Radiative recombination of electrons and holes of a semiconductor in the field of a strong electromagnetic wave

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We consider parametric transformation of intense electromagnetic radiation that causes quantum transitions of the electrons of a semiconductor between the edge of the valence band and the edge of the conduction band. The Bogolyubov-Bogolyubov (Jr.) method of excluding boson variables and the asymptotic Krylov-Bogolyubov-Mitropol'skii asymptotic averaging method are used to derive the kinetic equations for the distribution functions of the excitations produced by the external radiation in the electron-hole and photon subsystems. These equations are used to investigate the dependence of the semiconductor recombination-radiation power density on the frequency of the emitted photons and on the temperature. It is shown that the frequency dependence can be resonant; the number of resonance singularities and their positions are determined by the polarization of the exciting electromagnetic field. The results are the consequence of a radical restructuring of the carrier energy spectrum in the external intense electromagnetic-radiation field.

1. INTRODUCTION

The physical properties of quantum systems acted upon by intense laser radiation have attracted much attention in the last few years. Among the main peculiarities of laser action are the deformation and restructuring the carrier energy spectrum by virtual absorption and emission of laser photons. The most considerable spectrum distortions occur in various resonance situations, e.g., in cyclotron resonance,[1,2] when semiconductors are subjected to electromagnetic radiation of frequency close to the band gap[3–5] or to the energy gap between two conduction bands.[6] Under resonance conditions, even a relatively weak interaction is capable of radically altering the carrier energy spectrum and of exerting a substantial influence on the properties of the system as a whole.

In some cases the character of the energy-spectrum restructuring depends on the type of laser-field polarization, this being due to a manifestation of the spin properties of the electron subsystem.[2,5] This circumstance permits control of the physical characteristics of quantum systems by varying the type of the laser-radiation polarization. Such a variation is relatively easily effected in a real experiment. The study of the polarization dependences of physical phenomena that occur in the presence of resonant laser radiation of high intensity is therefore of considerable interest.

In the present paper we investigate theoretically the recombination radiation spectrum of a semiconductor in an external field produced by strong laser radiation of arbitrary polarization, described classically with the aid of the potential

$$\mathbf{A}_{ext}(\mathbf{r}, t)=a[\cos(\omega_0 t-\mathbf{k}_0\mathbf{r}),\ G\sin(\omega_0 t-\mathbf{k}_0\mathbf{r}),\ 0], \qquad (1)$$

where $a = \mathrm{const} > 0$ is the amplitude of the potential, ω_0 and $\mathbf{k}_0 = (0,0,\omega_0\varepsilon^{1/2}/c)$ are the frequency and the wave vector of the laser wave (ε is the dielectric constant of the medium and c is the speed of light in vacuum). The parameter G determines the type of external-field polarization: the field is elliptically dextropolarized at $0 < G < 1$ and elliptically levopolarized at $-1 < G < 0$; the values $G = 0$ and $G = \pm 1$ pertain to linear and circular polarization. We consider the case of parametric resonance, when the external-field frequency is connected with the semiconductor band gap ε_g by the equation[1)]

$$\omega_0=\varepsilon_g+\Delta, \quad 0<\Delta\ll\varepsilon_g. \qquad (2)$$

Laser radiation is taken to be strong if the condition[3]

$$\Omega\gg\omega_S,\ \tau_R^{-1} \qquad (3)$$

is met, where Ω is the frequency of the electron transitions between the edges of the valence and conduction bands under the influence of the external field; ω_S is the frequency of the carrier collisions with one another, as well as with phonons and impurity atoms; τ_R is the electron and hole radiative-recombination time. In our problem we have (in the customary units)[4]

$$\Omega\approx\rho\omega_0, \quad \rho=2e_0|\mathbf{E}_0|s/\hbar\omega_0^2\ll1, \quad s=(\varepsilon_g/2m)^{1/2}, \qquad (4)$$

where e_0 and m are the absolute value of the charge and the effective mass of the electron (hole), and $\mathbf{E}_0$ is the amplitude of the electric component of the laser field. For typical semiconductor parameters ($\varepsilon_g = 1$, $m = 0.1m_e$) and for $|\mathbf{E}_0| \sim 10^5$ W/cm, ρ reaches values of the order of 10^{-2}.

To calculate the rate of photon generation per unit semiconductor volume it is convenient to use the Bogolyubov-Bogolyubov (Jr) (B-B) method of excluding the boson amplitudes.[7,8] The use of the Furry representation[9,10] makes it possible to take into account the interaction of electrons or holes with the classical external field (1) even during the initial stage of the calculations.

According to the results, the recombination-radiation spectrum has resonant singularities and depends substantially on the type of exciting-field polarization. The emis-

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sion-spectrum singularities are attributed to the complicated structure of the quasienergy spectrum of the electron-hole excitations induced in the semiconductor by the strong field (1).

2. KINETIC EQUATION

Consider a semiconductor electron-hole subsystem S interacting with quantized photon and phonon fields Σ_1 and Σ_2, respectively, and also with the time-dependent classical external field (1). The Hamiltonians of the free electron-hole, photon, and phonon fields are of the form ($\alpha = 1,2$; $\omega_{k\lambda_2}$ are the eigenfrequencies of the phonon field)

$$H(S)=\sum_{p\sigma}\varepsilon_p(a_{p\sigma}^+a_{p\sigma}+b_{p\sigma}^+b_{p\sigma}),\quad \varepsilon_p=\frac{\varepsilon_g}{2}\left(1+\frac{2p^2}{m\varepsilon_g}\right)^{1/2},$$
$$H(\Sigma_\alpha)=\sum_{k\lambda_\alpha}\omega_{k\lambda_\alpha}c^+_{k\lambda_\alpha}c_{k\lambda_\alpha},\quad \omega_{k\lambda_1}=c|k|\varepsilon^{-1/2}. \tag{5}$$

Here $a_{p\sigma}(a^+_{p\sigma})$ and $b_{p\sigma}(b^+_{p\sigma})$ are the operators of annihilation (creation) of an electron and a hole in a state with quantum numbers ($\mathbf{p},\sigma$); they satisfy anticommutation relations of the type

$$[a_{p\sigma},a^+_{p'\sigma'}]_+=[b_{p\sigma},b^+_{p'\sigma'}]_+=\delta_{pp'}\delta_{\sigma\sigma'};$$

$c_{k\lambda_\alpha}(c^+_{k\lambda_\alpha})$ is a photon (at $\alpha = 1$) or phonon (at $\alpha = 2$) annihilation (creation) operator satisfying the commutation relations

$$[c_{k\lambda_\alpha},c^+_{k'\lambda'_{\alpha'}}]_-=\delta_{\alpha\alpha'}\delta_{kk'}\delta_{\lambda_\alpha\lambda'_\alpha},$$
$$[c_{k\lambda_\alpha},c_{k'\lambda'_{\alpha'}}]_-=[c^+_{k\lambda_\alpha},c^+_{k'\lambda'_{\alpha'}}]_-=0.$$

The summation with respect to $\mathbf{p}$ and $\mathbf{k}$ is over the usual quasidiscrete spectrum. The parameter $\lambda_1 = 1,2$ determines the polarization of the photons of field Σ_1, while the parameter λ_2 indicates the number of the phonon mode; $\sigma = \pm 1$ is the spin index. The symbols S, Σ_1, and Σ_2 stand here and elsewhere for operators acting on the wave functions of the system as functions of the aggregate of occupation numbers of the corresponding fields.

Denoting by $H_{\rm ext}(t,S)$ the operator of electron and hole interaction with the external field (1), we express the Hamiltonian of the system (S,Σ_1,Σ_2) in the Schrödinger representation of the dynamic quantities in the form

$$H=H(t,S,\Sigma_1,\Sigma_2)=H(S)+H_{ext}(t,S)$$
$$+\sum_{\alpha=1,2}\left\{H(\Sigma_\alpha)+\sum_{k\lambda_\alpha}[J_{k\lambda_\alpha}(S)c_{k\lambda_\alpha}+J^+_{k\lambda_\alpha}(S)c^+_{k\lambda_\alpha}]\right\}, \tag{6}$$

where $J_{k\lambda_\alpha}(S)$ are certain operator expressions bilinear in the electron and hole creation and annihilation operators. We assume henceforth that in the infinitely remote past there was not external field ($H_{\rm ext}(t,S)\to 0$ and $t\to -\infty$) and the system as a whole was in thermodynamic equilibrium with the heat bath.

We introduce the statistical operator D_t of the system (S,Σ_1,Σ_2), which satisfies the Liouville equation

$$i\,\partial D_t/\partial t=[H,\ D_t]_- \tag{7}$$

with the initial condition ($t_0\to -\infty$; $\beta = 1/k_B T$, k_B is the Boltzmann constant and T is the absolute temperature of the heat bath)

$$D_{t_0}=\rho(S)\prod_{\alpha=1,2}D(\Sigma_\alpha),$$
$$D(\Sigma_\alpha)=\exp[-\beta H(\Sigma_\alpha)]/\mathrm{Sp}_{(\Sigma_\alpha)}\exp[-\beta H(\Sigma_\alpha)],$$
$$\mathrm{Sp}_{(S)}\rho(S)=1. \tag{8}$$

We represent the formal solution of the Liouville equation in the form

$$D_t=U(t,\ t_0)D_{t_0}U^{-1}(t,\ t_0), \tag{9}$$

where $U(t,t_0) = U(t,t_0,S,\Sigma_1,\Sigma_2)$ is a unitary operator defined by the equation

$$i\partial U(t,\ t_0)/\partial t=HU(t,\ t_0),\quad U(t_0,\ t_0)=1.$$

The mean value of an arbitrary dynamic quantity defined in the Schrödinger representation by the operator $u(t,S,\Sigma_1,\Sigma_2)$ is

$$\langle u\rangle_t=\mathrm{Sp}_{(S,\Sigma_1,\Sigma_2)}u(t,\ S,\ \Sigma_1,\ \Sigma_2)D_t. \tag{10}$$

Substituting (9) in (10) we get

$$\langle u\rangle_t=\mathrm{Sp}_{(S,\Sigma_1,\Sigma_2)}u(t,\ S_t,\ \Sigma_{1t},\ \Sigma_{2t})D_{t_0}, \tag{11}$$

$$u(t,\ S_t,\ \Sigma_{1t},\ \Sigma_{2t})=U^{-1}(t,\ t_0)u(t,\ S,\ \Sigma_1,\ \Sigma_2)U(t,\ t_0). \tag{12}$$

Equations (9)–(12) effect a transition to the Heisenberg representation, which coincides at $t = t_0$ with the Schrödinger representation.

Whereas a dynamic quantity in the Schrödinger representation is defined by an operator $u(S,\Sigma_1,\Sigma_2)$ that does not depend explicitly on the time, the operator of this quantity in the Heisenberg representation obeys the equation of motion

$$i\partial u(S_t,\ \Sigma_{1t},\ \Sigma_{2t})/\partial t=[u(S_t,\ \Sigma_{1t},\ \Sigma_{2t}),\ H(t,\ S_t,\ \Sigma_{1t},\ \Sigma_{2t})]_-.$$

Choosing as the operator of the dynamic quantity $u(S,\Sigma_1,\Sigma_2)$ the photon-number operator $N_{k\lambda_1} = c^+_{k\lambda_1}c_{k\lambda_1}$ of a quantized electromagnetic field in a state with quantum numbers ($\mathbf{k},\lambda_1$), we get

$$\frac{\partial\langle N_{k\lambda_1}\rangle_t}{\partial t}=\frac{\partial}{\partial t}\mathrm{Sp}_{(S,\Sigma_1,\Sigma_2)}N_{k\lambda_1}(t)D_{t_0}$$
$$=i\,\mathrm{Sp}_{(S,\Sigma_1,\Sigma_2)}J_{k\lambda_1}(S_t)c_{k\lambda_1}(t)D_{t_0}+\text{c.c.} \tag{13}$$

We use the B-B method[7,8] to exclude the photon amplitudes from the right-hand side of (13). To this end we write the Heisenberg equations of motion in the integral form

$$c_{k\lambda_1}(t)=\overset{(I)}{c}_{k\lambda_1}(t)-iB_{k\lambda_1}(t),\quad c^+_{k\lambda_1}(t)=\overset{(I)+}{c}_{k\lambda_1}(t)+iB^+_{k\lambda_1}(t). \tag{14}$$

Here

$$\overset{(I)}{c}_{k\lambda_1}(t)=U_0^{-1}(t,t_0)c_{k\lambda_1}U_0(t,t_0)=c_{k\lambda_1}\exp[-i\omega_{k\lambda_1}(t-t_0)],$$
$$\overset{(I)+}{c}_{k\lambda_1}(t)=U_0^{-1}(t,t_0)c^+_{k\lambda_1}U_0(t,t_0)=c^+_{k\lambda_1}\exp[i\omega_{k\lambda_1}(t-t_0)] \tag{15}$$

are the photon annihilation and creation operators in the interaction representation. The transition of this operator is with the aid of the unitary transformation operator

$$U_0(t,t_0)=\exp\left\{-i\left[H(S)+\sum_{\alpha=1,2}H(\Sigma_\alpha)\right](t-t_0)\right\}; \quad (16)$$

$$B_{\mathbf{k}\lambda_1}(t)=\int_{t_0}^{t}d\tau\exp[-i\omega_{\mathbf{k}\lambda_1}(t-\tau)]J^+_{\mathbf{k}\lambda_1}(S_\tau),$$

$$B^+_{\mathbf{k}\lambda_1}(t)=\int_{t_0}^{t}d\tau\exp[i\omega_{\mathbf{k}\lambda_1}(t-\tau)]J_{\mathbf{k}\lambda_1}(S_\tau). \quad (17)$$

We denote by $N^{(eq)}_{\mathbf{k}\lambda_1}$ the Planck distribution functions of the equilibrium photons and phonons ($\alpha = 1,2$):

$$N^{(eq)}_{\mathbf{k}\lambda_\alpha}=[\exp(\beta\omega_{\mathbf{k}\lambda_\alpha})-1]^{-1}. \quad (18)$$

Substituting (14) in (13), we use relations of the type

$$\mathrm{Sp}_{(S,\Sigma_1,\Sigma_2)}J_{\mathbf{k}\lambda_1}(S_t)\overset{(I)}{c}_{\mathbf{k}\lambda_1}(t)D_{t_0}$$

$$=N^{(eq)}_{\mathbf{k}\lambda_1}\mathrm{Sp}_{(S,\Sigma_1,\Sigma_2)}[\overset{(I)}{c}_{\mathbf{k}\lambda_1}(t),J_{\mathbf{k}\lambda_1}(S_t)]_-D_{t_0} \quad (19)$$

and the equations

$$[c^{(I)}_{\mathbf{k}\lambda_1}(t),J_{\mathbf{k}\lambda_1}(S_t)]_-=i[B_{\mathbf{k}\lambda_1}(t),J_{\mathbf{k}\lambda_1}(S_t)]_-,$$

$$[\overset{(I)}{c}_{\mathbf{k}\lambda_1}(t),J^+_{\mathbf{k}\lambda_1}(S_t)]_-=-i[B^+_{\mathbf{k}\lambda_1}(t),J_{\mathbf{k}\lambda_1}(S_t)]_-, \quad (20)$$

and obtain the following exact equation

$$\frac{\partial\langle N_{\mathbf{k}\lambda_1}\rangle_t}{\partial t}=\int_{t_0}^{t}d\tau\exp[-i\omega_{\mathbf{k}\lambda_1}(t-\tau)]\mathrm{Sp}_{(S,\Sigma_1,\Sigma_2)}\{J_{\mathbf{k}\lambda_1}(S_t)J^+_{\mathbf{k}\lambda_1}(S_\tau)$$

$$+N^{(eq)}_{\mathbf{k}\lambda_1}[J_{\mathbf{k}\lambda_1}(S_t),J^+_{\mathbf{k}\lambda_1}(S_\tau)]_-\}D_{t_0}+\text{c.c.} \quad (21)$$

Identities of the type (19) follow from the spectral representations of the temporal correlation functions. Equations (20) are the consequence of commutation of the operators $c_{\mathbf{k}\lambda_1}(t)$ and $\mathbf{J}_{\mathbf{k}\lambda_1}(S_t)$, taken at the same instant of time.

We calculate the right-hand side of the obtained kinetic equation in the Furry representation.[9,10] To this end we write the operator $U(t,t_0)$ in the multiplicative form

$$U(t,t_0)=U_0(t,t_0)U_1(t,t_0)U_2(t,t_0),$$

$$U_1(t_0,t_0)=U_2(t_0,t_0)=1. \quad (22)$$

The unitary operators $U_1(t,t_0)$ and $U_2(t,t_0)$ are defined by the equations

$$i\partial U_1(t,t_0)/\partial t=H_{ext}(t,S_t^{(I)})U_1(t,t_0),$$

$$i\frac{\partial U_2(t,t_0)}{\partial t} \quad (23)$$

$$=\sum_{\alpha=1,2}\sum_{\mathbf{k}\lambda_\alpha}[J_{\mathbf{k}\lambda_\alpha}(S_t^{(F)})c^{(I)}_{\mathbf{k}\lambda_\alpha}(t)+J^+_{\mathbf{k}\lambda_\alpha}(S_t^{(F)})c^{(I)+}_{\mathbf{k}\lambda_\alpha}(t)]U_2(t,t_0),$$

in which

$$H_{ext}(t,S_t^{(I)})=U_0^{-1}(t,t_0)H_{ext}(t,S)U_0(t,t_0) \quad (24)$$

is the Hamiltonian of the semiconductor electron and hole interactions with the classical external field (1) in the interaction representation, and

$$J_{\mathbf{k}\lambda_\alpha}(S_t^{(F)})=U_1^{-1}(t,t_0)J_{\mathbf{k}\lambda_\alpha}(S_t^{(I)})U_1(t,t_0)$$

$$=U_1^{-1}(t,t_0)U_0^{-1}(t,t_0)J_{\mathbf{k}\lambda_\alpha}(S)U_0(t,t_0)U_1(t,t_0) \quad (25)$$

is an operator in the Furry representation. The connection between the operators of an arbitrary dynamic quanity u in different representation is given by the equations

$$u(t,S_t,\Sigma_{1t},\Sigma_{2t})=U_2^{-1}(t,t_0)u(t,S_t^{(F)},\Sigma_{1t}^{(F)},\Sigma_{2t}^{(F)})U_2(t,t_0)$$

$$=U_2^{-1}(t,t_0)U_1^{-1}(t,t_0)u(t,S_t^{(I)},\Sigma_{1t}^{(I)},\Sigma_{2t}^{(I)})U_1(t,t_0)U_2(t,t_0), \quad (26)$$

$$u(t,S_t^{(I)},\Sigma_{1t}^{(I)},\Sigma_{2t}^{(I)})=U_0^{-1}(t,t_0)u(t,S,\Sigma_1,\Sigma_2)U_0(t,t_0).$$

For operators acting only on Σ-field operators, the Furry representation coincides with the interaction representation $u(t,\Sigma_{1t}^{(F)},\Sigma_{2t}^{(F)})=u(t,\Sigma_{1t}^{(I)},\Sigma_{2t}^{(I)})$. If the operator of the dynamic quantity u in the Schrodinger representation does not depend explicitly on the time, in the Furry representation it obeys the equation of motion

$$i\frac{\partial u(S_t^{(F)},\Sigma_{1t}^{(F)},\Sigma_{2t}^{(F)})}{\partial t}=\Big[u(S_t^{(F)},\Sigma_{1t}^{(F)},\Sigma_{2t}^{(F)}),H(S_t^{(F)})$$

$$+H_{ext}(t,S_t^{(F)})+\sum_{\alpha=1,2}H(\Sigma_\alpha)\Big]_-. \quad (27)$$

The system statistical operator $D_t^{(F)}$ in the Furry representation is defined by the equations

$$D_t=U_0(t,t_0)U_1(t,t_0)D_t^{(F)}U_1^{-1}(t,t_0)U_0^{-1}(t,t_0)$$

$$=U_0(t,t_0)D_t^{(I)}U_0^{-1}(t,t_0), \quad (28)$$

$$D_t^{(F)}=U_1(t,t_0)U_2(t,t_0)D_{t_0}U_2^{-1}(t,t_0)U_1^{-1}(t,t_0)$$

and satisfies the Liouville equation

$$i\frac{\partial D_t^{(F)}}{\partial t}$$

$$=\Big[\sum_{\alpha=1,2}\sum_{\mathbf{k}\lambda_\alpha}\{J_{\mathbf{k}\lambda_\alpha}(S_t^{(F)})c^{(I)}_{\mathbf{k}\lambda_\alpha}(t)+J^+_{\mathbf{k}\lambda_\alpha}(S_t^{(F)})c^{(I)+}_{\mathbf{k}\lambda_\alpha}(t)\},D_t^{(F)}\Big]_-. \quad (29)$$

The explicit form of the operators $H_{ext}(t,S)$ and $\mathbf{J}_{\mathbf{k}\lambda_\alpha}(S)$ depends on the specific choice of the semiconductor model. The analysis that follows is based on the use of a very simple model defined by the equation[11-13]

$$\{i\gamma_0\partial/\partial t+s\gamma[i\partial/\partial\mathbf{r}-e_0\mathbf{A}(\mathbf{r},t)/c-e_0\mathbf{A}_{ext}(\mathbf{r},t)/c]$$

$$+R\Phi(\mathbf{r},t)-ms^2\}\Psi(\mathbf{r},t)=0. \quad (30)$$

Here Ψ, $\mathbf{A}$, and Φ are respectively the operators of the electron-hole, photon, and phonon fields in the Heisenberg representation; $\gamma\mu$ are Dirac matrices ($\mu = 0, 1, 2, 3$); R is a certain matrix that depends on the constants of the interaction of the phonons with the electrons and holes. Equation (30) describes the dynamics of the carriers of a semiconductor consisting of two isotropic and orbitally nondegenerate bands with extrema at one and the same quasimomentum space of the electron.

We present the operators $H_{ext}(t,S)$ and $J_{\mathbf{k}\lambda_1}(S)$ directly in the interaction representation. Within the framework of the chosen semiconductor model we have

$$H_{ext}(t,S_t^{(I)}) = -\frac{1}{c}\int d\mathbf{r}\, \mathbf{j}^{(I)}(\mathbf{r},t)\mathbf{A}_{ext}(\mathbf{r},t), \tag{31}$$

$$J_{\mathbf{k}\lambda_1}(S_t^{(I)}) = -(2V\omega_{\mathbf{k}\lambda_1}\varepsilon)^{-1/2}\int d\mathbf{r}\, \mathbf{e}_{\mathbf{k}\lambda_1}\mathbf{j}^{(I)}(\mathbf{r},t)e^{i\mathbf{kr}}, \tag{32}$$

where $\mathbf{e}_{\mathbf{k}\lambda_1}$ is the polarization vector of the photons of field Σ_1,

$$\mathbf{j}^{(I)}(\mathbf{r},t) = -e_0 s{:}\Psi^{(I)+}(\mathbf{r},t)\gamma_0\boldsymbol{\gamma}\Psi^{(I)}(\mathbf{r},t){:} \tag{33}$$

is the electric-current density operator constructed with the aid of the electron-hole field operator $\Psi^{(I)}(\mathbf{r},t)$. The latter obeys the equation (30) of the two-band model for free electrons and holes, and can be represented by the expansion

$$\Psi^{(I)}(\mathbf{r},t) = \sum_{\mathbf{p}\sigma}\left(\frac{ms^2}{V\varepsilon_{\mathbf{p}}}\right)^{1/2}[a_{\mathbf{p}\sigma}\varphi_{\mathbf{p}\sigma}^{(+1)}(\mathbf{r},t) + b_{\mathbf{p}-\sigma}^{+}\varphi_{\mathbf{p}\sigma}^{(-1)}(\mathbf{r},t)]. \tag{34}$$

The bispinor functions $\varphi_{\mathbf{p}\sigma}^{(\pm 1)}(\mathbf{r},t)$ satisfying Eq. (30) at $\mathbf{A}(\mathbf{r},t)\equiv 0$, $\mathbf{A}_{ext}(\mathbf{r},t)\equiv 0$ and $\Phi(\mathbf{r},t)\equiv 0$, are given in Refs. 14 and 15.

In the Furry representation the operator $J_{\mathbf{k}\lambda_1}$ takes the form

$$J_{\mathbf{k}\lambda_1}(S_t^{(F)}) = (2V\omega_{\mathbf{k}\lambda_1}\varepsilon)^{-1/2} \times \int d\mathbf{r}[e_0 s : \Psi^{(F)+}(\mathbf{r},t)\gamma_0\mathbf{e}_{\mathbf{k}\lambda_1}\boldsymbol{\gamma}\Psi^{(F)}(\mathbf{r},t) : -\mathbf{e}_{\mathbf{k}\lambda_1}\mathbf{j}_{vac}(\mathbf{r},t)]e^{i\mathbf{kr}} \tag{35}$$

and the electron-hole operator in the Furry representation obeys Eq. (30) with $\mathbf{A}(\mathbf{r},t)\equiv 0$, $\Phi(\mathbf{r},t)\equiv 0$ and can be obtained from (34) by formal replacement of the carrier wave functions $\varphi_{\mathbf{p}\sigma}^{(\pm 1)}(\mathbf{r},t)$ by the wave functions $\psi_{\mathbf{p}\sigma}^{(\pm 1)}(\mathbf{r},t)$, that describe the behavior of the electrons and holes in the classical external field (1) and satisfy the asymptotic condition

$$\psi_{\mathbf{p}\sigma}^{(\pm 1)}(\mathbf{r},t)\to\varphi_{\mathbf{p}\sigma}^{(\pm 1)}(\mathbf{r},t) \quad \text{as} \quad t\to-\infty.$$

Thus,

$$\Psi^{(F)}(\mathbf{r},t) = \sum_{\mathbf{p}\sigma}\left(\frac{ms^2}{V\varepsilon_{\mathbf{p}}}\right)^{1/2}[a_{\mathbf{p}\sigma}\psi_{\mathbf{p}\sigma}^{(+1)}(\mathbf{r},t) + b_{\mathbf{p}-\sigma}^{+}\psi_{\mathbf{p}\sigma}^{(-1)}(\mathbf{r},t)]. \tag{36}$$

The presence in (35) of a term containing the classical current density

$$\mathbf{j}_{vac}(\mathbf{r},t) = -e_0 s\sum_{\mathbf{p}\sigma}\frac{ms^2}{V\varepsilon_{\mathbf{p}}}\psi_{\mathbf{p}\sigma}^{(-1)+}(\mathbf{r},t)\gamma_0\boldsymbol{\gamma}\psi_{\mathbf{p}\sigma}^{(-1)}(\mathbf{r},t), \tag{37}$$

is due to the redistribution of the charges in the Dirac "vacuum sea" by the external field (1).[16]

After substituting (36) in (35) and introducing the notation ($j, j_1 = \pm 1$)

$$M_{\mathbf{p}\sigma;\mathbf{p}_1\sigma_1}^{(j,j_1)}(\mathbf{k},\lambda_1|t) = \frac{e_0 s}{(2V\omega_{\mathbf{k}\lambda_1}\varepsilon)^{1/2}}\frac{ms^2}{V(\varepsilon_{\mathbf{p}}\varepsilon_{\mathbf{p}_1})^{1/2}}\int d\mathbf{r}\,\psi_{\mathbf{p}\sigma}^{(j)+}(\mathbf{r},t) \times\gamma_0\mathbf{e}_{\mathbf{k}\lambda_1}\boldsymbol{\gamma}\psi_{\mathbf{p}_1\sigma_1}^{(j_1)}(\mathbf{r},t), \tag{38}$$

it is convenient to rewrite the operator $J_{\mathbf{k}\lambda_1}(S_t^{(F)})$ in the form

$$J_{\mathbf{k}\lambda_1}(S_t^{(F)}) = \sum_{\mathbf{p}\sigma}\sum_{\mathbf{p}_1\sigma_1}[a_{\mathbf{p}\sigma}^{+}a_{\mathbf{p}_1\sigma_1}M_{\mathbf{p}\sigma;\mathbf{p}_1\sigma_1}^{(+1,+1)}(\mathbf{k},\lambda_1|t) + b_{\mathbf{p}-\sigma}b_{\mathbf{p}_1-\sigma_1}^{+}M_{\mathbf{p}\sigma;\mathbf{p}_1\sigma_1}^{(-1,-1)}(\mathbf{k}\lambda_1|t) + a_{\mathbf{p}\sigma}^{+}b_{\mathbf{p}_1-\sigma_1}^{+}M_{\mathbf{p}\sigma;\mathbf{p}_1\sigma_1}^{(+1,-1)}(\mathbf{k},\lambda_1|t) + b_{\mathbf{p}-\sigma}a_{\mathbf{p}_1\sigma_1}M_{\mathbf{p}\sigma;\mathbf{p}_1\sigma_1}^{(-1,+1)}(\mathbf{k},\lambda_1|t)]. \tag{39}$$

The wave functions $\psi_{\mathbf{p}\sigma}^{(\pm 1)}(\mathbf{r},t)$ will henceforth be replaced by the approximate functions obtained in Refs. 14 and 15 by the Krylov-Bogolyubov-Mitropol'skiĭ asymptotic averaging method.[17] For simplicity we shall neglect the wave vectors $\mathbf{k}$ and $\mathbf{k}_0$.

We return now to Eq. (21). Its form in the Furry representation is

$$\frac{\partial\langle N_{\mathbf{k}\lambda_1}\rangle_t}{\partial t} = \int_{t_0}^{t} d\tau\exp[-i\omega_{\mathbf{k}\lambda_1}(t-\tau)]\mathrm{Sp}_{(S,\Sigma_1,\Sigma_2)}\{J_{\mathbf{k}\lambda_1}(S_t^{(F)})J_{\mathbf{k}\lambda_1}^{(F)+}(t,\tau) + N_{\mathbf{k}\lambda_1}^{(eq)}[J_{\mathbf{k}\lambda_1}(S_t^{(F)}),J_{\mathbf{k}\lambda_1}^{(F)+}(t,\tau)]_-\}D_t^{(F)} + \text{c.c.}, \tag{40}$$

where

$$J_{\mathbf{k}\lambda_1}^{(F)}(t,\tau) = U_2(t,\tau)J_{\mathbf{k}\lambda_1}(S_\tau^{(F)})U_2^{-1}(t,\tau).$$

We seek an expansion of the right-hand side of (40) in powers of the interactions of the subsystem S with the boson fields Σ_1 and Σ_2. Assuming these interactions to be weak, we retain in this expansion only the leading terms. Since the operator $J_{\mathbf{k}\lambda_\alpha}(S)$ is itself of first order in the interaction, it suffices to retain in the expansion of the operator $U_2(t,\tau)$ the zeroth-order terms, so as to obtain the expansion of the right-hand side of (40) accurate in terms of second order of smallness, inclusive: $U_2(t,\tau)\approx 1$. Putting $J_{\mathbf{k}\lambda_1}^{(F)}(t,\tau)\approx J_{\mathbf{k}\lambda_1}(S_\tau^{(F)})$ in (40) and introducing the reduced statistical operator

$$\rho_t^{(F)}(S) = \mathrm{Sp}_{(\Sigma_1,\Sigma_2)}D_t^{(F)},$$

we get

$$\frac{\partial\langle N_{\mathbf{k}\lambda_1}\rangle_t}{\partial t} = \int_{t_0}^{t} d\tau\exp[-i\omega_{\mathbf{k}\lambda_1}(t-\tau)]\mathrm{Sp}_{(S)}\{J_{\mathbf{k}\lambda_1}(S_t^{(F)})J_{\mathbf{k}\lambda_1}^{+}(S_\tau^{(F)}) + N_{\mathbf{k}\lambda_1}^{(eq)}[J_{\mathbf{k}\lambda_1}(S_t^{(F)}),J_{\mathbf{k}\lambda_1}^{+}(S_\tau^{(F)})]_-\}\rho_t^{(F)}(S) + \text{c.c.} \tag{41}$$

Upon substitution of (39) in (41), the right-hand side of the latter is expressed in terms of the mean values of the products of four Fermi operators. Let us calculate these mean values, pairing the operators in accordance with the Bloch–de Dominicis theorem.[18] As a result we get a kinetic equation containing only single-particle distribution functions of the electron-hole excitations

$$f_{\mathbf{p}\sigma}^{(a)} = \mathrm{Sp}_{(S)}\{a_{\mathbf{p}\sigma}^{+}a_{\mathbf{p}\sigma}\}\rho_t^{(F)}(S),\quad f_{\mathbf{p}\sigma}^{(b)} = \mathrm{Sp}_{(S)}\{b_{\mathbf{p}\sigma}^{+}b_{\mathbf{p}\sigma}\}\rho_t^{(F)}(S). \tag{42}$$

We can derive for these, in turn, kinetic equations of type

(41) by using the method developed in Refs. 7 and 8. The interactions of the electron-hole subsystem with the fields Σ_1 and Σ_2 make in the lowest order an additive contribution to the collision integrals of these equations, while the collision integrals themselves depend on the Planck distribution functions (18) of the equilibrium photons and phonons.

In the next higher approximation we omit from the right-hand sides of the kinetic equations for the functions $f_{\mathbf{p}\sigma}^{(a)}$, $f_{\mathbf{p}\sigma}^{(b)}$ and $\langle N_{\mathbf{k}\lambda_1}\rangle_t$ the rapidly oscillating terms, in accord with the averaging method of Ref. 17, and retain thus only the secular terms. Being interested in the electron-hole subsystem state that interacts with the external field (1), a state resulting form the relaxation, we seek stationary solutions of the kinetic equations for the functions (42). If the conditions

$$\rho\omega_0 < \omega_{\mathbf{k}\lambda_2}, \quad (\Delta/\rho\omega_0)^2 \ll \tau_R \omega_S \tag{43}$$

are met, we can neglect in the right-hand sides of these equations the interactions of the electrons and holes with the quantized electromagnetic field Σ_1 (Refs. 3 and 19). The solutions sought are then the Fermi distribution functions defined by the expression

$$f_{\mathbf{p}\sigma}^{(a)} = f_{-\mathbf{p}\sigma}^{(b)} = [\exp[\beta(E_{\mathbf{p}\sigma} - \omega_0/2)] + 1]^{-1}, \tag{44}$$

in which

$$E_{\mathbf{p}\sigma} = \omega_0(1 + \lambda_{\mathbf{p}\sigma}\,\mathrm{sign}\,\delta_{\mathbf{p}})/2, \quad \lambda_{\mathbf{p}\sigma} = (\delta_{\mathbf{p}}^2 + \rho_\sigma^2)^{1/2},$$
$$\delta_{\mathbf{p}} = 2\varepsilon_{\mathbf{p}}/\omega_0 - 1, \quad \rho_\sigma = \rho(1 + \sigma G)/2. \tag{45}$$

Using (44), we get ultimately

$$\frac{\partial\langle N_{\mathbf{k}\lambda_1}\rangle_t}{\partial t} = \frac{e_0^2 s^2}{2\omega_{\mathbf{k}\lambda_1}\varepsilon} \sum_{\sigma,\sigma_1=\pm1} \frac{1}{(2\pi)^2} \int d\mathbf{p} \Big(|(\mathbf{e}_{\mathbf{k}\lambda_1})_z|^2 \delta_{\sigma,-\sigma_1}$$
$$+ |e_{\mathbf{k}\lambda_1}^{(-\sigma)}|^2 \delta_{\sigma,\sigma_1}\Big) \{2[(N_{\mathbf{k}\lambda_1}^{(eq)} + 1)(1 - f_{\mathbf{p}\sigma}^{(a)}) f_{\mathbf{p}\sigma_1}^{(a)}$$
$$- N_{\mathbf{k}\lambda_1}^{(eq)} f_{\mathbf{p}\sigma}^{(a)} (1 - f_{\mathbf{p}\sigma_1}^{(a)})]$$
$$\times U_{\mathbf{p}\sigma_1}^2 V_{\mathbf{p}\sigma}^2 \delta(E_{\mathbf{p}\sigma} - E_{\mathbf{p}\sigma_1} + \omega_{\mathbf{k}\lambda_1} - \omega_0) + [(N_{\mathbf{k}\lambda_1}^{(eq)} + 1) f_{\mathbf{p}\sigma}^{(a)} f_{\mathbf{p}\sigma_1}^{(a)}$$
$$- N_{\mathbf{k}\lambda_1}^{(eq)} (1 - f_{\mathbf{p}\sigma}^{(a)})(1 - f_{\mathbf{p}\sigma_1}^{(a)})] U_{\mathbf{p}\sigma_1}^2 U_{\mathbf{p}\sigma}^2 \delta(E_{\mathbf{p}\sigma} + E_{\mathbf{p}\sigma_1} - \omega_{\mathbf{k}\lambda_1})$$
$$+ [(N_{\mathbf{k}\lambda_1}^{(eq)} + 1)(1 - f_{\mathbf{p}\sigma}^{(a)})(1 - f_{\mathbf{p}\sigma_1}^{(a)}) - N_{\mathbf{k}\lambda_1}^{(eq)} f_{\mathbf{p}\sigma}^{(a)} f_{\mathbf{p}\sigma_1}^{(a)}]$$
$$\times V_{\mathbf{p}\sigma_1}^2 V_{\mathbf{p}\sigma}^2 \delta(E_{\mathbf{p}\sigma} + E_{\mathbf{p}\sigma_1} + \omega_{\mathbf{k}\lambda_1} - 2\omega_0)\}. \tag{46}$$

Here

$$U_{\mathbf{p}\sigma} = [{}^1/_2(1 + |\delta_{\mathbf{p}}|/\lambda_{\mathbf{p}\sigma})]^{1/2}, V_{\mathbf{p}\sigma} = \mathrm{sign}\,\delta_{\mathbf{p}}[{}^1/_2(1 - |\delta_{\mathbf{p}}|/\lambda_{\mathbf{p}\sigma})]^{1/2}, \tag{47}$$
$$e_{\mathbf{k}\lambda_1}^{(\sigma)} = (\mathbf{e}_{\mathbf{k}\lambda_1})_x + i\sigma(\mathbf{e}_{\mathbf{k}\lambda_1})_y.$$

3. RECOMBINATION-RADIATION SPECTRUM

Equations (46) allows us to investigate the recombination-radiation spectrum of a semiconductor the an external field (1). The right-hand side of (46) difers from zero if $\omega_1 \equiv \omega_{\mathbf{k}\lambda_1} \approx \omega_0$. Therefore at temperatures that are not too high, when $k_B T \ll \omega_0$, we can omit from this side the terms proportional to the equilibrium-photon distribution function $N_{\mathbf{k}\lambda_1}^{(eq)}$, and confine ourselves thus to consideration of spontaneous emission.

The energy radiated in the frequency interval $(\omega_1, \omega_1 + d\omega_1)$ by a unit semiconductor volume per unit time is equal to $W(\omega_1)d\omega_1$, where

$$W(\omega_1) = \frac{\omega_1^3 \varepsilon^{1/2}}{2\pi^2 c^3} \sum_{\lambda_1=1,2} \frac{\partial\langle N_{\mathbf{k}\lambda_1}\rangle_t}{\partial t}. \tag{48}$$

According to (46), the function $W(\omega_1)$ can be represented as a sum of two terms: $W(\omega_1) = W_{x,y}(\omega_1) + W_z(\omega_1)$. We shall investigate below their behavior at temperatures $k_B T \ll \omega_1$.

If the electromagnetic wave (1) is elliptically polarized, we have $W_{x,y}(\omega_1) = 0$ in the regions $\omega_0 - \rho\omega_0(1 - |G|)/2 < \omega_1 < \omega_0$ and $\omega_0 < \omega_1 < \omega_0 + \rho\omega_0(1 - |G|)/2$. At the frequencies $\omega_0, \omega_0 \pm \rho\omega_0(1 - |G|)/2$ and $\omega_0 \pm \rho\omega_0(1 + |G|)/2$ the wave has resonance singularities. If the inequality $\Delta < \rho\omega_0$ holds, the resonant frequencies $\omega_0 - \rho\omega_0(1 \pm |G|)/2$ are located in the band gap of a semiconductor not acted upon by the electromagnetic wave. The resonance singularities of the function $W_{x,y}(\omega_1)$, which are present at the frequencies $\omega_0 \pm \rho\omega_0(1 - |G|)/2$ and $\omega_0 \pm \rho\omega_0(1 + |G|)/2$ in the case of linear polarization, merge. For circular polarization of the external field, resonances exist only at the frequencies ω_0 and $\omega_0 \pm \rho\omega_0$, and there is no radiation at all in the region $\omega_0 - \rho\omega_0 < \omega_1 \leqslant \varepsilon_g$.

The quantity $W_{x,y}(\omega_1)$ is responsible for emission of phonons whose polarization vector $\mathbf{e}_{\mathbf{k}\lambda_1}$ lies in the xy plane that is perpendicular to the propagation direction of the electromagnetic wave (1). If the vector $\mathbf{e}_{\mathbf{k}\lambda_1}$ is perpendicular to the xy plane, the spectrum of the radiation in the vicinity of the external-field frequency is described by the function $W_z(\omega_1)$. In the case of elliptic polarization this function is zero at $\omega_1 = \omega_0$, $\omega_0 - \rho\omega_0/2 < \omega_1 < \omega_0 - \rho\omega_0|G|/2$, $\omega_0 + \rho\omega_0|G|/2 < \omega_1 < \omega_0 + \rho\omega_0/2$, and has at $\omega_1 = \omega_0 \pm \rho\omega_0/2$, $\omega_0 \pm \rho\omega_0|G|/2$, resonance singularities. For linear polarization, the function $W_z(\omega_1)$ has only three resonance peaks at the frequencies ω_0 and $\omega_0 \pm \rho\omega_0/2$, separated by regions in which there is no radiation at all. For circular polarization, the spectrum of the radiation with $(\mathbf{e}_{\mathbf{k}\lambda_1})_x = (\mathbf{e}_{\mathbf{k}\lambda_1})_y = 0$ is described by a smooth curve that has no resonance singularities whatever, with $W_z(\omega_1) = 0$ when $\omega_1 \leqslant \omega_0 - \Delta/2 - [(\Delta/2)^2 + (\rho\omega_0/2)^2]^{1/2}$ and $\omega_0 \leqslant \omega_1 \leqslant \omega_0 - \Delta/2 + [(\Delta/2)^2 + (\rho\omega_0/2)^2]^{1/2}$.

Thus, in the vicinity of the external-field frequency the

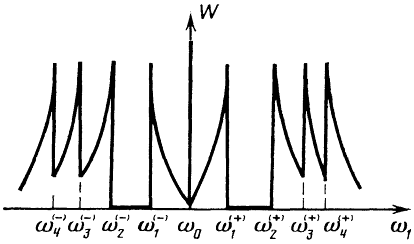

FIG. 1. Recombination-radiation spectrum of a semiconductor at $0 < |G| < 1/2$; $\omega_1^{(\pm)} = \omega_0 \pm \rho\omega_0|G|/2$, $\omega_2^{(\pm)} = \omega_0 \pm \rho\omega_0(1 - |G|)/2$, $\omega_3^{(\pm)} = \omega_0 \pm \rho\omega_0/2$, $\omega_4^{(\pm)} = \omega_0 \pm \rho\omega_0(1 + |G|)/2$.

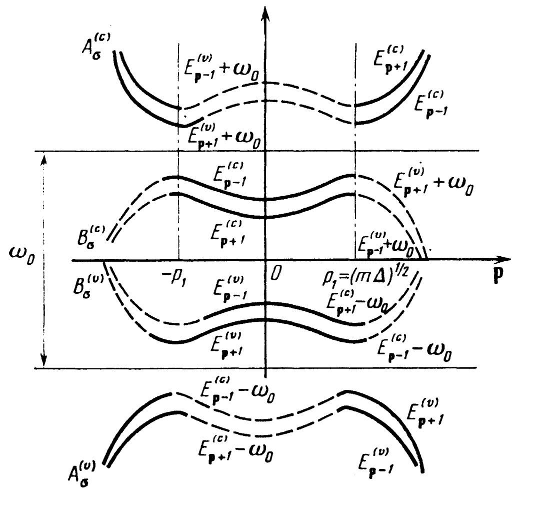

FIG. 2. Quasienergy spectrum of an electron in the field of a resonant electromagnetic wave at $0<G<1$. The solid lines show the valence band $E^{(v)}_{\mathbf{p}\sigma}=-E_{\mathbf{p}-\sigma}$ and the conduction band $E^{(c)}_{\mathbf{p}\sigma}=E_{\mathbf{p}\sigma}$; the dashed lines show the single-photon replicas of these bands.

spectrum of the spontaneous recombination radiation of a semiconductor can contain, generally speaking, nine resonance peaks whose positions and heights are determined by the type of polarization of the electromagnetic wave (Fig. 1). The radiation intensity at the frequencies $\omega_1<\omega_0-\rho\omega_0|G|/2$ increases with lowering of the absolute temperature, while at the frequencies $\omega_1\geqslant\omega_0-\rho\omega_0|G|/2$ it decreases also as $T\to 0$ and vanishes at zero. If the electromagnetic wave is linearly polarized, the foregoing results go over into the results of Ref. 20, which were confirmed experimentally in Ref. 21.

We examine now the quantum processes responsible for the presence of the foregoing singularities in the semiconductor emission spectrum. To this end we turn to the picture, obtained in Ref. 5, of the quasienergy spectrum of an electron in the external field (1) (Fig. 2). We recognize here that according to (44) the subbands $A^{(v)}_\sigma$ and $B^{(c)}_\sigma$ are completely occupied, while the subbands $A^{(c)}_\sigma$ and $A^{(v)}_\sigma$ are completely empty (see Ref. 3). The latter are populated when the temperature is raised. We assume for the sake of argument $0<G<1$.

Emission of photons of frequency $\omega_1=\omega_0+j\rho\omega_0(1+\sigma G)/2$ ($j=\pm1$, $\sigma=\pm1$) is due to two types of quantum transition of an electron: 1) at $j(|\mathbf{p}|-p_1)>0$ an electron crosses over from the conduction band $E^{(c)}_{\mathbf{p}\sigma}$ into the valence band $E^{(v)}_{\mathbf{p}-\sigma}$; 2) at $j(|\mathbf{p}|-p_1)<0$ an electron from the one-photon replica $E^{(v)}_{\mathbf{p}-\sigma}+\omega_0$ of the valence band $E^{(v)}_{\mathbf{p}-\sigma}$ crosses over to the one-photon replica $E^{(c)}_{\mathbf{p}\sigma}-\omega_0$ of the conduction band $E^{(c)}_{\mathbf{p}\sigma}$. The radiation in the propagation direction of the laser wave is circularly dextropolarized at frequencies $\omega_1=\omega_0+j\rho\omega_0(1-G)/2$ and is elliptically levopolarized at frequencies $\omega_1=\omega_0+j\rho\omega_0(1+G)/2$. This result is due to helicity conservation in this system. Emission of photons of frequency $\omega_1=\omega_0+j\rho\omega_0/2$ is also due to two types of quantum transition: 1) at $j(|\mathbf{p}|-p_1)>0$ an electron from the conduction band $E^{(c)}_{\mathbf{p}\sigma}$ goes to the valence band $E^{(v)}_{\mathbf{p}\sigma}$; 2) at $j(|\mathbf{p}|-p_1)<0$ an electron from the one-photon replica $E^{(v)}_{\mathbf{p}\sigma}+\omega_0$ of the valence band $E^{(v)}_{\mathbf{p}\sigma}$ goes to a one-photon replica $E^{(c)}_{\mathbf{p}\sigma}-\omega_0$ of the conduction band $E^{(c)}_{\mathbf{p}\sigma}$. The suppression of the emission at the frequencies $\omega_1=\omega_0+\rho\omega_0(1+\sigma G)/2$, $\omega_0+\rho\omega_0/2$ at $T\to 0$ is due to the fact that the subbands $A^{(c)}_\sigma$ from which the electron transitions stem are populated only at finite temperatures. The subbands $A^{(v)}_\sigma$ to which the electron goes over at finite temperatures are only partially occupied.

Two types of quantum transition are responsible for the emission at the frequencies $\omega_1=\omega_0+(1/2)\sigma\rho\omega_0 G\,\mathrm{sign}(|\mathbf{p}|-p_1)$ ($\sigma=\pm1$): 1) an electron from the conduction band $E^{(c)}_{\mathbf{p}\sigma}$ goes over to a one-photon replica $E^{(c)}_{\mathbf{p}-\sigma}-\omega_0$ of the conduction band $E^{(c)}_{\mathbf{p}-\sigma}$ 2) an electron from the one-photon replica $E^{(v)}_{\mathbf{p}\sigma}+\omega_0$ of the valence band $E^{(v)}_{\mathbf{p}\sigma}$ goes to the valence band $E^{(v)}_{\mathbf{p}-\sigma}$. Finally, the radiation of frequency $\omega_1=\omega_0$ is due to the transitions $E^{(c)}_{\mathbf{p}\sigma}\to E^{(c)}_{\mathbf{p}\sigma}-\omega_0$ and $E^{(v)}_{\mathbf{p}\sigma}+\omega_0\to E^{(v)}_{\mathbf{p}\sigma}$. These processes constitute Compton scattering of electrons by a resonant electromagnetic wave (see, e.g., Ref. 22).

4. CONCLUSION

The presence of resonance radiation peaks is due to the appearance of Van Hove singularities in the electron energy spectrum at $|\mathbf{p}|=p_1$ (Ref. 23). The divergence of the radiation intensity at the resonance frequencies can be eliminated by taking into account the dissipative processes, for example the transfer of the energy of the excited electrons to the phonon subsystem. Allowance for electron-phonon interaction causes the electronic states to acquire a finite half-width Γ, and the singular functions $\delta(x)$ in (46) to be replaced by $\Gamma/\pi(\Gamma^2+x^2)$. As a result, the height of the resonant-radiation peaks is found to be finite.[20] For standard values of the semiconductor parameters ($\varepsilon_g=1$ eV, $m=0.1m_e$, $\varepsilon=16$, $\Gamma=10^{11}\ \mathrm{s}^{-1}$) and at $|\mathbf{E}_0|\sim 10^5$ V/cm and $k_B T\sim\rho\omega_0$ the value of W at the resonance points ($\omega_1\neq\omega_0$) reaches values on the order of 10^2 erg/cm^3.

The estimate presented is valid if the following inequalities hold[19]

$$\frac{\varepsilon\omega_0}{2mc^2}\frac{\Delta}{\rho\omega_0}\frac{\omega_0}{\Gamma},\qquad \left(\frac{\varepsilon\Delta}{mc^2}\right)^{1/2}\ll 1.$$

In the opposite case we must allow in the calculation for the finite wavelengths of the radiation incident on the semiconductor and of the outgoing radiation.[24] This allowance leads to additional anisotropy of the radiated electromagnetic energy. The optimal, from the experimental standpoint, is observation of radiation propagating in the same direction as the exciting strong laser field.

[1)]We use in this article a system of units in which $\hbar=1$.

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