Turbulent conductivity of a magnetoactive plasma

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Anomalous heating of a plasma was investigated under conditions such that the lower hybrid pump-wave intensity exceeds the three-wave parametric-instability excitation threshold. It is shown that scattering of charged particles by superthermal fluctuations of the electrostatic field is a nonlinear stabilization mechanism and causes pump-power dissipation in the plasma.

INTRODUCTION

Relaxation and transport processes in a plasma are most fully described by a Boltzmann kinetic equation which treats the charged-particle collisions. In fact, since important macroscopic plasma properties such as the electric conductivity and the viscosity and heat-conduction coefficients are governed by particle collisions,[1,2] their calculation calls for solution of a kinetic equation with a collision integral. A correct determination of the latter for various plasma systems is thus a most important problem of plasma theory.

It is shown in Ref. 1 that in that nonequilibrium-plasma collision interval in which the level of wave fluctuations exceeds the thermal-noise level, account must be taken of the dynamic polarization of the medium (in the absence of this polarization, the charged-particle collisions are described by the Landau collision integral). References 3 and 5 are devoted to a derivation of the collision integral with allowance for the plasma dynamic polarization. In particular, the collision integral of an isotropic plasma placed in a high-frequency (HF) electric field close to the plasma frequency was obtained in Ref. 3. This integral is expressed in terms of the spectral density of the electrostatic-field fluctuations and increases anomalously near the parametric-instability threshold, since the plasma field fluctuations increase anomalously near the threshold. The conductivity of a plasma acted upon by an external HF pump field was investigated in Refs. 4 and 5 with allowance for dynamic polarization in the absence of parametric instability.

The mechanism by which HF waves are absorbed in a magnetoactive plasma have been actively investigated in recent years with the aim of using them for supplementary plasma heating in devices intended for controlled thermonuclear fusion. One of the frequency bands in which effective HF power dissipation mechanisms exist in a thermonuclear plasma is the lower hybrid band (frequencies of the order of several GHz). A phenomenon extensively used for plasma heating in this frequency region is parametric resonance. The experimental results cannot be attributed only to Coulomb collisions and to Čerenkov damping.[6,7] Indeed, since the plasma becomes turbulent when parametric instability develops, the anomalous heating can be caused by scattering of charged particles from turbulent pulsations of an electric field that is described by an effective collision frequency ν_{eff} or by a turbulent conductivity σ_{turb} ($\sigma_{\text{turb}} \propto \nu_{\text{eff}}$).

The HF conductivity of a magnetoactive plasma near the parametric instability of a lower-hybrid pump was investigated in Ref. 8. The analysis was based on linear fluctuation theory, so that σ_{turb} diverged as the HF field pump field E_0 approached the threshold field strength E_{thr} (for excitation of parametric instability). It is quite clear that this divergence of the HF conductivity is due to the unrestricted growth of the fluctuation-field intensity in the plasma, a growth predicted in the region near threshold (critical fluctuations by the linear theory.[2,3] Actually, however, no such growth occurs, or when the field fluctuations are high enough nonlinear instability-saturation mechanisms come into play. This should produce in the plasma a stationary superthermal fluctuation level much higher that that of the thermal fluctuations.[9] The HF conductivity of the plasma has then a finite value that should depend susbtantially on the pump field.

In this paper we develop, on the basis of the Klimontovich-Silin[10–12] microscopic approach, a theory of the fluctuations of a magnetoactive plasma under conditions such that the parametric instability of the lower-hybrid wave is saturated. The results are used to investigate the anomalous heating of a plasma when the pump intensity exceeds the threshold for excitation of three-wave parametric instability, the latter taken to be the decay of the pump wave into a lower-hybrid wave and an ion-sound wave. It is shown that scattering of charged particles from superthermal plasma fluctuations, which is described by the effective collision frequency, is a nonlinear stabilization mechanism. We have used here the analogy between the role of ordinary binary collisions and particles scattering from fluctuations. The idea of this approach to stabilization was developed in Refs. 13–15.

We have calculated σ_{turb} and found it to be proportional to the pump-wave intensity, contrary to the linear theory[3,8] that predicts unlimited growth of the conductivity near threshold. We show that σ_{turb} can be considerably greater than the conductivity due to binary collisions. The results can be used to interpret experimental data on lower-hybrid heating of a plasma.

1. SPECTRAL DENSITY OF ELECTROSTATIC-FIELD FLUCTUATIONS IN A MAGNETOACTIVE PLASMA

The procedure for deriving the equations for a fluctuating electric field and for the spectral density of fluctuations in a magnetoactive plasma placed in an HF pump field is described in detail in Ref. 8. Following this paper, we start with a system of equations for the microscopic distribution

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function of particles of species α:

$$F_\alpha(\mathbf{r},\mathbf{p},t)=\sum_i \delta(\mathbf{r}-\mathbf{r}_{\alpha i}(t))\,\delta(\mathbf{p}-\mathbf{p}_{\alpha i}(t))$$

($\mathbf{r}_{\alpha i}$ and $\mathbf{p}_{\alpha i}$ are the radius vector and momentum of the ith particle of species α) and for the microscopic electrostatic field $\mathbf{E}(\mathbf{r},t)$:

$$\hat{L}F_\alpha+e_\alpha\mathbf{E}\partial F_\alpha/\partial\mathbf{p}=0, \qquad (1)$$

$$\operatorname{div}\mathbf{E}=4\pi\sum_\alpha e_\alpha\int d\mathbf{p}\,F_\alpha(\mathbf{r},\mathbf{p},t). \qquad (2)$$

We have introduced here the operator

$$\hat{L}=\frac{\partial}{\partial t}+\mathbf{v}\frac{\partial}{\partial\mathbf{r}}+\frac{e_\alpha}{c}[\mathbf{v}\mathbf{B}_0]\frac{\partial}{\partial\mathbf{p}}, \qquad (3)$$

where e_α is the charge of the particle of species α and $\mathbf{B}_0$ is a constant magnetic field, assumed to be directed along the z axis. We represent the electric field as the sum of an HF pump field $\mathbf{E}_0$ and a fluctuating field δE:

$$\mathbf{E}(\mathbf{r},t)=\mathbf{E}_0(t)+\delta\mathbf{E}(\mathbf{r},t). \qquad (4)$$

We define the fluctuation of the distribution function as

$$\delta f_\alpha(\mathbf{r},\mathbf{p},t)=F_\alpha(\mathbf{r},\mathbf{p},t)-\langle F_\alpha(\mathbf{r},\mathbf{p},t)\rangle,$$

where $\langle F_\alpha(\mathbf{r},\mathbf{p},t)\rangle = n_\alpha f_\alpha(\mathbf{r},\mathbf{p},t)$, f_α is the single-particle distribution defined as the microscopic density averaged over a Liouville distribution, and n_α is the equilibrium density of the particles of species α. It follows from (1) that δf_α satisfies the equation

$$\left\{\hat{L}+e_\alpha\mathbf{E}_0(t)\frac{\partial}{\partial\mathbf{p}}\right\}(\delta f_\alpha-\delta f_\alpha^{\text{sour}})=-e_\alpha n_\alpha\delta\mathbf{E}\frac{\partial f_\alpha}{\partial\mathbf{p}}, \qquad (5)$$

where δf is the distribution-function fluctuation with the interaction between particles neglected. For the fluctuating electric field we obtain from (2)

$$\operatorname{div}\delta\mathbf{E}=4\pi\sum_\alpha e_\alpha\int d\mathbf{p}\,\delta f_\alpha. \qquad (6)$$

We assume now that the plasma is spatially homogeneous and average (1) over a Liouville distribution. This yields for the single-particle distribution function the equation

$$\left\{\frac{\partial}{\partial t}+e_\alpha\left(\mathbf{E}_0(t)+\frac{1}{c}[\mathbf{v}\mathbf{B}_0]\right)\frac{\partial}{\partial\mathbf{p}}\right\}f_\alpha=-\frac{e_\alpha}{n_\alpha}\frac{\partial}{\partial\mathbf{p}}\langle\delta f_\alpha\,\delta\mathbf{E}\rangle. \qquad (7)$$

The right hand side of (7) contains the average product of the distribution-function fluctuation and the fluctuating electrostatic field, taken at one and the same point of space and one and the same instant of time. This quantity determines the collision integral

$$I_\alpha(\mathbf{p},t)=-\frac{e_\alpha}{n_\alpha}\frac{\partial}{\partial\mathbf{p}}\langle\delta f_\alpha\,\delta\mathbf{E}\rangle. \qquad (8)$$

Solving (5) by the Fourier-transform method and transforming to a reference frame in which $k_y=0$ we obtain the following expression for δf_α:

$$\begin{aligned}
&\delta f_\alpha(\omega,\mathbf{k},\mathbf{p}'+\Delta\mathbf{p})\\
&=\delta f_\alpha^{\text{sour}}(\omega,\mathbf{k},\mathbf{p}'+\Delta\mathbf{p})-e_\alpha n_\alpha\sum_{m,n=-\infty}^{\infty}J_m(a_\alpha)J_n(a_\alpha)\\
&\times\int_0^\infty d\tau\exp\left\{-i(\omega+n\omega_0-k_\parallel v_\parallel')\tau-i\frac{k_\perp v_\perp'}{\Omega_\alpha}\right.\\
&\times\left.[\sin(\varphi+\Omega_\alpha\tau)-\sin\varphi]\right\}\\
&\times\left[k_\parallel\frac{\partial}{\partial p_\parallel}+\cos(\varphi+\Omega_\alpha\tau)k_\perp\frac{\partial}{\partial p_\perp}\right]f_\alpha(p_\perp',p_\parallel')\\
&\times\delta E[\omega+(n-m)\omega_0,\mathbf{k}], \qquad (9)
\end{aligned}$$

where the subscripts $\parallel$ and $\perp$ denote respectively vector components parallel and perpendicular to the field. Here Ω_α is the gyrofrequency of the particles of species α, φ is the azimuthal angle in velocity space, J_n is a Bessel function of real argument and of order n,

$$a_\alpha=\mu\gamma_\alpha,\quad \mu=E_0k_\perp c/(\omega_0B_0),\quad \gamma_\alpha=\Omega_\alpha^2/(\omega_0^2-\Omega_\alpha^2).$$

In the derivation of (9), we specified the pump field perpendicular to the magnetic field in the dipole approximation, i.e.,

$$\mathbf{E}_0(t)=E_0\mathbf{y}\cos\omega_0 t,$$

and put $\mathbf{p}'=\mathbf{p}-\Delta\mathbf{p}$, where $\Delta\mathbf{p}$ is the momentum change due to the HF field.

Substituting (9) in (6) we obtain the following system of equations for the fluctuating field

$$\delta\mathbf{E}(\omega,\mathbf{k})+\sum_\alpha\sum_{m,n=-\infty}^{\infty}\chi_\alpha^n J_n(a_\alpha)J_m(a_\alpha)\delta\mathbf{E}(\omega+(n-m)\omega_0,\mathbf{k})=\delta\mathbf{E}^{\text{sour}}(\omega,\mathbf{k}), \qquad (10)$$

where $\delta\mathbf{E}^{\text{sour}}$ is connected with $\delta f_\alpha^{\text{sour}}$ by the Poisson equation (6). The linear susceptibility $\chi_\alpha^n\equiv\chi_\alpha(\omega+n\omega_0,\mathbf{k})$ of the plasma is given by

$$\chi_\alpha^n=\frac{4\pi e_\alpha^2 n_\alpha}{k^2}\int d\mathbf{p}\sum_{m=-\infty}^{\infty}\frac{J_m^2(k_\perp v_\perp/\Omega_\alpha)}{\omega-k_\parallel v_\parallel-m\Omega_\alpha+n\omega_0}\times\left(\frac{m\Omega_\alpha}{v_\perp}\frac{\partial}{\partial p_\perp}+k_\parallel\frac{\partial}{\partial p_\parallel}\right)f_\alpha(p_\perp,p_\parallel). \qquad (11)$$

Assuming the arguments a_α of the Bessel functions to be small, we retain the terms $m, n=0, \pm1$ in the sum over m and n. In this approximation it is easy to obtain, using (10), an expression for the electrostatic-field-fluctuation spectral density averaged over the period $2\pi/\omega_0$ of the HF field[8]:

$$\begin{aligned}
&\overline{\langle\delta\mathbf{E}\,\delta\mathbf{E}\rangle_{\omega,\mathbf{k}}}\\
&=|\varepsilon_E(\omega,\mathbf{k})|^{-2}\left\{\langle\delta\mathbf{E}^2\rangle_{\omega,\mathbf{k}}^0+\frac{\mu^2}{4}(\chi_e^0)^2\left[\frac{\langle\delta\mathbf{E}^2\rangle_{\omega+\omega_0,\mathbf{k}}^0}{|\varepsilon_1(\omega+\omega_0,\mathbf{k})|^2}\right.\right.\\
&\left.\left.+\frac{\langle\delta\mathbf{E}^2\rangle_{\omega-\omega_0,\mathbf{k}}}{|\varepsilon_{-1}(\omega-\omega_0,\mathbf{k})|^2}\right]\right\}, \qquad (12)
\end{aligned}$$

where the superior bar denotes averaging over the HF-field period,

$$\langle \delta E^2 \rangle^0_{\omega,\mathbf{k}} = (2\pi)^3 \sum_\alpha \frac{8e_\alpha^2 n_\alpha}{k^2} \sum_{n=-\infty}^{\infty} \mathrm{Re} \int_0^\infty dp_\perp p_\perp \int_{-\infty}^{\infty} dp_\parallel$$

$$\times \int_0^\infty d\tau \exp[i(\omega - k_\parallel v_\parallel - n\Omega_\alpha)\tau] J_n^2(k_\perp v_\perp/\Omega_\alpha) f_\alpha(p_\perp, p_\parallel) \quad (13)$$

is the spectral correlation function of the particles interacting in the magnetoactive plasma in the absence of a pump field (cf. Ref. 10),

$$\varepsilon_E(\omega, k)$$

$$= \varepsilon_0(\omega, \mathbf{k}) + \frac{\mu^2}{4} \chi_e^0 \chi_i^0 \left[\frac{1}{\varepsilon_1(\omega+\omega_0, \mathbf{k})} + \frac{1}{\varepsilon_{-1}(\omega-\omega_0, \mathbf{k})} \right]$$

$$\varepsilon_n(\omega + n\omega_0, \mathbf{k}) = 1 + \sum_\alpha \chi_\alpha^n, \qquad n=0, \pm 1. \quad (14)$$

Furthermore, it was recognized in the derivation of (12) that the correlator of the source fluctuations satisfies the equation[10]

$$[\hat{L} + e_\alpha \mathbf{E}_0(t)\, \partial/\partial \mathbf{p}] \langle \delta f_\alpha \delta f_\beta \rangle^{\text{ист}}_{x,t,x',t'} = 0, \qquad x = (\mathbf{r}, \mathbf{p}) \quad (15)$$

with initial condition

$$\langle \delta f_\alpha \delta f_\beta \rangle^{\text{ист}}_{x,t,x',t'} |_{t=t'} = n_\alpha \delta_{\alpha\beta} \delta(x-x') f_\alpha(x,t). \quad (16)$$

It can be seen from (12) that in a magnetoactive plasma acted upon by an HF pump field the electrostatic-field fluctuation density averaged over the period of the HF field is expressed in terms of the spectral correlation function of the noninteracting particles obtained neglecting the pump field, and in terms of the plasma functions that depend on the pump intensity and frequency.

2. COLLISION INTEGRAL IN A MAGNETIC PLASMA

Using the definitions introduced above, we express the collision integral in terms of the Fourier transforms of the distribution functions (4) and of (6) the electric field. It is convenient for this purpose to express (8) in the form[8]

$$I_\alpha(\mathbf{p}' + \Delta\mathbf{p})$$

$$= -\frac{e_\alpha}{n_\alpha} \mathrm{Re} \int \frac{d\omega}{2\pi} \int \frac{d\mathbf{k}}{(2\pi)^3} \frac{\partial}{\partial \mathbf{p}'} \langle \delta f_\alpha(\omega, \mathbf{k}, \mathbf{p}' + \Delta\mathbf{p}) \delta \mathbf{E}(\omega, \mathbf{k}) \rangle. \quad (17)$$

We average the collision integral (17) over the period $2\pi/\omega_0$ of the HF field, assuming that the velocity of the charged-particle oscillations in the pump field is much less than their thermal velocity, i.e.,

$$e_\alpha E_0 / (m_\alpha \omega_0 v_{T\alpha}) \ll 1.$$

Here m_α and $m_{T\alpha}$ are respectively the mass and thermal velocity of a particle of species α. Since the collision integral is the partial derivative of the single-particle distribution with respect to time,

$$I_\alpha(\mathbf{p}' + \Delta\mathbf{p}) = \partial f_\alpha(\mathbf{p}' + \Delta\mathbf{p})/\partial t,$$

the main contribution to the averaged collision integral is made by the slowly varying part of the distribution function. Therefore

$$\overline{I_\alpha(\mathbf{p}' + \Delta\mathbf{p})} = I_\alpha(\mathbf{p}'),$$

$$I_\alpha(\mathbf{p}') = -\frac{e_\alpha}{n_\alpha} \mathrm{Re} \int \frac{d\omega}{2\pi} \int \frac{d\mathbf{k}}{(2\pi)^3} \frac{\mathbf{k}}{k^2} \frac{\partial}{\partial \mathbf{p}'}$$

$$\times \overline{\langle \delta f_\alpha(\omega, \mathbf{k}, \mathbf{p}' + \Delta\mathbf{p}) \delta E(\omega, \mathbf{k}) \rangle}. \quad (18)$$

Substituting in (18) the explicit expressions for the fluctuations (9) of the distribution functions and (10) of the electrostatic field and integrating the result with respect to the azimuthal angle φ, we get

$$I_\alpha(p_\perp, p_\parallel) = \sum_{n=-\infty}^{\infty} \int d\mathbf{k} (\hat{L}_{\alpha n} D_{\alpha n} \hat{L}_{\alpha n} + \hat{L}_{\alpha n} A_{\alpha n}) f_\alpha(p_\perp, p_\parallel). \quad (19)$$

We have introduced in (19) the notation

$$I_\alpha(p_\perp, p_\parallel) = \frac{1}{2\pi} \int_0^{2\pi} d\varphi\, I_\alpha(\mathbf{p}' + \Delta\mathbf{p}), \qquad \hat{L}_{\alpha n} = k_\parallel \frac{\partial}{\partial p_\parallel} + \frac{n\Omega_\alpha}{v_\perp} \frac{\partial}{\partial p_\perp}.$$

The quantities $D_{\alpha n}$ and $A_{\alpha n}$ in the collision integral (19) are respectively the diffusion coefficient in velocity space and the dynamic-friction coefficient, and are given by

$$D_{\alpha n} = \frac{e_\alpha^2}{16\pi^3 k^2} J_n^2 \left(\frac{k_\perp v_\perp}{\Omega_\alpha} \right) \int d\omega \langle \delta \mathbf{E}\, \delta \mathbf{E} \rangle_{\omega,\mathbf{k}} \delta(\omega - k_\parallel v_\parallel - n\Omega_\alpha), \quad (20)$$

$$A_{\alpha n} = \frac{e_\alpha^2}{2\pi^2 k^2} J_n^2 \left(\frac{k_\perp v_\perp}{\Omega_\alpha} \right)$$

$$\times \int d\omega \frac{\mathrm{Im}\, \varepsilon_E(\omega, \mathbf{k})}{|\varepsilon_E(\omega, \mathbf{k})|^2} \Big\{ \delta(\omega - k_\parallel v_\parallel - n\Omega_\alpha)$$

$$- \frac{\mu}{2} \chi_e^0 \left[\frac{\delta(\omega + \omega_0 - k_\parallel v_\parallel - n\Omega_\alpha)}{\varepsilon_1(\omega + \omega_0, \mathbf{k})} - \frac{\delta(\omega - \omega_0 - k_\parallel v_\parallel - n\Omega_\alpha)}{\varepsilon_{-1}(\omega - \omega_0, \mathbf{k})} \right] \Big\}.$$

Note that in the absence of a pump field ($E_0 = 0$) expressions (20) and (21) go over into the familiar expressions for the diffusion and dynamic friction coefficients in a magnetoactive plasma (see, e.g., Eqs. (50.10) and (50.11) of Ref. 10).

It can be seen from (21) that a weak field does not change substantially the dynamic-friction coefficient. On the other hand, in view of the anomalous increase of the fluctuations under parametric-instability conditions, the velocity-space diffusion coefficient (20) can increase substantially. We therefore assume below that the main contribution to the collision integral (19) of a parametrically unstable plasma is made by the first term, due to the particle diffusion in velocity space.

In the next section, the general relations obtained above for the collision integral of a homogeneous magnetoactive plasma will be used to describe the electstatic-oscillation damping due to charged-particle scattering by superthermal fluctuations. The mechanism of this scattering is physically similar to binary particle collisions, since the plasma collective fluctuations can be regarded as a gas of quasiparticles, i.e., plasmons. Thus, wave scattering by fluctuations can be characterized by some effective scattering frequency for the charged particles of the plasma, expressed in terms of the plasma turbulent conductivity, determined in turn by the collision integral. Since the effective collision frequency can considerably exceed the binary collision frequencies of the charged particles, the damping of the plasma waves by the fluctuations may turn out to be the mechanism that ensures effective dissipation of the HF pump power in the plasma.

3. EFFECTIVE CHARGED-PARTICLE COLLISION FREQUENCY IN A TURBULENT PLASMA

We introduce, as the property descriptive of charged-particle scattering by turbulent fluctuations of an electrostatic field in a plasma, the effective collision frequency $\nu_{\rm eff}$. The plan for deriving an equation for $\nu_{\rm eff}$ is the following. Using the analogy between the roles of binary scattering and particle scattering by fluctuations,[14] we express $\nu_{\rm eff}$ in terms of the turbulent plasma conductivity $\sigma_{\rm turb}$:

$$\nu_{\rm eff}=(m_e\omega_0^2/e^2n_e)\,\sigma_{\rm turb}\,. \qquad (22)$$

The conductivity of a plasma placed in an HF pump field can be obtained from the energy-balance equation

$$\frac{1}{2}\sigma_{\rm turb}E_0^2=\sum_\alpha n_\alpha\int d\mathbf{p}\,\frac{p^2}{2m_\alpha}I_\alpha(\mathbf{p}), \qquad (23)$$

where $I_\alpha(\mathbf{p})$ is the collision integral of the charged particles in the plasma and is defined by (19). Once instability has set in, the intensity of the fluctuation fields in the plasma increases, and the main contribution to the collision integral is made by the term associated with the particle diffusion in velocity space. As a result we have

$$\nu_{\rm eff}(E_0)=\frac{2m_e\omega_0^2}{e^2n_eE_0^2}\int\frac{d\omega}{2\pi}\int\frac{d\mathbf{k}}{(2\pi)^3}\frac{\omega}{4\pi}\widetilde{\langle\delta\mathbf{E}\delta\mathbf{E}\rangle}_{\omega,\mathbf{k}}\sum_\alpha \operatorname{Im}\chi_\alpha^0, \qquad (24)$$

where $\widetilde{\langle\delta\mathbf{E}\delta\mathbf{E}\rangle}_{\omega,\mathbf{k}}$ is the spectral density of the turbulent electrostatic-field fluctuations in the plasma under parametric-instability conditions, i.e., at $E_0>E_{\rm thr}$. Since the spectral density of the fluctuations depends in this case on the effective collision frequency, expression (24) is the equation for $\nu_{\rm eff}$ in a turbulent plasma.

For specific calculations, we confine ourselves to a determination of the effective collision frequency for a case of practical interest, when the pump frequency ω_0 is close to the lower hybrid resonance of a magnetized plasma

$$\omega_{LH}=\omega_{pi}(1+\omega_{pe}^2/\Omega_e^2)^{-1/2},$$

where ω_{pe} and ω_{pi} are the Langmuir frequencies of the plasma electrons and ions, respectively.

The parametric instabilities produced in such a system were tabulated in Refs. 16–21. In a plasma with relative plasma parameters

$$\Omega_i\ll\omega_{pi}\lesssim\omega_0\ll\omega_{pe}\ll\Omega_e,$$

parametric instability can set in with respect to pump-wave decay into a lower hybrid wave and an ion-sound wave:

$$\omega_0=\omega_{lk}+\omega_{sk}. \qquad (25)$$

Here

$$\omega_{lk}=\omega_{LH}\left(1+\frac{m_i}{m_e}\frac{k_\parallel^2}{k^2}\right)^{1/2},\qquad \omega_{sk}=kv_s,\qquad v_s=\left(\frac{T_e}{m_i}\right)^{1/2}$$

The pump can be regarded as homogeneous in this case if[8]

$$v_s/v_A=\beta^{1/2}\ll1\qquad(v_A=c\Omega_i/\omega_{pi}).$$

The decay condition (25) is satisfied if $\omega_0\gtrsim3\omega_{LH}$ and $T_e\gg T$.[16] The parametric instability then has a resonant character, since the decay waves are natural modes of the plasma. The excitation threshold of the instability is given by

$$E_{\rm thr}^2=\frac{16\omega_0^2B_0^2}{k^2c^2}\frac{\gamma_0\gamma_{-1}}{\omega_{sk}\omega_{lk}}(kr_{De})^2, \qquad (26)$$

where γ_0 and γ_{-1} are the Landau damping decrements of the ion-sound and lower-hybrid waves, respectively, and r_{De} is the electron Debye radius.

If $\omega_0<3\omega_{LH}$ the condition

$$k_\parallel/k>3(m_e/m_i)^{1/2}$$

for the existence of weakly damped ion-sound oscillations is not met and the decay (25) is forbidden. The pump wave can decay into a lower-hybrid wave and an ion quasimode,[17] the latter due to the presence of a pump field in the plasma. Since the ion quasimode is a strongly damped wave, a decay in which it takes part is nonresonant and will not be considered here.

At low frequencies the fluctuation spectrum (12) contains broad maxima due to random motion of the charged particles, as well as narrow maxima corresponding to fluctuations due to collective oscillations. The spectral distribution of the fluctuations near the natural frequencies (i.e., in the plasma transparency region, where $|\operatorname{Im}\varepsilon_E|\ll|\operatorname{Re}\varepsilon_E|$ and $|\operatorname{Im}\varepsilon_{-1}|\ll|\operatorname{Re}\varepsilon_{-1}|$) can be represented in the form

$$\langle\delta\mathbf{E}\delta\mathbf{E}\rangle_{\omega,\mathbf{k}}=\sum_i I_\mathbf{k}^i\delta(\omega-\omega_{ik}). \qquad (27)$$

Here ω_{ik} are the natural frequencies of the plasma, i.e., $\omega_{ik}=\omega_{sk},\omega_{lk}$ respectively for the ion-sound and the lower-hybrid waves, and $I_\mathbf{k}^i$ is the intensity of the fluctuation oscillations at the frequency ω_{ik}.

Since it follows from (12) that near the natural frequencies

$$I_\mathbf{k}^i\propto[\operatorname{Im}\varepsilon_E(\omega_{ik},\mathbf{k}_i)]^{-1}$$

the intensity of the fluctuations increases without limit in the kinetic-intensity region determined from the condition $\operatorname{Im}\varepsilon_E(\omega_{ik},\mathbf{k}_i)=0$ (these are called critical fluctuations[22]). It was shown in Ref. 8 that in the case of the decay (25) we have

$$\operatorname{Im}\varepsilon_E(\omega,\mathbf{k})\propto(1-E_0^2/E_{\rm thr}^2), \qquad (28)$$

where $E_{\rm thr}$ is defined in (26). Below the threshold of this instability, the HF plasma conductivity is

$\sigma \propto \ln(1 - E_0^2/E_{\rm thr}^2)$. Linear fluctuation theory predicts thus that the density W of the electromagnetic-wave energy absorbed in the plasma should increase without limit near the threshold of the parametric instability ($E_0 \to E_{\rm thr}$), since $W \propto \sigma$.

We now show that allowance for the additional wave damping due to charged-particle scattering by superthermal fluctuations leads to saturation of the level of the latter and hence to a finite saturation of the plasma HF conductivity. The spectral distribution of the electrostatic field fluctuations $\langle \widetilde{\delta \mathbf{E} \delta \mathbf{E}} \rangle_{\omega, \mathbf{k}}$ near the natural frequencies of the plasma, with allowance for $\nu_{\rm eff}$, is obtained from Eq. (12) in which the natural frequencies of the plasma are taken to be $\tilde{\omega}_{ik} = \omega_{ik} + i\nu_{\rm eff}$:

$$\langle \widetilde{\delta \mathbf{E} \delta \mathbf{E}} \rangle_{\omega, \mathbf{k}} = \pi [\tilde{I}_{\mathbf{k}}^s \delta(\omega - \tilde{\omega}_{sk}) + \tilde{I}_{\mathbf{k}}^l \delta(\omega - \tilde{\omega}_{lk})]. \tag{29}$$

$\tilde{I}_{\mathbf{k}}^s$ and $\tilde{I}_{\mathbf{k}}^l$ in (29) are the turbulent-fluctuation intensities at the natural ion-sound ($\tilde{\omega}_{sk}$) and lower-hybrid (ω_{lk}) frequencies respectively, and are given by

$$\tilde{I}_{\mathbf{k}}^s = |\operatorname{Im} \tilde{\varepsilon}_E|^{-1} \left(\frac{\partial \operatorname{Re} \tilde{\varepsilon}_E}{\partial \omega_{sk}} \right)^{-1} \langle \delta \mathbf{E}^2 \rangle^0_{\omega_{sk}, \mathbf{k}} + \frac{\mu^2}{4} [\chi_e^0 (\omega_{sk})]^2$$
$$\times \frac{\pi \delta(\omega_{sk} - \omega_0 + \omega_{lk})}{|\operatorname{Im} \tilde{\varepsilon}_E| \, |\operatorname{Im} \tilde{\varepsilon}_{-1}| \, (\partial \operatorname{Re} \tilde{\varepsilon}_E / \partial \omega_{sk}) \, (\partial \operatorname{Re} \tilde{\varepsilon}_{-1} / \partial \omega_{lk})} \langle \delta \mathbf{E}^2 \rangle^0_{\omega_{lk}, \mathbf{k}}, \tag{30a}$$

$$\tilde{I}_{\mathbf{k}}^l = \frac{|\varepsilon_0(\omega_{sk})|^2}{|\operatorname{Im} \tilde{\varepsilon}_E| \, |\operatorname{Im} \tilde{\varepsilon}_{-1}| \, (\partial \operatorname{Re} \tilde{\varepsilon}_E / \partial \omega_{lk}) \, (\partial \operatorname{Re} \tilde{\varepsilon}_{-1} / \partial \omega_{lk})}$$
$$\times \langle \delta \mathbf{E}^2 \rangle^0_{\omega_{lk}, \mathbf{k}} + \frac{\mu^2}{4} [\chi_e^0 (\omega_{sk})]^2$$
$$\times \frac{\pi \delta(\omega_{lk} - \omega_0 + \omega_{sk})}{|\operatorname{Im} \tilde{\varepsilon}_E| \, |\operatorname{Im} \tilde{\varepsilon}_{-1}| \, (\partial \operatorname{Re} \tilde{\varepsilon}_E / \partial \omega_{lk}) \, (\partial \operatorname{Re} \tilde{\varepsilon}_{-1} / \partial \omega_{lk})} \langle \delta \mathbf{E}^2 \rangle^0_{\omega_{sk}, \mathbf{k}}, \tag{30b}$$

where

$$\operatorname{Im} \tilde{\varepsilon}_E = \left[4 \left(1 + \frac{\omega_{pe}^2}{\Omega_e^2} \right) \frac{(\gamma_0 + \nu_{\rm eff})(\gamma_{-1} + \nu_{\rm eff})}{\omega_{sk} \omega_{lk}} - \frac{\mu^2}{4} (k r_{De})^{-2} \right]$$
$$\times \left[2 (k r_{De})^2 \left(1 + \frac{\omega_{pe}^2}{\Omega_e^2} \right) \frac{\gamma_{-1} + \nu_{\rm eff}}{\omega_{lk}} \right]^{-1}, \tag{31}$$

$$\operatorname{Im} \tilde{\varepsilon}_{-1} = 2 \frac{\gamma_{-1} + \nu_{\rm eff}}{\omega_{lk}} \left(1 + \frac{\omega_{pe}^2}{\Omega_e^2} \right). \tag{32}$$

It was assumed in the derivation of (30) that the real part of the dielectric constant is independent of $\nu_{\rm eff}$. It is easily verified that $\operatorname{Im} \tilde{\varepsilon}_E$ is finite at $E_0 > E_{\rm thr}$.

Thus, if expression (29) for the spectral density of the fluctuations is used in (24), we obtain a nonlinear equation for $\nu_{\rm eff}$ in a turbulent plasma. The existence of a solution of this equation means that the interaction between perturbations in a nonequilibrium plasma produces a stationary turbulent state.

We note that the turbulent-collision frequency was calculated in Ref. 12 for an isotropic plasma under the assumption that the contribution made to the Langmuir-wave dissipation by turbulent ion-sound fluctuations is equal to $\gamma + \nu_{\rm eff}(E_0)$, where γ is the rate of Landau damping of the Langmuir oscillations.

4. PLASMA CONDUCTIVITY FAR ABOVE THRESHOLD

We consider the regime far above threshold, where the effective collision frequency significantly exceeds the Landau-damping rates of the ion-sound and of the lower-hybrid waves:

$$\nu_{\rm eff}(E_0) \gg \gamma_0, \ \gamma_{-1}. \tag{33}$$

It must be borne in mind, however, that the nonlinear equation (24) for $\nu_{\rm eff}$ is valid only if

$$\nu_{\rm eff}(E_0) < \omega_{sk}. \tag{34}$$

Substituting expression (29) for the field-fluctuation spectral density in Eq. (24) and integrating with respect to ω and $\mathbf{k}$, we obtain a nonlinear equation for $\nu_{\rm eff}$. The form of this equation is determined by the structures of the terms $\tilde{I}_{\mathbf{k}}^s$ and $\tilde{I}_{\mathbf{k}}^l$ in expression (29) for the spectral density of the field fluctuations, i.e., by the amplitudes of the fluctuations at the ion-sound and lower-hybrid frequencies, respectively. Solving this equation under conditions (25), (33), and (34), it is easy to verify that $\nu_{\rm eff}$ is governed mainly by the ion-sound fluctuation level. We have then

$$\nu_{\rm eff} = 32 (2\pi)^{1/2} \frac{m_i}{m_e} (k_0 r_{De})^4 \frac{\gamma_0}{\omega_{sk}} \left(\frac{\gamma_{-1}}{\omega_{lk}} \right)^2 \left(\frac{E_0}{E_{\rm thr}} \right)^2 \omega_{sk}, \tag{35}$$

where k_0 is the wave number that satisfies the decay condition (25). It can be seen from (35) that the effective collision frequency is proportional to the pump-field strength and is inversely proportional to the square of the external magnetic field strength. From (33) and (34) follow constraints on those pump-field strength values for which expression (35) is valid:

$$10^{-2} \frac{m_e}{m_i} (k_0 r_{De})^{-4} \left(\frac{\omega_{lk}}{\gamma_{-1}} \right)^2 < \frac{E_0^2}{E_{\rm thr}^2} < \left(\frac{m_e}{m_i} \right)^{5/6}$$
$$\times (n_e r_{De})^{-2/3} \left(\frac{\omega_{lk}}{\gamma_{-1}} \right)^{5/3} (k_0 r_{De})^{-10/3} \frac{\omega_{sk}}{\gamma_0} \left(\frac{\omega_{pe}}{\Omega_e} \right)^{8/3}. \tag{36}$$

It follows from (23) and (35) that the density of the electromagnetic-field energy absorbed by the plasma has the following dependence on the pump amplitude:

$$W \propto \nu_{\rm eff} E_0^2 \propto E_0^4.$$

Substituting in (35) typical values of the parameters of the hot plasma attainable in existing tokamaks, we can readily show that the effective collision frequency governed by the turbulent conductivity of the plasma can exceed the frequency of the electron-ion collisions by several orders of magnitude. This results in an effective HF-power dissipation mechanism that accounts qualitatively for the lower-hybrid heating process.

Notice must be taken of the similarity of our present results to those of Ref. 23, in which a quasilinear theory of parametric decay instabilities in a magnetoactive plasma has been developed. It is shown in Ref. 23 that quasilinear saturation of the parametrically excited noise is due to deformation of the electron distribution function. The turbulent-plasma HF conductivity that determines the anomalous absorption of the HF pump field was calculated and found to be proportional to E_0^2 [cf. Eq. (35)]. In Ref. 24 it is indicated that under conditions of saturated parametric ion-cyclotron instability the oscillation energy-density level is proportional to E_0^4. This agrees with the experimental results for anomalous absorption of electromagnetic waves in a collisionless magnetoactive plasma (see Ref. 25), which have shown that in various parametric instabilities the power absorbed above threshold is proportional to E_0^4.

Nonlinear saturation of parametric decay to the pump wave into lower-hybrid and ion-sound waves is the subject also of Ref. 26–28, in which the most effective saturation mechanism was assumed to be the secondary instability of the excited lower-hybrid wave. Let us compare expression (23) for the absorbed power with the corresponding expression derived from Eqs. (12), (13), and (21) of Ref. 27 in the case $\omega_0 \gtrsim 3\omega_{LH}$. For typical thermonuclear-plasma parameters ($n_0 = 10^{14}$ cm^{-3}. $T_e = 5$ keV, $B_0 = 50$ kG, $kr_{De} = 25$ and $\mu = 5\cdot 10^{-2}$) the power absorbed by the plasma as a result of charged-particle scattering by fluctuations is of the same order as that obtained in Ref. 27. Note that as the magnetic field is increased or the plasma density decreased, the saturation mechanism we are proposing becomes more effective. Note also that the absorption of the power pumped into the plasma becomes more effective with decreasing pump frequency, in accord with the conclusion of Ref. 27.

The nonlinear stage of parametric instability near lower-hybrid resonance was investigated in Refs. 18–20. In Ref. 18 the nonlinear saturation mechanism was taken to be induced scattering by ions, which is substantial in an isothermal plasma. In Ref. 19 was proposed that the parametric instability in the frequency region $\omega_0 \lesssim 2\omega_{LH}$ is suppressed by induced scattering from the electrons. Since we are considering a nonisothermal plasma with $\omega_0 \gtrsim 3\omega_{LH}$, a direct comparison of our present results with the conclusions of Refs. 18–20 is impossible. We note only that the mechanisms proposed in these references for the saturation of parametric instability were found to be less effective than the secondary-decay mechanism of Ref. 27.

CONCLUSIONS

As noted in Refs. 20 and 21, whereas a linear theory of parameteric instabilities in lower-hybrid plasmas heating has been well developed, nonlinear saturation of the fluctuation and the associated heating are still far from understood. Experimental investigations of lower-hybrid plasma heating gave rise to a number of conjectures concerning the importance of fluctuations. It was emphasized in Ref. 6 that electron and ion scattering by turbulent pulsations of an electric field can be the cause of the observed rapid plasma heating by a fast magnetosonic wave (FMSW). It was noted at the same time that the heating observed cannot be attributed to Coulomb collisions and to Čerenkov or cyclotron damping of the excited FMSW. Similar conclusions were drawn in Ref. 7 from investigations of anomalous electron heating in induced *ls* scattering in a lower-hybrid wave field. To explain the observed heating, the effective collision frequency should exceed the pair-collision frequency by an order of magnitude, a premise confirmed by measurements of the electric conductivity and of the electron diffusion along the magnetic field with increasing HF field. Experiment revealed a correlation between the heating, on the one hand, and the excitation of ion-sound noise, on the other. Strong scattering of the lower-hybrid wave by low-frequency density fluctuations ($\delta n/n \leqslant 0.4$) was observed in Ref. 9 There the investigations pointed to turbulent heating of the particles. This enabled the authors of Ref. 29 to suggest that the HF power absorbed by the resonant electrons is rapidly dissipated as a result of the anomalously high coefficient of diffusion in velocity space.

Investigations of lower-hybrid heating in the Alcator A tokamak [30] have shown that the lower-hybrid waves can be strongly scattered by density fluctuations at the edge of the plasma pinch. Decay spectra and a strong increase of the low-frequency oscillations during the time of the HF pulse were observed, as was a linear dependence of the power of the lower-hybrid decay wave on the pump power P_0 in the range 200 W $< P_0 <$ 100 kW. The authors pointed to a possibility of attributing the observations to scattering by density fluctuations.

Microwave-scattering[31] and CO_2-laser emission scattering[32] methods were used in a number of recent experimental investigations of fluctuations in a tokamak plasma. It was shown that the fluctuations have a maximum at the edge of the plasma pinch, where their amplitudes can reach $\delta n/n \approx 1$. In this regime, the rates of the particle and energy transport processes are determined by Bohm diffusion, i.e., intense fluctuations cause strong scattering of the HF radiation incident on the plasma. The amplitudes of the low-frequency density fluctuations measured in the Alcator-A tokamak reached 100% at the diaphragm radius. It was found that the fluctuation intensity is linear in the plasma HF pump power in a wide range of P_0.

A numerical simulation,[33] on a microscopic scale, of the lower-hybrid plasma heating has shown that the magnitude and sign of the charged-particle acceleration depend on the strength of the electric field with which the particle is in phase. If a particle moving along the gyro-orbit is no longer in phase with the wave by the time the next resonance sets in, its acceleration during the succeeding instant of time is independent of the preceding acceleration, so that heating takes place on the average. Such a decorrelation can occur if HF noise (fluctuations) are present in the plasma.

The theory developed in the present paper for HF conductivity of a magnetoactive plasma is thus in qualitative agreement with the experimental data and with the results of numerical simulation.

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