Neutrino oscillations in a variable-density medium and ν-bursts due to the gravitational collapse of stars

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Under certain conditions, the propagation of a beam of oscillating neutrinos in a variable-density medium takes the form of an almost complete transformation of the initial type of neutrino into another type. The depth of the oscillations is then negligible. The transformation can occur in the cores and envelopes of collapsing stars.

1. INTRODUCTION

The interaction between neutrinos and matter modifies the picture of ν-oscillations.[1-3] The effect of the medium is analogous in a number of significant respects to the coherent regeneration of K mesons, and also to the appearance of a refractive index.

In a medium of constant density, the overall character of the oscillations is the same as in vacuum, i.e., there is a change in only the oscillation length and depth. A number of astrophysical[1,4,5] and geophysical[1,3,6] applications of these oscillations have been considered in the constant-density approximation.

In the present paper, we examine qualitatively new effects that appear in a variable-density medium.

2. EQUATION FOR THE TRANSFORMATION PROBABILITY; RESONANCE CONDITION

We shall consider the mixing of two types of neutrino. To be specific, we shall suppose that they are ν_e and ν_μ. Suppose the ν_e are created in the source. We shall seek the probability $P(t)$ of an oscillatory $\nu_e \to \nu_e$ transformation in a time t (or distance $r \approx ct$ from the source). The equations describing the evolution of the wave functions of the ν_e and ν_μ in the medium[1] (essentially, the Schroedinger equations) can be used to show that the probability $P(t)$ is the solution of the following equation:

$$MP - \dot{M}\dot{P} + M(M^2 + 4\overline{M}^2)\dot{P} - 2\overline{M}^2\dot{M}(2P-1) = 0 , \tag{1}$$

where

$$M = (2\pi/l_\nu)(\cos 2\theta - l_\nu/l_0), \quad 2\overline{M} = (2\pi/l_\nu)\sin 2\theta, \tag{2}$$

$l_\nu = 4\pi E/\Delta m^2$ is the oscillation length in vacuum, E is the neutrino energy, $\Delta m^2 = m_1^2 - m_2^2$ is the difference between the squares of the masses, θ is the mixing angle, l_0 is a characteristic length of the medium, given by

$$l_0^{-1} = \frac{\rho}{m_N} \sum_i X_i \frac{\Delta f_i(0)}{2\pi E}, \tag{3}$$

$\Delta f_i(0) = f_{ie}(0) - f_{i\mu}(0)$ is the difference between the ν_e and ν_μ forward-scattering amplitudes for the ith component of the medium ($i = e,p,n$), X_i is the abundance of the ith component per nucleon, ρ is the density of the medium, and m_N is the nucleon mass. In accordance with (3), we have $l_0^{-1} \sim G_F\rho/m_N$, where G_F is the Fermi constant.

The initial conditions for (1) are:

$$P(0) = 1, \quad \dot{P}(0) = 0, \quad \ddot{P} = -2\overline{M}^2. \tag{4}$$

If the density is constant ($\dot{M} = 0$), we find from (1) that

$$\ddot{P} + (M^2 + 4\overline{M}^2)\dot{P} = 0. \tag{5}$$

This equation has a periodic solution of the form

$$P = 1 - A \sin^2(\pi r/l_m)$$

with oscillation length

$$l_m = 2\pi(M^2 + 4\overline{M}^2)^{-1/2} \tag{6}$$

and oscillation depth determined by the mixing angle in the medium:[1]

$$A = \sin^2 2\theta_m = \overline{M}^2 l_m^2/\pi^2. \tag{7}$$

We recall that the mixing angle θ_m relates the ν_e and ν_μ states with the neutrino eigenstates ν_1^m and ν_2^m in the medium. In the medium states with the specific masses of ν_1 and ν_2 are not eigenstates of the Hamiltonian and themselves oscillate).

Using (2) and (6), we can rewrite (7) in the form

$$\sin^2 2\theta_m = \sin^2 2\theta[(\cos 2\theta - l_\nu/l_0)^2 + \sin^2 2\theta]^{-1}. \tag{8}$$

from which it follows that, for low values of $\sin^2 2\theta$, the dependence of $\sin^2 2\theta_m$ on l_ν/l_0 ($l_\nu/l_0 \sim \rho E$) exhibits a resonance. When

$$l_\nu/l_0 = \cos 2\theta , \tag{9}$$

$\sin^2 2\theta_m$ is a maximum, i.e., $\sin^2 2\theta_m = 1$. The quantity $\sin^2 2\theta_m$ falls rapidly as l_ν/l_0 departs from $\cos 2\theta \approx 1$. Condition (9), for which the mixing angle in the medium is equal to 45° ($\sin^2 2\theta_m = 1$), will be referred to as the resonance condition. Correspondingly, the values of ρ and E for which (9) is satisfied will be referred to as the resonance values. From (9) and (3), we have

$$\rho_R = m_N \Delta m^2 \cos 2\theta/2^{1/2}G_F E. \tag{10}$$

The width $\Delta\rho_R$ of the resonance layer will be defined as the density interval around ρ_R in which $\sin^2 2\theta_m > 1/2$. From (8) we have

$$\Delta\rho_R = \rho_R \tan 2\theta. \tag{11}$$

The resonance becomes narrower as the mixing angle decreases. Similarly, we may introduce a resonance energy E_R and a resonance width $\Delta E_R = E_R \tan 2\theta$.

The physical meaning of the resonant behavior of $\sin^2 2\theta_m$ is as follows. Suppose that a constant-density layer intercepts a neutrino flux with a continuous energy spectrum. Neutrinos with energy $E = E_R$ (ρ) will then oscillate with maximum oscillation depth (despite the small mixing in vacuum). The energy dependence of the oscillation amplitude is determined by the resonance curve (8): $\sin^2 2\theta_m = f[l_v(E)]$.

In the case of a variable-density medium, the oscillation depth does not depend on θ_m alone, and even in the resonance layer ($\rho = \rho_R$), it may turn out to be low (see below).

3. MEDIUM WITH SLOWLY-VARYING DENSITY (ADIABATIC REGIME); NONOSCILLATORY TRANSFORMATION IN THE ν-BEAM

Consider a layer of a medium satisfying the following conditions:

(1) The density distribution $\rho(r)$ has no singularities and the derivative $d\rho/dr$ is a smooth function.

(2) The variation of density with r is relatively slow and such that, for a given mixing angle, the width Δr_R of the resonance layer in the r direction is less than the oscillation length

$$\Delta r_R = (d\rho/dr)^{-1}\Delta\rho_R \geqslant l_m^R/2. \tag{12}$$

This will be referred to as the adiabatic condition.

(3) The density distribution $\rho(r)$ is a monotonic function of ρ_0 up to $\rho_{min} \simeq 0$. We shall also assume that the resonance density for neutrinos of given energy E falls into the interval $\rho_0 - \rho_{min}$, so that the neutrinos generated in the region with $\rho = \rho_0$ will traverse the resonance layer.

Under this conditions, the solution $P(r)$ is universal with respect to the density distribution. The universality can conveniently be expressed in terms of the following dimensionless parameters. Let

$$n = [\rho(r) - \rho_R]/\Delta\rho_R, \tag{13}$$

where ρ_R and $\Delta\rho_R$ are determined by the values of E, θ, and Δm^2. We shall measure distance in units of n rather than r. We note that, at resonance, $n = 0$, and, as $\rho \to \infty$, $n \to \infty$, while for $\rho \to 0$, $n \to -(\tan 2\theta)^{-1}$. The initial conditions are set at

$$n_0 = [\rho(r_0) - \rho_R]/\Delta\rho_R, \tag{14}$$

where n_0 is the number of resonance layers that can be fitted between the point at which the neutrino is created and the resonance layer. When $\rho(r_0) \gg \rho_R$, we have

$$n_0 \approx \rho(r_0)/\rho_R \sin 2\theta \approx 1/\sin 2\theta_m^\circ$$

[see also (8)], i.e., n_0 is equal to the reciprocal of the mixing parameter at the point at which the ν is created, and increases with distance from the resonance layer and/or with decreasing $\sin^2 2\theta$.

We also introduce the further variable

$$m = (d\rho/dr)^{-1}\Delta\rho_R/l_m^R, \tag{15}$$

i.e., the number of oscillation lengths in the resonance layer. This number increases as the distribution $\rho(r)$ becomes shallower or $\sin^2 2\theta$ decreases.

When conditions (1)–(3) are satisfied, the solution $P(r)$ is a function of n, n_0, and m, i.e., $P \simeq P(n,n_0,m)$, and is not very dependent on the density distribution $\rho(r)$.

Let us examine some of the properties of the function $P(n,n_0,m)$:

(a) P is an oscillating function of n with period T; by definition, $T \simeq 1/m$ in the resonance layer and, as we depart from resonance, $T \sim 1/[m(n^2 + 1)^{1/2}]$.

(b) P oscillates around its mean value

$$\bar{P}(n, \, n_0) = [1 + n_0(n_0^2 + 1)^{-1/2}n(n^2 + 1)^{-1/2}]/2; \tag{16}$$

(c) The amplitude of the P oscillations is $A_P = |P_{max} - P_{min}|$ and is a maximum in the resonance layer

$$A_P^R \approx (n_0^2 + 1)^{-1/2} \tag{17}$$

and decreases with distance from the layer. A_P is practically independent of m.

(d) At exit from the layer, for $\rho \to 0$,

$$\bar{P}_0 \to [1 - n_0(n_0^2 + 1)^{-1/2}\cos 2\theta]/2. \tag{18}$$

Consider the limit of large n_0. Increasing n_0 means that the point at which the ν is created becomes more distant (in ρ) from the resonance layer. The oscillation amplitude is then shown by (17) to decrease ($A_P \sim 1/n_0$) and the average of $P(n,n_0)$ tends to the asymptotic value given by

$$\bar{P}_a(n) = [1 + n(n^2 + 1)^{-1/2}]/2. \tag{19}$$

At exit, $\bar{P}_a \to \sin^2\theta$ (see Fig. 1).

Thus, for small mixing angles θ in vacuum and large n_0 (so that the ν-flux is generated quite far from the resonance layer), the neutrino propagation process takes the form of a virtually nonoscillatory transformation of one type of neutrino into the other.

The interpretation of these results is as follows. The neutrino oscillations in the medium take place around the eigenstates ν_1^m and ν_2^m, i.e., the neutrino oscillates, as in the vacuum, around states of particular mass of ν_1 and ν_2. If the density of the medium varies, there is also a variation in the
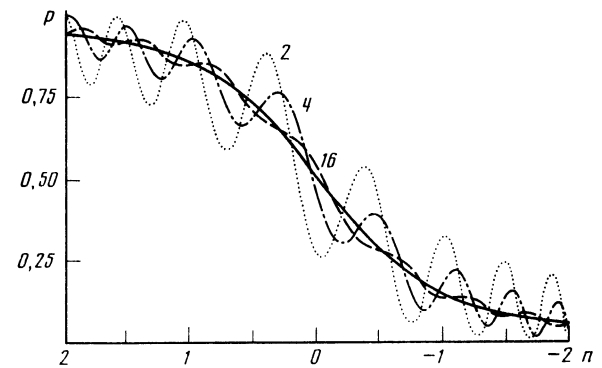

FIG. 1. Transformation probability $P(n)$ for different values of n_0 (indicated against the curve). Solid curve—asymptotic form of $\bar{P}(n)$ (nonoscillatory transformation).

eigenstates ν_1^m and ν_2^m or, more precisely, there is a change in the mixture of ν_e and ν_μ. When $\rho \gg \rho_R$, it can be shown from (8) that $\theta_m \simeq \pi/2$, i.e., ν_2^m becomes practically identical with ν_e. When $\rho = \rho_R$, we have $\theta_m = \pi/4$ and ν_2^m contains equal admixtures of ν_e and ν_μ. When $\rho \ll \rho_R$, we have $\theta_m = \theta$ and, when θ is small, ν_2^m consists mostly of ν_μ. When the density changes from $\rho \gg \rho_R$ to $\rho \ll \rho_R$, the basis ν_1^m, ν_2^m rotates through 90° relative to the basis ν_e, ν_μ. The adiabatic condition then shows that the neutrino state $\nu(t)$ is altered as a result of the variation in density: $\nu(t)$ follows ν_1^m and ν_2^m, while the impurities ν_1^m and ν_2^m in $\nu(t)$ undergo only a small change.

Let us suppose that the neutrinos ν_e are generated in a region with $\rho_0 \gg \rho_R$ and then cross layers with continuously decreasing density ($\rho \to 0$). Initially, $\theta_m^0 \simeq 90°$, $\sin^2 2\theta_m^0 \simeq \sin^2 2\theta (\rho_R/\rho_0)^2 \ll 1$, and $\nu(0) = \nu_e \simeq \nu_2^m$. As the density tends to zero, the state ν_2^m rotates through 90°, as noted above, and because of the adiabatic property, ν_2^m rotates together with $\nu(t)$. When θ is small, $\nu(t) \simeq \nu_2^m \simeq \nu_\mu$ in the final state. Thus, ν_e becomes transformed into ν_μ. As the distance of the point of creation of the neutrinos from the resonance layer increases (in ρ), $\sin^2 2\theta_m^0$ decreases, and the difference between $\nu(t)$ and ν_2^m also decreases. There is an attendant reduction in the oscillation depth. In the limit of large ρ_0, the function $\nu(t)$ becomes practically identical with ν_2^m, i.e., the eigenstate in the medium, and, consequently, there are no oscillations. This limiting case corresponds to a nonoscillatory transformation.

4. APPLICATIONS OF OSCILLATION EFFECTS

We shall now formulate the general conditions for the above effects to produce observable consequences.

(1) Both amplification of oscillations and significant changes in the properties of the ν-beam in the adiabatic regime are due to the crossing of the resonance layer by the neutrinos. The resonance condition is satisfied for a particular sign of l_ν/l_0 or $\Delta f(0) \cos 2\theta/\Delta m^2$ [see (8)]. Since replacing ν with $\bar\nu$ produces a change in the sign of $\Delta f(0)$, the resonance condition is satisfied in a given medium only for a neutrino or an antineutrino. If the oscillation effects are amplified in the ν-channel, they are suppressed in the $\bar\nu$-channel, and vice versa.

(2) the matter effect occurs in a charge-asymmetric medium. If the particle and antiparticle densities are equal, we have $\Sigma_i n_i \Delta f_i(0) = 0$.

(3) The typical scale over which the influence of matter is significant is $l \gtrsim l_0 = A/G_F\rho$, where $A = 3.5 \times 10^4$ km. Hence, it follows that the thickness of the medium must be greater than $d = l\rho > 3.5 \times 10^9$ g/cm^2.

These conditions and the adiabatic condition are satisfied in the sun as well as in the envelopes and cores of collapsing stars.

5. NEUTRINO FLUXES FROM COLLAPSING STARS

The effects examined above can occur in a wide range of values of Δm^2 and $\sin^2 2\theta$ in the outer layers of the cores (above the neutrino sphere) and in the envelopes of collapsing stars.

For the purposes of estimates, we shall use the density distribution in the envelope at the beginning of the collapse, by analogy with the situation prevailing in white dwarfs:

$$\rho \approx \rho_0 [(R_B/r) - 1]^3, \qquad (20)$$

where $\rho_0 \simeq (8-10) \times 10^5$ g/cm^3 and $R_B = 5 \times 10^8$ cm. The maximum density in the interior is $\rho_{max} \simeq 10^9$ g/cm^3. For massive stars, there is, in addition to (20), an extended hydrogen envelope with $\rho \simeq 10^{-9}$ g/cm^3. The envelope may begin to expand during the collapse process with velocity $v \simeq 5000$ km/s.

For collapsing cores, we shall use the model density distribution and the neutron and electron densities given in Ref. 7. The density in the neutrino sphere will be taken to be $\rho \simeq 10^{12}-10^{13}$ g/cm^3.

We have used these density distributions to calculate the ν-parameter ranges (see Fig. 2) for which the adiabatic transformation conditions (1)-(3) of Section 3 are satisfied. The upper limits for Δm^2 are determined by the maximum density. The lower limits for $\sin^2 2\theta$ follow from the adiabatic conditions. The values of Δm_{max}^2 for $\nu_e \leftrightarrow \nu_\mu$ and $\nu_e \leftrightarrow \nu_s$ (ν_s is the sterile state) are different:

$$\Delta m_{max}^2 (\nu_e \leftrightarrow \nu_\mu) = 3 \cdot 10^{-2} \Delta m_{max}^2 (\nu_e \leftrightarrow \nu_s).$$

This difference is largely due to the strong neutronization of the core ($n_n \gg n_e$). We note that in the case of the ($\nu_e \leftrightarrow \nu_s$)-oscillations, the ratio l_ν/l_0 changes sign for $\rho \simeq 10^9$ g/cm^3 in the core. Here, we have a cancellation of the effects on neutrons, on the one hand, and on electrons and protons, on the other. This means that, if most of the conversion in the core is to the neutrino, most of the conversion in the envelop will be to the antineutrino.

Since, under the above conditions, the neutrino state is practically the same as one of the eigenstates in the medium, there is no loss of coherence due to the spreading of the wave packets corresponding to the different ν_i^m.
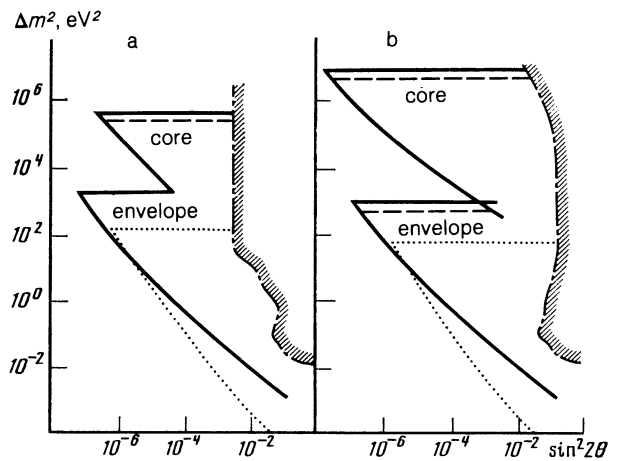
FIG. 2. Range of ν-parameters in which the cores and envelopes of collapsing stars exhibit strong adiabatic transformation: a—$\nu_e \leftrightarrow \nu_\mu$, b—$\nu_e \leftrightarrow \nu_s$ oscillations. Below the shaded line—effect at exit from the envelope: $P_0 < (\sin^2 2\theta)/2$, dotted line—effect of an expanding envelope at the end of the ν-burst ($t = 10$ s); dot-dash lines—experimental limits.

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For the ranges of values of the parameters Δm^2, $\sin^2 2\theta$ given in Section 3 (Fig. 2), the factor representing the suppression of the flux of neutrinos of the original type approaches something between $(1/4)\sin^2 2\theta$ and $(1/2)\sin^2 2\theta$.

We must now consider the consequences of the oscillation effect in matter from the point of view of detection of ν-bursts due to collapses.

(a) Suppose that conditions (1)–(3) are satisfied in the $\bar{\nu}_e \leftrightarrow \bar{\nu}_\alpha$ channel, where $\bar{\nu}_\alpha = \bar{\nu}_\mu$ or $\bar{\nu}_\tau$. In that case, the $\bar{\nu}_e$ flux will be almost completely transformed into the $\bar{\nu}_\alpha$ flux, and vice versa; $\bar{\nu}_e$ and $\bar{\nu}_\alpha$ will exchange their spectra. It is assumed that the $\bar{\nu}_\mu$ and $\bar{\nu}_\tau$ have harder spectra than the $\bar{\nu}_e$ (the total fluxes are roughly equal for the entire burst). The exchange of the spectra between the $\bar{\nu}_e$ and $\bar{\nu}_\alpha$ means that there is a substantial increase in the number of events in scintillation counters recording $\bar{\nu}_e$ $(\sigma_\nu \sim E_\nu^2)$ (Ref. 8).

(b) If the resonance condition is satisfied for the neutrinos, $\nu_e \to \nu_\alpha$ $(\nu_\alpha = \nu_\mu, \nu_\tau)$, an effect analogous to that just described may be expected in systems based on radiochemical methods (ν_e detection).

The $\nu_\alpha \leftrightarrow \nu_e$ or $\bar{\nu}_\alpha \leftrightarrow \bar{\nu}_e$ spectrum exchange in the stellar core leads to a substantial release of energy in the envelope because of $\nu_e e$- or $\bar{\nu}_e e$-scattering. This may play a definite part in the mechanism responsible for the shedding of the envelope.

(c) Resonance in the $\nu_e \to \nu_s$ or $\bar{\nu}_e \to \bar{\nu}_s$ channels. The sterile neutrino fluxes appear to be very low. The effect due to the precession of spin in the magnetic field is negligible for $H \lesssim 10^{12}$ G and $\Delta m^2 \lesssim 10^3$ eV^2. The $\nu_e \to \nu_s$ or $\bar{\nu}_e \to \bar{\nu}_s$ transformations will therefore give rise to strong (by several orders of magnitude) suppression of the ν_e or $\bar{\nu}_e$ fluxes.

We emphasize the importance of simultaneous experiments on the detection of ν_e and $\bar{\nu}_e$.

We note that, since l_ν/l_0 has opposite signs in cores and envelopes, a strong effect may be present for both $\bar{\nu}_e$ and ν_e with different energies.

In the case of mixing of three or more neutrino types, it is possible that two or more differences Δm^2 will fall into the range of strong adiabatic conversion. A combination of the above effects may then be observed.

We note that matter effects may not remain constant in the course of a ν-burst ($\Delta t \simeq 5$–20 s). This time dependence will reflect the variation in the structure of the core and envelope.

[1)]Apart from θ_m, the oscillation depth is also found to depend on the initial conditions. The formula (7) then corresponds to the initial condition (4).

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