Wind instability and helical perturbations of magnetized relativistic jets

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Wind instability, defined as the vortex analog of the Kelvin-Helmholtz instability in inhomogeneous flow with a velocity profile, is discussed. It is shown that, when a magnetic field and supersonic streaming are present (under both relativistic and nonrelativistic conditions), wind instability may give rise to the development of helical perturbations with characteristic wavelength and azimuthal number on the surface of the jet. The corresponding growth-rate maximum is determined by a resonance between the surface wave and vortices located at a distance from the separation boundary that is a definite fraction of the perturbation wavelength. The results obtained are used to interpret the wave picture observed in cosmic outflows from quasars and active galactic cores.

1. INTRODUCTION

The instability of a tangential discontinuity stabilized by a magnetic field, which simulates a common form of shear flow with a large local velocity gradient, has been used to explain a wide range of phenomena, including those occurring in cosmic plasmas. In particular, in this paper, we shall consider the wave structure of one of the most interesting objects, namely, cosmic outflows (jets) joining the cores of active galaxies to radiating radio clouds. This structure is usually associated with the Kelvin-Helmholtz instability that develops on the surface of a magnetized jet (see the review given in Ref. 1 and the references cited therein). However, this explanation encounters a number of difficulties because the Kelvin-Helmholtz instability growth rate does not have a maximum in the long-wave region.[1)] Nonlinear mechanisms are then brought in to escape from this difficulty.[1]

We shall show that the Miles-Phillips[3,4] vortex shear instability, currently used to explain wind-driven waves on the sea surface, is more suitable as a way of explaining the observed wave picture than the Kelvin-Helmholtz instability. We shall use the phrase, "wind instability," to emphasize that we shall be dealing with the instability of a free separation boundary. For a magnetized nonrelativistic plasma, this instability was examined in Ref. 5 in the case of an incompressible medium.

The motion in jets is most likely to be supersonic, and this has a significant effect on the developing wave picture. L. D. Landau has shown[6] that, under symmetric conditions with $u > u_{\rm crit} = 2^{1/2}c_s$, the tangential discontinuity becomes stabilized for waves propagating along the discontinuity ($2u$ is the discontinuity in velocity and c_s the velocity of sound). This result has recently found a striking confirmation in the case of a two-dimensional system (shallow water experiments[7]), for which there are no perturbations propagating at an angle to the flow, and complete stabilization is attained. According to Syrovatskiĭ,[8] in the three-dimensional case, there is a growing wave instability for waves propagating at an angle θ to the velocity discontinuity vector.

Even for discontinuities below the critical Landau value, but greater than $3^{1/2}c_s$, the maximum γ occurs in the region of nonzero angles $\theta_{\max} > 0$ (although, for $u < u_{\rm crit}$, we have $\gamma(\theta = 0) \neq 0$]. When looked upon as a function of velocity, $\gamma(\theta = 0)$ passes through a maximum for $2u = 3^{1/2}c_s$ (see Appendix). We shall show below that qualitatively similar behavior occurs in the case of wind instability, but there may be significant detailed differences. For cosmic jets, the analysis is usually relativistic, although the observed velocities have so far been subrelativistic (with the exception of jets ~ 1 pc long, for which ultrarelativistic motion has been definitely observed).

The high degree of polarization of the emitted radiation can be regarded as evidence for the presence of a regular magnetic field in jets and, for many situations that are of interest to us (for example, one-sided jets), this field is longitudinal (oriented along the jet).[1]

We shall examine the wind instability of a relativistic jet that is partially stabilized by a magnetic field. The analysis will be confined to planar geometry, but the parameters that we shall obtain can be relevant for a cylindric jet. This has been confirmed in a particular case for which an analytic solution has been found for the latter.

In Section 2 we give a formulation of the problem and obtain a dispersion relation for the wind instability of a relativistic flow in the case of a plane separation boundary. In Section 3 we analyze the growth rate and its angular dependence, and show that the supersonic jet must take the form of a helix. The sense of the helix is random, and the angle of twist is $\pi/4$. In Section 4 we consider a limiting case in which cylindric geometry can be analyzed. The results are used in Section 5 to examine observed wave perturbations in cosmic outflows. The angular dependence of the Kelvin-Helmholtz instability growth rate (tangential discontinuity) is examined in the Appendix, both in the relativstic and nonrelativistic limits.

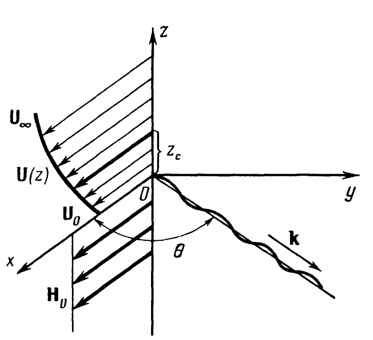

FIG. 1. Mutual disposition of the flow velocity vector, the magnetic field, and the direction of wave propagation for a plane separation boundary between the media.

2. DERIVATION OF DISPERSION RELATION FOR SURFACE WAVES

Consider two media separated by the $z = 0$ plane (Fig. 1). The region $z > 0$ is occupied by plasma of density ρ_0 and velocity $U(z)$ along the X axis, which can be described by the following equations of relativistic hydrodynamics:

$$\partial T_{ik}/\partial x_k=0, \tag{2.1}$$

where T_{ik} is the energy-momentum tensor

$$T_{ik}=(P+\mathscr{E})u_iu_k+P\delta_{ik}. \tag{2.2}$$

P is the pressure, $\mathscr{E}$ is the internal energy density, and u_i the 4-velocity. For $z<0$, the plasma of density ρ_0 lies in a uniform longitudinal magnetic field $\mathbf{H}_0$. The velocity profile for $z<0$ is assumed to be $U(z)\equiv$const, and we transform to a frame of reference in which $U(z)\equiv0$. The plasma is then described by the magnetohydrodynamic equations for a perfect fluid (including the displacement current):

$$\rho\left\{\frac{\partial \mathbf{v}}{\partial t}+(\mathbf{v}\cdot\nabla)\mathbf{v}\right\}=-\nabla P+\frac{1}{4\pi}[(\operatorname{curl}\mathbf{H})\times\mathbf{H}]-\frac{1}{4\pi c}\left[\frac{\partial \mathbf{E}}{\partial t}\times\mathbf{H}\right],$$

$$\frac{\partial\rho}{\partial t}+\operatorname{div}(\rho\mathbf{v})=0,\qquad \frac{\partial \mathbf{H}}{\partial t}=\operatorname{curl}(\mathbf{v}\times\mathbf{H}), \tag{2.3}$$

$$\operatorname{curl}\mathbf{E}=-\frac{1}{c}\frac{\partial \mathbf{H}}{\partial t},\qquad \operatorname{div}\mathbf{E}=0,\qquad \operatorname{div}\mathbf{H}=0.$$

We examine the instability of the separation boundary between the media against small-amplitude sinusoidal perturbations propagating at all possible angles to the flow velocity $U(z)$. The boundary conditions are the continuity of pressure and of displacement ($\zeta=\bar{\zeta}$) across the separation boundary.

We linearize (2.1) and (2.3), and take the perturbation in the form

$$v\sim v(z)\exp[i(k_xx+k_yy-\omega t)]$$

and so on, where $k_x = k\cos\theta$, $k_y = k\sin\theta$, and θ is the angle between the direction of propagation of the wave and $U(z)$. From (2.3), we have

$$\bar\rho_0\frac{\partial \mathbf{v}}{\partial t}=-\nabla\bar{p}+\frac{1}{4\pi}[(\operatorname{curl}\mathbf{h})\times\mathbf{H}_0]-\frac{1}{4\pi c}\left[\frac{\partial \mathbf{e}}{\partial t}\times\mathbf{H}_0\right],$$

$$\frac{\partial\bar\rho}{\partial t}+\rho_0\operatorname{div}\mathbf{v}=0,\qquad \frac{\partial \mathbf{h}}{\partial t}=\operatorname{curl}(\mathbf{v}\times\mathbf{H}_0), \tag{2.4}$$

$$\operatorname{curl}\mathbf{e}=-\frac{1}{c}\frac{\partial \mathbf{h}}{\partial t},\qquad \operatorname{div}\mathbf{e}=0,\qquad \operatorname{div}\mathbf{h}=0.$$

The zero subscript in (2.4) labels the unperturbed variables and lower-case letters represent deviations from unperturbed values ($\mathbf{H}=\mathbf{H}_0+\mathbf{h}$, $\mathbf{E}=\mathbf{e}$, and so on).

From (2.4), we obtain the equation for the amplitude of the change in the magnetic pressure $\bar{p}_H = H_0h_x/4\pi$:

$$\bar{p}_H''-\bar{p}_H\left(k^2-\frac{\omega^2}{\bar{v}_A^2}-\frac{\omega^2}{c^2}\right)=0, \tag{2.5}$$

where $\bar{v}_A\equiv H_0(4\pi\bar\rho_0)^{-1/2}$ is the Alfven velocity and primes represent differentiation with respect to z. It is assumed in the derivation of (2.5), and this assumption will be maintained throughout, that the velocity of sound is $\bar{c}_s=0$ for $z<0$ because, in the case in which we are interested, $\bar{c}_s\ll\bar{v}_A$, c. The relationship between e_y and v_z and the equation for e_y can be obtained from (2.4):

$$e_y''-e_y\left(k^2-\frac{\omega^2}{c^2}\right)=\frac{\omega^2}{c}\frac{4\pi\bar\rho_0}{H_0}v_z, \tag{2.6}$$

$$e_y^{\mathrm{IV}}-e_y''\left(2k^2-2\frac{\omega^2}{c^2}-\frac{\omega^2}{\bar{v}_A^2}-\frac{\omega^4}{c^2(\bar{v}_A^2k_x^2-\omega^2)}\right)+e_y\left(k^2-\frac{\omega^2}{\bar{v}_A^2}-\frac{\omega^2}{c^2}\right)\left(k^2-\frac{\omega^2}{c^2}-\frac{\omega^4}{c^2(\bar{v}_A^2k_x^2-\omega^2)}\right)=0. \tag{2.7}$$

If we confine our attention to waves that decay as they propagate into the medium, we obtain the following two solutions from (2.7) for $z<0$:

$$e_{y1}\sim\exp\left[\left(k^2-\frac{\omega^2}{c^2}-\frac{\omega^2}{\bar{v}_A^2}\right)^{1/2}z\right], \tag{2.8}$$

$$e_{y2}\sim\exp\left\{\left[k^2-\frac{\omega^2}{c^2}-\frac{\omega^4}{c^2(\bar{v}_A^2k_x^2-\omega^2)}\right]^{1/2}z\right\}. \tag{2.9}$$

Substituting (2.8) and (2.9) in (2.6), we obtain the relationship between e_y and v_z:

$$e_{y1}=-\frac{\bar{v}_A^2}{c}\frac{4\pi\bar\rho_0}{H_0}v_{z1}, \tag{2.10}$$

$$e_{y2}=\frac{c(\omega^2-\bar{v}_A^2k_x^2)}{\omega^2}\frac{4\pi\bar\rho_0}{H_0}v_{z2}. \tag{2.11}$$

The following relation can be deduced from (2.4):

$$\bar{p}_H'=\frac{i}{\omega}\bar\rho_0(\omega^2-k_x^2\bar{v}_A^2)v_z-\frac{i\omega H_0}{4\pi c}e_y. \tag{2.12}$$

Using (2.10) and (2.11), we obtain

$$\bar{p}_{H1}'=\frac{i}{\omega}\bar\rho_0v_{z1}[\omega^2(1+\bar{v}_A^2/c^2)-k_x^2\bar{v}_A^2], \tag{2.13}$$

$$\bar{p}_{H2}'=0. \tag{2.14}$$

The latter relation shows that oscillations in the electric field e_{y2} are unrelated to oscillations in the magnetic pressure $\bar{p}_H$, and therefore do not deform the separation boundary between the media.

Substituting the solution of (2.5) in (2.13) and using the fact that $v_z = -i\omega\zeta$ on the undisturbed separation boundary, we obtain

$$\bar{p}_H=\bar{\rho}_0\left[\omega^2\left(1+\frac{\bar{v}_A}{c^2}\right)^2-k_x^2\bar{v}_A^2\right]\left(k^2-\frac{\omega^2}{c^2}-\frac{\omega^2}{\bar{v}_A^2}\right)^{-1/2}\zeta. \quad (2.15)$$

For $z>0$, the linearized equations (2.1) take the form

$$\begin{aligned}
&iW_0\Gamma^2(z)(k_xU(z)-\omega)v_x+W_0\Gamma^2(z)U'(z)v_z\\
&\qquad+i\varepsilon c_s^2\left(k_x-\frac{\omega U(z)}{c^2}\right)=0,\\
&W_0\Gamma^2(z)(k_xU(z)-\omega)v_y+k_yc_s^2\varepsilon=0,\\
&iW_0\Gamma^2(z)(k_xU(z)-\omega)v_z+c_s^2\varepsilon'=0,\\
&\varepsilon[k_xU(z)-\omega]-\varepsilon\frac{c_s^2}{c^2}U(z)\left[k_x-\frac{\omega U(z)}{c^2}\right]\\
&\qquad+W_0(k_xv_x+k_yv_y-iv_z')=0,
\end{aligned} \quad (2.16)$$

where $W_0=P_0+\mathscr{E}_0$ is the enthalpy density in the flow and $\Gamma(z)\equiv[1-U^2(z)/c^2]^{-1/2}$ is the Lorentz factor.

From (2.16), we obtain the connection between the pressure fluctuations and the displacement ζ of the boundary in the flow, using the fact that $v_z=i(k_xU_0-\omega)\zeta$:

$$\frac{c_s^2}{c^2}\varepsilon=\frac{W_0}{c^2}(k_xU_0-\omega)\left[\frac{v_z'}{v_z}\bigg|_{z=0}(k_xU_0-\omega)-k_xU_0'\right]\frac{\Gamma_0^2}{\alpha_0^2}\zeta, \quad (2.17)$$

where $U_0\equiv U(0)$, $\Gamma_0\equiv\Gamma(0)$, $\alpha_0\equiv\alpha(0)$, and $\alpha(z)$ is the relativistic analog of the transverse wave number for $z>0$:

$$\alpha^2(z)\equiv\left[k_x-\frac{\omega U(z)}{c^2}\right]^2\Gamma^2(z)+k_y^2-\frac{[k_xU(z)-\omega]^2}{c_s^2}\Gamma^2(z). \quad (2.18)$$

The relativistic analog of the Rayleigh equation is also found to follow from (2.16):

$$v_z''+v_z'\Phi(z)-v_z\left[\frac{U''(z)+U'(z)\Phi(z)}{U(z)-V-i\delta}+\alpha^2(z)\right]=0, \quad (2.19)$$

where $V\equiv\omega/k_x$,

$$\Phi(z)\equiv2\left\{\frac{k_x^2[U(z)-V]U'(z)}{c_s^2}+\frac{k_y^2U(z)U'(z)}{c^2}+k_x^2\left[1-\frac{VU(z)}{c^2}\right]\frac{VU'(z)}{c^2}\right\}\frac{\Gamma^2(z)}{\alpha^2(z)},$$

and $\delta>0$ is a small term governing the Landau-Lin rule for bypassing the singularity at $\mathrm{Im}\,\omega=0$:

$$U(z_c)=V. \quad (2.20)$$

The singularity appears because of resonance between vortices in the flow and the surface waves.[2)] Actually, it is precisely the vortex perturbations that are transported with the flow velocity in the case of an unmagnetized isentropic flow. It is also readily seen that the resonance corresponds to synchronism between surface waves and vortices, if we follow the current lines, which is most simply done in the reference frame in which the surface wave is at rest. (For a more detailed discussion of the special case of wind waves, see Ref. 4 and the review article in Ref. 10.)

Equating pressure fluctuations on the perturbed separation boundary, we obtain the "dispersion relation"

$$\frac{W_0}{c^2}\frac{\Gamma_0^2}{\alpha_0^2}(U_0-V)\left[\frac{v_z'}{v_z}\bigg|_{z=0}(U_0-V)-U_0'\right]=\bar{\rho}_0\left[V^2\left(1+\frac{\bar{v}_A^2}{c^2}\right)-\bar{v}_A^2\right]\beta^{-1}, \quad (2.21)$$

where

$$\beta\equiv\left(k^2-\frac{\omega^2}{c^2}-\frac{\omega^2}{\bar{v}_A^2}\right)^{1/2}. \quad (2.22)$$

The quantity $(v_z'/v_z)_{z=0}$ (in 2.21) must be determined from (2.19).

When $U(z)=\mathrm{const}$, equation (2.21) becomes identical with the dispersion relation for the Kelvin-Helmholtz instability of a relativistic tangential discontinuity, partially stabilized by a magnetic field. The pole responsible for resonance is then absent from the Rayleigh equation.

3. WIND-INSTABILITY GROWTH RATE OF SURFACE WAVES

When $\bar{\rho}_0c^2\gg W_0$, the dispersion relation (2.21) leads to the following expressions for the growth rate $\gamma=\mathrm{Im}\,\omega$ and phase velocity $v_{\mathrm{ph}}=\mathrm{Re}\,\omega/k$ of surface waves:

$$\gamma\approx\frac{1}{2}\frac{W_0}{\bar{\rho}_0c^2}\frac{\beta}{\alpha_0^2}k_x\Gamma_0^2\frac{(U_0-v_A^*)^2v_A^*}{\bar{v}_A^2}\mathrm{Im}\frac{v_z'}{v_z}\bigg|_{z=0}, \quad (3.1)$$

$$v_\Phi\approx v_A^*\cos\theta, \quad (3.2)$$

where $v_A^*\equiv\bar{v}_A(1+\bar{v}_A^2/c^2)^{-1/2}$.

The quantity $\mathrm{Im}(v_z'/v_z)|_{z=0}$ in (3.1) can be found from (2.19). To do this, let us consider the equation conjugate to (2.19)[3)]:

$$Z''-[Z\Phi(z)]'-Z\left[\frac{U''(z)+U'(z)\Phi(z)}{U(z)-V+i\delta}+\alpha^2(z)\right]=0. \quad (3.3)$$

Multiplying (2.19) by Z and (3.3) by v_z, subtracting (3.3) from (2.19), integrating the resulting difference with respect to z between 0 and $+\infty$, and taking the pole into account, we obtain

$$\mathrm{Im}\frac{v_z'}{v_z}\bigg|_{z=0}=-\pi\left[\frac{U''(z_c)}{|U'(z_c)|}+\Phi(z_c)\,\mathrm{sign}\,U'(z_c)\right]\times\mathrm{Re}\frac{v_z(z_c)Z(z_c)}{v_z(0)Z(0)}. \quad (3.4)$$

If we now apply the WKB method to (2.19) and (3.3) under the usual conditions for its validity, i.e., $\alpha'^2(z)\ll\alpha^4(z)$, $|\alpha''(z)|\ll\alpha^3(z)$ (Ref. 11), we find that

$$v_z(z)\approx Z(z)\sim\alpha^{-1/2}(z)\exp\left\{-\int_0^z\alpha(z)\,dz\right\}. \quad (3.5)$$

Substituting (3.5) and (3.4) in (3.1), we obtain the following expressions for the wind-wave growth rate:

$$\begin{aligned}\gamma(k,\theta)\approx&-\frac{\pi}{4}\frac{W_0}{\rho_0c^2}\frac{|\sin2\theta|}{\alpha_0\alpha(z_c)}k^2\Gamma_0^2\frac{(U_0-v_A^*)^2v_A^*}{\bar{v}_A^2}\\
&\times\left[\frac{U''(z_c)}{|U'(z_c)|}+2\frac{v_A^*|U'(z_c)|}{c^2-v_A^{*2}}\right]\exp\left\{-2k\int_0^{z_c}\left\{\cos^2\theta\Gamma^2(z)\right.\right.\\
&\left.\left.\times\left[\left(1-\frac{v_A^*U(z)}{c^2}\right)^2-\frac{(U(z)-v_A^*)^2}{c_s^2}\right]+\sin^2\theta\right\}^{1/2}dz\right\}, \quad (3.6)\end{aligned}$$

which is valid for large enough $k(kz_c \gg 1)$. In the nonrelativistic limit $c \to \infty$, the expressions given by (3.6) and (3.2) become[12]:

$$\gamma(k,\theta) \approx -\frac{\pi}{4}\frac{\rho_0}{\bar\rho_0}\frac{(U_0-\bar v_A)^2 U''(z_c)}{\bar v_A |U'(z_c)|} \times \frac{|\sin 2\theta|}{[1-\cos^2\theta(U_0-\bar v_A)^2/c_s^2]^{1/2}} \times \exp\left\{-2k\int_0^{z_c}\left[1-\cos^2\theta\frac{(U(z)-\bar v_A)^2}{c_s^2}\right]^{1/2} dz\right\}, \quad (3.7)$$

$$v_\Phi \approx \bar v_A \cos\theta. \quad (3.8)$$

We note that, in contrast to the usual shear flows, there is no need for a change in the volocity profile in the case of the wind instability because of the presence of the free boundary. The instability condition (positive growth rate) reduces to the inequality $U''(z_c) < 0$.

When the compressiblity of the plasma can be neglected which is determined by the condition $|U_0 - \bar v_A| = |U_0 - U(z_c)| \ll c_s$, we find from (3.7) that $\gamma \sim |\sin 2\theta|$. [The factor $\sin\theta$ originates from the transverse projection of the wave number for $z<0$ (2.22), whereas $\cos\theta$ comes from the dispersion law for surface Alfven waves (3.8).] Hence it follows that the most rapidly growing are the perturbations propagating at $\theta_{max} \simeq \pi/4$ to the flow velocity (Fig. 2a). We note that the flow is ultrasonic relative to the medium for $z<0$: $\bar v_A = U(z_c) \gg \bar c_s$, which is the condition for the validity of (3.7).

Since, for relativistic flows, the expression for the growth rate (3.6) contains the factor $|\sin 2\theta|$, waves with wave vector **k** pointing at an angle θ_{max} to $\mathbf{U}(z)$ are also the most rapidly growing. The angle θ_{max} depends on the relationship between the characteristic parameters of the media.

The expression given by (3.6) is rather complicated in the general case, but it can be substantially simplified in the case of the ultrahard equation of state in the flow ($c_s = c$), for which it takes the form

$$\gamma(k,\theta) \approx -\frac{\pi}{4}\frac{W_0}{\bar\rho_0 c^2}\Gamma_0^2\frac{(U_0-v_A^*)^2 v_A^*}{\bar v_A^2}\frac{|\sin 2\theta|}{1-(v_A^{*2}/c^2)\cos^2\theta} \times\left[\frac{U''(z_c)}{|U'(z_c)|}+2\frac{v_A^*|U'(z_c)|}{c^2-v_A^{*2}}\right] \times \exp\left\{-2kz_c\left(1-\frac{v_A^{*2}}{c^2}\cos^2\theta\right)^{1/2}\right\} \quad (3.9)$$

(this formula is reproduced for purely illustrative purposes). When $kz_c \gg 1$, (3.9) shows that γ decreases exponentially with increasing k. For small enough k, the wind-instability growth rate increases exponentially with increasing[4)] k (Fig. 2b) (Ref. 5). The maximum value of γ is reached for[5)]

$$k = k_{max} \approx \frac{1}{2z_c}\left(1-\frac{\bar v_A^2}{\bar v_A^2+c^2}\cos^2\theta\right)^{-1/2}. \quad (3.10)$$

Hence, in the region $k = k_{max}$, the maximum of γ with respect to θ is attained when

$$\theta = \theta_{max} \approx \text{arc tg}\,[c/(\bar v_A^2+c^2)^{1/2}]. \quad (3.11)$$

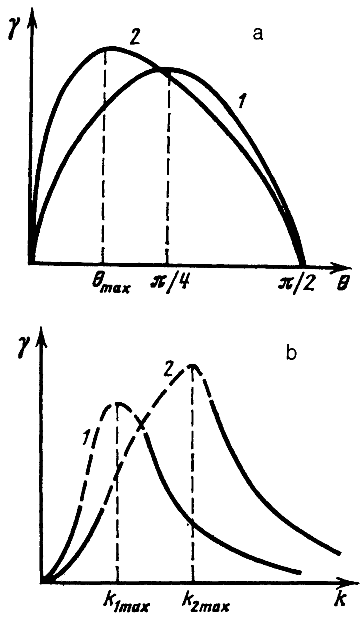

FIG. 2. Wind-instability growth rate for supersonic flows: a—as a function of the angle θ between the wave vector k and the flow velocity $U(z)$ for $k = \text{const}$; b—as a function of the wave number k for $\theta = \text{const}$. Curves 1 and 2 refer to the nonrelativistic and relativistic flow conditions, respectively. For large k, the asymptotic expressions are: $\gamma \sim \exp(-2kz_c)$ (1) and $\gamma \sim \exp\{-2kz_c[1-(v^{*2}/c^2)\cos^2\theta]^{1/2}\}$ (2).

It is clear from this result that the relativistic nature of the motion leads to the preferential growth of waves propagating at an angle $0 < \theta_{max} < \pi/4$ to the flow velocity $\mathbf{U}(z)$ (see Fig. 2a).

4. WIND INSTABILITY OF A CYLINDRIC JET

Waves propagating at an angle to the flow velocity $\mathbf{U}(r)$ on a plane separation boundary, i.e., $\sim\exp[i(k_x x + k_y y - \omega t)]$, correspond to helical perturbations of the cylindric jet, i.e., $\sim\exp[i(kx + m\varphi - \omega t)]$, where $m = 0, 1, 2, \ldots$ is the azimuthal number. Analysis of these helical waves for an arbitrary relationship among the chracteristic parameters of the media is mathematically too difficult. However, the instability growth rate can readily be obtained in the limiting cases of incompressible plasma in the flow ($c_s \to \infty$) and zero sound velocity in the jet ($\bar c_s = 0$).

Consider a jet of radius R in a plasma flow. The velocity profile is $U = U(r) \ll c$ for $r > R$, and $U \equiv 0$ for $r < R$ (Fig. 3). The magnetic pressure in the jet can be deduced from (2.4) and is found to be

$$\bar p_H'' + \frac{1}{r}\bar p_H' - \bar p_H\left[\frac{\bar v_A^2 - V^2}{\bar v_A^2}k^2 + \frac{m^2}{r^2}\right] = 0. \quad (4.1)$$

In the present section, primes represent derivatives with respect to r and $V \equiv \omega/k$. The solution of (4.1) that is bounded at $r = 0$ is

$$\bar p_H = \bar p_H(R) I_m^{-1}[(1-V^2/\bar v_A^2)^{1/2}kR] I_m[(1-V^2/\bar v_A^2)^{1/2}kr], \quad (4.2)$$

where $I_m(x)$ is the modified Bessel function.

On the perturbed surface of the jet ($r = R + \zeta$),

$$\bar p_H' = -\bar\rho_0 k^2(\bar v_A^2 - V^2)\zeta. \quad (4.3)$$

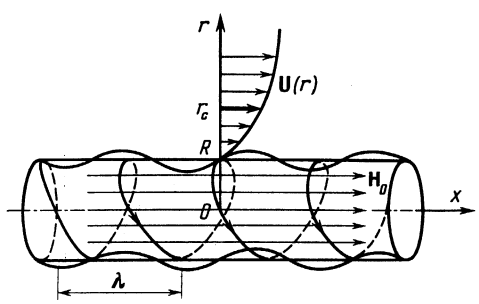

FIG. 3. Cylindric jet stabilized by external pressure and magnetic field.

Hence, using (4.2), we find that

$$\bar{p}_H|_{r=R+\zeta}=-\bar{\rho}_0 k(\bar{v}_A^2-V^2)\frac{I_m[(1-V^2/\bar{v}_A^2)^{1/2}kR]}{I_m'[(1-V^2/\bar{v}_A^2)^{1/2}kr]_{r=R}}\zeta. \quad (4.4)$$

The pressure in the flow is

$$p|_{r=R+\zeta}=\rho_0 V\left[V\left(\frac{v_r'}{v_r}\Big|_{r=R}+\frac{1}{R}\right)+U'(R)\right]\frac{k^2R^2}{k^2R^2+m^2}\zeta. \quad (4.5)$$

Since the pressures must be equal at $r=R+\zeta$, we obtain the dispersion relation

$$\rho_0 V\left[V\left(\frac{v_r'}{v_r}\Big|_{r=R}+\frac{1}{R}\right)+U'(R)\right]\frac{k^2R^2}{k^2R^2+m^2}$$
$$=\bar{\rho}_0 k(V^2-\bar{v}_A^2)I_m\left[\left(1-\frac{V^2}{\bar{v}_A^2}\right)^{1/2}kR\right]I_m'^{-1}$$
$$\times\left[\left(1-\frac{V^2}{\bar{v}_A^2}\right)^{1/2}kr\right]_{r=R}. \quad (4.6)$$

When $\bar{\rho}_0 \gg \rho_0$, it follows from (4.6) that

$$V\approx\bar{v}_A \quad (4.7)$$

and we can use the asymptotic expansions for $I_m(x)$ and $I_m'(x)$ for small values of the argument. Equation (4.6) then assumes the form

$$\rho_0 V\left[V\left(\frac{v_r'}{v_r}\Big|_{r=R}+\frac{1}{R}\right)+U'(R)\right]\frac{R}{k^2R^2+m^2}$$
$$=\bar{\rho}_0\frac{1}{m}(V^2-\bar{v}_A^2). \quad (4.8)$$

The Rayleigh equation for v_r in the flow is

$$v_r''+\frac{Q(r)}{r}v_r'-v_r\left[\frac{U''(r)-U'(r)\Psi(r)/r}{U(r)-V-i\delta}+\frac{\Psi(r)}{r^2}+k^2+\frac{m^2}{r^2}\right]=0, \quad (4.9)$$

where

$$\Psi(r)=\frac{k^2r^2-m^2}{k^2r^2+m^2},\qquad Q(r)=\frac{k^2r^2+3m^2}{k^2r^2+m^2}.$$

Calculations similar to those given in Section 3 enable us to use (4.9) to determine the imaginary part of the quantity $(v_r'/v_r)_{r=R}$ in (4.8).

For $U'(r_c)>0$, we find that

$$\mathrm{Im}\,\frac{v_r'}{v_r}\Big|_{r=R}\approx-\pi\left[\frac{U''(r_c)}{U'(r_c)}-\frac{1}{r_c}\frac{(kr_c)^2-m^2}{(kr_c)^2+m^2}\right]\frac{K_m^2(kr_c)}{K_m^2(kR)}, \quad (4.10)$$

where r_c determines the position of the resonance layer $U(r_c)=\mathrm{Re}\,V$. The growth rate γ of the surface wave for $\bar{\rho}_0\gg\rho_0$ then follows from (4.8):

$$\gamma_m(k)\approx\frac{1}{2}\frac{\rho_0}{\bar{\rho}_0}\bar{v}_A\frac{kRm}{k^2R^2+m^2}\,\mathrm{Im}\,\frac{v_r'}{v_r}\Big|_{r=R}. \quad (4.11)$$

Using the uniform expansion of the modified Bessel function $K_m(x)$ for $m\to\infty$, which, however, is accurate enough down to $m\sim1$ (similarly to the Stirling formula for $m!$), we obtain the following expression for the growth rate from (4.10) and (4.11):

$$\gamma_m(k)\approx-\frac{\pi}{2}\frac{\rho_0}{\bar{\rho}_0}\bar{v}_A\frac{mkR}{(kR)^2+m^2}\left[\frac{U''(r_c)}{U'(r_c)}-\frac{1}{r_c}\frac{(kr_c)^2-m^2}{(kr_c)^2+m^2}\right]$$
$$\times\left[\frac{(kR)^2+m^2}{(kr_c)^2+m^2}\right]^{1/2}\left(\frac{R}{r_c}\right)^{2m}\left(\frac{m+[(kr_c)^2+m^2]^{1/2}}{m+[(kR)^2+m^2]^{1/2}}\right)^{2m}$$
$$\times\exp\{-2[((kr_c)^2+m^2)^{1/2}-((kR)^2+m^2)^{1/2}]\}$$
$$\text{for}\quad k(r_c-R)\gg1,\quad\text{if}\quad m\gg1. \quad (4.12)$$

This yields

$$\gamma_m(k)\sim m\exp\{-2k(r_c-R)\}\quad\text{for}\quad m^2\ll(kR)^2, \quad (4.13)$$
$$\gamma_m(k)\sim m^{-1}(R/r_c)^{2m}\quad\text{for}\quad m^2\gg(kR)^2, \quad (4.14)$$

from which it is clear that γ is the maximum for

$$m=m_{max}\approx kR. \quad (4.15)$$

The wave-vector component k_y on the plane separation boundary corresponds to m/R in cylindric geometry. Thus, knowing $\theta_{\max}$ on the plane, we can estimate $m_{\max}$:

$$m_{max}\approx k_xR\,\tan\theta_{\max}. \quad (4.16)$$

The conditions $c_s\to\infty$ and $\bar{c}_s=0$, for which the results obtained in this section are valid, are the same as the conditions for which $\theta_{\max}\simeq\pi/4$ (see Section 3). Substituting $\theta_{\max}\simeq\pi/4$ in (4.16), we obtain

$$m_{max}\approx k_xR. \quad (4.17)$$

Comparison of (4.17) and (4.15) shows that the two estimates for $m_{\max}$ lead to similar results. We shall therefore use (4.16) whenever the instability of a cylindric jet cannot be analyzed.

5. ASTROPHYSICAL ESTIMATES

According to (3.10), the wind-instability growth rate has a maximum with respect to k in the long-wave region. This enables us to explain the presence of a distinct scale of perturbations, observed on the surface of many jets, e.g., in the quasar 3C273, the Hercules A radiogalaxy, and so on.

As in the case of the Kelvin-Helmholtz instability (see Appendix), the most rapidly growing perturbations in the ultrasonic flow are those propagating at an angle to the flow velocity on the plane separation boundary (3.11), or helical waves on the surface of the cylindric jet (3.15).

Recent observations performed with the VLA radio-telescope have revealed a helical structure in one of the two asymmetric jets in Hercules A.[13]

The formula $m_{\max} \simeq k_{\max} R$ (4.15) enables us to estimate $m_{\max}$ when $c_s \gg \bar{v}_A \gg \bar{c}_s$ and the flow is nonrelativistic. For example, if we determine the jet radius R and wavelength $\lambda_{\max} = 2\pi/k_{\max}$ from the radio image of Hercules A, we find that $m_{\max} \simeq 2$–3.

According to (3.10) and (3.11), the relativistic character of the motion leads to a shift of the maximum of the wind-instability growth rate toward shorter wavelengths

$$k_{max} \approx (1+\bar{v}_A^2/2c^2)^{1/2}(2z_c)^{-1} \quad (5.1)$$

and smaller azimuthal numbers

$$m_{max} \approx 2\pi(1+\bar{v}_A^2/c^2)^{-1/2} R/\lambda_{max}, \quad (5.2)$$

which follows from (4.16), (3.10), and (3.11).

We have assumed in our analysis of wind instability that the velocity profile was formed only in the lower-density medium. It is shown in Ref. 14 for the nonrelativistic case that the presence of a profile in the higher-density medium gives rise to a different instability picture. In particular, dispersion of the phase velocities of the surface waves becomes significant, and the narrow resonance layer is replaced by a wide resonance region[6)]

$$z_c(k) \approx \frac{\bar{\rho}_0}{\rho_0} \frac{U_0^2}{U_0'^2} k, \qquad \bar{\rho}_0 \ll \rho_0. \quad (5.3)$$

At short distances from the core (of the order of a few kpc), the jet density is probably greater by a few orders of magnitude than the density of the ambient medium but, at large distances, the jet becomes less dense than the intergalactic plasma (because of expansion).[1]

The planar model considered in Sections 2 and 3 can thus be used for a jet segment close to the core (the velocity profile being formed in the ambient medium) or a distant and highly rarefied segment (velocity profile being formed inside the jet).

It is important to note that data on the physical conditions prevailing in jets are very unreliable.[1] We do not even know whether these jets are continuous flows, as implied by the name, or how they are maintained, although a strong magnetic field is known to be present ($H \sim 10^{-3}$–10^{-5} Gs, if equally distributed) on a kpc scale.[1,15]

Estimates are usually based on a number of alternative variants, for example, the jet density is taken to be greater than, of the same order as, or lower than, the density of the ambient medium, and so on. We shall confine our attention to the conditions examined above, which do not cover all possibilities, but provide us with a fairly simple analytic result. Our aim will be to consider the possibility of a helical structure and the differences between wind and Kelvin-Helmholtz instabilities that may enable us to resolve previous contradictions relating to the scale of perturbations observed in jets.[2]

The condition $\bar{v}_A \gg \bar{c}_s$ can probably be satisfied in kiloparsec jets. Because $\rho c_s^2 \sim \bar{\rho}\bar{v}_A^2$, the assumption of confinement of a denser jet by external pressure[1] leads to the inequality $c_s \gg \bar{v}_A$, used above. We note that the ambient medium must then be much hotter than the usual galactic gas. This has led to a discussion in the literature of whether the jet is maintained by pressure or external magnetic field (which may be due to currents flowing on the jet surface),[7)] or it is assumed that the jet is not confined, but expands into the ambient medium. Different situations may occur for different outflows and at different distances from the active core.[15]

The sufficient condition for the resonance responsible for the wind instability to occur is that a surface wave can propagate on the jet surface. This is probably assured by the presence of a magnetic field, and presupposes some jet confinement mechanism and also the presence of a velocity profile in the flow, for which hydrodynamics must be applicable. On the other hand, the scale of the resulting perturbations is not very sensitive to the relationship between the parameters of the jet and of the ambient medium (this is in contast to the wind-instability growth rate).

We note that the hydrodynamic approximation is valid, at any rate, for distances $L \lesssim 30$ kpc from the core of an active galaxy. The main component of the interstellar medium in this region is probably the coronal plasma with density $n \sim 10^{-2.5}$ cm^{-3} and temperature $T \sim 10^{5.7}$ K (Ref. 18). These parameters vary little with distance from the galactic core for $L \lesssim 30$ kpc but, thereafter, there is a sharp reduction in density.[19] Hence, for $L \lesssim 30$ kpc, the Coulomb mean free path

$$l_{\text{Coul}} \approx 16\pi n v_T^4 \omega_p^{-4} \Lambda^{-1} \sim 1 \text{ pc}$$

($\Lambda \simeq 10$ is the Coulomb logarithm, v_T the thermal velocity of ions, and ω_p the plasma frequency) is much less than the characteristic wavelength $\lambda \simeq 2$–3 kpc of these perturbations and the characteristic transverse size of the outflow.

In the region of a hot spot, the plasma is characterized by a high temperature and low density. The Coulomb mean free path is much greater than the typical linear dimensions of the outflow, but the Larmor radius of the particles is significantly smaller than the characteristic dimensions, so that the plasma can be described by collisionless magnetohydrodynamics.

We note in conclusion that a highly nonlinear stage of wind instability development may occur at large distances from the core and may cause the jet to decay into individual plasma condensations, actually observed in the case of, for example, NGC 6251.

APPENDIX

Instability of a relativistic ultrasonic tangential discontinuity

The Kelvin-Helmholtz instability of an ultrasonic relativistic flow was investigated numerically in Ref. 20.[8)] On the other hand, transformation to the symmetric frame of reference can be used to carry out a fairly simple analytic solution in the case of a velocity discontinuity in a homogeneous medium. We shall reproduce it both in order to compare with wind stability and because it may be of interest in itself. Thus, the monotonic rise in the Kelvin-Helmholtz growth rate is limited only by dissipation[14] and leads to the growth of the shortest-wave perturbations that can be responsible for small-scale structures or turbulent spreading of the separation boundary.

In the symmetric reference frame, the equation for $V=\omega/k_x$ is

$$(u+V)^2\left\{\Gamma^2k_x^2\left[\left(1+\frac{uV}{c^2}\right)^2-\frac{(u+V)^2}{c_s^2}\right]+k_y^2\right\}^{-1/2}$$
$$=-(u-V)^2\left\{\Gamma^2k_x^2\left[\left(1-\frac{uV}{c^2}\right)^2-\frac{(u-V)^2}{c_s^2}\right]+k_y^2\right\}^{-1/2}. \quad (A1)$$

Squaring both sides of this equation and canceling factors that do not contain imaginary roots, we obtain the following biquadratic equation for V:

$$V^4\left(\Gamma^2\frac{u^2}{c^4}-\frac{1}{c^2}-\frac{\Gamma^2}{c_s^2}\right)+2V^2\left(\frac{1}{\cos^2\theta}+\Gamma^2\frac{u^2}{c_s^2}\right)$$
$$+\left[\frac{2u^2}{\cos^2\theta}-\Gamma^2u^4\left(\frac{1}{c_s^2}-\frac{1}{c^2}\right)\right]=0, \quad (A2)$$

and, hence,

$$\omega=\pm k_x\left\{\frac{1}{\cos^2\theta}+\Gamma^2\frac{u^2}{c_s^2}\mp\left[\left(\frac{1}{\cos^2\theta}+\Gamma^2\frac{u^2}{c_s^2}\right)^2\right.\right.$$
$$\left.\left.+\Gamma^2\left(\frac{u^2}{c^2}+\frac{u^2}{c_s^2}-2\frac{u^4}{c^4}\right)\left[\frac{2}{\cos^2\theta}-\Gamma^2\left(\frac{u^2}{c_s^2}-\frac{u^2}{c^2}\right)\right]\right]^{1/2}\right\}^{1/2}$$
$$\times\left[\Gamma\left(\frac{1}{c^2}+\frac{1}{c_s^2}-2\frac{u^2}{c^4}\right)^{1/2}\right]^{-1}. \quad (A3)$$

The upper signs in (A3) correspond to growing oscillations.

For $u<u_{\rm crit}=2^{1/2}c_s(1+c_s^2/c^2)^{-1/2}$, we have stabilization of waves propagating at angles

$$\theta<\theta_\Gamma=\arccos\frac{2^{1/2}c_s}{u\Gamma}\left(1-\frac{c_s^2}{c^2}\right)^{-1/2}$$

to the flow, but waves traveling at larger angles remain unstable. The preferential growth of waves propagating at $\theta_{\max}>0$ to the flow begins for velocities $u>u^*$ ($u^*<u_{\rm crit}$). When $c_s\ll c$, we find that u^* is close to its nonrelativistic values $3^{1/2}c_s/2$ and $\theta_{\max}\simeq\arccos u^*/u\Gamma$. For the ultrarelativistic equation of state,

$$\theta_{max}=\arccos\frac{\{3(6-u^2/c^2)^{1/2}-(7-2u^2/c^2)\}^{1/2}}{(u/c)\Gamma(1+4u^2/c^2)^{1/2}},\quad c_s=\frac{c}{3^{1/2}}.$$

In the nonrelativistic limit, we obtain the familiar result $u_{\rm crit}=2^{1/2}c_s$ (see Introduction). The maximum of γ for $u>u^*$ shifts toward larger θ with increasing u, but remains constant in magnitude: $\gamma_{\max}=kc_s/2$.

[1)]In the planar case, the maximum can occur only for fairly short waves as a result of competition between the Kelvin-Helmholtz growth rate and the viscous damping. In the cylindric case, numerical calculations reveal the presence of a poorly-defined maximum at wavelengths greater than the radius of the cylinder.[2]

[2)]If we take into account the magnetic field in the flow in the presence of shear ($z>0$) then, as was shown in the nonrelativistic case, this produces a splitting of the resonance layer by an amount equal to the Alfven velocity v_A (Ref. 9), which is significant for $v_A\gg\bar{v}_A$.

[3)]The term containing the first derivative v_z' in (2.19) can also be avoided by using the exponential replacement.

[4)]This result is also confirmed by solutions of the Rayleigh equation for profiles of the form $U(z)=\bar{v}_A+(U_0-\bar{v}_A)(1-z/z_c)^\alpha$, $\alpha=2n+1$; $n=0, 1, 2, \ldots$. For $kz_c\gg1$, we then have $\gamma\sim(kz_c)^\beta/z_c$, where $\beta=|1-2\alpha|$ (Ref. 5). Although these profiles contain a point of inflection that coincides with the critical point z_c, the estimate γ agrees with the general case.

[5)]We note that, because capillary-gravitational waves exhibit significant dispersion in the region of the phase velocity mainimum, where the threshold for wind waves is also found to occur,[3,4] the very general nature of the resonance that produces the maximum growth rate even in the absence of dispersion[5,12] remained virtually ignored for a long period of time.

[6)]In this case, we have $\gamma(k)\sim(\bar{\rho}_0/\rho_0)^2k^3$ for small $k(kz_c\ll1)$, and there is also a maximum of γ for $k_{\max}\simeq(\sqrt{3}/2)(\rho_0/\bar{\rho}_0)^{1/2}(U_0'/U_0)$.

[7)]It is possible that the jet is confined by an azimuthal magnetic field (see, for example, Ref. 16 and Benford's paper[17]). This field may not be readily observed because it is localized in a weakly radiating "cocoon" region, which certain data suggest surrounds the jet.[1]

Surface currents (azimuthal in our case and longitudinal in pinch geometry) should spread on the scale $\delta\sim R/{\rm Re}_m\ll R$ because the medium is nonideal and the magnetic Reynolds number ${\rm Re}_m\equiv4\pi\sigma U_0R/c^2$ is very large for well-conducting interstellar and intergalactic media. However, δ may become significantly greater if the jet gives rise to turbulence that reduces the conductivity of the medium.[17]

[8)]For a numerical analysis of the Kelvin-Helmholtz instability in a relativistic cylindric jet, see Refs. 21 and 22.

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