

Critical behavior of the kinetic coefficients of anisotropic composites

A. S. Skal

A. F. Ioffe Physicotechnical Institute, Academy of Sciences of the USSR

(Submitted 28 May 1985; resubmitted 9 December 1985)

Zh. Eksp. Teor. Fiz. **90**, 2057–2064 (June 1986)

Formulas are obtained for calculating the effective anisotropic kinetic Hall and Seebeck coefficients. It is shown that the decrease of the anisotropy of both the electrical conductivity and the Hall effect can be described near the threshold by two new critical exponents, which characterize the transitions to the quasi-two-dimensional and quasi-one-dimensional states. The anisotropy of the thermopower is characterized by the same exponents as for the conductivity when the components have similar thermal conductivities, but at large thermal-conductivity ratios the anisotropy of the thermopower does not fall off on the nonmetallic side of the transition. The transition from the quasi-two-dimensional state to the two-dimensional state occurs at a two-dimensional threshold, while the transition from the quasi-one-dimensional to the one-dimensional state occurs at the punch-through threshold, which depends on the size of the sample.

It is natural to begin a study of the galvanomagnetic and thermoelectric properties of anisotropic systems by considering the simplest model of uniaxial anisotropy: a geometrically isotropic distribution of the components, one of which is characterized by a tensor

$$\hat{\sigma}_1 = \begin{pmatrix} \sigma_{1x} & 0 & 0 \\ 0 & \sigma_{1y} & 0 \\ 0 & 0 & \sigma_{1z} \end{pmatrix}, \quad \begin{aligned} \sigma_{1x} &= \sigma_{1z} = \text{const}, \\ a &= \tau_{1y}/\sigma_{1x}, \quad 0 < a < \infty, \end{aligned} \quad (1)$$

while the other is characterized by a scalar σ_2 (a is the anisotropy parameter).

The first paper on anisotropic percolation was the study by Shklovskii,¹ which was based on the model of a "single-core" superlattice in which a critical exponent λ was introduced:

$$\begin{aligned} A(a, p) &= G_y^{\text{eff}}(a, p)/G_x^{\text{eff}}(a, p) \sim 1 + \text{const}(p-p_c)^\lambda, \\ \lambda &= \xi - \nu, \end{aligned} \quad (2)$$

where $G_x^{\text{eff}}(a, p)$ and $G_y^{\text{eff}}(a, p)$ are the effective normalized conductances along the directions of the coordinate axes; ν is the exponent of the correlation length, which characterizes the straight-line distance between sites of the superlattice for the case of an infinite cluster, ξ is the exponent of the macrobond^{2,3} joining sites of the superlattice along the cluster. The main result of Ref. 1 is that the anisotropy falls off faster than the conductivity as the threshold is approached; here $\lambda > 0$ and depends only on the dimensionality of the space and not on the type of lattice or on the anisotropy parameter. It is known that in a three-dimensional space $\nu_3 = 0.8$ and $\xi_3 = 1$ (Refs. 2 and 3), and therefore $\lambda_3 = 0.2$ (the subscript gives the dimensionality of the space), whereas in a two-dimensional space $\nu_2 = 1.33$ and $\xi_2 = 1$ (Ref. 3), leading to a contradiction: $\lambda < 0$. Kirkpatrick⁴ conjectured that no model that applies to a three-dimensional space and not to a two-dimensional space can be correct. However, the fractal model that Kirkpatrick considers preferable suffers from the lack of a principal characteristic of disordered systems—the element of randomness, Renormalization-group calculations

in the fractal model gave $\lambda_2/\nu_2 = 0.64$ (Ref. 5) and $\lambda_2/\nu_2 = 0.306$, $\lambda_3/\nu_3 = 0.263$ (Ref. 6); Vinogradov and Sarychev⁷ obtained $\lambda_2/\nu_2 = 0.75$ and $\lambda_3 = 0.3$; Straley⁸ calculated $\lambda = 1$ for a Cayley tree, which gives $\lambda = 3$ in the high-dimensional limit.

As we see from these published data, the estimated values of the exponent λ vary widely and are in need of refinement. The problem of the thermopower in metal-insulator composites was considered in Ref. 9. In Ref. 10 it was shown that the thermopower calculated in Ref. 9 is equal to the thermopower for a pure metal, and therefore the conclusion that the exponents of the anisotropy of the electrical conductivity and thermopower is not justified. The problem of the Hall effect in composites has not been considered in the framework of percolation theory. In the present study the electrical conductivity and the Hall and Seebeck coefficients are calculated by the same computer program for a $20 \times 20 \times 20$ simple cubic lattice. The uncut and cut bonds were modeled with the aid of a random-number generator; the uncut bonds are assigned a conductivity according to formula (1). The system of Kirchhoff equations was solved for the potential differences along the x and y axes, and periodic boundary conditions were imposed on the other faces. The anisotropy parameter was varied over the range $0.001 < a < 1000$.

The Hall coefficients were calculated using Eq. (1) of Ref. 11, which is valid in weak magnetic fields in three-dimensional space and in any classical magnetic fields in two-dimensional space¹² ($\sigma_1/\sigma_2 \rightarrow \infty$) and which in the anisotropic case assumes the form

$$\begin{aligned} R^{\text{eff}}(a, p) &= \int_V \mathbf{j}'(a, \mathbf{r}) [\mathbf{j}(a, \mathbf{r}) \mathbf{H}] R(\mathbf{r}) dV / G_x^{\text{eff}}(a, p) U_x G_y^{\text{eff}}(a, p) U_y H L_z, \end{aligned} \quad (3)$$

where $\mathbf{j}(a, \mathbf{r})$ and $\mathbf{j}'(a, \mathbf{r})$ are the current densities obtained from the solution of the equation $\text{div}[\hat{\sigma}(\mathbf{r}) \nabla U] = 0$ with potential differences U_x and U_y specified, respectively, at the boundaries $x = 0, x = L_x$ and $y = 0, y = L_y$, of a parallel-

epiped with sides L_x, L_y, L_z ; $\hat{\sigma}(\mathbf{r}), R(\mathbf{r})$ are the local values of the conductivity and Hall coefficient, $H \parallel z$ is the magnetic field, and L_z is the length of the sample along H . In a two-dimensional space the Hall coefficient can be expressed explicitly in term of the conductivity [see Eq. (38) of Ref. 13], but in this equation an indeterminacy arises at $\sigma_{1x}\sigma_{1y} = \sigma_{2x}\sigma_{2y}$ and makes it inapplicable in the limit of strong anisotropy $\sigma_{1y} \rightarrow 0$, whereas Eq. (3) is free from this shortcoming and is more general.

The thermopower was studied for composites with a tensor character of the thermal conductivity $\kappa(\mathbf{r})$ and with Seebeck coefficients α_1 and α_2 for components 1 and 2, but component 2, in contrast to Ref. 9, was described by the scalars $\sigma_2 \neq 0$ and $\kappa_2 \neq 0$. On the assumption that the temperature field of the external source in the sample is not changed by the thermocurrents, formula (1) of Ref. 10 assumes in the anisotropic case the form ($i = x, y, z$)

$$\alpha_i^{\text{eff}}(a, b, p) = \alpha_1 + \int_S (\alpha_2 - \alpha_1) j_{n,i}(a, S) T(b, S) dS / G_i^{\text{eff}}(a, p) U_i(T_1 - T_2) L_x, \quad (4)$$

where $j_{n,i}(a, S)$ is the projection of the current density onto the normal to the intercomponent surface S ; $T(b, S)$ is the temperature on this surface ($b = \kappa_{1y}/\kappa_{1x}$) as obtained from the solution of the system of Kirchoff's equations with boundary conditions $T = T_1$ at $x = 0$ and $T = T_2$ at $x = L_x$; L_x is the length of the sample along the electric field. In the case when the anisotropy of the electrical and thermal conductivities are the same, this formula implies

$$\alpha_i^{\text{eff}}(a, b, p) = \alpha_1 + (\alpha_2 - \alpha_1) \frac{\kappa_i^{\text{eff}}(a, b, p) / G_i^{\text{eff}}(a, p) - \kappa_2 / \sigma_2}{\kappa_{1x} / \sigma_{1x} - \kappa_{1y} / \sigma_{1y} - \kappa_{1z} / \sigma_{1z}}, \quad (5)$$

The isotropic analog of this formula is also obtained in Refs. 13 and 14.

The calculated family of functions $A(a, p)$ is shown in a log-log plot in Fig. 1. The property that the anisotropy decreases as $a \rightarrow 1$ is reflected in the fact that all the curves converge to the coordinate origin. As the concentration

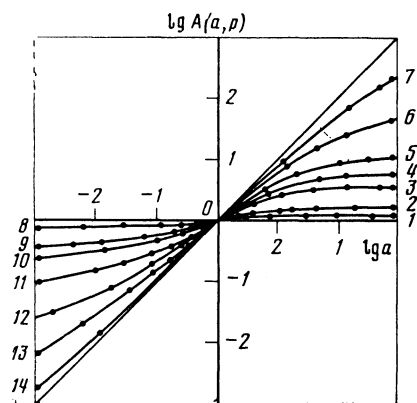


FIG. 1. The function $A(a, p)$ versus a for various parameters p : 1) 0.36, 2) 0.4, 3) 0.5, 4) 0.6, 5) 0.7, 6) 0.8, 7) 0.93, 8) 0.34, 9) 0.36, 10) 0.4, 11) 0.5, 12) 0.55, 13) 0.6, 14) 0.8.

changes from p_c to unity and at small anisotropy, the functions $A(a, p)$ are straight line segments with slopes that change gradually from 0° to 45° for $a > 1$ and from 180° to 225° for $a < 1$. As the anisotropy increases, two new regularities appear: The curves flatten out, but they remain inclined to the abscissa near $p \sim 1$. These features can be explained on the basis of an infinite-cluster model, but because there is at present a parallel development of several models in percolation theory, one naturally wonders which of them to believe, since each gives a plausible explanation for the decrease in anisotropy at the threshold. Each of the models was created for solving a definite group of problems. For example, the fractal model lends itself well to renormalization-group calculations, while the model of a single-core superlattice is suitable for calculations of the effective kinetic coefficients,^{15,16} magnetic alloys,¹⁷ etc. in cases where one must use local values to calculate an effective value. The value of these models is that they permit an asymptotic solution of many problems which would require a great deal of computer time, and in the anisotropic-conductivity problem they have provided a framework for the numerical calculation. The question of which of the models gives a good description of disordered systems must be answered by experiment, e.g., computer modeling.

It is more correct to speak not of the topology of the infinite cluster in general terms but of the topology from the standpoint of some particular problem. Studies of the distribution functions of the contributions of the volume elements to the kinetic coefficients¹⁸⁻²⁰ have shown that the electrical conductivity "sees" only the hot points (the bottlenecks between finite clusters of the conducting component) and "suspects" nothing of the active points of the Hall emf (the parts where the large longitudinal and transverse currents intersect), whereas both kinds of points participate in the Hall effect. Active points of the thermopower appear in the case when the thermal conductivities of the components are sharply different, and vanish when they become similar. Evidently, the other kinetic coefficients, for which the current percolation processes are nonuniform over the sample, also "see" their own singular points. Because the critical interval is small and the wide parts of the infinite cluster can change little on passage through it, all the changes are due to the singular points whose volume fraction goes to zero at the threshold and which are spaced a distance equal to the correlation length of the infinite cluster. The singular points can exist only because the wide segments which connect them into the superlattice do not contribute to the kinetic coefficients.

Figure 2 shows a schematic of the joint superlattice formed by the two superlattices of the longitudinal and transverse conductivities. Each of the latter contains its own hot points (the filled-in circles), and the macrobonds have a variable width. At the bottlenecks where the macrobonds intersect are the active points of the Hall emf (the open circles). The joint superlattice for the conductivity, Hall emf, and thermopower contains at its sites the active points of the Hall emf and on its bonds the hot points of the conductivity, which coincide with the active points of the thermopower. In



FIG. 2. Schematic of the "backbone" of an infinite cluster. The horizontal and vertical hatching indicates the superlattices for the longitudinal and transverse current, the filled-in circles are the hot points, the wavy lines are the macrobonds at whose intersections (at the bottlenecks) are found the active points (the open circles).

the case when there are no active points of the thermopower, the phase transition occurs without a superlattice. In comparing the result of Refs. 18–20 with the older papers,^{2,3} it should be noted that the single-core situation is replaced by a hot-point resistance described by other critical exponents, but instead there is now no contradiction in connection with the dimensionality of the space: the model is valid for spaces from two to six dimensions (six is the critical dimensionality). While Refs. 2 and 3 discussed only one side of the transition, $p > p_c$, for which all the other cited "backbone" models were also developed (the backbone is the part of the infinite cluster with the "dead ends" deleted), now a unified picture is obtained for both sides of the threshold. The "singular" points consist of component 2 below the threshold and component 1 above the threshold, and they are located in approximately the same places.

At the threshold there remains, in a finite volume, a single hot point, which then goes over from component 2 to component 1, and an infinite cluster forms. Here $G_x^{\text{eff}}(a, p)$ and $G_y^{\text{eff}}(a, p)$ are equal, since they are determined by this point. In the isotropic case the number of hot points grows with distance from the threshold, while in the anisotropic case a deformation of the superlattice occurs: the number of hot points grows more slowly for $a > 1$, since along the anisotropy axis they become "cold" and go out of play, but for $a < 1$ they grow faster still. The increase or decrease in the number of hot points is due to a rearrangement of the current paths corresponding to the segments of the sloping straight lines which converge at the origin in Fig. 1, whereas the plateau effect is due to a stabilization of the current paths and to the fact that an anisotropic superlattice has formed. Because there is a scatter in the Joule heat evolved by the hot points, the closer one is to the threshold the faster the plateauing occurs—there are fewer hot points; conversely, the farther one is from the threshold the longer the rearrangement goes on—there is a larger number to choose from. We can introduce the limiting value of the anisotropy parameter, $a_{\text{lim}}(p)$, beginning with which the anisotropic cluster is completely formed. We see from Fig. 1 that the curves in the first and third quadrants are asymmetric; this leads to

two universal curves characterizing the transition to the quasi-two-dimensional and quasi-one-dimensional states:

$$A(a, p) = \begin{cases} f_1(p) \sim 1 + \text{const}(p-p_c)^{\lambda_{31}}, & a > a_{1,np}(p), \\ f_2(p) \sim 1 - \text{const}(p-p_c)^{\lambda_{32}}, & a < a_{2,np}(p), \end{cases} \quad (6)$$

where $\lambda_{31} = 0.45 \pm 0.1$ and $\lambda_{32} = 0.15 \pm 0.1$ are the exponents of the quasi-one-dimensional and quasi-two-dimensional transitions, respectively. In a two-dimensional space there is only one exponent λ_{21} , but the number of anisotropy exponents increases with increasing dimensionality. The scatter in the value of λ in the literature cited is due to the fact that λ was calculated for small anisotropy (for example, in the renormalization-group calculations $a = 1$ is the point of stability of the solution), where there is no universal conductivity function and where a critical exponent should apparently not exist at all.

Let us now return to the second regularity observed in the functions $A(a, p)$ —to the fact there is no plateau at concentrations close to unity. Such a change in the behavior of the curves might be due to internal causes, e.g., the appearance of a new threshold. For $a \rightarrow 0$ the two types of behavior of the curves are separated by a two-dimensional threshold, at which the transition from the quasi-two-dimensional state to the two-dimensional state occurs, while for $a \rightarrow \infty$ a new threshold appears, which, unlike the usual thresholds, is not universal, since it depends on the volume of the sample. When the volume of the sample goes to infinity, one can find a finite cluster consisting of component 2 such that the straight line joining the opposite contacts along the anisotropy axis passes through the cluster. This means that the electrical current must bend around this cluster, and the influence of the smaller conductivity, which leads to the plateau effect should show itself. Thus, in infinite samples, all the curves at concentrations less than unity are bounded and flatten out, while in finite samples there is a threshold which we shall call the punch-through threshold. At this threshold a transition from the quasi-one-dimensional to the one-dimensional state occurs. Although this threshold is not universal, it is important in physical effects which depend on the size of the sample: in thin films and in p - n junctions.²¹

The effective-medium theory^{13,22,23} has been developed only for an infinite sample. Furthermore, to use this theory one must solve a transcendental equation in concentration at every point in the interval $p_c < p < 1$. Only near $p \sim 1$, where the linear approximation is valid, are explicit formulas obtained in the limit of strong anisotropy; the formulas obtained in all three of the papers cited agree to within constants:

$$A(a, p) = \begin{cases} 0.948/(1-p)^2, & a(1-p)^2 \gg 1, \\ a, & a(1-p)^2 \ll 1. \end{cases} \quad (7)$$

These formulas agree qualitatively with our calculations, although a quantitative comparison near $p \sim 1$ cannot be made because our system lies above the punch-through threshold p_p . Formulas (7) imply that at smaller anisotropies the functions $A(a, p)$ increase linearly with a . At larger a the curves flatten out, but the transition between the asymptotes of formula (7) has not been obtained. It is known that the

effective-medium approximation gives a good description of the kinetic coefficients over the entire range of concentration except the critical. Here again there is surprisingly good agreement of the ordinates of the plateaus with the numerical calculations over a wide range of concentrations, $0.5 < p < 1$, i.e., where the linear approximation is no longer applicable.

The anisotropic conductivity as $a \rightarrow \infty$ is characterized by two mechanisms: one, which dominates for $p < p_p$, consists of a transition between filaments and leads to a plateau, while the other, which dominates for $p > p_p$, characterizes the conductivity along the filaments and can be described by an exact formula. For this we denote by N the number of sites along the directions x and z and by k the number along y . The probability that there is a line consisting of conducting sites along the anisotropy axis is p^k , and in an area of N^2 sites there are $N^2 p^k$ such lines. The punch-through threshold occurs when $N^2 p^k = 1$, and consequently, $p_p = N^{-2/k}$. The following formula takes both mechanisms into account:

$$G_y^{\text{eff}}(a, p) = \begin{cases} 0.918\sigma_x/(1-p)^2, & 0.5 < p < p_p, \\ 0.918\sigma_x/(1-p)^2 + p^k\sigma_y, & p \sim p_p, \\ p^k\sigma_y, & p_p < p < 1, \\ \sigma_y \exp[-k(1-p)], & 1-p \ll 1. \end{cases} \quad (8)$$

Near p_p this formula describes a slope to the plateau that is smaller the closer the concentration to the punch-through threshold.

The first question that arises in the problem of the anisotropic Hall effect is how many independent limiting effective Hall coefficients can be obtained in the case of a uniaxial anisotropy with conductivity tensor (1) and local Hall coefficients R_1 and R_2 for components 1 and 2. It is easy to show that by virtue of the symmetry of formula (3) with respect to the x and y axes, one can obtain four limiting independent effective Hall coefficients: $(R_{\parallel}^{\text{eff}}(0, p), R_{\parallel}^{\text{eff}}(\infty, p))$ when the magnetic field is parallel to the anisotropy axis, and $(R_{\perp}^{\text{eff}}(0, p), R_{\perp}^{\text{eff}}(\infty, p))$ when it is perpendicular to it. Because only one active point of the Hall emf remains in a finite volume at the threshold for quasi-two-dimensional and quasi-one-dimensional states, we have $R_{\parallel}^{\text{eff}}(a, p_c)$

$= R_{\perp}^{\text{eff}}(a, p_c)$, and a formula analogous to (6) is valid:

$$B(a, p) = R_{\parallel}^{\text{eff}}(a, p)/R_{\perp}^{\text{eff}}(a, p) \sim \begin{cases} 1 + \text{const}(p-p_c)^{g_{31}}, & a \rightarrow \infty, \\ 1 - \text{const}(p-p_c)^{g_{32}}, & a \rightarrow 0, \end{cases} \quad (9)$$

where $g_{31} = 0.2 \pm 0.1$ and $g_{32} = 0.1 \pm 0.1$ describe the transitions to the quasi-one-dimensional and quasi-two-dimensional states, respectively.

In the isotropic case there are four exponents of the Hall coefficient.²⁰ We shall call these exponents the first-order exponents, in distinction to g_{31} and g_{32} (and λ_{31} and λ_{32}), which we shall call the second-order exponents, because they describe a small correction to the kinetic coefficients. Figure 3 shows the concentration dependence of the Hall coefficients for quasi-one-dimensional and quasi-two-dimensional systems; the curves imply that $R_{\parallel}^{\text{eff}}(a, p) > R_{\perp}^{\text{eff}}(a, p)$ for $a > 1$ and $R_{\perp}^{\text{eff}}(a, p) > R_{\parallel}^{\text{eff}}(a, p)$ for $a < 1$. The decrease of $B(a, p)$ can be explained with the aid of an anisotropic superlattice, as was done for the conductivity, only here one should add that the quasi-one-dimensional ($i = 1$) and quasi-two-dimensional ($i = 2$) superlattices are characterized by their own exponents: ν_{3i} is the exponent of the correlation length, l_{3i} is the exponent of the hot points, e_{3i} is the exponent of the active points of the Hall emf, and m_{3i} is the exponent of the active points of the thermopower. Because the first-order exponents do not vary with anisotropy, the relations²⁰ among them become

$$t = d\nu_{3i} - l_{3i}, \quad f = t + e_{3i} - d\nu_{3i}, \quad h = d\nu_{3i} - m_{3i}, \quad (10)$$

where f is the critical exponent of the Hall coefficient, h is the exponent of the thermopower, and d is the dimensionality of the space.

In the quasi-two-dimensional case there is one more transition that occurs when the concentration reaches the two-dimensional threshold p_{2c} : for $p < p_{2c}$ new critical exponents arise for the first-order kinetic coefficients, while for $p > p_{2c}$ the familiar exponents of the two-dimensional problem remain. This transition is also characterized by its own correlation length, which differs from the correlation length at the three-dimensional threshold. For $p < p_{2c}$ the current

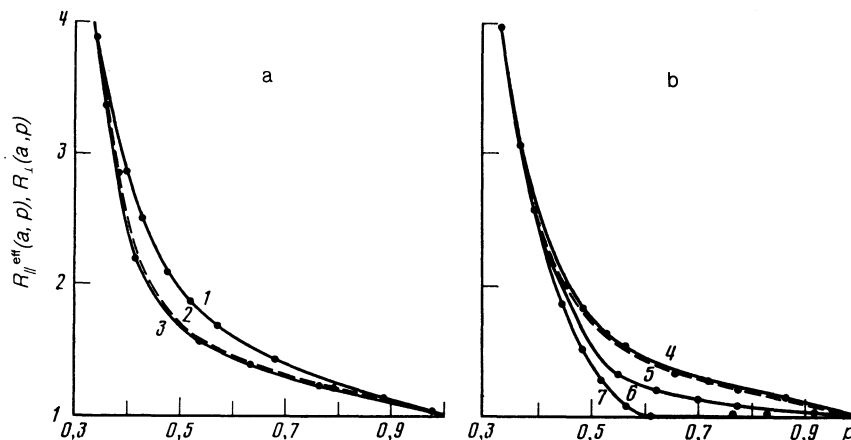


FIG. 3. Concentration dependence of the effective anisotropic Hall coefficients $R_{\parallel}^{\text{eff}}(a, p)$ and $R_{\perp}^{\text{eff}}(a, p)$ for $a > 1$ (a) and $a < 1$ (b): 1) $R_{\parallel}^{\text{eff}}(200, p)$; 2) $R_{\parallel}^{\text{eff}}(1, p) = R_{\perp}^{\text{eff}}(1, p)$; 3) $R_{\perp}^{\text{eff}}(200, p)$; 4) $R_{\parallel}^{\text{eff}}(0.01, p)$; 5) $R_{\perp}^{\text{eff}}(1, p) = R_{\parallel}^{\text{eff}}(1, p)$; 6) $R_{\parallel}^{\text{eff}}(0.01, p)$; 7) $R_{\perp}^{\text{eff}}(0.001, p)$.

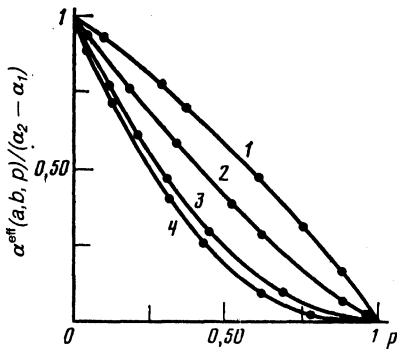


FIG. 4. Concentration dependence of the effective anisotropic Seebeck coefficient $\alpha^{\text{eff}}(a,b,p)$: 1) $a = 0.033$, $b = 2$; 2) $a = 0.1$, $b = 2$; 3) $a = 30$, $b = 0.5$; 4) $a = 10$, $b = 0.5$, $\sigma_2/\min(\sigma_{1x}, \sigma_{1y}, \alpha_{1z}) = 0.01$, $\kappa_2/\min(\kappa_{1x}, \kappa_{1y}, \kappa_{1z}) = 0.05$.

flows mainly through two-dimensional clusters, and it is only at the hot points consisting of component 2 that it jumps over into the adjacent plane through the poorly conducting component-1 part. For $p > p_{2c}$ hot points of conducting component 1 form at these same places, and therefore the correlation lengths (the average distances between these points) above and below the threshold are equal. Therefore, the difference in the exponents of the kinetic coefficients below and above the threshold are related to the geometry of the singular points. This is especially pronounced in the Hall effect: for $p < p_{2c}$ the coefficient $R_{\parallel}^{\text{eff}}(a,p)$, grows with increasing distance from the threshold, while for $p > p_{2c}$ it remains constant^{12,24} (see also curve 7 in Fig. 3b).

Let us assume that the local electrical and thermal conductivities of components 1 are tensors of the type in Eq. (1), while those of component 2 are scalars, as are the Seebeck coefficients α_1 and α_2 of components 1 and 2. It was shown in Ref. 10 that the critical exponents of the thermopower exist only in two limiting cases: when the thermal conductivities of the components are about the same, and when the thermal conductivities of the components are sharply different. In the first case the first-order exponents—those of the electrical conductivity and thermopower—are equal, and their anisotropy exponents λ_{31} and λ_{32} are therefore also equal. In the second case the coefficient $\alpha^{\text{eff}}(a,b,p)$ for $p < p_c$ is equal to a constant and does not depend on the electrical conductivity; therefore, there is no

decrease of the anisotropy near the threshold. For $p > p_c$ the thermal and electrical percolation begins, and these two processes cause the anisotropy at the threshold to decrease still faster. The problem of evaluating these anisotropy exponents requires additional calculations and remains an open question. Figure 4 shows the concentration dependence of the effective Seebeck coefficient $\alpha^{\text{eff}}(a,b,p)/(\alpha_2 - \alpha_1)$.

I am grateful to Yu. A. Firsov, V. V. Bryksin, and V. A. Prigodin for discussion of this study and very valuable comments and to the Computer Center of the Leningrad Institute of Nuclear Physics and specifically to N. A. Poltavskaya for assistance in the calculations and for good software.

¹B. I. Shklovskii, Phys. Status Solidi B **85**, K111 (1978).

²A. S. Skal and B. I. Shklovskii, Fiz. Tekh. Poluprovodn. **8**, 1586 (1974) [Sov. Phys. Semicond. **8**, 1029 (1975)].

³P. G. de Gennes, J. Phys. (Paris) **37**, L1 (1976).

⁴S. Kirkpatrick, AIP Conf. Proc. **40**, 99 (1977).

⁵C. J. Lobb, D. J. Frank, and M. Tinkman, Phys. Rev. B **23**, 2262 (1981).

⁶J. Vannimenus and M. Knezevic, J. Phys. C **17**, 4927 (1984).

⁷A. P. Vinogradov and A. K. Sarychev, Zh. Eksp. Teor. Fiz. **85**, 1144 (1983) [Sov. Phys. JETP **58**, 665 (1983)].

⁸J. P. Straley, J. Phys. C **13** 4335 (1980).

⁹J. P. Troadee and D. J. Bediau, J. Phys. C **16**, 119 (1983).

¹⁰A. S. Skal, Zh. Eksp. Teor. Fiz. **88**, 516 (1985) [Sov. Phys. JETP **61**, 302 (1985)].

¹¹A. S. Skal, Dokl. Akad. Nauk SSSR **260**, 602 (1981) [Sov. Phys. Dokl. **26**, 872 (1981)].

¹²A. S. Skal, Fiz. Tverd. Tela (Leningrad) **27**, 1407 (1985) [Sov. Phys. Solid State **27**, 849 (1985)].

¹³B. Ya. Balagurov, Zh. Eksp. Teor. Fiz. **85**, 568 (1983) [Sov. Phys. JETP **58**, 331 (1983)].

¹⁴V. Halpern, J. Phys. C **16**, L217 (1983).

¹⁵L. Friedman and M. Pollak, Philos. Mag. **B 44**, 487 (1981).

¹⁶H. Böttger and V. V. Bryksin, Phys. Status Solidi B **113**, 9 (1982).

¹⁷T. S. Lubensky, Phys. Rev. B **15**, 311 (1977).

¹⁸A. S. Skal, Zh. Tekh. Fiz. **51**, 2443 (1981) [Sov. Phys. Tech. Phys. **26**, 1445 (1981)].

¹⁹A. S. Skal, Philos. Mag. **B 45**, 335 (1982).

²⁰A. S. Skal, J. Phys. C **18**, 3483 (1985).

²¹V. N. Gusyatnikov and M. E. Raïkh, Fiz. Tekh. Poluprovodn. **18**, 1077 (1984) [Sov. Phys. Semicond. **18**, 670 (1984)].

²²J. Bernasconi, Phys. Rev. B **9**, 4575 (1984).

²³V. N. Prigodin and A. N. Samukhin, Fiz. Tverd. Tela (Leningrad) **26**, 1344 (1984) [Sov. Phys. Solid State **26**, 817 (1984)].

²⁴M. E. Levishtein, M. S. Shur, and A. L. Éfros, Zh. Eksp. Teor. Fiz. **69**, 2203 (1977) [Sov. Phys. JETP **42**, 1120 (1977)].

Translated by Steve Torstveit