Instability of one-dimensional drift solitons with allowance for the vector nonlinearity

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We prove numerically the instability of a one-dimensional drift soliton to transverse perturbations. We show that the vector nonlinearity does not eliminate the instability.

The instability of a one-dimensional soliton potential drift wave to transverse perturbations was proven in Ref. 1. A one-dimensional soliton decays into two-dimensional solitons which are stable to infinitely small perturbations[2] and which survive collisions with one another.[3,4] However, the instability of the one-dimensional soliton was shown in Ref. 1 with allowance for only one of the drift wave nonlinearities—the scalar one. Recently[5] it was asserted that when one takes the other nonlinearity—the vector one—into account, the one-dimensional soliton turns out to be stable and the largest margin of stability is realized in the case of perturbations which are strictly perpendicular to the magnetic field ($k_{\parallel} = 0$). We solve here the problem of the stability of a one-dimensional soliton numerically in order to verify that statement and to study in more detail the effect of the vector nonlinearity on the evolution of such a soliton.

We normalize the variables to the characteristic dimensions of a two-dimensional soliton moving with a velocity $u = u^* + \delta u$, $\delta u > 0$ in a plasma with an inhomogeneous density and electron temperature. Here $u^* = (r_B/L_n)c_s$ is the drift velocity, L_n and L_T the characteristic inhomogeneity lengths, and $r_B = c_s/\omega_{Bi}$ the effective Larmor radius. The soliton wave number k_0 and the amplitude Φ_0 of the potential are given by the relations

$$(k_0 r_B)^2 = \delta u/u \equiv a, \quad e\Phi_0/T_e = -2(L_T/L_n)(u/u^*)a,$$

so that the equation for the drift wave potential[1] in dimensionless variables $\tau = k_0 \delta u t$, $\xi = k_0 x$, $\Psi = \Phi/\Phi_0$, $\eta = k_0(y - ut)$ takes the form

$$\frac{\partial}{\partial\tau}(\Psi - a\Delta\Psi) + \frac{\partial}{\partial\eta}(\Delta\Psi - \Psi + \Psi^2) + bJ(\Psi, \Delta\Psi) = 0, \qquad (1)$$

where J is the Jacobian and b a parameter which equals

$$b = 2k_0 L_T a \gg a \qquad (2)$$

and determines the relative contribution of the vector nonlinearity.

Experimental values of the parameters of a tokamak plasma, given in Ref. 5 are

$$r_B \sim 0.1 \text{ cm}, \quad L_n \sim 10 \text{ cm}, \quad e\Phi_0/T_e \leqslant 0.03, \qquad (3)$$

whence follows $a \sim 10^{-2}$ and $b \lesssim 1$. We can then neglect in (1) the term $a\Delta\Psi$. We set the parameter consecutively equal to $b = 0$ (purely scalar nonlinearity), $b = 1$, 2, and 4 (with a margin towards a strong vector nonlinearity). We studied for those values of the parameters numerically the evolution of a one-dimensional soliton for transverse perturbations with a wavelength larger than the size of the soliton by a factor four, i.e., we solved Eq. (1) with the initial condition

$$\Psi = 1.5\,\mathrm{ch}^{-2}(\eta/2) + 0.2\sin(\pi\xi/8). \qquad (4)$$

In all cases considered we observed a decay of a one-dimensional soliton into a set of a finite number (along each wavelength of the perturbation) of axially symmetric two-dimensional drift solitons. The scenario of the decay was the following: those parts of the one-dimensional soliton with an amplitude which exceeded the average value when the perturbation was taken into account moved faster (as the velocity is proportional to the amplitude) and the soliton was distorted. Afterwards when the transverse perturbation became sufficiently strong two-dimensional self-compression mechanism was triggered and two-dimensional solitons were formed.

In the case of a purely scalar nonlinearity ($b = 0$, Fig. 1) along each wavelength of the perturbation one fast and

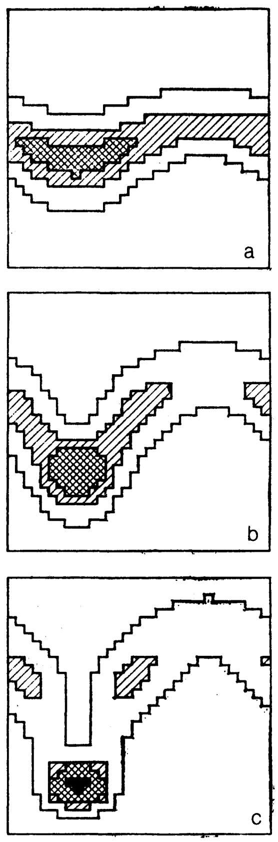

FIG. 1. Decay of a one-dimensional drift soliton into two-dimensional ones. The axis is directed downwards. The 0.5, 1, 1.5, and 3.5 level lines of the dimensionless potential Ψ are shown. We have blackened the regions $\Psi > 3.5$. The linear dimensions of the figure are the same as the wavelength of the perturbation ($\lambda = 16$). The dimensionless time $\tau = 3$ (a), 7 (b), and 10 (c). The parameter $b = 0$ (purely scalar nonlinearity).

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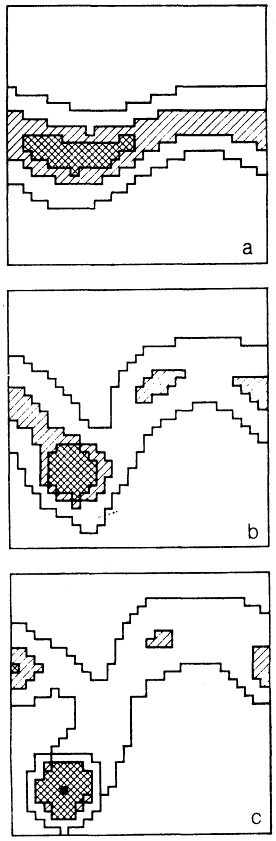

FIG. 2. The same as in Fig. 1 but taking the vector nonlinearity into account ($b = 2$).

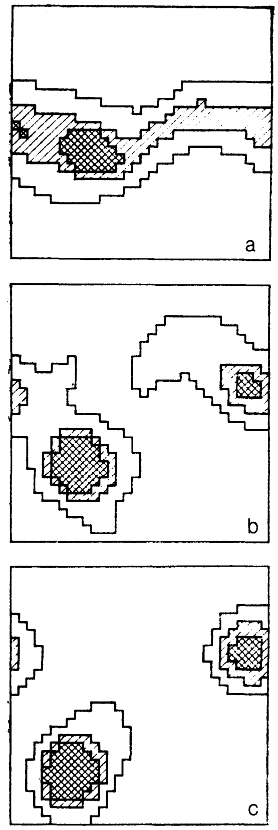

FIG. 3. The same as in Fig. 1 but with a stronger vector nonlinearity ($b = 4$).

two slow two-dimensional solitons were formed the latter arranged symmetrically relative to the trajectory of the fast soliton (owing to the symmetry of the problem under reflection $\xi \to -\xi$ when $b = 0$).

When the vector nonlinearity is taken into account ($b \neq 0$, Figs. 2 and 3) the symmetry of Eq. (1) relative to the above mentioned transformation is violated and, of course, the scenario of the decay also becomes asymmetrical. The slow soliton situated to the right of the trajectory of the fast one becomes larger and the left-hand one is very weakly realized in the case of a stronger nonlinearity and is not formed at all when $b = 4$.

The rate at which the two-dimensional solitons are formed when the vector nonlinearity is taken into account turns out to be faster than in the purely scalar case (see Figs. 1–3). The vector nonlinearity in the range of values of the parameter b which we considered ($0 \leqslant b \leqslant 4$) did thus not check or slow down the decay of the one-dimensional soliton and turned out to affect mainly the symmetry of the process, the number and relative arrangement of the stable, axially symmetric two-dimensional drift solitons which are formed as the result of the decay.

We note that the range of values of the parameter b considered here is rather wide and it certainly covers the typical range in which this parameter changes in tokamak plasmas ($b \lesssim 1$). The conclusion reached in Ref. 5 that the vector nonlinearity has a stabilizing effect in the problem of the stability of drift solitons in such a plasma is therefore erroneous. The misunderstanding seems to us to be due to two reasons. First, when substituting the plasma parameters in inequality (3.13) of Ref. 5 the authors of that paper made a mistake and their subsequent conclusion that for the given plasma parameters the vector nonlinearity has a strong effect is incorrect. Second, the dispersion equation for the perturbations derived in Ref. 5 in the case of a moderate vector nonlinearity (in our notation for $b_\perp \equiv bk_\perp/k_0 \sim 1$) is extended to the case of a strong vector nonlinearity, $b_\perp \gg 1$, which requires special justification.

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