Hydrodynamic effects in the electric conductivity of two-dimensional metals

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Hydrodynamic effects of a new type, due to singularities of electron-phonon relaxation in two-dimensional (layered) metallic compounds, are predicted. It is shown that the experimental data on the thermoelectric power and on the electric conductivity of acceptor compounds of graphite can be explained within the framework of the hydrodynamic approach.

In the hydrodynamic mechanism of electric conduction, frequent normal collisions impart to the system of the metal quasiparticles a common drift whose velocity $\mathbf{u}(\mathbf{r})$ is determined by collisions in which quasimomentum is lost in the bulk or on the surface of the metal. The order of magnitude of the conductivity is[1]

$$\sigma \equiv (e^2 n/p_F) l_{tr}, \quad l_{tr}^{-1} \approx (d^2/l_N)^{-1} + l_V^{-1}. \tag{1}$$

Here l_N and l_V are respectively the electron mean free paths relative to normal collisions and collisions that do not conserve the quasimomentum, d is the sample thickness, $l_V \gg l_N$ and $d \gg l_N$. The temperature dependence of the resistance has a minimum, since $\partial\sigma/\partial T > 0$ in the region defined by the inequalities

$$l_N \ll d \ll (l_N l_V)^{1/2}, \tag{2}$$

and in which the dissipation is determined by the friction of the quasiparticle liquid against the wall.

Conditions (2) are difficult to meet in the case of $3D$ uncompensated metals, for owing to the large size of the Fermi surface (FS) the electron-phonon umklapp processes in these metals are relatively easily produced[1)] right down to very low temperatures.[2] On the other hand, the carrier density in many layered metallic compounds is low, and accordingly the size of the FS is small compared with that of the Brillouin zone; charge transport makes these metals uncompensated.[4,5] For example, the FS of the graphite acceptor compound $C_{16}FeCl_3$ contains two cylinders with radii $p_1 = 0.08\ \Delta, p_2 = 0.05\ \Delta$, where Δ is the side of the hexagonal boundary of the Brillouin zone.[6] U-processes are thus practically impossible, even if the thermal momentum of the phonon is $q_T \approx p_1$. Two-dimensional metals may therefore be favorable objects for the display of hydrodynamics effect. We shall show that one can obtain here qualitatively new hydrodynamic effects that have no analog in the $3D$ case. What makes $2D$ bodies unique is the substantial difficulty encountered by the relaxation of the momentum-odd distribution-function component, owing to the "blocking" of the Bloch diffusion on the FS and to manifestation of superdiffusion processes.[7]

We shall show first that the available experimental data on the thermoelectric power and on the electric conductivity of acceptor graphite compounds can be explained in the hydrodynamic approach. Assume that at low temperatures such that $q_T \ll \Delta$ the following conditions are met:

$$l_V \gg l_{ep}{}^N, \quad l_{pe} \ll l_{pd} \ll l_{pp}, \tag{3}$$

where the subscripts e and p refer to electrons and phonons, respectively, and l_{pd} is the mean free path with respect to collisions of phonons with structure defects; the nature of the latter is immaterial to us. Conditions (3) ensure a joint drift of electrons and of a group of phonons with momenta $q < 2p_1$, the only phonons that can collide with electrons. The entrainment thermoelectric power can be easily calculated by determining the heat transported by this group of phonons and using the known relation between the thermoelectric power and the Peltier coefficient. The contribution of one phonon mode to the thermoelectric power is

$$S = \left(\frac{T}{\omega_m}\right)^2 e^{-1} \int_0^{\omega_m/T} \frac{x^3\,dx}{4[1+(p_2/p_1)^2]\,\mathrm{sh}^2(x/2)}, \tag{4}$$

where $\omega_m = 2p_1 s$ is the maximum energy of the phonons interacting with the electrons, and s is the speed of sound. This expression agrees quite accurately with experiment[8] (according to Ref. 6, $\omega_m = 420$ K for the longitudinal acoustic mode). At $T > 150$ K (corresponding to $q_T > p_1$) the thermoelectric power [Eq. (4)] saturates at $S = 31\ \mu$V/K, as against the experimental $S = 37\ \mu$V/K (see Fig. 1). Saturation at the $S \approx e^{-1}$ level attests unambiguously to total entrainment of the phonons. Note that the thermoelectric power (4) does not depend on the mechanism that retards the joint drift of the electrons and phonons.

The typical temperature dependence of the resistivity

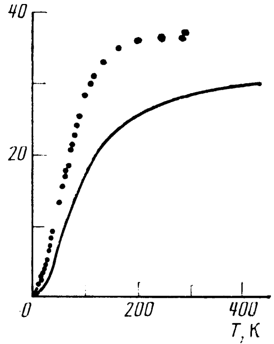

FIG. 1. Temperature dependence of the thermoelectric power of $C_{16}FeCl_3$. The experimental values were taken from Ref. 8, and the curve is calculated from Eq. (4).

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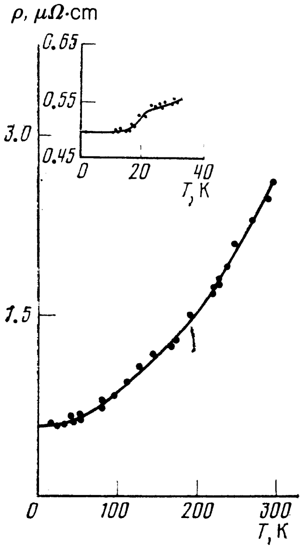

FIG. 2. Temperature dependence of the resistivity of the compound $C_{16}AsF_5$ (Ref. 9). The inset shows $\rho(T)$ for the low-temperature region in enlarged scale.

$\rho(T)$ of acceptor compounds of graphite (Fig. 2) can be explained in natural fashion by assuming the drift-deceleration mechanism to be collisions between the phonons and the structure defects. Umklapp processes would yield a more ragged plot. Favoring the phonon-defect mechanism is also the experimentally observed substantial dependence the temperature behavior of $\Delta\rho = \rho(T) - \rho(0)$ on the properties of the sample of $\Delta\rho \gg \rho(0)$ (Ref. 9). We have

$$\Delta\rho(T) \approx \frac{p_F}{ne^2}\frac{l_{pe}}{l_{ep}l_{pd}}. \tag{5}$$

At $q_T > p_F$ we have $l_{pe}/l_{ep} \propto T$.

Next, a "step" is observed on the $\rho(T)$ plot in the region of the residual resistivity; this also agrees with the predictions of the hydrodynamic theory.[1] (The step is due to disruption of the joint drift, and its location is determined from the condition $l_{ep} \approx l_i$, where l is the electron-impurity length.)

We consider now the electric conductivity of a $2D$ metal plate, let the conducting layers be perpendicular to the plate surface, and let the electric field $\mathbf{E}$ be applied along the layers. The drift distribution in the two-dimensional metal sets in within a time determined by the superdiffusion, by the normal electron-electron collisions, and by other quasimomentum-conserving processes. We have therefore at $d \gg l_N$ the usual hydrodynamic situation (1). We shall show that in the two-dimensional case with $d \ll l_N$ effects of hydrodynamic type are also possible, although the quasiparticles no longer have a drift-governed distribution. We use as an example the simplest possible case that admits of a sufficiently complete solution of the problem. Let the FS be singly connected and convex, and let the inequalities $l_N \gg l_i \gg d$, $q_T \ll p_F$ hold. It suffices then to retain the electron–phonon collision integral part $\hat{\nu}_s$ that is principal in terms of the problem parameters q_T/p_F and $s/v_F \ll 1$, which describes the electron diffusion on the FS with allowance for phonon exchange between states having opposite momenta.[7] The kinetic equation for the nonequilibrium increment to the electron distribution function $\chi(\mathbf{p},z)$ takes then the form

$$\hat{\nu}_s\chi + \frac{\chi}{l_i} + \vartheta\frac{\partial\chi}{\partial z} = -e\mathbf{E}\mathbf{n},$$
$$\hat{\nu}_s\chi = -{}^1/_2 \operatorname{div}\left[D(\nabla\chi|_{\mathbf{p}} - \nabla\chi|_{-\mathbf{p}}\right], \tag{6}$$

where $\vartheta \ll 1$ is the angle between the electron velocity and the plate surface, $\mathbf{n} = \mathbf{v}/v$, and D is the coefficient of diffusion over the FS. For the χ component that is even in $\mathbf{p}$ we obtain at $l_B = p_F^2/D \ll l_i$ ($\tau_B = l_B/v_F$ is the time of Bloch diffusion through the entire FS) the following equation:

$$-\vartheta\hat{\nu}_s^{-1}\vartheta\frac{\partial^2\chi_a}{\partial z^2} + \frac{\chi_a}{l_i} = -e\mathbf{E}\mathbf{n}. \tag{7}$$

The first term in the left-hand part of (7) describes a certain diffusion (viscosity), but the "diffusion coefficient" is an operator that acts on functions of $\mathbf{p}$. We present for this process a physical interpretation which we shall find useful and allows us to write the solution under conditions more general than those under which Eq. (7) is valid.

A nonequilibrium phonon-emitting electron traveling at a small angle ϑ to the surface goes off after a time $\tau(\vartheta)$ into a region of angles much larger than ϑ, after which it collides rapidly with the surface, on which it relaxes. The emitted phonons, however, being absorbed by an electron having an opposite momentum (there is no other possibility in the $2D$ case), also take the electron out of the angle region $\approx\vartheta$. The result of these processes—the vanishing of this electron in the small-angle region and the simultaneous appearance of a hole having an opposite momentum—is similar to the Andreev reflection. The hole moves in the opposite direction but, having a positive charge, it draws, just as the electron, energy from the field. Succeeding collisions transform the hole again into electron of the same velocity as the initial one. The effective mean free path of the electron is thus substantially increased by the collisions with the phonons. It can be derived as the total path covered by a particle that executes one-dimensional Brownian motion with step length $v_F\tau(\vartheta)\vartheta$ (projected along the z axis) and with a step time $\tau(\vartheta)$ prior to relaxation on the surface or in the bulk:

$$\bar{\chi} = -e\mathbf{E}\mathbf{n}l_{ef}(\vartheta), \qquad |l_{ef}^{-1}| \approx \left[\frac{d^2}{6\vartheta^2 l(\vartheta)} + \frac{d}{2|\vartheta|}\right]^{-1} + (l^*)^{-1}, \tag{8}$$
$$l(\vartheta) = v_F\tau(\vartheta), \qquad \tau(\vartheta) = \tau_B\vartheta^2 + \tau_1,$$
$$(l^*)^{-1} = l_i^{-1} + l^{-1}(\vartheta_c)\alpha + l_N^{-1}(\vartheta_c) + \tilde{l}^{-1},$$
$$\alpha = l_{pe}/l_{pd}.$$

The result (8) for the function $\overline{\chi}$ averaged over the plate thickness is qualitatively correct if $d \ll l_N$ (in the opposite limiting case we have the usual hydrodynamics), and is written in such a way that it corresponds to the asymptotically exact solution of Eq. (7) (for the limiting cases $\vartheta^4 \gg$ and $\vartheta^4 \ll d^2/l_B l_i$) in the region of its validity, provided the electrons are diffusely reflected from the surface:

$$\chi(z = \pm d/2,\ \vartheta \lessgtr 0) = 0.$$

The second term in the square brackets in (8) is of the form usual for the electric conductivity of a thin plate, and corresponds to collisionless motion of an electron towards the surface. The time $\tau(\vartheta)$ of departure from the region $\approx\vartheta$ is determined by the diffusion over the FS and is therefore proportional to ϑ^2 right up to angles $\vartheta \approx (\tau_1/\tau_B)^{1/2} \approx q_T/p_F$, while at smaller angles the departure takes places within the time τ_1 of one elementary electron-phonon collision. Complete bulk relaxation takes place over a path length l^*; contributing to $(l^*)^{-1}$ are all the processes that intermix the isolated electron–phonon subgroups.[7,8] (The term $\tilde{l}^{-1}$ takes into account momentum-nonconserving collisions other than *e-i* and *p-d*, for example electron-electron collisions with umklapp.) The characteristic angular dimension ϑ_c over which the function $\bar{\chi}$ changes (this angle influences substantially the superdiffusion time[10] τ_{sd} and consequently l_N) corresponds obviously to the region of the most effective "gliding" electrons and is given by relation $\vartheta_c^2 l(\vartheta_c) l^*(\vartheta_c) \approx d^2$ or else by the usual $3D$ relation $\vartheta_c l^*(\vartheta_c) \approx d$ (one must choose the expression that yields the larger value of ϑ_c). These relations and the expressions for $l^*(\vartheta_c)$ (for the *q/p* supperdiffusion mechanism, for example, $l_{sd} = v_F\tau_{sd} \approx l_1(\vartheta_c p_F/q_T)^6$ (Ref. 10) and $l_1 = v_F\tau_1$) constitute a system of equations for l^* and ϑ_c.

It can be seen from (8) that hydrodynamic-type effects that result from the "Andreev reflection" are possible if $d(l_i^{-1} + l_B^{-1}) \ll 1$, i.e., for thin samples for which the Fuchs formula $l_{tr} \approx d \ln l/d$ is valid in the $3D$ case. Hydrodynamics holds in this case for a small group of electrons near $\vartheta = 0$, while l_{ef} increases rapidly with decreasing ϑ. Thus, the region in which hydrodynamic effects exist is much larger in the $2D$ than in the $3D$ case [see (2)]. From (8) we obtain the following expression for the transport length $(l_N \gg d)$:

$$l_{tr} \approx d\left\{\frac{l^*}{(l'l^*)^{1/2}+d} + \frac{1}{2}\ln\left[\left(\frac{l^*}{d}\right)^2 + 1\right]\right\}, \tag{9}$$

$$(l')^{-1} = (l^*)^{-1} + [l_1 + d(l_B/l^*)^{1/2}]^{-1}.$$

We point out, on the basis of (1) and (9), the characteristic manifestations of the effects considered, and the conditions for their observation. A specific manifestation of hydrodynamics is that the resistance may not have the temperature minimum that occurs in general at $d \ll l_N$. It is easy to verify, however, that many of the processes that determine the length l^* lead to a minimum,2) viz., scattering of electrons by impurities (in the region where $\partial\sigma/\partial T > 0, \sigma \propto T^{5/4}$ or $\sigma \propto T^{3/2}$), scattering of phonons by defects (if l_{pd} does not decrease rapidly with increasing temperature), or *q/p* superdiffusion ($\sigma \propto T^{4/5}$). However, for example, the superdiffusion mechanism connected with the phonon-phonon collisions[7] does not lead to a minimum. If there is no minimum, a manifestation of the hydrodynamic situation may be satisfaction of the inequality $l_{tr} \gg d$ (although it can also attest to high specularity of the surface scattering).

The necessary conditions for the existence of hydrodynamic effects, at least at some thicknesses d, are

$$l_1(l_i^{-1} + \tilde{l}^{-1}) \ll 1, \quad \alpha \ll 1.$$

It follows from (3) and (5) that for acceptor compounds of graphite, having a ratio $\rho(300)/\rho(0) \gg 1$, these conditions are met at least at $q_T \gtrsim p_F$. For these substances, the resistance minimum connected with ordinary hydrodynamics is best observed in a temperature region where $q_T \gtrsim p_F$ (but at temperatures not so high that the phonon-phonon U processes eliminate the phonon drag). Conditions (2) acquire in this case the form ($l_V = l_{ep}/\alpha$ is determined by *p-d* scattering)

$$1 \ll d/l_1 \ll \alpha^{-1/2}. \tag{10}$$

Observation of the minimum due to the "Andreev reflection" is possible at $q_T < p_F$ when $l_N \approx l_{sd} \gg l_1$; Eq. (9) leads to the following conditions:

$$1 \ll (d/l_1)p_F/q_T \ll \min\left[(l_i/l_1)^{3/2}, \quad (p_F/q_T)^2 (l_i/l_1)^{1/2}, \right.$$
$$\left.(p_F/q_T)^3 \alpha^{-1/2}, \ (p_F/q_T)^5\right], \tag{11}$$

which are apparently less stringent than (10).

Note that in metals with a singly connected convex FS there is no *q/p* superdiffusion,[7] therefore some of the restrictions on the observation of new hydrodynamic effects are lifted. There is no need for the first inequality of (11), nor for the temperature constraint $q_T < p_F$.

1) Nonetheless, a resistance minimum that is probably of hydrodynamic origin was recently observed in potassium.[3]

2) With respect to the resistance minimum, the following refinement is in order. An inverse temperature dependence of the conductivity ($\partial\sigma/\partial T > 0$) is possible in two different situations. In the first, the hydrodynamic effects determine the value of the conductivity $l_{tr} \approx d(l^*/l')^{1/2}$, $l' \ll l^*$ at $l_N \gg d$ and $l_{tr} \approx d^2/l_N$ at $l_N \ll d$), and the conductivity is substantially dependent on the temperature. In the second, the conductivity is independent of temperature in the principal approximation ($l_{tr} \approx l_i$ or $l_{tr} \approx d \ln(l_i/d)$), and the weak inverse temperature dependence is due to relatively infrequent normal collisions. We discuss below only the first situation (to which, in essence, the minimum condition (2) also pertains). We note, however, that in the $2D$ case the weak inverse temperature dependence is manifested under more relaxed conditions than in the $3D$ case—this is seen already from the fact that the Fuchs logarithm in (9) does not contain the electron-phonon length $l(\vartheta_c)$.

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