Classical size effects in the electrical conductivity of semiconductors with degenerate valence bands under specular-scattering conditions

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We have calculated the electrical conductivity of thin semiconductor films with complex degenerate valence bands (p-type Ge) under the assumption of specular scattering of holes from the surface, when the film thickness is comparable to the mean free path of the holes. We have shown that the process of conversion of light holes into heavy holes (and the reverse) upon collision with the surface causes the electrical conductivity of the film to depend on its thickness, despite the fact that the scattering is specular. This effect is interpreted to be a consequence of the appearance of an effective force of friction between the light and heavy components of the hole gas due to the conversion of holes at the surface. In a time-varying electric field the processes of hole conversion at the surface lead to additional absorption of field energy. We show that the real part of the film electrical conductivity can have a finite value, even if no holes are scattered in the interior of the film.

1. INTRODUCTION

It is well-known that the electrical conductivity of thin films is smaller than that of bulk samples, since the current carriers, in addition to bulk scattering, also undergo scattering by the surface.[1] In the case investigated by Fuchs[1] (which assumed one kind of carrier—electrons—and a scalar relaxation time in the volume), effects due to the finite size were present only to the extent that the surface scattering of electrons was diffusive. For specular scattering, in which case the component of electron momentum parallel to the surface of the film is conserved along with the energy, the electrical conductivity of the film did not depend on its thickness.

In this paper we will calculate the electrical conductivity of thin semiconductor films with complex valence bands (p-type Ge) under specular-scattering conditions of the current carriers by the surface. The energy spectrum of the carriers in such semiconductors consists of two bands with different effective masses (light and heavy holes). It turns out that in this case a size dependence is present which affects the carrier mean free path, although scattering of the holes by the surface is specular. The point is that in the course of scattering by the surface, processes in which a light hole converts to a heavy hole (and conversely) can occur with nonzero probability (see Fig. 1). Since the mobilities of light and heavy holes are different, these processes give rise to an effective frictional force between the light and heavy hole gases. The presence of this force leads to lowered electrical conductivity of the film. Since the effectiveness of this friction increases as the film thickness is decreased (because a larger and larger number of holes can reach the surface), the hole-conversion processes at the surface lead to a dependence of the film resistivity on its thickness. For a film thickness a large compared to the mean free path l, the relative decrease in the conductivity is of order $l/a \ll 1$. This quantity is simply the fraction of carriers which can reach the film surface and can undergo conversion. For $a \ll 1$ the electrical conductivity saturates, and its value for these thicknesses is smaller than that of bulk samples by roughly a factor of 2 (for comparable masses of light and heavy holes).

Since the component of momentum parallel to the surface does not relax for specular scattering, in a static electric field the resistive mechanism investigated here, being due to conversion of holes at the surface, cannot by itself be responsible for a finite film resistance. This is correct even if the film thickness is small compared to the mean free path, so that the holes collide with the surface more often than they scatter in the interior. In other words, if we exclude scattering of holes in the interior (by impurities or by acoustic phonons), the conductivity of the system becomes infinite. This is easy to understand. In reality, since the total charge of a system of holes is not zero, in a static electric field applied parallel to the plane of the film a nonzero momentum will be "pumped" into the system; since this momentum

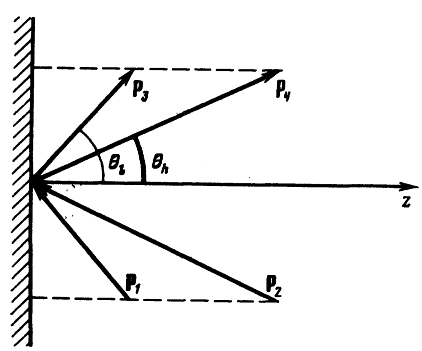

FIG. 1. A schematic of specular scattering of light and heavy holes at a surface, taking into account the conversion of one type of hole into another. The momenta of the light hole ($\mathbf{p}_1$,$\mathbf{p}_3$) and of the heavy hole ($\mathbf{p}_2$,$\mathbf{p}_4$) before and after scattering are shown. The equation of the surface is $z = 0$.

cannot relax (if bulk scattering is excluded) the system cannot establish a stationary state. Stated simply, the system of charges as a whole will be accelerated without limit by the field, which implies an infinite conductivity.

Let us now consider the case of a time-varying electric field of frequency ω. As we already noted, conversion of holes at the surface leads to the appearance of a frictional force between the light and heavy components of the hole gas. It is clear that in a time-varying field this frictional force will cause absorption of field energy in addition to absorption due to collisions in the film bulk. To avoid misunderstanding, let us emphasize that the frequency of the field is assumed to be small compared to the characteristic separation between the light and heavy hole bands, so that the field does not cause interband transitions. Under these circumstances, whereas the resistance of the film in a static field is controlled by collisions in the interior, the situation for a time-varying field is quite different; since the momentum communicated to the system of charges by the external field is zero when averaged over a period of the field, it is clear that for a sufficiently high field frequency the real part of the conductivity, which corresponds to absorption of field energy, can have a finite value even if scattering in the interior of the film is absent. In this case the absorption of energy is wholly determined by the mechanism under discussion here, i.e., conversion of holes at the surface. For example, in the case where $\omega^{-1} \ll \tau_f \ll \tau$ (τ is the mean free time relative to scattering in the bulk, $\tau_f \sim a/v$ is the time of flight, $\bar{v}$ is a characteristic hole velocity), the dissipative part of the conductivity is

$$\operatorname{Re}\sigma \sim ne^2\tau_f/m(\omega\tau_f)^2,$$

where m is a hole mass and n is the hole concentration (we here assume that the masses of the light and heavy holes are the same order of magnitude). This formula has the usual form; the role of the mean free time is played by the time of flight.

The resistivity mechanism we investigate in this paper, which is due to the conversion of particles at the surface accompanied by a change in their mass, is in many ways analogous to the resistivity mechanism connected with electron-electron collisions between particles with different masses in a three-dimensional gas.[2] As shown in Ref. 2, taking into account electron-electron collisions between different-mass particles leads to the appearance of a frictional force between the different components of the gas due to the difference in their mobilities. Collision of a hole with the surface, in the course of which it converts to a hole with a different mass, plays a role analogous to collision of particles with different masses in the three-dimensional gas.

Let us discuss still more briefly the relationship between the problem investigated in this paper and the problem of size effects in the conductivity of many-valley semiconductors (n-type germanium). At first glance it seems that these problems are completely equivalent from a physics standpoint, since in a many-valley semiconductor scattering of electrons by the surface can be accompanied by a change in their mass (both in the course of inter-valley transfer and in the course of intra-valley scattering). However, it is well-known that in many-valley semiconductors and semimetals the effect of finite size on the mean free path is absent for specular scattering of electrons by a surface (see the review in Ref. 3). The point is that in the case of many-valley semiconductors the conductivity is a tensor, and when an electric field is applied parallel to the plane of the film a transverse concentration gradient arises consisting of carriers from different valleys, and a transverse electric field appears so as to reduce the transverse carrier currents to zero. Thus a transverse field arises of just the right size to ensure that the size effect is absent for specular scattering.[3] In the problem we investigate in this paper the conductivity is a scalar, and when an electric field is applied parallel to the plane of the film, neither an electric field nor a hole concentration gradient will appear in the transverse direction. For this reason, it is found that size effects are present even for specular scattering of carriers.

The film thickness will be assumed to be large compared to the wavelength of the holes, so that the behavior of the carriers can be described by the Boltzmann kinetic equation. The presence of the film boundaries is taken into account with the help of boundary conditions imposed on the light- and heavy-hole distribution functions. The derivation of these boundary conditions is given in the Appendix.

In Section 2 we obtain a general expression for the film electrical conductivity in a time-varying field for an arbitrary film thickness and arbitrary ratio of light and heavy hole masses.

In Section 3 we calculate the film electrical conductivity in a static electric field in certain special cases for which an analytic investigation is possible (the cases of small and large film thicknesses, and small ratio of hole masses).

Finally, in Section 4 we calculate the real part of the film electrical conductivity in a time-varying field in certain limiting cases.

2. GENERAL EXPRESSION FOR THE FILM ELECTRICAL CONDUCTIVITY

In the spherical approximation the energies of light and heavy holes are determined by the expressions

$$\varepsilon_l(\mathbf{k}) = \hbar^2k^2/2m_l, \qquad \varepsilon_h(\mathbf{k}) = \hbar^2k^2/2m_h,$$

in which m_l and m_h are assumed to be positive. A nonequilibrium state of light and heavy holes is described with the help of the momentum distribution functions $f_l(\mathbf{k}), f_h(\mathbf{k})$[4,5] (see also the Appendix). As shown in the Appendix, the boundary condition for the distribution functions for the case of specular scattering of holes at the surface takes the form ($z=0$):

$$f_l(\mathbf{p}_3) = W_l^l f_l(\mathbf{p}_1) + W_h^l f_h(\mathbf{p}_2), \quad f_h(\mathbf{p}_4) = W_l^h f_l(\mathbf{p}_1) + W_h^h f_h(\mathbf{p}_2), \tag{1}$$

where

$$W_l^{\,l} = W_h^{\,h} = 1 - W, \quad W = W_l^{\,h} = W_h^{\,l} = \frac{3\sin 2\theta_l \sin 2\theta_h}{1+3\cos^2(\theta_l-\theta_h)}. \tag{2}$$

The relative arrangement of the vectors $\mathbf{p}_1$, $\mathbf{p}_2$, $\mathbf{p}_3$, $\mathbf{p}_4$ is shown in the Fig. 1. The angles θ_l and θ_h (see Fig. 1) are connected by the relation

$$\sin\theta_h=\mu^{1/2}\sin\theta_l,\quad \mu=m_l/m_h<1, \tag{3}$$

which follows from the equality of the longitudinal components of hole momenta. The quantity W is the probability of conversion of a light hole into a heavy one (and conversely) upon collision with the surface. It is small for small mass ratios, which is related to the fact that in this case the available range of angles for the heavy holes is small (see Fig. 1).

In order to find the film conductivity, we must find a solution to the kinetic equations for the light and heavy hole distribution functions and subject them to the boundary conditions (1) at both surfaces of the film. Since in the problem investigated here spatial gradients can be large, of the order of a carrier mean free path, in the collision-integral term which includes bulk scattering we are not in general allowed to introduce a relaxation time which depends only on energy. This is connected with the fact that the equation for the first angular moment of the distribution function, which determines the current, is coupled to the equations for the other moments. Therefore we will assume that the scattering probability in the volume has the simple isotropic form; $\omega_{\mathbf{kk}'}=\omega_0\delta[\varepsilon(\mathbf{k})-\varepsilon(\mathbf{k}')]/V$, where ω_0 does not depend on angle and V is the volume of the system, while the momenta $\mathbf{k}$, $\mathbf{k}'$ pertain either to one and the same energy band (for intraband transitions) or to different bands (for interband scattering). This scattering probability was used in the first paper of Pikus[4] to describe kinetics in p-Ge. Then the relaxation times for light and heavy holes are found to be equal to one another, and are given by the expression[4]

$$1/\tau_l=1/\tau_h=1/\tau=w_0(2\varepsilon)^{1/2}(m_l^{3/2}+m_h^{3/2})/\pi^2\hbar^3. \tag{4}$$

The kinetic equations for the hole distribution functions take the form (the z-axis is perpendicular to the plane of the film)

$$\begin{aligned}\frac{\partial f_l}{\partial t}+v_{lz}\frac{\partial f_l}{\partial z}+\frac{e\mathbf{E}(t)\mathbf{p}}{m_l}\frac{\partial f_0}{\partial\varepsilon}&=-\frac{f_l-f_0}{\tau},\\ \frac{\partial f_h}{\partial t}+v_{hz}\frac{\partial f_h}{\partial z}+\frac{e\mathbf{E}(t)\mathbf{p}}{m_h}\frac{\partial f_0}{\partial\varepsilon}&=-\frac{f_h-f_0}{\tau},\end{aligned} \tag{5}$$

where v_{lz}, v_{hz} are the z-components of velocity for light and heavy holes, respectively, f_0 is the equilibrium distribution function which depends only on energy and is the same (for a given energy) for light and heavy holes, $\mathbf{E}(t)=\mathbf{E}_0\exp(-i\omega t)$, and $\mathbf{E}_0$ is an electric field vector parallel to the plane of the film. We note that the collision integral reduces to the simple form shown in (5) (for an isotropic scattering probability) only in the case that $\bar{f}_l(k)$, $\bar{f}_h(k)$ equal f_0, where the bar denotes an average over directions of the momentum $\mathbf{k}$. As we will see, in the present problem this is the case even when we include spatially-inhomogeneous terms in the distribution functions. Under these circumstances, when the approximations we have assumed relating to bulk scattering hold, the solution we obtain below is exact.

Solutions to Eq. (5) have the form

$$f_l=f_0+f_l^{(1)}\exp(-i\omega t),\quad f_h=f_0+f_h^{(1)}\exp(-i\omega t), \tag{6}$$

where

$$f_l^{(1)}=-\frac{e\mathbf{E}_0\mathbf{p}\tau}{m_l}\frac{\partial f_0}{\partial\varepsilon}\frac{1}{(1-i\omega\tau)}+A(v_{lz})\exp\left\{-\frac{z(1-i\omega\tau)}{l_l\cos\theta_l}\right\},$$

$$f_h^{(1)}=-\frac{e\mathbf{E}_0\mathbf{p}\tau}{m_h}\frac{\partial f_0}{\partial\varepsilon}\frac{1}{(1-i\omega\tau)}+B(v_{hz})\exp\left(-\frac{z(1-i\omega\tau)}{l_h\cos\theta_h}\right). \tag{7}$$

In Eqs. (7) $l_l=v_l\tau$, $l_h=v_h\tau$ are the mean free paths of light and heavy holes respectively, θ_l is the angle between the light-hole velocity vector and the positive direction of the z-axis (and analogously—θ_h). When we subject the functions (7) to the boundary conditions (1) on both surfaces of the film ($z=\mp a/2$), we obtain, using (2),

$$A=\alpha e\mathbf{E}_0\mathbf{p}\tau\frac{\partial f_0}{\partial\varepsilon}\left(\frac{1}{m_l}-\frac{1}{m_h}\right)\frac{1}{(1-i\omega\tau)},\quad B=-A\frac{\operatorname{sh}\gamma}{\operatorname{sh}\beta}, \tag{8}$$

$$\alpha=W\operatorname{sh}\beta[2\operatorname{sh}\beta\operatorname{sh}\gamma+W(e^{-\gamma}\operatorname{sh}\beta+e^{-\beta}\operatorname{sh}\gamma)]^{-1}, \tag{9}$$

$$\beta=\frac{a(1-i\omega\tau)}{2l_h|\cos\theta_h|},\quad \gamma=\frac{a(1-i\omega\tau)}{2l_l|\cos\theta_l|}. \tag{10}$$

In Eqs. (8)–(10) (and all equations that follow) it is necessary to assume that θ_l and θ_h are connected by the relation (3).

Knowing the distribution function, we calculate the current density

$$\mathbf{j}(t)=\frac{1}{a}\int_{-a/2}^{a/2}dz\frac{2e}{(2\pi)^3}\left(\int f_l\mathbf{v}_l\,d^3\mathbf{k}+\int f_h\mathbf{v}_h\,d^3\mathbf{k}\right). \tag{11}$$

Then for the light-hole conductivity we obtain

$$\sigma_l(\omega)=\frac{n_le^2}{m_l}\left\langle\frac{\tau}{(1-i\omega\tau)}\left[1-\frac{3l_l(1-\mu)}{2a(1-i\omega\tau)}\int_0^{\pi/2}d\theta_l\sin^3\theta_l\cos\theta_l\times 2\alpha\operatorname{sh}\gamma\right]\right\rangle, \tag{12}$$

where n_l is the concentration of light holes. The angular brackets in (12) imply an average over the Maxwell distribution (for a nondegenerate hole gas);

$$\langle D\rangle={}^4/_3\pi^{-1/2}\int_0^\infty e^{-\varepsilon'}(\varepsilon')^{3/2}D(\varepsilon')d\varepsilon',\quad \varepsilon'=\varepsilon/k_BT. \tag{13}$$

Analogously, for the heavy-hole conductivity we obtain

$$\sigma_h(\omega)=\frac{n_he^2}{m_h}\left\langle\frac{\tau}{(1-i\omega\tau)}\left[1+\frac{3l_h(1-\mu)}{2\mu a(1-i\omega\tau)}\int_0^{\arcsin\mu^{1/2}}d\theta_h\sin^3\theta_h\cos\theta_h\cdot 2\alpha\operatorname{sh}\gamma\right]\right\rangle, \tag{14}$$

where n_h is the concentration of heavy holes. We note that the hole concentrations are expressed by the usual formulae which involve equilibrium distribution functions (it is easy to convince oneself of this with the help of Eqs. (6)–(8)), and thus do not depend on the coordinates. In Eq. (14) the integration over θ_h is taken only up to the cutoff angle arcsin $\mu^{1/2}$, since for larger angles the conversion probability of heavy holes to light holes equals zero. Using relation (3), let us proceed from the θ_h integration in (14) to the integration over θ_l. Then the integral in Eq. (14) can be expressed in terms of the integral in Eq. (12). Taking into account also that $l_h = \mu^{1/2} l_l$, $n_l = \mu^{3/2} n_h$, we finally obtain

$$\sigma(\omega) = \frac{n_h e^2}{m_h} \left\langle \frac{\tau}{(1-i\omega\tau)} \right\rangle + \frac{n_l e^2}{m_l} \left\langle \frac{\tau}{(1-i\omega\tau)} \left\{ 1 - \frac{3 l_l (1-\mu)^2}{2a(1-i\omega\tau)} \times \int_0^{\pi/2} d\theta_l \sin^3\theta_l \cos\theta_l \times 2W \operatorname{sh}\beta \operatorname{sh}\gamma [2 \operatorname{sh}\beta \operatorname{sh}\gamma + W(e^{-\gamma} \operatorname{sh}\beta + e^{-\beta} \operatorname{sh}\gamma)]^{-1} \right\} \right\rangle \quad (15)$$

for the total hole conductivity. We note once more that in the integral (15) the functions of the angle θ_h must be converted into functions of angle θ_l by using relation (3).

The expression we have obtained for the electrical conductivity is correct for any ratio of hole masses ($\mu < 1$) and any film thickness.

3. STATIC ELECTRICAL CONDUCTIVITY

Let us study first the electrical conductivity of a system in a constant electric field ($\omega = 0$). We will for the present assume that the ratio of light to heavy hole masses is of order unity. Then the mean free paths of light and heavy holes are of the same order of magnitude ($l_l \sim l_h \sim l$) and there are two distinct parameter regimes; the regimes of large and small film thicknesses.

1) In the case of large film thickness ($a \gg l$), $\gamma, \beta \gg 1$ in Eq. (15), and we obtain for the electrical conductivity of the system

$$\sigma = \sigma^{(0)} - \sigma_l^{(0)} (l_l/a)(1-\mu)^2 \mu^{1/2} I_1(\mu), \quad (16)$$

where

$$\sigma^{(0)} = \sigma_h^{(0)} + \sigma_l^{(0)}, \quad \sigma_h^{(0)} = \frac{n_h e^2 \langle\tau\rangle}{m_h}, \quad \sigma_l^{(0)} = \frac{n_l e^2 \langle\tau\rangle}{m_l} \quad (17)$$

is the hole electrical conductivity for an unbounded sample. In writing out (16) we have allowed for the fact that the hole mean free path does not depend on energy. For the integral $I_1(\mu)$ we have the following expression (in the integral (15) we have introduced the variable of integration $x = \sin^2\theta_l$):

$$I_1(\mu) = 9 \int_0^1 dx \frac{x^2 (1-x)^{1/2} (1-\mu x)^{1/2}}{1 + 3[(1-x)^{1/2}(1-\mu x)^{1/2} + \mu^{1/2} x]^2},$$

$$I_1(0) = 6\left(\frac{6}{5} - \frac{16\pi}{27 \cdot 3^{1/2}} \right). \quad (18)$$

As is clear from Eq. (16), for large film thicknesses the relative decrease in the conductivity is small and equals the fraction of carriers which reach the film surface.

2) In the small-film-thickness case ($a \ll l$), $\beta, \gamma \ll 1$. Expanding the function under the integral sign in (15) for small values of β and γ, and introducing the variable of integration $x = \sin^2\theta_l$, we obtain

$$\sigma = \sigma^{(0)} - \frac{3}{2} \sigma_l^{(0)} (1-\mu)^2 I_2(\mu),$$

$$I_2(\mu) = \frac{1}{2} \int_0^1 x\, dx [(1-x)^{1/2} + \mu^{1/2} (1-\mu x)^{1/2}]^{-1}. \quad (19)$$

Integrating leads to the result

$$I_2(\mu) = -\frac{1}{(1-\mu^2)} \left\{ \frac{1}{3} + \frac{1}{3\mu^{1/2}} [(1-\mu)^{3/2} - 1] - \frac{1}{(1+\mu)} [1 - \mu^{1/2}(1 - (1-\mu)^{1/2})] + \frac{\mu^{1/2}}{2(1+\mu)^{3/2}} \times \ln\left[\frac{((1+\mu)^{1/2} + \mu^{1/2})((1+\mu)^{1/2} - 1)(1 + (1-\mu^2)^{1/2})}{((1+\mu)^{1/2} - \mu^{1/2})((1+\mu)^{1/2} + 1)(1 - (1-\mu^2)^{1/2})} \right] \right\}. \quad (20)$$

Thus, the relative decrease in the electrical conductivity (for $\mu \sim 1$) as we go from large to small thicknesses is of order unity.

Let us now turn to the case of a small ratio between light and heavy hole masses ($\mu \ll 1$). Here, the mean free path of heavy holes is small compared with that of light holes: $l_h = \mu^{1/2} l_l$.

For large film thicknesses Eq. (16) is correct, in which the integral $I_2(\mu)$ must be evaluated at $\mu = 0$. It is easy to see from Eq. (15) that this expression is correct for $a \gg l_h$ (when we can neglect the second term in the denominator of the integrand in (15) compared to the first). So,

$$\sigma = \sigma_h^{(0)} + \sigma_l^{(0)} [1 - \mu^{1/2} I_1(0) l_l/a], \quad a \gg l_h. \quad (21)$$

Let us now investigate the thin-film limit $a \ll l_h$. We introduce a new variable of integration $t = \cos\theta_l$ into the integral (15). As it turns out, characteristic values of t in (15) for $a \ll l_h$ are such that $\beta \ll 1$, $\gamma \ll 1$. Expanding the integrand in (15) in these small quantities, we obtain

$$\sigma = \sigma_h^{(0)} + \sigma_l^{(0)} \times \left\{ 1 - \frac{3}{2} \int_0^1 dt (1-t^2) W \left[\frac{a}{l_l t} + W\left(1 + \frac{\mu^{1/2}}{t}\right) \right]^{-1} \right\}, \quad (22)$$

the probability of hole conversion (2) now is

$$W = 12 \mu^{1/2} t (1-t^2)(1+3t^2)^{-1}, \quad \mu \ll 1. \quad (23)$$

In writing down (22), (23), we have taken into account the smallness of the angle θ_h for small mass ratios μ (see Eq. (31)). It is easy to show that integrating over the region $t \sim 1$ leads to the value 2/3 for the integral in (22), so that the expression in the square brackets in Eq. (22) reduces to zero. Therefore, we will calculate the integral (22) in terms of corrections to the value 2/3. It is easy to see that the basic corrections are determined by the region $t \ll 1$. Starting from this, we transform the integrand in the following way:

$$W\left[\frac{a}{l_l t}+W\left(1+\frac{\mu^{1/2}}{t}\right)\right]^{-1}$$

$$=1-\left(\frac{a}{l_l}+\mu^{1/2}W\right)\left[\frac{a}{l_l}+W(t+\mu^{1/2})\right]^{-1}$$

and substitute for W its value in the limit $t\ll 1$. Then from Eq. (22) we obtain

$$\sigma=\sigma_h^{(0)}+\sigma_l^{(0)}\cdot\frac{3}{2}\int_0^1 dt\left(\frac{a}{l_h}+12\mu^{1/2}t\right)\left[\frac{a}{l_h}+12t(t+\mu^{1/2})\right]^{-1}. \quad (24)$$

Finally, from Eq. (24) we have

$$\sigma=\sigma_h^{(0)}+\sigma_l^{(0)}\left[\frac{\pi}{8}\left(\frac{3a}{l_h}\right)^{1/2}+\frac{3}{2}\mu^{1/2}\ln\left(\frac{l_h}{a}\right)^{1/2}\right],$$

$$\mu l_h\ll a\ll l_h, \quad (25a)$$

$$\sigma=\sigma_h^{(0)}+\sigma_l^{(0)}\cdot\frac{3}{2}\mu^{1/2}\ln\left(\frac{1}{\mu}\right)^{1/2},\quad a\ll\mu l_h. \quad (25b)$$

Of course, we note that Eq. (25b) is also a consequence of using Eqs. (19), (20), in which μ is assumed to be small.

Let us integrate these results for $\mu\ll 1$. As can be shown from Eqs. (12), (14), for small μ the deviation of the conductivity of heavy holes from its bulk value $\sigma_h^{(0)}$ is always parametrically small compared with the deviation of the light-hole conductivity from its bulk value $\sigma_l^{(0)}$. This is related to the fact that for small mass ratios only a small fraction of the heavy holes undergo conversion due to collision with the surface (those which arrive almost normal to the surface). However, light holes can undergo conversion when they are incident at any angle. Thus, for small μ we can assume that the entire difference between the electrical conductivity of the film and that of a bulk sample is related to conversion of light holes to heavy holes at the surface. So long as the film thickness is large compared to the mean free path of light holes l_l, only a fraction of the light holes, equal to $l_l/a\ll 1$, undergoes conversion due to collisions with the surface; in this case, the conversion probability is small; $W{\sim}\mu^{1/2}$ (see Eq. (23)). Therefore, for $a\gg l_l$ our result takes the form (21). For $l_l\gg a$ all the light holes reach the film surface, and since the probability for conversion into heavy holes after one collision is small, a light hole in the interval between two consecutive collisions in the bulk is specularly reflected many times first by one, then by the other surface of the film is succession. In this case the probability for conversion to a heavy hole increases. The frequency of collisions with the film surfaces for a typical light hole (which arrives at the surface at an angle ~ 1) equals l_l/a. This large factor (for $a\ll l_l$) is precisely what appears in Eq. (21). Therefore, as long as $a\gg l_h$ the majority of light holes still do not undergo conversion into heavy holes within the time between two consecutive bulk collisions.

At still smaller thicknesses ($a\ll l_h$) the conversion length for a typical light hole (equal to $\sim a/\mu^{1/2}$) becomes smaller than its mean free path l_l. Let us show that the main contribution to the light-hole conductivity comes from holes which are incident at small angles to the surface. Despite the fact that their number is small, their contribution turns out to be decisive due to their long effective mean free paths. In fact, the effective mean free path l_t of a light hole arriving at an angle t to the surface is determined by the condition $\mu^{1/2}tl_t t/a\sim 1$ (the conversion probability for holes after one collision is $\sim\mu^{1/2}t$; see Eq. (23)). The number of such holes is $\sim n_l t$. Then their contribution to the electrical conductivity is $\sim\sigma_l^{(0)}a/l_h t$. Substituting the minimum angle $t_{\min}$ into this formula, where $t_{\min}$ is determined from the condition $l_{t_{\min}}\sim l_l$, we find (see Eq. (25a)) that the main contribution comes from the light holes which arrive at the surface at the small angle $(a/l_h))^{1/2}$.

Under these circumstances, for small hole-mass ratios the film electrical conductivity in the limit of small film thicknesses is smaller than that of a bulk sample by a quantity equal to the electrical conductivity of light holes in a bulk sample.

4. TIME-VARYING ELECTRIC FIELD. REAL PART OF THE ELECTRICAL CONDUCTIVITY

Let us determine the frequency dependence of the real part of the conductivity.

1) We first investigate the case of small mass ratios μ. Let the film thickness lie in the interval $\mu l_h\ll a\ll l_h$, so that in a static field Eq. (25a) is valid (i.e., the quantity a is such that the terms with the logarithm in this formula can be neglected). Then for $\omega\tau(T)\ll 1$ (τ is evaluated at the thermal energy) Eq. (25a) is correct. Assume now that the condition $1\ll\omega\tau(T)\ll l_h/a$ is fulfilled. When this relation holds, the moduli of the complex quantities β and γ in the integral (15) are small compared to unity. Expanding the integrand in Eq. (15) in these small quantities, and introducing the variable of integration $t=\cos\theta_1$, we obtain

$$\sigma(\omega)=\frac{n_l e^2}{m_l}\left\langle\frac{\tau}{(1-i\omega\tau)}\right.$$
$$\left.\times\left\{1-\frac{3}{2}\int_0^1 dt(1-t^2)W\left[\frac{a}{l_l t}(1-i\omega\tau)+W\right]^{-1}\right\}\right\rangle$$
$$+\frac{n_h e^2}{m_h}\left\langle\frac{\tau}{(1-i\omega\tau)}\right\rangle, \quad (26)$$

where the probability W is determined by Eq. (23). Just as in the case of a static field (see Section 3), integrating near the region $t\sim 1$ leads to a value of 2/3 for the integral in Eq. (26), so that the expression in square brackets reduces to zero. We will calculate the integral in Eq. (26) by taking into account corrections to the value 2/3 (they are determined by the region $t\ll 1$). For this case we will transform the integrand exactly as was done in Section 3. Ultimately we obtain

$$\mathrm{Re}\,\sigma(\omega)=\frac{\pi}{8}\left(\frac{3}{2}\right)^{1/2}\frac{n_l e^2}{m_l}\left\langle\frac{\tau^*}{(\omega\tau^*)^{1/2}}\right\rangle+\frac{n_h e^2}{m_h}\left\langle\frac{\tau}{(\omega\tau)^2}\right\rangle,$$
$$\frac{1}{\tau(T)}\ll\omega\ll\frac{1}{\tau^*(T)}, \quad (27)$$

where $\tau^* = a/\mu^{1/2}v_l$ is the conversion time of a typical light hole (see Section 3) due to collisions with the film surfaces ($\tau^*/\tau \sim a/l_h \ll 1$). If, however, the frequency of the field is such that ($\omega\tau(T) \gg l_h/a$), then in the denominator of the integrand given in Eq. (15) we can neglect the second term compared to the first, and we obtain

$$\operatorname{Re}\sigma(\omega) = I_1(0)\frac{n_l e^2}{m_l}\left\langle\frac{\tau^*}{(\omega\tau^*)^2}\right\rangle + \frac{n_h e^2}{m_h}\left\langle\frac{\tau}{(\omega\tau)^2}\right\rangle,$$

$$\omega \gg \frac{1}{\tau^*(T)}. \tag{28}$$

If $\omega^*(t) \ll 1$, then the length traversed by a light hole within one period of the electric field is larger than its conversion length to a heavy hole. Therefore, the majority of light holes (in addition to those which are incident on the surface at a small angle) respond as if the field were static, which is the cause of the weaker frequency dependence of Re σ (see Eq. (27)). However, when the length traversed by a typical light hole within one field period becomes smaller than its conversion length, the formula for Re σ, which describes the contribution to absorption connected with conversion of light holes at the surface, takes the usual form; the role of a mean free time is played by the conversion time (see Eq. (28)). We note that for $a \ll \mu^{1/2} l_h$, beginning with frequencies $\omega \sim 1/(\mu\tau^*\tau^2)^{1/3} \ll 1/\tau^*$, the quantity Re σ is determined by the hole conversion mechanism at the surface.

2) Let us now investigate the case $\mu \sim 1$ ($l_l \sim l_h \sim l$) and small film thicknesses $a \ll 1$. Let the field frequency be such that $1 \ll \omega\tau(T) \ll l/a$. Then the moduli of the quantities β and γ in the integral (15) are small compared to unity. Let us expand the integrand in Eq. (15) in a series in these quantities; in the denominator of the integrand we save only terms proportional to the first power of β and γ, and also terms containing the product of the first power of these quantities; in the numerator, we save only terms $\sim\beta\gamma$. Finally, after some simple transformations, we obtain

$$\operatorname{Re}\sigma(\omega) = \frac{n_h e^2}{m_h}\left\langle\frac{\tau}{(\omega\tau)^2}\right\rangle\left\{1+\mu^{1/2}\left[1-\frac{3}{2}(1-\mu)^2 I_2(\mu)\right]\right\}$$
$$+\frac{(1-\mu)^2}{8\mu^{1/2}} I_3(\mu)\frac{n_l e^2\langle\tau_f\rangle}{m_l}, \qquad \frac{1}{\tau(T)} \ll \omega \ll \frac{1}{\tau_f(T)}, \tag{29}$$

where $\tau_f = a/v_l$ is the time of flight, and

$$I_3(\mu) = \int_0^{\pi/2} d\theta_l \frac{\sin\theta_l[1+3\cos^2(\theta_l+\theta_h)]}{\cos\theta_h(\cos\theta_l+\mu^{1/2}\cos\theta_h)^2},$$

$$\sin\theta_h = \mu^{1/2}\sin\theta_l, \quad \theta_h < \pi/2.$$

This integral for $\mu \sim 1$ has a value of order unity. We will not concern ourselves with evaluating it. Finally, in the region $l/a \ll \omega\tau(T)$, after transforming the functions sh β and sh γ, taking into account the smallness of the ratio a/l, we obtain

$$\operatorname{Re}\sigma(\omega) = \frac{3}{2}(1-\mu)^2\frac{n_l e^2}{m_l}\left\langle\frac{\tau_f}{(\omega\tau_f)^2} I_4(\mu,\rho)\right\rangle,$$

$$\omega \gg \frac{1}{\tau_f(T)}, \tag{30}$$

$$I_4(\mu,\rho) = \int_0^{\pi/2} d\theta_l \sin^3\theta_l \cos\theta_l W(1-W)\left\{(1-W)^2+\left(\frac{W}{2}\right)^2\right.$$
$$\left.\times\left[\operatorname{ctg}\left(\frac{\rho}{2\cos\theta_l}\right)+\operatorname{ctg}\left(\frac{\rho}{2\mu^{1/2}\cos\theta_h}\right)\right]^2\right\}^{-1}. \tag{31}$$

Here $\rho = \omega\tau_f \gg 1$. As we can show, for large ρ the integral (31) does not depend on ρ, and has a value of order unity (for $\mu \sim 1$).

Thus, in the case $\mu \sim 1$, Re σ decreases in the usual way in the frequency interval $\tau^{-1} \ll \omega \ll (\tau\tau_f)^{-1/2}$ (energy absorption in this case is due to collisions in the interior). Then there is a wide plateau in the frequency interval $(\tau\tau_f)^{-1/2} \ll \omega \ll \tau_f^{-1}$ (see Eq. 29)), for which absorption is due to conversion of holes at the surface. For these frequencies, we can wholly exclude bulk scattering ($\tau \to \infty$); nevertheless, the magnitude of the energy absorption is finite. The constancy of Re σ in this frequency interval is easily understood, because the length traversed by a hole in one period of the field is large compared to the film thickness for $\omega \ll \tau_f^{-1}$ (for $\mu \sim 1$ this length plays the role of a conversion length). Finally, when the length traversed by a hole in one field period becomes smaller than the conversion length, the formula for Re σ (see Eq. (30)) again takes the usual form; the time of flight plays the role of the mean free path.

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APPENDIX

Let us determine the form of the boundary conditions which the light- and heavy-hole distribution functions must satisfy at the surface.

A hole state in a complex band is characterized by an angular momentum projection M along the direction of its quasimomentum (i.e., its helicity); the values $M = \pm 1/2$ correspond to the light-hole band, while the values $M = \pm 3/2$ hold for the heavy hole band. Each band is doubly degenerate. The wave function of a hole state can be written in the form[6]

$$\Psi_M(\mathbf{k}) = \exp(i\mathbf{kr})\chi_M(\mathbf{k}), \qquad \chi_M(\mathbf{k}) = \sum_\lambda D^{(3/2)}_{\lambda M}(\mathbf{k})u_\lambda, \tag{A1}$$

where u_λ is an eigenfunction of the matrix J_z ($\lambda = \pm 1/2, \pm 3/2$),

$$D^{(3/2)}_{\lambda M} = \exp(-i\lambda\varphi)\, d^{(3/2)}_{\lambda M}(\theta)\exp(-iM\psi)$$

is a finite-rotation matrix,[7] ρ, θ are the polar angles of the vector $\mathbf{k}$; the angle is arbitrary, and we will henceforth set it equal to zero.

As was shown in Ref. 8, in order to describe a nonequilibrium state of carriers in a complex band it is necessary in general to use the density matrix $f_{MM'}(\mathbf{k})$. The elements of this matrix with $|M| = |M'| = 3/2$ describe the heavy holes. The nondiagonal (in the bands) elements with $|M| \neq |M'|$ are small in the absence of an alternating field which can cause interband transitions.[8] Because we do not treat these transitions in this paper, henceforth the density matrix elements with $|M| \neq |M'|$ will not be included. Then there exist two density matrices $f^{(l)}_{MM'}$ and $f^{(h)}_{MM'}(\mathbf{k})$ (light and heavy holes), consisting of four elements.

If we take bulk scattering into account in the Born approximation, then in an unbounded sample there exist closed equations for the momentum distribution functions of light and heavy holes.[8] The distribution function is the trace of the corresponding density matrix.

$$f_l(\mathbf{k})=\frac{1}{2}\sum_{M=\pm 1/2} f^{(l)}_{MM}(\mathbf{k}),\quad f_h(\mathbf{k})=\frac{1}{2}\sum_{M=\pm 3/2} f^{(h)}_{MM}(\mathbf{k}). \quad \text{(A2)}$$

However, in a bounded sample (film) the equations for the distribution functions turn out to be unclosed, since the boundary conditions can in principle mix the diagonal ($M=M'$) and nondiagonal ($|M|=|M'|, M\neq M'$) density matrix elements. Therefore, it is necessary to determine boundary conditions for the light and heavy hole density matrices.

For later use, we will require the value of the scattering amplitude $F^{M'\mathbf{p}'}_{M\mathbf{p}}$ at the surface from an initial state $M\mathbf{p}$ to a final $M'\mathbf{p}'$. To determine this quantity, let us write down the wave function for a state of the system in which a wave is incident on the surface corresponding to a light hole having momentum $\mathbf{p}_1$ and helicity M_1, after which a reflected wave appears corresponding to a light hole with momentum $\mathbf{p}_3$ and a heavy hole with momentum $\mathbf{p}_4$ (see Fig. 1):

$$\Psi_1=\exp(i\varkappa\mathbf{r}_\parallel)\left[\frac{\exp(-iq_1z)}{|v_{lz}|^{1/2}}\chi_{M_1}(\mathbf{p}_1)+\frac{\exp(iq_1z)}{|v_{lz}|^{1/2}}\times\sum_{M=\pm M_1}F^{M\mathbf{p}_3}_{M_1\mathbf{p}_1}\chi_M(\mathbf{p}_3)+\frac{\exp(iq_2z)}{|v_{hz}|^{1/2}}\sum_{M=\pm 3/2}F^{M\mathbf{p}_4}_{M_1\mathbf{p}_1}\chi_M(\mathbf{p}_4)\right]. \quad \text{(A3)}$$

The momenta $\mathbf{p}_1$, $\mathbf{p}_2$, $\mathbf{p}_3$, $\mathbf{p}_4$ have the same component $\hbar\varkappa$ along the surface, while their z-components equal $-\hbar q_1$, $-\hbar q_2$, $\hbar q_1$, $\hbar q_2$ respectively. Analogously, the wave function corresponding to the process of scattering a heavy hole with momentum $\mathbf{p}_2$ and helicity M_1 takes the form

$$\Psi_2=\exp(i\varkappa\mathbf{r}_\parallel)\left[\frac{\exp(-iq_2z)}{|v_{hz}|^{1/2}}\chi_{M_1}(\mathbf{p}_2)+\frac{\exp(iq_2z)}{|v_{hz}|^{1/2}}\times\sum_{M=\pm M_1}F^{M\mathbf{p}_4}_{M_1\mathbf{p}_2}\chi_M(\mathbf{p}_4)+\frac{\exp(iq_1z)}{|v_{lz}|^{1/2}}\sum_{M=\pm 1/2}F^{M\mathbf{p}_3}_{M_1\mathbf{p}_2}\chi_M(\mathbf{p}_3)\right]. \quad \text{(A4)}$$

In Eqs. (A3), (A4) v_{lz}, v_{hz} are the z-components of the light and heavy hole velocities while $\mathbf{r}_\parallel=(x,y)$.

The boundary condition for the light- and heavy-hole density matrices for the case of specular scattering can be written in the form ($z=0$)

$$f^{(l)}_{MM'}(\mathbf{p}_3)=\sum_{M_1,M_1'=\pm 1/2}K^{MM'}_{M_1M_1'}(\mathbf{p}_3,\mathbf{p}_1)f^{(l)}_{M_1M_1'}(\mathbf{p}_1)+\sum_{M_1,M_1'=\pm 3/2}K^{MM'}_{M_1M_1'}(\mathbf{p}_3,\mathbf{p}_2)f^{(h)}_{M_1M_1'}(\mathbf{p}_2),$$
$$f^{(h)}_{MM'}(\mathbf{p}_4)=\sum_{M_1,M_1'=\pm 1/2}K^{MM'}_{M_1M_1'}(\mathbf{p}_4,\mathbf{p}_1)f^{(l)}_{M_1M_1'}(\mathbf{p}_1)+\sum_{M_1,M_1'=\pm 3/2}K^{MM'}_{M_1M_1'}(\mathbf{p}_4,\mathbf{p}_2)f^{(h)}_{M_1M_1'}(\mathbf{p}_2), \quad \text{(A5)}$$

$$K^{MM'}_{M_1M_1'}(\mathbf{p}',\mathbf{p})=F^{M\mathbf{p}'}_{M_1\mathbf{p}}(F^{M'\mathbf{p}'}_{M_1'\mathbf{p}})^*. \quad \text{(A6)}$$

The kernel K, which is responsible for the surface scattering, is written in terms of the scattering amplitudes in a way analogous to the way the collision-integral kernel which describes bulk scattering is written in the kinetic equations for the bulk density matrix (see Refs. 8, 9). The coefficient of proportionality in (A6) was chosen to be unity because the quantity $K^{MM}_{M_1M_1}(\mathbf{p}',\mathbf{p})$, which is obviously the transition probability from the state $M_1\mathbf{p}$ into the state $M\mathbf{p}'$, must equal the ratio of the z-component of the current density for the wave incident on the surface. This ratio precisely equals the squared modulus of the corresponding scattering amplitudes, according to our definition of these amplitudes (see (A3), (A4)).

Let us now determine the scattering amplitude. In the parabolic spectral region (where the hole energy is small compared to energy involved in scattering into other energy bands) the wave functions Ψ_1, Ψ_2 satisfy zero boundary conditions at the surface; $\Psi_1(z=0)=\Psi_2(z=0)=0$.[10,11] Using these conditions, we obtain from (A3) and (A4) two systems of four equations each:

$$\varphi_{M_1}(\pi-\theta_l)+\sum_{M=\pm M_1}F^{M\mathbf{p}_3}_{M_1\mathbf{p}_1}\varphi_M(\theta_l)+\left(\frac{|v_{lz}|}{|v_{hz}|}\right)^{1/2}\sum_{M=\pm 3/2}F^{M\mathbf{p}_4}_{M_1\mathbf{p}_1}\varphi_M(\theta_h)=0, \quad \text{(A7)}$$

$$\varphi_{M_1}(\pi-\theta_h)+\sum_{M=\pm M_1}F^{M\mathbf{p}_4}_{M_1\mathbf{p}_2}\varphi_M(\theta_h)+\left(\frac{|v_{hz}|}{|v_{lz}|}\right)^{1/2}\sum_{M=\pm 1/2}F^{M\mathbf{p}_3}_{M_1\mathbf{p}_2}\varphi_M(\theta_l)=0, \quad \text{(A8)}$$

where $\varphi_M(\theta)$ is a four-component column vector whose λ-component is $p^{(3/2)}_{\lambda M}(\theta)$; the angles θ_l, θ_h are shown in Fig. 1. Using properties of the finite-rotation matrices,[7] we find from the system (A7), (A8) that

$$F^{M'\mathbf{p}'}_{M\mathbf{p}}=(-1)^{M'-M}F^{-M'\mathbf{p}'}_{-M\mathbf{p}}. \quad \text{(A9)}$$

Using this property, we obtain for $|M_1|=|M_1'|$

$$\sum_M K^{MM}_{M_1M_1'}(\mathbf{p}',\mathbf{p})\propto\delta_{M_1M_1'}, \quad \text{(A10)}$$

where $M=\pm 1/2$ if the momentum $\mathbf{p}'$ belongs to the light-hole band and $M=\pm 3/2$ if it belongs to the heavy-hole band. Taking the trace of equation (A5) and using property (A10), we obtain Equation (1) of the main text, in which

$$W_l^{\,l}=\sum_{M=\pm 1/2}|F^{M\mathbf{p}_3}_{M_1\mathbf{p}_1}|^2,\quad W_l^{\,h}=\sum_{M=\pm 3/2}|F^{M\mathbf{p}_4}_{M_1\mathbf{p}_1}|^2,\quad |M_1|={}^1/_2,$$

$$W_h^{\,l}=\sum_{M=\pm 1/2}|F^{M\mathbf{p}_3}_{M_1\mathbf{p}_2}|^2,\quad W_h^{\,h}=\sum_{M=\pm 3/2}|F^{M\mathbf{p}_4}_{M_1\mathbf{p}_2}|^2,\quad |M_1|={}^3/_2.$$

From the systems (A7), (A8) we finally obtain expression (2) for the probabilities W^k_i. We note that the equality of the transition probabilities (see (2)) is a consequence of the principle of detailed balance. Using (1), (2), we can easily show that the total light- and heavy-hole current incident on the surface equals the total hole current reflected from the surface.

Thus, we have established that the boundary condition (1) contains only the distribution functions (A2) and does not mix in the nondiagonal components of the density matrix. This fact is a consequence of property (A9) of the scattering amplitude, which in turn is a consequence of the zero boundary condition which the wave functions Ψ_1, Ψ_2 satisfy. It would seem that more general boundary conditions on the wave functions do not result in closed boundary conditions for the distribution functions, since they also contain the nondiagonal components of the density matrix.

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