A non-linear modification of the field of an external source in a rarefied magnetized plasma

V. N. Gol'dberg, E. A. Mareev, V. A. Ugrinovskiĭ, and Yu. V. Chugunov

Institute of Applied Physics, Academy of Sciences of the USSR
(Submitted 11 October 1985)
Zh. Eksp. Teor. Fiz. **90**, 2013–2022 (June 1986)

We study the effect of a striction nonlinearity on the structure of the quasistationary field excited by an external source of electromagnetic waves in a rarefied magnetized plasma. We obtain a contracted equation for the electric field of the source near the resonance surface; we find a set of its analytical solutions. We give a numerical analysis of the boundary problem for the corresponding equation. The results obtained enable us to trace the nature of the nonlinear modification of the field as function of the parameter $(E_u/E_p)(h/r_D)$.

The heightened interest in the theory of electromagnetic-wave emission by a source in a magnetoactive plasma is due to the many applications in the fields of microwave heating, diagnostics of laboratory and cosmic plasmas, and so on. The peculiar electrodynamic properties of a plasma, and especially the presence of electrostatic eigenoscillations (resonances) and spatial dispersion, lead to the occurrence of special features in the structure of the field of a source and are accompanied by a change in its impedance properties as compared to vacuum properties.[1] In the high-frequency case such changes are most important when the frequency of the radiation falls in the region where resonances are excited while the characteristic dimension of the emitter is small compared to the electromagnetic wavelength. The source field is then characterized by a steep increase on the characteristic surface (in a uniform plasma—on the resonance cone) while the impedance is characterized by the presence of a real part caused by the effective excitation of the plasma resonance. We study in the present paper the effect of a striction-type nonlinearity on the structure of the field of the source under those conditions. We note that the power level of the sources in experiments which have been performed or are being planned is sufficient for the appearance of nonlinear effects.[2–4] However, there is only a relatively small number of papers devoted to the problem of the emission of waves by a source in a magnetoactive plasma with nonlinearity taken into account. In our view, this is due to the complexity of the problem: a consistent solution of the problem of the particle distribution and the electric field structure in the vicinity of the emitter is very complicated even in an isotropic plasma.[5] Some problems of the effect of strictional nonlinearity on the field structure have been discussed in the literature for sources of a special shape.[6,7]

We show in the present paper that even a weak nonlinearity may lead to an appreciable redistribution of the field of a compact source under resonance conditions. We use here the transition to the contracted equation for the electric field near the resonance surface which was proposed for a study of lower-hybrid plasma heating problems (Refs. 8–10).[1)] We discuss the conditions under which the solution of the boundary problem for the equation obtained describes the field of a source placed in a plasma. We give the results of a numerical solution of the corresponding boundary problem. We find a set of analytical solutions of the equation studied.

1. We consider the field of the source of an external current which oscillates with a frequency ω and which is located in a magnetized plasma ($\omega_{He} \gg \omega_{pe}$; ω_{pe} and ω_{He} are the electron Langmuir and gyrofrequencies). We assume everywhere in what follows that the characteric size h of the source is small compared to the wavelength of the electromagnetic mode.

This enables us to use near it the electrostatic approximation. Writing down the hydrodynamic equations for the hf plasma motions we find an equation connecting the amplitude ψ of the hf potential with the magnitude of the low-frequency (quasineutral) perturbations δn of the plasma density (cf. Ref. 7):

$$\frac{2i}{\omega}\Delta\frac{\partial\psi}{\partial t}+\Delta_\perp\psi+\frac{\partial}{\partial z}\left[(1-n_0-\delta n+isn_0)\frac{\partial\psi}{\partial z}\right]+3\frac{v_T^2}{\omega^2}\Delta\frac{\partial^2\psi}{\partial z^2}=-4\pi\rho_{\rm ext}. \qquad (1)$$

Here $\rho_{\rm ext}(\mathbf{r}) = -(i/\omega)\operatorname{div}\mathbf{j}_{\rm ext}$; n_0 is the equilibrium density of the electrons in the plasma, normalized, like δn, to the critical density $n_c = m\omega^2/4\pi e^2$;v_T is the electron thermal velocity; we assume that $s = \nu/\omega \ll 1$ (ν is the electron collision frequency). The z-axis is directed along the external magnetic field; $\Delta_\perp = \partial^2/\partial x^2 + \partial^2/\partial y^2$. We shall look for stationary field and density distributions under conditions where a striction-type nonlinearity dominates. The expression for small charged-particle-density perturbations of the which are caused in a magnetized plasma only by the presence of the E_z field component has then the form

$$\delta n=-n_0E_p^{-2}|\partial\psi/\partial z|^2, \qquad (2)$$

where $E_p = [4m\omega^2(T_e + T_i)/e^2]^{1/2}$ is the plasma field. Neglecting collisions[2)] we get from (1) and (2) an equation for the potential:

$$\Delta_\perp\psi+(1-n_0)\frac{\partial^2\psi}{\partial z^2}+\frac{n_0}{E_p^2}\frac{\partial}{\partial z}\left[\frac{\partial\psi}{\partial z}\left|\frac{\partial\psi}{\partial z}\right|^2\right]+3\frac{v_T^2}{\omega^2}\Delta\frac{\partial^2\psi}{\partial z^2}=-4\pi\rho_{\rm ext}. \qquad (3)$$

If the charged-particle density exceeds the critical density, i.e., when the longitudinal permittivity is negative, quasipo-

 0038-5646/86/061180-06$04.00

tential waves of frequency ω can propagate in the plasma. Neglecting nonlinear and dispersive terms in Eq. (3), it follows in this case that in the vicinity of the source a resonance cone is excited with an apex angle $\theta = \arctan \mu$, $\mu = (n_0 - 1)^{-1/2}$.

We note that we obtained Eq. (3) assuming spatial dispersion to be weak: $h \gg v_T/\omega$. Therefore, if the external charge distribution is sufficiently smooth, the dispersive correction becomes important at distances $r \approx h(h\omega/v_T)^2 \gg h$ from the source while for $r \gtrsim h$ it is sufficient to take into account the nonlinear correction.[3)] Under the action of the nonlinearity there arises an interaction between two sets of characteristics of the wave equation passing through the region occupied by the source. This process can be studied only numerically when one considers hyperbolic equations, except for the simplest cases.[11] Outside the interaction region the solution can be simplified, but the boundary conditions are decided by this region.

We can simplify Eq. (3) outside the interaction region by using a quasiuniform field near the characteristic surface (the resonance cone). Since in the neighborhood of the resonance cone the field is maximal and decreases slowly with increasing distance from the source, it just here that the nonlinear and dispersive effects are most important. It is convenient to change to a frame of reference with the τ-axis along the characteristic surface and the Δ-axis at right angles to it, i.e., turn the coordinate system around the y-axis:

$$\Delta = z \sin\theta - x\cos\theta, \quad \tau = z\cos\theta + x\sin\theta. \tag{4}$$

Neglecting the term $\partial^2\psi/\partial\tau^2$ and writing $E = -\partial\psi/\partial\Delta$ we transform Eq. (3) to the form

$$\frac{\partial E}{\partial \tau} + \frac{E}{2\tau} - \alpha\frac{\partial^3 E}{\partial\Delta^3} - \beta\frac{\partial}{\partial\Delta}(E|E|^2) = 0, \tag{5}$$

where $\alpha = 3/2\mu r_D^2, \beta = \mu/2n_0E_p^2$. The boundary condition in τ is obtained by starting from the solution of the wave equation (3) in the interaction region. The simplest case is when the characteristic value of the field E_u near the source is sufficiently small: $E_u^2/E_p^2 \ll 1$. It is clear from Eq. (3) that the non-linear interaction is then small and at distances $r \gtrsim h$ from the source its field is linear. However, in that case, too the non-linearity "accumulating" along the resonance surface can lead to a considerable change in the linear structure of the field. To prove this statment we study Eq. (5) with the boundary condition $E(\tau = \tau_0, \Delta) = E_0(\Delta)$, where E_0 is the linear cold field specified for $\tau_0 \gtrsim h$, of a compact source near the resonance cone. As an example of a source we consider in what follows an electric monopole: $\rho_{\text{ext}} = qh\pi^{-2}(r^2 + h^2)^{-2}$ so that the boundary condition has the form

$$E_0 = \frac{q\mu^{1/2}}{2\cdot 2^{1/2}\tau_0^{1/2}}(\Delta + ih)^{-3/2}. \tag{6}$$

We use for the boundary conditions in Δ the conditions of emission as $\Delta \to \infty$ and of a decrease in the field as $\Delta \to -\infty$ for the asymptotic form of the plasma wave which appears against the background of the cold solution.

2. When solving the boundary value problem (5),(6) it is advisable to write these equations in terms of dimensionless variables

$$\Delta' = \Delta/h, \quad \tau' = \tau\alpha/h^3, \quad E' = Eh^3/q_{\text{eff}}\alpha^{1/2},$$

where

$$q_{\text{eff}} = q\mu^{1/2}/2\sqrt{2}, \quad \sigma = q_{\text{eff}}^2\beta/h^4$$

(we omit the primes in what follows),

$$\frac{\partial E}{\partial \tau} + \frac{E}{2\tau} - \frac{\partial^3 E}{\partial\Delta^3} - \sigma\frac{\partial}{\partial\Delta}(E|E|^2) = 0, \tag{7}$$

$$E_0 = \tau_0^{-1/2}(\Delta + i)^{-3/2}. \tag{8}$$

Equation (7) contains a single dimensionless parameter which under the conditions of the problem is small: $\sigma \ll 1$. The parameter τ_0 in Eq. (8) satisfies the conditions $\alpha/h^2 \lesssim \tau_0 \ll 1$.

We analyze first of all the simplest case when the evolution of the boundary field (8) is mainly determined by nonlinearity or the dispersion. Estimates show that the degree of the influence of the nonlinearity is determined by the parameter σ/τ_0 and equals in dimensional variables

$$\frac{\mu h}{24 n_0 \tau_0}\frac{q^2}{h^4 E_p^2}\frac{h^2}{r_D^2}.$$

If the factor σ/τ_0 is sufficiently large ($\sigma/\tau_0 \gg 10$), one can in, neglect the region where the larger part of the energy of the boundary field is concentrated the third derivative in Eq. (7). Writing $E = \tau^{-1/2}Ae^{i\varphi}$ it is convenient in that case to change to a set of equations for the amplitude and phase of the field:

$$\frac{\partial A}{\partial\tau} - \frac{3\sigma}{\tau}A^2\frac{\partial A}{\partial\Delta} = 0, \tag{9}$$

$$\frac{\partial\varphi}{\partial\tau} - \frac{\sigma}{\tau}A^2\frac{\partial\varphi}{\partial\Delta} = 0. \tag{10}$$

It follows from Eq. (9) that the amplitude A is constant on the characteristics given by the relation $d\Delta/d\tau = -3\sigma A^2/\tau$. Hence one easily gets an expression which explicitly determines the function $A^2(\Delta,\tau)$:

$$A^2 = [1 + (\Delta + 3\sigma A^2 \ln(\tau/\tau_0))^2]^{-3/2}. \tag{11}$$

In accordance with the boundary condition at $\tau = \tau_0$ we have $A^2 = (1 + \Delta^2)^{-1/2}$. It is clear from Eq. (11) that as τ increases the nonlinearity leads to a steeper boundary distribution of the field amplitude. When $\tau_c = \tau_0 \exp(4\sqrt{2}/9\sigma)$ the derivative $\partial A^2/\partial\Delta$ becomes infinite (in the point $\Delta_c = -15/9$), and then the function $A^2(\Delta)$ becomes tripled-valued. However, as one approaches τ_c the spatial dispersion becomes important so that it is no longer possible to neglect the term $\partial^3E/\partial\Delta^3$. It follows from Eq. (11) that the dispersion term becomes comparable in magnitude with the nonlinear term when $\sigma/\tau_m \approx [1 - 3\sigma\ln(\tau_m/\tau_0)]^{-2}$. Hence, we have for $\tau_m \approx \sigma$ small σ ($\sigma \ll 1$, but $\sigma \gg \tau_0 \gg \sigma e^{-(3\sigma)^{-1}}$). It will become clear from numerical calculations that as τ increases further a solitary wave splits off from the main field distribution in the region of the steep front, and the mildly sloping front becomes jagged due to the appearance of a plasma wave against the background of the cold solution.

It is important to note that, if $\sigma \ll 1$, when one gets away from the source the nonlinearity accumulates rather slowly ($\tau_c, \tau_m \gg \tau_0$). This justifies taking the linear field of the source as the boundary condition.

We consider a second limiting case when $\sigma/\tau_0 \ll 10$, i.e., we can neglect the nonlinear term in Eq. (7) which can be written in the form

$$\partial E/\partial\tau + E/2\tau - \partial^3 E/\partial\Delta^3 = 0. \tag{12}$$

Taking the Fourier transform we get a solution which satisfies the boundary value condition (8):

$$E = \frac{2\exp(-3i\pi/4)}{\pi^{1/2}\tau^{1/2}} \int_0^\infty k^{1/2} \exp\{-i[k^3(\tau-\tau_0) - k(\Delta+i)]\} dk. \tag{13}$$

One shows easily that if $\tau_0 \ll 1$ the solution of the boundary problem (13) is the same as the exact expression for the field of the source near the characteristic. For small τ ($\tau_0 \ll \tau \ll 1$) the magnitude of the integral (13) is completely determined by the vicinity of the point $k = 0$; this leads to Eq. (8) for the cold field. Therefore, if $\tau \ll 1$, the dispersion is unimportant and the field is determined by the characteristic size of the source. If $\tau \gtrsim 1$, using the steepest-descent method, we can find the asymptotic expansion of the integral (13) for $|\Delta| \gg \tau^{1/3}$:

$$E = \tau^{-1/2}\Delta^{-3/2} - \frac{2}{\sqrt{3}}\tau^{-1} \exp\left[-\left(\frac{\Delta}{3\tau}\right)^{1/2}\right] \exp\left[\frac{2i}{3\sqrt{3}} \frac{\Delta^{3/2}}{\tau^{1/2}}\right], \quad \Delta > 0, \tag{14}$$

$$E = \exp\left[\frac{3i\pi}{2}\right]\tau^{-1/2}|\Delta|^{-3/2} - \frac{2}{\sqrt{3}}\tau^{-1} \exp\left[i\frac{|\Delta|^{1/2}}{(3\tau)^{1/2}}\right] \exp\left[-\frac{2}{3\sqrt{3}} \frac{|\Delta|^{3/2}}{\tau^{1/2}}\right], \quad \Delta < 0, \tag{15}$$

From the expressions obtained it is clear that in the region $\Delta > 0$ a plasma wave appears with a characteristic wavelength $\lambda \sim \tau^{1/3} r_D$. The dispersion is thus substantial when the wavelength of the plasma wave becomes larger than or of the order of the size of the source. There is then formed a region occupied by field oscillations which broadens in a self-similar way as one gets away from the source. The role of the nonlinearity here reduces to the fact that with increasing τ this region is displaced relative to the linear distribution to the side of negative Δ, but the rate of this process decreases with increasing τ and the total displacement turns out to be small. Indeed, we consider the solution (14) in the range $\tau^{1/3} \ll \Delta \ll \tau$ where the amplitude of the plasma wave is much larger than the magnitude of the cold field:

$$E = \tau^{-1}\left[\Omega^{-1} - \frac{2}{\sqrt{3}}\exp\left(\frac{2i}{3\sqrt{3}}\Omega\right)\right] \approx -\frac{2}{\sqrt{3}}\tau^{-1}\exp\left(\frac{2i}{3\sqrt{3}}\Omega\right).$$

Here $\Omega = |\Delta|^{3/2}/\tau^{1/2}$. We shall look for a solution of the nonlinear equation (7) in the form

$$E = -\frac{2}{\sqrt{3}}\tau^{-1}\exp\left\{\frac{2i}{3\sqrt{3}} \frac{(\Delta - f(\tau))^{3/2}}{\tau^{1/2}}\right\}. \tag{16}$$

One easily checks that the equation is satisfied, provided $f(\tau) = 4\sigma/3\tau + \text{const}$. Since the dispersive structure develops at $\tau \approx 1$, we put const $= -4\sigma/3$, which gives a rough estimate of the magnitude of the total displacement in Δ: $|\Delta_{\text{disp}}| = \sigma$. We see that $|\Delta_{\text{displ}}| \ll 1$; this fact is connected with the decrease of the field when one gets away from the source.

3. The analysis given above of limiting cases, while making possible to comprehend the basic features of the behavior of the field of a compact source, at the same time enables us to proceed to a more detailed computer study of the problem. It is convenient for what follows to eliminate the term $E/2\tau$ from Eq. (7) by putting $E = u\tau^{-1/2}$. Equations (7),(8) take the form

$$\frac{\partial u}{\partial\tau} - \frac{\partial^3 u}{\partial\Delta^3} - \frac{\sigma}{\tau}\frac{\partial}{\partial\Delta}(u|u|^2) = 0, \qquad u_0(\Delta) = (\Delta+i)^{-1/2}. \tag{17}$$

We note the presence of a number of integrals of Eq. (17):

$$I_1 = \int_{-\infty}^{\infty} u\, d\Delta, \qquad I_2 = \int_{-\infty}^{\infty} |u|^2 d\Delta,$$

$$I_3 = \int_{-\infty}^{\infty} \left\{\left|\frac{\partial u}{\partial\Delta}\right|^2 - \frac{\sigma}{2\tau}|u|^4\right\} d\Delta, \tag{18}$$

$$I_4 = \int_{-\infty}^{\infty} \left\{u \int u^* d\Delta - u^* \int u\, d\Delta\right\} d\Delta.$$

As to its physical meaning, $I_1\tau^{-1/2}$ is the potential difference between the points $\Delta = \pm\infty$, $I_2\tau^{-1}$ the energy of the field, $I_3\tau^{-1}$ the Hamiltonian, and $I_4\tau^{-1}$ the energy flux through a plane $\tau = \text{const}$ per unit length along y. All these quantities decrease with increasing τ because the field decreases when one moves away from the source along the characteristic. We did not succeed in finding other integrals of Eq. (17). Apparently this indicates the impossibility of solving this equation by the inverse-scattering method.

The results of a numerical analysis of the boundary problem (7),(8) are given in Figs. 1 to 6. The boundary

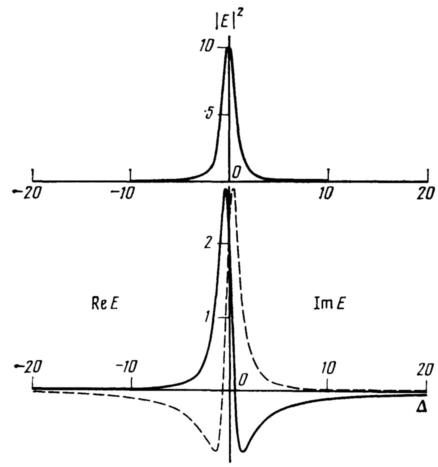

FIG. 1. The functions $|E|^2(\Delta)$ (top), ReE (bottom, full drawn), and Im E (bottom, dashed) for $\tau_0 = 0.1$, $\tau = \tau_0$.

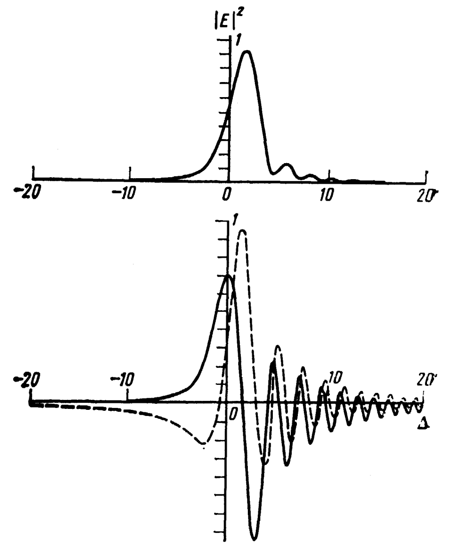

FIG. 2. The distributions $|E|^2(\Delta)$, $\operatorname{Re} E(\Delta)$, and $\operatorname{Im} E(\Delta)$ for $\tau = 0.5$ and $\sigma = 0$.

condition (8) was given at $\tau_0 = 0.1$ (Fig. 1). We show in the graphs the distributions of the real and imaginary parts of the field and also the square of the modulus $|E|^2$ as functions of Δ for $\tau = 0.5$ for five values of σ: $\sigma = 0, 0.5, 1, 2$, and 5. We note that under the conditions of the problem σ is a small parameter but since the behavior of the system is qualitatively determined by the ratio σ/τ_0 we considered also for the convenience of the numerical calculations $\sigma \gtrsim 1$. The case $\sigma = 0$ illustrates the development of the wave structure on the background of the cold solution in accordance with the analytical expressions (13) to (15). It is clear that as τ increases the region occupied by the oscillations increases and their relative amplitude grows as does the wavelength of the plasma wave. There also appear oscillations in the $|E|^2$ distribution, and the maximum of this distribution shifts in the direction of positive Δ. A consideration of the cases $\sigma/\tau_0 = 5$ and $\sigma/\tau_0 = 10$ corresponding to weak and moderate nonlinearity shows that the effect of the latter leads to a shift in the

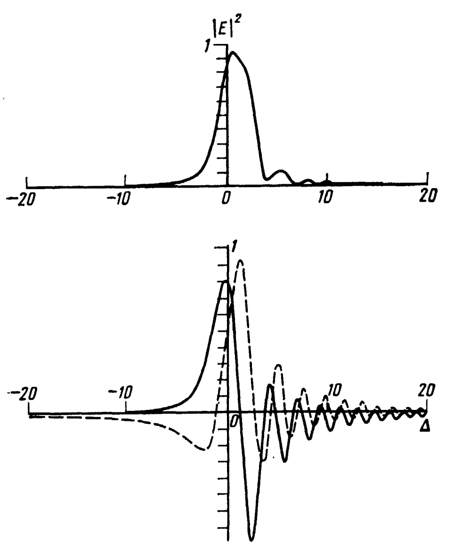

FIG. 3. The same as in Fig. 2 for $\sigma = 0.5$.

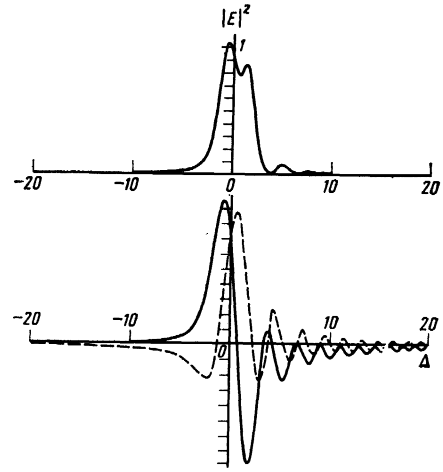

FIG. 4. The same as in Fig. 2 for $\sigma = 1$.

pattern of the field in the direction of negative Δ. In the region of the field maximum characteristic deformations appear in the distribution and when $\sigma/\tau_0 = 10$ the maximum splits. The relative amplitude of the plasma wave as compared to the linear case decreases. When the nonlinearity increases ($\sigma/\tau_0 = 20$) the tendencies noted here manifest themselves more clearly: the distortions in the region of the maximum of $|E|^2$ become deeper and the relative amplitude of the plasma wave decreases. Finally, in the case of a strong nonlinearity ($\sigma/\tau_0 = 50$) a solitary wave splits off from the main part of the distribution in the region of the field maximum; this solitary wave is shifted in the direction of negative Δ, but as τ increases the rate at which it is shifted decreases. Additional analysis showed that as σ/τ_0 increases further the number of solitary waves formed from the given boundary distribution increases. A characteristic feature of the solitary solutions (Fig. 6) is that here the real and the imaginary parts of the field are shifted in phase relative to one another by an amount which is independent of τ and is determined by the boundary condition. This fact enables us to find an analytical expression for the solitary solutions observed in the numerical experiment.[4)]

We shall look for a solution of Eq. (7) in the form

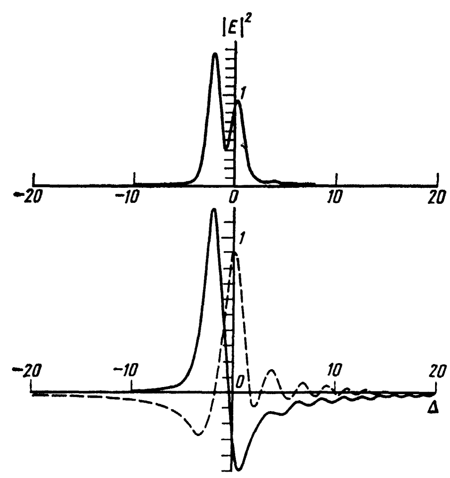

FIG. 5. The same as in Fig. 2 for $\sigma = 2$.

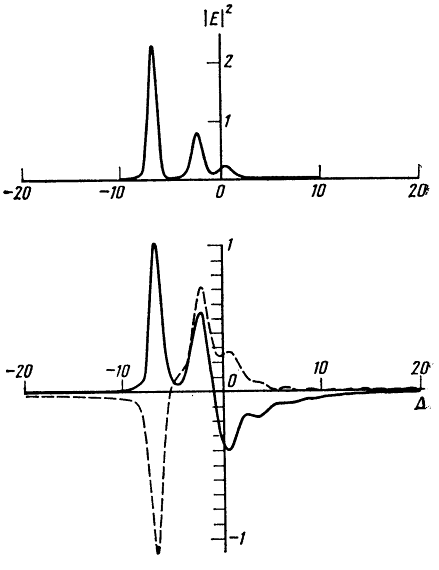

FIG. 6. The same as in Fig. 2 for $\sigma = 5$.

$$E=\tau^{-1/2}F(\zeta)\exp\left\{ik_0\frac{\Delta+c_2\ln(\tau/\tau_2)}{\tau^{1/2}}\right\}e^{i\theta_0},$$

$$\zeta=\frac{\Delta+c_1\ln(\tau/\tau_1)}{\tau^{1/2}}, \tag{19}$$

where $c_{1,2}$, $\tau_{1,2}$, k_0, and θ_0 are real numbers and F is the required function. We require that the conditions $2|c_{1,2}| \gg |\Delta + c_{1,2}\ln(\tau/\tau_{1,2})|$ be satisfied. After substituting (19) into (7) we then get an equation for the function F:

$$F'''+3ik_0F''-(c_1+3k_0^2)F'-i(k_0^3+k_0c_2)F \\ +\sigma[ik_0F|F|^2+(F|F|^2)']=0. \tag{20}$$

If c_1 and c_2 satisfy the conditions $c_1 = p^2 - 3k_0^2, c_2 = 3p^2 - k_0^2$, where p is a real constant, Eq. (20) takes the form

$$(F''-p^2F+\sigma F|F|^2)'+3ik_0(F''-p^2F+{}^1/_3\sigma F|F|^2)=0. \tag{21}$$

One easily finds for the case $k_0 \ll 1$ a solution of Eq. (21) which decreases at infinity: $F = (2/\sigma)^{1/2}p \operatorname{sech} p\zeta$ such that for $k_0 = 0$ the field has the form

$$E_1=\frac{p}{\tau^{1/2}}\left(\frac{2}{\sigma}\right)^{1/2}\operatorname{sech}\left\{\frac{p}{\tau^{1/2}}\left(\Delta+p^2\ln\frac{\tau}{\tau_1}\right)\right\}e^{i\theta_0}. \tag{22}$$

It is clear that when τ increases the solitary solution (22) decreases as $\tau^{-1/2}$ and its width increases $\propto \tau^{1/2}$. The center of mass of the solitary wave is displaced along the nonlinear characteristic $\Delta = -p^2\ln(\tau/\tau_1)$ and with increasing τ the rate of displacement decreases. The characteristic features of the solitary solutions, observed in the numerical analysis, are thus well described by Eq. (22). The parameters p, τ_0, and θ_0 are determined by the boundary condition.

Using (21) we find for the case $k_0 \gg 1$ the solution of Eq. (7) in the form of a wave packet:

$$E_2=\frac{p}{\tau^{1/2}}\left(\frac{6}{\sigma}\right)^{1/2}\operatorname{sech} p\zeta\exp\left\{ik_0\frac{\Delta+c_2\ln(\tau/\tau_2)}{\tau^{1/2}}\right\}\exp(i\theta_0). \tag{23}$$

We note that when $k_0 > p/\sqrt{3}$ the group velocity of the corresponding wave packet is negative, i.e., in the opposite direction to the direction of the velocity of the solitary waves of the type (22). However, in a numerical analysis of the problem (7), (8) packets such as (23) do not occur, since the field distribution at the boundary is rather smooth. Apparently, such solutions can be realized by the evolution of fields of more complex sources, e.g., of an electric quadrupole.

It is expedient to write the solutions (22),(23) also in dimensional variables:

$$E_1=E_m\left(\frac{r_D}{\tau}\right)^{1/2}\operatorname{sech}\left\{\frac{E_m}{E_p}\left(\frac{r_D}{6n_0\tau}\right)^{1/2}\left(\frac{\Delta}{r_D}+\frac{E_m^2}{E_p^2}\frac{\mu}{4n_0}\ln\frac{\tau}{\tau_1}\right)\right\} \\ \times\exp(i\theta_0), \tag{24}$$

$$E_2=E_m\left(\frac{3r_D}{\tau}\right)^{1/2}\operatorname{sech}\left\{\frac{E_m}{E_p}\left(\frac{r_D}{6n_0\tau}\right)^{1/2} \times\left[\frac{\Delta}{r_D}+\left(\frac{E_m^2}{E_p^2}\frac{\mu}{4n_0}-3k_0^2\right)\ln\frac{\tau}{\tau_1}\right]\right\} \\ \times\exp\left\{ik_0\left(\frac{2r_D}{3\mu\tau}\right)^{1/2} \times\left[\frac{\Delta}{r_D}+\left(\frac{E_m^2}{E_p^2}\frac{3\mu}{4n_0}-k_0^2\right)\ln\frac{\tau}{\tau_2}\right]\right\}\exp(i\theta_0). \tag{25}$$

Here $E_m, k_0, \theta_0, \tau_{1,2}$ are real constants.

Summarizing the results of the numerical calculations and the corresponding analytical discussion we can state that even a weak nonlinearity leads to an appreciable change in the structure of the quasistationary field of a source in a rarefied magnetoactive plasma. An obvious result of these changes must be the redistribution of the regions where *hf* energy is absorbed in the plasma volume surrounding the source.

APPENDIX

We showed above that if the parameter σ/τ_0 is large or small, the problem (7),(8) simplifies right from the start. The solution may also turn out to be simpler if it has a self-similar character.

Equation (7) allows a self-similar change of variables

$$\xi=(\Delta-\Delta_0)\tau^{-1/3},\quad E=\tau^{-1/6}\eta(\xi), \tag{A.1}$$

where Δ_0 is an arbitrary number; the equation takes then the form

$$\frac{d^3\eta}{d\xi^3}+\frac{1}{3}\xi\frac{d\eta}{d\xi}-\frac{\eta}{6}+\sigma\frac{d}{d\xi}(\eta|\eta|^2)=0. \tag{A.2}$$

Of most interest is the case when the linear cold field can also be written in self-similar variables. This occurs in the three-dimensional case only for a very special choice of the boundary conditions. In the two-dimensional case, however, the situation turns out to be more favorable. We consider, e.g., a source which is axisymmetric in the Δ,τ-plane: $\rho_{\text{ext}} = qh(2\pi)^{-1}(h^2 + r^2)^{-3/2}$ where q is the charge per unit length along y. In appropriate dimensionless variables the field near the resonance surface satisfies Eq. (7) without the term $E/2\tau$ while $\sigma = \mu^2q^2\beta/h^2$. The self-similar substitution (A.1) leads to an ordinary differential equation which can easily be integrated once over ξ:

$$\frac{d^2\eta}{d\xi^2}+\frac{\xi}{3}\eta+\sigma\eta|\eta|^2=\text{const}. \qquad (A.3)$$

The value of the constant can be determined by using the fact that as $\Delta\to\infty$ the solution is the same as the cold field: $E_{\text{cold}} = -i\xi^{-1}$ (here $\Delta^0 = -i$) whence const $= -i/3$. Using the substitution $\eta = -i\cdot 3^{-1/3}\eta', \xi = -3^{-1/3}\xi'$ we are led to the equation

$$d^2\eta'/d\xi'^2-\xi'\eta'+\sigma\eta'|\eta'|^2=1. \qquad (A.4)$$

For real ξ', Eq. (A.4) has been studied before in connection with the problem of the deformation of the region of plasma resonance in a linear layer of an isotropic plasma under the action of a longitudinal hf field.[12] In an obvious way one can use the results of that analysis to study the field of a point source in a magnetized plasma [for this Eq. (A.4) is also valid with $\xi'\sim\Delta/\tau^{1/3}$] while for a distributed source one needs an analytical continuation into the region of complex ξ'.

[1] We note that in Refs. 7 to 10 only systems with a planar geometry were studied where in the linear approximation the field does not decrease along the resonance surface.

[2] It is clear from Eq. (1) that collisions may be neglected under the conditions $s\ll(r_D/h)^2, s\ll(E_u/E_p)^2$.

[3] When there is a surface charge present and also in the case $n_0\to 1((n_0-1)\lesssim(r_D/h)^2; r_D = v_T/\omega_{pe}$ is the electron Debye radius) one needs special considerations; here one must take spatial dispersion into account already for $r\gtrsim h$.

[4] A similar circumstance was noted in Ref. 10 for solitary solutions formed as the result of the nonlinear evolution of a wavepacket.

[1] A. A. Andronov and Yu. V. Chugunov, Usp. Fiz. Nauk **116**, 79 (1975) [Sov. Phys. Usp. **18**, 343 (1975)].

[2] R. L. Stenzel and W. Gekelman, Phys. Fluids **20**, 1316 (1977).

[3] J. R. Wilson and K. L. Wong, Phys. Fluids **23**, 566 (1980).

[4] G. A. Markov, V. A. Mironov, and A. M. Sergeev, Pis'ma Zh. Eksp. Teor. Fiz. **29**, 672 (1979). [JETP Lett. **29**, 617 (1979)].

[5] Ya. L. Al'pert, A. V. Gurevich, and L. P. Pitaevskiĭ, Iskusstvennye sputniki v razrezhennoĭ plazme (Artificial satellites in a rarefied plasma) Nauka, Moscow, 1964. [English translation published by Consultants Bureau, New York].

[6] W. S. Wang and H. H. Kuehi, Phys. Fluids **22**, 1707 (1979).

[7] N. S. Erokhin, M. V. Kuzelev, S. S. Moiseev, A. A. Rukhadze, and A. B. Shvartsburg, Neravnovesnye i rezonansnye protsessy v plasmennoĭ radiofizike (Non-equilibrium and resonance processes in plasma radiophysics) Nauka, Moscow, 1982.

[8] C. J. Morales and Y. C. Lee, Phys. Rev. Lett. **35**, 930 (1975).

[9] A. N. Kozyrev, A. D. Piliya, and V. I. Fedorov, Fiz. Plazmy **5**, 322 (1979) [Sov. J. Plasma Phys. **5**, 180 (1979)].

[10] C. F. Carney, A. Sen, and F. Chu, Phys. Fluids **22**, 940 (1979).

[11] G. B. Whitham, *Linear and nonlinear waves*, Wiley, New York, 1974 (Russ. Trl. Mir, Moscow, 1977, p. 180).

[12] V. B. Gil'denburg and G. M. Fraĭman, Zh. Eksp. Teor. Fiz. **69**, 1601 (1975) [Sov. Phys. JETP **42**, 816 (1975)].

Translated by D.ter Haar