms of which the field $A^a_i(\eta)$ is expressed (see formulas (18)–(33)) is the solution of a nonlinear equation (in conformal coordinates, the latter reduces to the Liouville equation). The topological classification should ensure stability of the solution. Together with the fulfillment of the quasiclassicality criterion $S\gg\hbar$ for $Q>1$ this gives grounds for hope that the quasiclassical approximation can be successfully applied (on the basis of the solution described) to the calculation of the remaining integrals (13) in terms of which the hadron correlators (1) are expressed.

[1)]The subclass of two-dimensional fields is distinguished from other field configurations in that it is this subclass which contains a topological section whose homotopy classes are characterized by the number Q (see Sec. 4).

[2)]The method of steepest descent in the presence of Grassmann variables is considered, e.g., in Ref. 6.

[3)]This question is considered in more detail in Ref. 1.

[1]G. Iroshnikov, Proceedings of the International Seminar "Group Theoretical Methods in Physics", Yurmala, 1985, VNU Science Press BV, Utrecht (to be published); G. S. Iroshnikov, Yad. Fiz. **44**, (1986) [Sov. J. Nucl. Phys. **44** (1986)].

[2]S. Samuel, Nucl. Phys. **B149**, 517 (1979).

[3]G. S. Iroshnikov, Yad. Fiz. **38**, 512 (1983) [Sov. J. Nucl. Phys. **38**, 304 (1983)].

[4]J. Ishida and A. Hosoya, Prog. Theor. Phys. **62**, 544 (1979).

[5]N. V. Borisov and P. P. Kulish, Teor. Mat. Fiz. **51**, 335 (1982) [Theor. Math. Phys. **51**, 535 (1982)].

[6]P. Raymond, Field Theory: A Modern Primer, Benjamin/Cummings, Reading, Mass. (1981), p. 214 [Russ. transl., Nauka, Moscow (1984)].

[7]S. Ryang and J. Ishida, Prog. Theor. Phys. **66**, 685 (1981).

[8]G. S. Iroshnikov, Dokl Akad. Nauk SSSR **280**, 88 (1985) [Sov. Phys. Dokl. **30**, 47 (1985)].

[9]P. Becher, M. Böhm, and H. Joos, Gauge Theories of Strong and Electroweak Interactions, Wiley, N. Y. (1983), p. 182.

[10]A. A. Slavnov and L. D. Faddeev, Vvedenie v kvantovuyu teoriyu kalibrovochnykh poleĭ (Introduction to the Quantum Theory of Gauge Fields), Nauka, Moscow (1978), p. 62 [English translation published by Benjamin/Cummings, Reading, Mass. (1980)].

Translated by P. J. Shepherd