Lifetime hierarchy of charmed and beautiful hadrons

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The preasymptotic corrections to the total hadron widths of decays of charmed and beautiful hadrons are considered. Principal attention is paid to effects connected with spectator quarks. A new feature is the inclusion in the analysis of the so-called hybrid logarithms that appear in the momentum region $R^{-2} < p^2 < m_Q^2$, where m_Q is the mass of the heavy quark and R is the confinement region. The preasymptotic corrections for charmed hadrons are found to be of the order of unity, so that the estimates are semiquantitative in character. The lifetime hierarchies of the D and F mesons and of the corresponding baryons are reliably established. The difference between the lifetimes of beautiful hadrons should certainly not exceed several percent.

1. INTRODUCTION

Noticeable progress has been recently made in the theoretical understanding of effects that are preasymptotic in the mass of the heavy (charmed) quark in hadronic decays of charmed particles. While the first attempts[1–3] to describe the experimentally observed scatter of the total widths were more readily qualitative, recent approaches are more quantitative and lead to estimates that can be theoretically monitored. The following preasymptotic mechanisms have been discussed in the literature:

1) Annihilation mechanism.[2] Contemporary estimates[4] yield for D- and F-meson decays a ratio $\Delta\Gamma_{\text{ann}}/\Gamma_0 \approx 0.2$, where Γ_0 is the hadron width in the parton model:

$$\Gamma_0 = \frac{G_F^2 m_c^5}{64\pi^3}, \quad \left(\frac{N_c+2}{N_c}\Gamma_0\right)^{-1} \approx 7\cdot 10^{-13}\ \text{s}. \tag{1}$$

Here G_F is the Fermi constant, N_c the number of colors, and m_c the c-quark mass (we neglect the deviation of $\cos\theta_C$ from unity, where θ_C is the Cabibbo angle).

2) Interference mechanism in the decay of the D^+ meson and of certain baryons. The possibility of interference between the initial spectator quark and the final quark was noted in Ref. 1. Further elaboration is contained in Refs. 5–7.

3) The $cd \to us$ scattering in baryon decay.[7]

4) Overall enhancement of the nonleptonic Hamiltonian with $\Delta C = 1$. It has been assumed that the overall enhancement is described by factors $C_\pm$ that take into account the hard gluon exchange (see, e.g., Okun's book[8]). In the principal logarithmic approximation, these factors, which are calculated in Ref. 9, differ very little from unity and lead to a modest enhancement ($\sim 10\%$). According to later arguments, however, when soft gluons are taken into account the overall enhancement of the nonleptonic Hamiltonian is substantially greater and can reach a factor $\gtrsim 1.6$. This circumstance was designated as a rule for discarding the $1/N_c$ contribution[7,10] (see also Ref. 4).

We have shown in our preceding paper[7] that the combination of effects 2)–4) leads to the following relation for the hadron widths

$$\Gamma(\Lambda_c^+) > \Gamma(D^0) \approx \Gamma(F) > \Gamma(D^+) \approx \Gamma_0. \tag{2}$$

According to Ref. 7, each step in this hierarchy differs from the preceding by a factor 1.5–1.8. The signs of the preasymptotic corrections are reliably determined. The absolute value of the effect, however, contains an uncertainty due to nonleading corrections in α_s and m_c^{-1}, as well as connected with the procedure for determining the hadron matrix element (see below). To give an idea of the matrix-element uncertainty, we point out that in the case of Λ_c^+, where the crudest approximation is used,[7] calculation of the hadron matrix element in the potential model[1)] yields for the parameter $\xi = \Gamma(\Lambda_c^+)/\Gamma_0 - 1$ double the value obtained in Ref. 7.

The main lesson learned from the analysis of Ref. 7 is the following. For charmed particles, the preasymptotic corrections, which are proportional to m_c^{-2}, are large, of the order of unity.[7] It is difficult therefore to expect for charmed-hadron decays results of high accuracy within the framework of the employed expansion in terms of m_c^{-1}. The situation with the expansion parameter improves considerably for b-quark-containing hadrons (beautiful hadrons), in which the preasymptotic corrections are significantly smaller. We shall show that in beautiful hadrons the preasymptotic corrections must certainly be less than ten percent.

Our purpose here is twofold. First, we return to consideration of preasymptotic effects in decays of charmed particles. Besides the D and F mesons and the Λ_c baryon we consider three other weakly decaying charmed baryons. The analysis of Ref. 7 is supplemented by allowance for "hybrid" logarithmic renormalizations (see Ref. 11) that stem from gluons with momenta $R^{-2} < p^2 < m_Q^2$ and have heretofore not been discussed in the literature. To be sure, this effect does not influence the qualitative conclusions of Ref. 7, and merely reinforces somewhat the hierarchy (1) of the decay widths. Second, we shall analyze the preasymptotic corrections in a beautiful-hadron decays.

Let us formulate briefly the main statements.

We consider baryons consisting of one heavy quarks Q and two light ones q ($q = u$, d, or s). Assuming that all

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quarks are in the S wave and neglecting excitations, the number of such baryons is 15. (For a detailed classification in the case $Q = c$ see, e.g., Close's book.[12]) Here we are interested only in those baryons which cannot decay as a result of strong or electromagnetic interactions. The expected number of such baryons is apparently four[2)] (more accurately, four charmed and four beautiful). Three of them[3)]

$$(Qud),\ (Qus),\ (Qds), \tag{3}$$

in which the spin and flavor variables of the light quarks are antisymmetrized, make up an $SU(3)$ antitriplet. This group includes the most thoroughly investigated Λ_c baryon. The fourth baryon

$$(Qss) \tag{4}$$

is part of a sextet; in all cases $J^P = \frac{1}{2}^+$.

Analysis of the preasymptotic effects leads us to the hadron-width hierarchy:

$$\Gamma(css) > \Gamma(cds) > \Gamma(cus) \approx \Gamma(\Lambda_c^+) \tag{5}$$

[see also Eq. (2)], with each inequality denoting here a factor 1.5–2.

In the case of beautiful hadrons, the analogous chain takes the form

$$\Gamma(bud) \approx \Gamma(bus) > \Gamma(b\bar{d}) \approx \Gamma(b\bar{s}) > \Gamma(bds) > \Gamma(bss) \approx \Gamma(b\bar{u}). \tag{6}$$

The lifetime difference is here much smaller. Incidentally, it should be immediately stipulated that the inequality chain (6) must be approached with some caution, since these relations do not allow for annihilation diagrams and for some other effects of like type. (A more detailed discussion is given in Secs. 5 and 6.) Of greatest importance to us is that the different lifetimes are an order-of-magnitude smaller in the family of beautiful hadrons than in the family of charmed particles. Indeed, the parameter that determines the spread of the hadron widths is $16\pi^2 f_D^2/m_D^2$ for charmed hadrons and $16\pi^2 f_B/m_B^2$ for beautiful ones (here f_D and f_B are the masses of these mesons). Whereas

$$16\pi^2 f_D^2/m_D^2 \approx 1.6, \tag{7}$$

for b quarks we have

$$16\pi^2 f_B^2/m_B^2 \approx 0.08. \tag{8}$$

An order of magnitude is used because of the mass ratio m_D^2/m_B^2, and the factor $\approx \frac{1}{2}$ is due to the f_B^2/f_D^2 ratio.[14,11] A natural estimate for $\Delta\Gamma$ (b hadrons)/Γ_0 would therefore be $\sim 10\%$. It will be shown below, however, that owing to numerical factors (logarithmic renormalization and others) additional cancellation and suppression of the preasymptotic corrections take place. Literally our estimates yield $\Delta\Gamma$ (b hadrons)/$\Gamma_0 \sim 1\%$. To ensure against possible indeterminacies (which will be listed below), we formulate a conservative expected value of the spreads of the hadron widths $\Delta\Gamma$ of the particles B_u, B_d, Λ_b, $\Xi_b^{(A)}$, Ω_b:

$$\Delta\Gamma \lesssim 0.05\Gamma_{0b}, \tag{9}$$

where Γ_{0b} is the average width of the beautiful hadrons. A new mechanism, $B\bar{B}$ mixing, comes into play for B_s^0 mesons. The B_s^0 mesons therefore drop out of the general analysis. We note here only that the difference between the widths $\Delta\Gamma$ is in this case also $\lesssim 0.06\Gamma_{0b}$.

The plan of the article is the following: In Sec. 2 we obtain the effective Lagrangian that determines the power-law corrections to the charmed-hadron widths. In Sec. 3 is calculated the logarithmic renormalization of this Lagrangian for momenta smaller than the mass of the heavy quark (called hybrid logarithms). In Sec. 4 are discussed the matrix elements of the effective Lagrangian in the charmed hadrons. Beautiful hadrons are considered in Sec. 5. Section 6 contains the concluding remarks and deductions.

2. EFFECTIVE LAGRANGIAN FOR THE DECAY WIDTHS OF CHARMED HADRONS

The principles of the theory of preasymptotic effects in the hadronic widths of D and F mesons and of the corresponding baryons were discussed in detail in Ref. 7. We shall note here only the main points and focus on the aspects heretofore not considered ($\Xi_c^{(A)}$ and Ω_c baryons, hybrid logarithms).

As shown in Ref. 7, the parton result for the decay width of the c quark, as well as the asymptotic corrections [see mechanisms 1) and 4) listed in the Introduction] can be obtained from a certain effective Lagrangian. The latter is the imaginary part of an operator that appears in second order in the weak-interaction Lagrangian L_W:

$$L_{eff} = \mathrm{Im}\left[i \int d^4x\, e^{iqx} T\{L_W(x), L_W(0)\} \right]. \tag{10}$$

The decay width of hadron X is here

$$\Gamma_X = \frac{1}{m_X}\langle X | L_{eff} | X \rangle. \tag{11}$$

By applying to L_{eff} an analog of the operator expansion in inverse powers of the heavy-quark mass yields an expansion of the width Γ_X in powers of m_Q^{-1}. It must be stipulated that we are not referring here to a real Wilson expansion operator in a Euclidean region. In our problem the kinematics is pseudo-Euclidean and we must resort additionally to the duality concept (see Ref. 7 for details). The result of the pseudo-Euclidean kinematics is, in particular, the fact that besides the logarithmic renormalizations considered in the present paper there exist also power-law IR effects of the type considered by Bander *et al.*[2] Since these effects are small, they are of no significance whatever for charmed hadrons. A more detailed discussion is contained in Secs. 5 and 6.

The weak Lagrangian with $\Delta C = 1$ is of the form

$$L_W = \frac{G_F}{\sqrt{2}} \left\{ \frac{C_+ + C_-}{2} (\bar{s}\Gamma_\mu c)(\bar{u}\Gamma_\mu d) + \frac{C_+ - C_-}{2} (\bar{u}\Gamma_\mu c)(\bar{s}\Gamma_\mu d) \right\}, \tag{12}$$

where $\Gamma_\mu = \gamma_\mu(1+\gamma_5)$, and the coefficients C_+ and C_- reflect the renormalizations connected with gluons having momenta $m_c < p < m_W$ (see, e.g., Ref. 8):

$$C_- = C_+^{-2} = [\alpha_s(m_c)/\alpha_s(m_W)]^{12/25}. \tag{13}$$

The numerical values at $\alpha_s(m_W) = 0.4$ and $\alpha_s(m_c)$

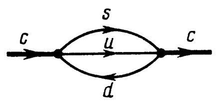

FIG. 1.

$= 0.25$ (which corresponds to $\Lambda_{QCD} \approx 100$ MeV) are

$$C_- \approx 1.58, \quad C_+ \approx 0.80. \qquad (14)$$

The leading term of the expansion of L_{eff} in powers of m_c^{-1} is obtained by closing three light-quark lines in the correlator (10) (see Fig. 1), and takes the form

$$L_{eff}^{(0)} \approx 1.1 \frac{C_+^2 + C_-^2}{2} \frac{G_F^2 m_c^5}{64\pi^3} \frac{\bar{c}c}{2}. \qquad (15)$$

With allowance for the relation $\langle c|\bar{c}c|c\rangle = 2m_c$, this term yields the known expression for the nonleptonic decay width of a "free" c quark:

$$\Gamma_c \equiv \Gamma(c \to \text{hadron}) \approx 1.1 \frac{C_+^2 + C_-^2}{2} \frac{G_F^2 m_c^5}{64\pi^3}. \qquad (16)$$

The coefficient $1.1(C_+^2 + C_-^2)/2$ reflects the overall enhancement of the c quark nonleptonic decays, and the neglected contribution of order N_c^{-1} have already been eliminated from it.[7,10]

The next expansion terms of L_{eff} are obtained by "opening" one of the light-quark lines (see Fig. 2). With allowance for these terms, L_{eff} takes the form

$$L_{eff} = L_{eff}^{(0)} + L_{eff}^{(a)} + L_{eff}^{(b)} + L_{eff}^{(c)}, \qquad (17)$$

where

$$L_{eff}^{(a)} = \frac{G_F^2 m_c^2}{4\pi} \left\{ \frac{C_+^2 + C_-^2}{2} (\bar{c}\Gamma_\mu c)(\bar{d}\Gamma_\mu d) + \frac{C_+^2 - C_-^2}{2} (\bar{c}\Gamma_\mu d)(\bar{d}\Gamma_\mu c) \right\}, \qquad (17a)$$

$$L_{eff}^{(b)} = -\frac{G_F^2 m_c^2}{8\pi} \left\{ \left(\frac{C_+ + C_-}{2}\right)^2 \left(\bar{c}\Gamma_\mu c - \frac{2}{3}\bar{c}\gamma_\mu\gamma_5 c\right) \bar{u}\Gamma_\mu u + {}^1/_4 (5C_+^2 + C_-^2 - 6C_+C_-)(\bar{c}_i\Gamma_\mu c_k - {}^2/_3 \bar{c}_i\gamma_\mu\gamma_5 c_k)(\bar{u}_k\Gamma_\mu u_i) \right\}, \qquad (17b)$$

$$L_{eff}^{(c)} = -\frac{G_F^2 m_c^2}{8\pi} \left\{ \left(\frac{C_+ - C_-}{2}\right)^2 (\bar{c}\Gamma_\mu c - {}^2/_3 \bar{c}\gamma_\mu\gamma_5 c)(\bar{s}\Gamma_\mu s) + {}^1/_4 (5C_+^2 + C_-^2 + 6C_+C_-) \times (\bar{c}_i\Gamma_\mu c_k - {}^2/_3 \bar{c}_i\gamma_\mu\gamma_5 c_k)(\bar{s}_k\Gamma_\mu s_i) \right\}. \qquad (17c)$$

The first two terms $L_{eff}^{(a)}$ and $L_{eff}^{(b)}$ of (17), which correspond to diagrams 2a and 2b, were obtained in Ref. 7, while the third term with open s-quark line (Fig. 2c) is obtained from the corresponding term with the u quark by substituting $C_- \to -C_-$, as readily seen from the Lagrangian (12).

The terms (17a)–(17c) describe the effects 2)–4) (Ref. 7). Thus, for example, when averaged over the D^+ meson, the operator (17a), which contains a d quark, describes the (destructive) interference between the d quark produced in c-quark decay and the spectator d quark contained in the D^+ meson. The same operator averaged over Λ_c^+ describes the contribution of the $cd \to us$ scattering. The operators with u and s quarks describe the interference between the spectator quarks in baryons and the c quark produced in the decay (this interference is destructive for the u quark and constructive for s). Note that the Lagrangian (17) does not contain the annihilation mechanism in the D^0 and F decays. The reason is that we have neglected the masses of the light quarks in the loops of the graphs of Fig. 2, and if gluon emission is disregarded the contribution of the annihilation mechanism is proportional to the light-quark mass. The annihilation-mechanism role will be discussed briefly in Sec. 6.

The normalization point μ of all the operators in (17a)–(17c) is of the order m_c. On the other hand, the characteristic virtualities of the quarks in D and F mesons and in baryons is of the order of $\mu \sim R^{-1}$, where R is the confinement radius. In the calculation of the corresponding effects it is necessary to take into account the evolution of the operators from $\mu \sim m_Q$ to $\mu \sim R^{-1}$. It was observed in Ref. 11 that this gives rise to hybrid terms of type $\alpha_s \ln(m_Q R)$, which are infrared relative to the heavy quark and ultraviolet relative to the light ones. They will be summed in the principal logarithmic approximation in Sec. 3.

3. LOGARITHMIC RENORMALIZATION OF L_{eff} IN THE LOW-TEMPERATURE REGION

To estimate the hadronic matrix elements of the Lagrangian (17) we shall use the nonrelativistic picture of the constituent quarks. Clearly, this picture is valid for quarks having a virtuality μ determined by the radius R within which the quarks are confined in the hadron: $\mu \sim R^{-1}$. In the calculation of the matrix element L_{eff} over a state with virtuality $\mu \sim R^{-1}$, the gluons exchanges introduce corrections $O[(\alpha_s \ln m_c R)^k]$ that must be summed by the renormalization-group method.

As applied to the Lagrangian (17), these renormalizations reduce to the following. Operators in which are multiplied colorless structures that are bilinear in the heavy and light quarks, e.g., $(\bar{c}\Gamma c)(\bar{q}\Gamma' q)$, are not renormalized in the region considered, regardless of the explicit form of Γ and Γ'. Operators in which the color flows over from the heavy quark to the light one, i.e., $\mathcal{O} = (\bar{c}\Gamma t^a q)(\bar{q}\Gamma' t^a c)$, are renormalized on account of diagrams of the type shown in Fig. 3, and their matrix elements in the state with virtually μ are determined by the universal formula (which is independent of Γ and Γ')

$$\langle \mathcal{O}'(m_c)\rangle = \langle \mathcal{O}(\mu)\rangle [\alpha_s(\mu)/\alpha_s(m_c)]^{9/2b}, \qquad (18)$$

where the parentheses contain the normalization point, the angle brackets designate a matrix element n a state with

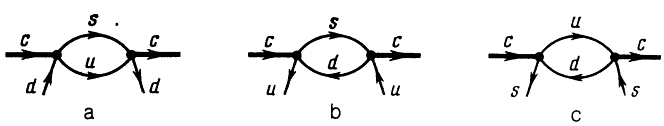

FIG. 2.

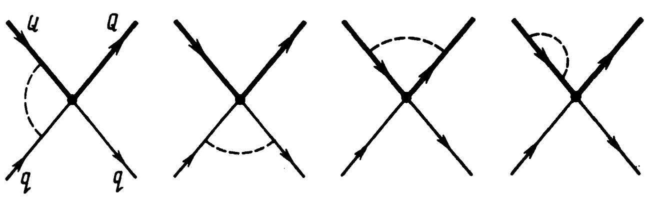

FIG. 3.

characteristic virtuality μ, and b is the first coefficient of the β function, viz., $b = 11 - 2n_f/3$ (n_f is the number of unfrozen quark types). More details concerning the calculation of the factor $[\alpha_s(\mu)/\alpha_s(m_c)]^{9/2b}$ (more accurately, of its analog), can be found in Ref. 11.

The prime sign on the operator $\mathscr{O}$ in (18) means that it is necessary to separate from the operator the structure that has with respect to the light quarks the form of the colored quark current

$$j^a=\bar{u}\gamma_\mu t^a u+\bar{d}\gamma_\mu t^a d+\bar{s}\gamma_\mu t^a s, \qquad (19)$$

where t^a is an $SU(3)$ generator in the fundamental representation. This structure is additionally renormalized on account of the "penguin" diagram shown in Fig. 4:

$$\langle j^a(m_c)\rangle=\langle j^a(\mu)\rangle[\alpha_s(\mu)/\alpha_s(m_c)]^{-2/b}. \qquad (20)$$

Allowance for the renormalization described by Eqs. (18) and (29) leads to the following (rather unwieldy) expression for the effective Lagrangian (17) in terms of operators normalized at the point $\mu \sim R^{-1}$:

$$\frac{64\pi^3}{G_F^2 m_c^5} L_{eff}=1,1\frac{C_+^2+C_-^2}{2}\frac{\bar{c}c}{2}+\frac{16\pi^2}{m_c^3}$$
$$\times\{{}^1/_2[C_+^2+C_-^2+{}^1/_3(1-\varkappa^{1/2})(C_+^2-C_-^2)]$$
$$\times(\bar{c}\Gamma_\mu c)(\bar{d}\Gamma_\mu d)+{}^1/_2(C_+^2-C_-^2)\varkappa^{1/2}(\bar{c}\Gamma_\mu d)(\bar{d}\Gamma_\mu c)$$
$$+{}^1/_3(C_+^2-C_-^2)\varkappa^{1/2}(\varkappa^{-2/9}-1)(\bar{c}\Gamma_\mu t^a c)j_\mu^a-{}^1/_8[(C_++C_-)^2$$
$$+{}^1/_3(1-\varkappa^{1/2})(5C_+^2+C_-^2-6C_+C_-)](\bar{c}\Gamma_\mu c-{}^2/_3\bar{c}\gamma_\mu\gamma_5 c)\bar{u}\Gamma_\mu u$$
$$-{}^1/_8\varkappa^{1/2}(5C_+^2+C_-^2-6C_+C_-)$$
$$\times(\bar{c}_i\Gamma_\mu c_k-{}^2/_3\bar{c}_i\gamma_\mu\gamma_5 c_k)\bar{u}_k\Gamma_\mu u_i-{}^1/_8[(C_+-C_-)^2$$
$$+{}^1/_3(1-\varkappa^{1/2})(5C_+^2+C_-^2+6C_+C_-)](\bar{c}\Gamma_\mu c-{}^2/_3\bar{c}\gamma_\mu\gamma_5 c)$$
$$\times\bar{s}\Gamma_\mu s-{}^1/_8\varkappa^{1/2}(5C_+^2+C_-^2+6C_+C_-)(\bar{c}_i\Gamma_\mu c_k-{}^2/_3\bar{c}_i\gamma_\mu\gamma_5 c_k)$$
$$\times(\bar{s}_k\Gamma_\mu s_i)-{}^1/_6\varkappa^{1/2}(\varkappa^{-2/9}-1)(5C_+^2+C_-^2)(\bar{c}\Gamma_\mu t^a c-{}^2/_3\bar{c}\gamma_\mu\gamma_5 t^a c)j_\mu^a\}, \qquad (21)$$

where

$$\varkappa=\alpha_s(\mu)/\alpha_s(m_c). \qquad (22)$$

We took it into account also that $b = 9$.

4. DECAY-WIDTH HIERARCHY OF CHARMED HADRONS

To estimate the total nonleptonic widths of charmed hadrons it is necessary to calculate matrix elements of type (11) for the effective Lagrangian (21). We consider these matrix elements under the assumption that the virtuality μ of the quarks in the hadrons corresponds to $\alpha_s(\mu) \approx 1$, so that $\varkappa \approx 4$. (This assumption was introduced in Ref. 15 in connection with an analysis of strange-particle decays; see also Ref. 16.) We neglect also the weak renormalization of the current j^a [see Eq. (20)]. This means that we neglect in (21) the two terms containing the factor $\varkappa^{-2/3} - 1$. Of course, numerical estimations of all the coefficients in (21) will be useful only for approximations. In fact, the coefficients $C_\pm$ can contain nonleading corrections $O(10\%)$. The factor $\varkappa$ is known to the same if not worse accuracy. In the best case, therefore, when there are no special compensating factors, we can count on a coefficient accuracy on the order of 20%. In number of matrix elements, the contributions proportional to C_+ and C_- cancel one another in part, and in these cases the uncertainty of the coefficient can reach a factor 1.5.

The matrix element of the operator $\bar{c}c$ for all the charmed hadrons is universal in our nonrelativistic approximation and amounts to $2m_c$. The contribution of the leading operator (15) to the widths is therefore also universal and amounts to

$$\Gamma_c=1,1\frac{C_+^2+C_-^2}{2}\Gamma_0\approx1.7\Gamma_0. \qquad (23)$$

We illustrate our estimate of the remaining matrix element with the D^+ meson as the example. In main outline, the procedure duplicates the approach of Ref. 7; the difference lies only in the allowance for the logarithmic renormalization.

In accord with the old practice[17] we assume that the operators $(\bar{c}\Gamma_\mu d)(\bar{d}\Gamma_\mu c)$ and $(\bar{c}_i\Gamma_\mu d_k)(\bar{d}_k\Gamma_\mu c_i)$ renormalized at the point $\mu \sim R^{-1}$ can be factored,[4)] so that

$$\langle D^+|(\bar{c}\Gamma_\mu d)(\bar{d}\Gamma_\mu c)|D^+\rangle$$
$$=3\langle D^+|(\bar{c}_i\Gamma_\mu d_k)(\bar{d}_k\Gamma_\mu c_i)|D^+\rangle=f_D^2 m_D^2\varkappa^{-4/9}, \qquad (24)$$

where f_D is the constant of the transition of a D meson into an axial current:

$$\langle D^+|\bar{c}\gamma_\mu\gamma_5 d|0\rangle=-if_D p_\mu, \qquad (25)$$

with the operator in (25) normalized at the point m_c (or a higher one) the difference between the normalization points of the operators in (25) and (24) leads to the appearance of the factor $\varkappa^{-4/9}$ in (24). (This factor is calculated in Ref. 11.)

Instead of using the factorization (or vacuum "interlining") technique, we can estimate the matrix elements (24) in a nonrelativistic model that is nonrelativistic in both quarks. In this model we have, for example,

$$\langle D^+|(\bar{c}\Gamma_\mu d)(\bar{d}\Gamma_\mu c)|D^+\rangle=-N_c 2m_c V^{-1}(1+4\mathbf{S}_c\mathbf{S}_d)=12m_c V^{-1}, \qquad (26)$$

where $\mathbf{S}_c$ and $\mathbf{S}_d$ are the spin operators of the s and $\bar{d}$ quarks,

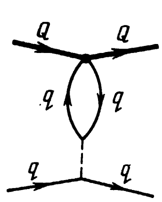

FIG. 4.

and $V = |\psi(0)|^{-2}$ is the volume occupied by the $\bar{d}$ quark in the meson. Comparing (24) and (26), we can estimate V^{-1} from

$$2m_c V^{-1} \approx f_D^2 m_D^2 \varkappa^{-4/9}/6. \qquad (27)$$

We shall use this estimate of the parameter V^{-} also when considering baryon matrix elements, although this approximation is apparently valid only accurate to ≈ 2. The last circumstance is indicated by an analysis, carried out by A. M. Badalyan and co-workers, of the three-particle problem in the nonrelativistic model with a funnel-type potential. The quantity $\int |\psi(r_{12}=0, r_{13})|^2 d^3 r_{13}$, which plays the role of V^{-1} when suitably renormalized, turned out to be twice as large as in (27).

Using relation (24), we obtain the correction to the total nonleptonic width of D^+ on account of the four-quark operators in (21):

$$\frac{\Delta\Gamma(D^+)}{\Gamma_0} \approx \frac{16\pi^2 f_D^2}{m_D^2} \frac{\varkappa^{-4/9}}{2} \Big\{ (C_+^2 - C_-^2)\varkappa^{1/2} + \frac{1}{3}\Big[C_+^2 + C_-^2 + \frac{1}{3}(1-\varkappa^{1/2})(C_+^2 - C_-^2)\Big]\Big\} \approx -1.0. \qquad (28)$$

(We have substituted here[14] $f_D \approx 160$–170 MeV, which yields $f_D^2/m_D^2 \approx 10^{-2}$.) It can be seen from this estimate that the logarithmic renormalizations enhance somewhat the correction $\Delta\Gamma(D^+)$ (without allowance for these renormalizations, i.e., at $\varkappa = 1$, we get[7] $\Delta\Gamma(D^+)/\Gamma_0 \approx -0.6$). It can be seen that the lifetime ratio $\tau(D^+)/\tau(D^0) \approx 2$ is perfectly understandable, at least semiquantitatively, within the framework of the approach discussed.

Recall (see Ref. 7) that $\Gamma(D^0)$ agrees with the estimate (23) apart from annihilation effects [that increase $\Gamma(D^0)$ by $\approx 20\%$ compared with (23), see Ref. 4].

We proceed now to discuss the baryon matrix elements of $L_{\rm eff}$. It can be easily noted that expression (21) contains paired operators that differ in permutation of the color variables of the light quark and the c quark. In view of the color antisymmetry of the wave function of the baryon, the matrix elements of the operators in these pairs differ only in sign. Next, within the framework of the nonrelativistic picture, the quark operators can be replaced by the nonrelativistic expressions:

$$(\bar{c}\Gamma_\mu c)(\bar{d}\Gamma_\mu d) \to 2m_c V^{-1}(1 - 4\mathbf{S}_c\mathbf{S}_d),$$
$$(\bar{c}\Gamma_\mu c - {}^2/_3 \bar{c}\gamma_\mu\gamma_5 c)(\bar{q}\Gamma_\mu q) \to 2m_c V^{-1}(1 - {}^4/_3 \mathbf{S}_c\mathbf{S}_q), \qquad (29)$$

where the spin part stems from a product of axial currents, and the spin-independent part from a product of vector currents. Actually, spin part is significant only in a baryon of type (4), i.e., css, in which the ss diquark has a spin $S_{ss} = 1$. In this case

$$\langle \mathbf{S}_c(\mathbf{S}_{s1} + \mathbf{S}_{s2})\rangle = \langle \mathbf{S}_c \mathbf{S}_{ss}\rangle = -1, \quad \langle \Omega_c | 1 - {}^4/_3 \mathbf{S}_c\mathbf{S}_s | \Omega_c\rangle = 2 + {}^4/_3.$$

In baryons of type (3), the spin of the heavy quark does not correlate with the spin of any of the light ones, and the spin terms in operators (26) are inessential.

Thus, using the estimate (27) and substituting the numerical coefficients, we obtain the corrections to the total nonleptonic widths of charmed baryons in the following form:

$$\Delta\Gamma(\Lambda_c)/\Gamma_0 \approx 0.54_d - 0.18_u = 0.36,$$
$$\Delta\Gamma(cus)/\Gamma_0 \approx 0.55_s - 0.18_u = 0.37,$$
$$\Delta\Gamma(cds)/\Gamma_0 \approx 0.54_d + 0.55_s \approx 1.1, \qquad (30)$$
$$\Delta\Gamma(css)/\Gamma_0 \approx {}^{10}/_3 \cdot 0.55_s \approx 1.8,$$

[cf., also Eq. (28)]. The subscripts d, u, and s designate those operators in (21) which contain the corresponding quarks responsible for the given contribution. Using (30), we can compare the contributions of the various mechanisms. The contribution of the $cd \to us$ scattering (subscript d) is almost identical with the (constructive) interference of the spectator quark with the decay scattering (subscript s). The smallest contribution is that of the (destructive) u-quark interference (u). The factor 10/3 in the last line of (30) corresponds to the contribution of two s quarks with allowance for the spin correlation. Relations (30) obviously lead to the chain of inequalities (5).

Note that if we discard in the contribution of the four-quark operators, just as in Γ_c (16), the terms containing the factor N_c^{-1} (with the logarithmic factors fixed), the numerical estimates change somewhat:

$$\Delta\Gamma(D^+)/\Gamma_0 \approx -1.5.\ \Delta\Gamma(\Lambda_c)/\Gamma_0 \approx 0.52,\ \Delta\Gamma(cus)/\Gamma_0 \approx 0.58,\ \Delta\Gamma(cds)/\Gamma_0 \approx 1.0,\ \Delta\Gamma(css)/\Gamma_0 \approx 1.9.$$

This change, however, is within the limits of the expected accuracy of the described approach and obviously does not change the character of discussed charmed-particle lifetime hierarchy.

5. PREASYMPTOTIC EFFECTS IN DECAYS OF BEAUTIFUL HADRONS

The Lagrangian of the nonleptonic weak interaction for b-hadron decays via the $b \to c$ transition is of the form

$$L_W = 2^{-1/2} V_{cb} G_F \{ {}^1/_2 (C_+ + C_-)(\bar{c}\Gamma_\mu b)[\bar{d}\Gamma_\mu u + \bar{s}\Gamma_\mu c] + {}^1/_2 (C_+ - C_-)(\bar{c}_i \Gamma_\mu b_k)[\bar{d}_k \Gamma_\mu u_i + \bar{s}_k \Gamma_\mu c_i]\}, \qquad (31)$$

where V_{cb} is the Kobayashi-Maskawa matrix element that describes the $b \to c$ transition (we have neglected V_{ub} compared with V_{cb} and put $V_{cs} = V_{ud} = 1$). The coefficients C_+ and C_- are now determined by the constant $\alpha_s(m_b) \approx 0.16$:

$$C_- = C_+^{-2} = (\alpha_s(m_b)/\alpha_s(m_W))^{12/23},$$
$$C_- \approx 1.28, \quad C_+ \approx 0.88. \qquad (32)$$

The width of the $b \to c\bar{c}s$ decay is substantially suppressed by the phase volume prepared with the width $b \to c\bar{u}d$ (by an approximate factor of two) in view of the presence of two massive c quarks in the final state. At the same time, the contribution of the term $(\bar{c}\Gamma_\mu b)(\bar{s}\Gamma_\mu c)$ in the preasymptotic corrections described by the diagram of Fig. 5 is not so strongly suppressed. In fact, the suppression factor in the diagram of Fig. 5 is the two-particle phase volume at $q^2 = m_b^2$. More accurately speaking, allowance for the c-quark mass in the diagram of Fig. 5 leads to the factor

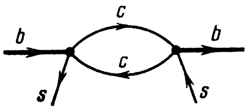

FIG. 5.

$$v\frac{3-v^2}{2}=(1-4m_c^2/m_b^2)^{1/2}(1+2m_c^2/m_b^2)=1-2m_c^4/m_b^4+\dots.$$

It is therefore apparently reasonable to neglect the c-quark mass in the discussion of the corrections that are preasymptotic in m_b^{-1}. In this approximation, the four-quark part of $L_{\rm eff}$ can be easily recalculated from (21) (the u quark in b-hadron decays corresponds to a d quark in c-hadron decays, while the d and s quarks correspond to the u quark in c-hadron decays). As a result, writing down only the four-quark part of $L_{\rm eff}$ (which is the cause of the different widths of the different beautiful hadrons), we have

$$\begin{aligned}L_{eff}^{(4)}=&|V_{cb}|^2(G_F^2m_b^2/4\pi)\\ &\times\{{}^1/_2[C_+^2+C_-^2+{}^1/_3(1-\varkappa^{1/2})](C_+^2-C_-^2)(\bar b\Gamma_\mu b)(\bar u\Gamma_\mu u)\\ &+{}^1/_2(C_+^2-C_-^2)\varkappa^{1/2}(\bar b\Gamma_\mu u)(\bar u\Gamma_\mu b)\\ &+{}^1/_3(C_+^2-C_-^2)\varkappa^{1/2}(\varkappa^{-2/9}-1)(\bar b\Gamma_\mu t^a b)j_\mu^a\\ &-{}^1/_8[(C_++C_-)^2+{}^1/_3(1-\varkappa^{1/2})(5C_+^2+C_-^2-6C_+C_-)](\bar b\Gamma_\mu b\\ &-{}^2/_3\bar b\gamma_\mu\gamma_5 b)(\bar d\Gamma_\mu d+\bar s\Gamma_\mu s)-{}^1/_8\varkappa^{1/2}(5C_+^2+C_-^2-6C_+C_-)\\ &\times(\bar b_i\Gamma_\mu b_k-{}^2/_3\bar b_i\gamma_\mu\gamma_5 b_k)(\bar d_k\Gamma_\mu d_i+\bar s_k\Gamma_\mu s_i)\\ &-{}^1/_6\varkappa^{1/2}(\varkappa^{-2/9}-1)(5C_+^2+C_-^2-6C_+C_-)\\ &\times(\bar b\Gamma_\mu t^a b-{}^2/_3\bar b\gamma_\mu\gamma_5 t^a b)j_\mu^a\},\end{aligned}\qquad(33)$$

where in the present section

$$\varkappa=\alpha_s(\mu\sim R^{-1})/\alpha_s(m_b)\approx 6.2\qquad(34)$$

and in the exponents of $\varkappa$ we neglect for simplicity the unfreezing of the virtual c quarks at momenta lower than m_b, i.e., we put $b=9$. Recall that $t^a=\lambda^a/2$ are generators,

$$j_\mu^a=\sum_{u,d,s}\bar q\gamma_\mu t^a q.$$

The matrix elements of the effective Lagrangian (33) are estimated in the same way as for charmed hadrons. We assume therefore for f_B the value

$$f_B\approx 110\ \text{MeV}\qquad(35)$$

which is midway between the two predictions of Ref. 14 (see also Ref. 11). We use for the normalization the lepton width of the b hadrons

$$\Gamma_l\equiv\Gamma(B\to l\nu_l X)\approx 0.6|V_{cb}|^2\frac{G_F^2m_b^5}{192\pi^3},\qquad(36)$$

where the block factor 0.6 (Ref. 8) reflects the fact that the c quark has a nonzero mass in the final state. The value of Γ_l is, according to the experimental data,[18] approximately 13% of the total width.

Calculations similar to those of the preceding section lead to the following estimates:

$$\begin{aligned}&\Delta\Gamma(B_d^0)\approx\Delta\Gamma(B_s^0)\approx 0,\quad \Delta\Gamma(B^-)/\Gamma_l\approx -0.1,\quad \Delta\Gamma(bud)/\Gamma_l\\ &\qquad\approx\Delta\Gamma(bus)/\Gamma_l\approx 0.07_u-0.03_{d,s}\approx 0.04,\\ &\Delta\Gamma(bds)/\Gamma_l\approx -2\cdot 0.03\approx -0.06,\\ &\Delta\Gamma(bss)/\Gamma_l\approx {}^{10}/_3(-0.03)\approx -0.1,\end{aligned}\qquad(37)$$

corresponding to a chain of relations (6). If these results are taken literally, the scatter in the nonleptonic widths of beautiful hadrons should not exceed $\approx 1\%$ of the total width.

A few explanatory remarks are in order here. First, we have neglected in the estimates those terms of (33) which are proportional to $\varkappa^{-2/9}-1$. The coefficient of these terms is numerically suppressed by an approximate factor of 6 compared with the normal order of magnitude, a common feature of penguin contributions.[15] Note also that j^a is an $SU(3)$ singlet. The corresponding terms in (33) therefore do not cause the widths of the three mesons to differ from one another, nor do they cause differences between the widths of the three baryons (3). The terms proportional to j^a can only make the widths of the mesons different from those of the baryons (3) and/or different from the width of the (bss) baryon. However, as already noted, the effect is negligibly small numerically. Second, the Lagrangian (33) takes no account of the annihilation mechanism, since it does not reflect effects connected with gluon exchange and with the c-quark mass in the final state.[5)] The annihilation contributions to nonleptonic decays, induced by exchange of soft (nonperturbative) gluons), were investigated in the case of the D^0 meson in Ref. 4 by the method of the QCD sum rules. It was found that this mechanism increases the nonleptonic D^0 width by about 20%. If this is so, the relative role of this mechanism in the B_d^0 and B_s^0 decays can be easily estimated:

$$\frac{\Delta\Gamma_{ann}}{\Gamma}\approx 0.2\frac{f_B^2/m_B^2}{f_D^2/m_D^2}\approx 0.01.\qquad(38)$$

Approximately the same value can be obtained from $b\bar d(b\bar s)$ annihilation, if we lift the chiral suppression by "turning-on" of the c-quark mass rather by lifting the chiral suppression.

Finally, the third source of the uncertainty is the effect of Bander *et al.*[2] viz., emission of a magnetic-type hard gluon by the initial light quark [see Fig. 6, which shows by way of example the corresponding interference-mechanism correction for $\Gamma(B^-)$]. If we mentally remove in Fig. 6 the dashed line representing the gluon, we obtain a skeleton diagram that generates the first term of the Lagrangian (33). This term leads in turn to the estimate (37) for $\Delta\Gamma(B^-)/\Gamma_l$. The Lagrangian (33) takes also into account the "dressing" by gluons in the leading logarithmic approximation.

The gluon of Fig. 6 does not yield a logarithm. However, as noted by Bander *et al.*,[2] in the imaginary part of the diagram this gluon yields, in view of the infrared behavior of

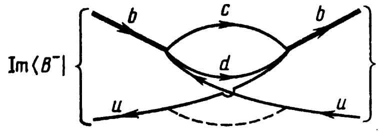

FIG. 6.

the u-quark propagator, a power-law enhancement of the type

$$(\alpha_s/4\pi)\,(M_B/\langle E\rangle)^2 \tag{39}$$

compared with (33). Here $\langle E\rangle$ is the average energy of the $\bar{u}$ quark in the B^- meson, and $\langle E\rangle \gtrsim 300$ MeV. Generally speaking, it must be concluded on the basis of (39) that the effect of Bander *et al.* can change the estimates (37) by approximately two or three times and change the annihilation contribution to B_d^0 and B_s^0 by several percent. Note, however, that in Ref. 18 the effect of Bander *et al.* was calculated for $B^0\overline{B}^0$ mixing and it was found that in this case the numerical coefficient in (39) is actually of order $\alpha_s/16\pi$. An even smaller factor, of order $\alpha_s/32\pi$, enters in the ratio of the annihilation diagram with a hard gluon to that without a gluon.[2] If this were so also in other cases, the effect of Bander *et al.* would be insignificant in the family of beautiful particles. (It can be neglected, at any rate, for charmed particles.)

According to (37), the nonleptonic width of Ω_b (bss) is 1.3% smaller than the average width Γ_{0b} of the beautiful hadrons.[6)] Taking into account, however, our remarks concerning the crude nature of the model used to estimate the matrix element $\langle \Omega_b | L_{\rm eff} | /\Omega_b\rangle$, as well as the other uncertainties listed above, we adhere to our cautious standpoint and formulate the conservation prediction that the lifetime difference in the family of beautiful hadrons is $O(5\%)$.

6. CONCLUSION

We have investigated preasymptotic effects in inclusive nonleptonic widths of beautiful and charmed hadrons. Can it be stated that a theory of these effects exists at present?

The answer to this question is both yes and no. A theory based only on first principles of QCD does not exist and will hardly be developed in the nearest future, in view of the essentially pseudo-Euclidean kinematics of the investigated processes. We have nonetheless succeeded in developing a systematic approach based on the abundant experience gained in QCD. The most substantial additional assumption invoked by us is quark-hadron duality as $m_Q \to \infty$. This duality certainly takes place asymptotically. We are thus able to estimate the interference effects in mesons and baryons, and Qq scattering in baryons, in the form of expansions in terms of m_Q^{-1}. For beautiful hadrons these preasymptotic corrections are small (of order of 1% or possibly several percent), so that the results for the spread of the widths can in principle be made quantitative with an accuracy that might be theoretically known beforehand. Unfortunately, the problem has not yet been completely solved, since the four-fermion operators of (33) are averaged over the b-hadronic states very crudely. This aspect, however, will undoubtedly be improved. For B_s^0 and B_d^0 mesons, it is necessary also to calculate the contribution of the annihilation mechanism and to evaluate in all cases the effect of Bander *et al.*,[2] in analogy with the procedure used in Ref. 18.

The conclusion that the lifetimes of all the beautiful hadrons are equal within $O(5\%)$ is of practical importance for the analysis of the problem of CP invariance in a $(B^0 - \overline{B}^0)$ meson system.

As for charmed hadrons, the parameter of the expansion in terms of m_c^{-1} turns out here to be of the order of unity and we can hardly expect a quantitative description of inclusive nonleptonic widths. Nonetheless, the qualitative picture reflected in Eqs. (2) and (5) should hold. It is instructive to compare the hierarchy (2) and (5) obtained in our approach with the presently available experimental information on the lifetimes of D and F mesons and baryons.

Figure 7 shows the experimental data (heavy points), borrowed from the review by Caso and Touboul.[20] The solid and dashed lines indicate the statistical and systematic errors of Ω_c^0. The baryon Ξ_c^0 has not yet been observed in experiment.

The vertical dotted lines to the left of the experimental "whiskers" represent somewhat arbitrarily our predicted lifetimes. We used Eqs. (28) and (30) for the power-law preasymptotic corrections, and simultaneously took into account the overall enhancement of the c-quark decay width (23). We write thus, say for the nonleptonic D^+-meson width,

$$\Gamma(D^+) = \Gamma_0(1.7-1.0) \approx 0.7\Gamma_0.$$

The experimental lifetime of D^0 was used for normalization [i.e., to fix the parameter m_c in Γ_0, see Eq. (1)]. By way of estimate, we obtained by the described method the ratios $\Gamma_{\rm tot}(X)/\Gamma_{\rm tot}(D^0)$, where $\Gamma_{\rm tot}(X) = \Gamma_{\text{non-lept}}(X) + \Gamma_{sl}(X)$ and $\Gamma_{sl}(X) \approx \frac{2}{3}\Gamma_0 \approx G_F^2 m_c^5/96\pi^3$. It is these ratios which are shown in Fig. 7 as theoretical expectation values of the total lifetimes.

It can be seen that on the whole the theoretical picture accords with experiment. It would nonetheless be desirable to "stretch" the experimental values of τ_F and τ_{Λ_c} slightly upward and to lower substantially $\tau_{\Xi_c^+}$, and especially τ_{Ω_c}. Within the framework of our approach Ω_c^0 is the shortest-

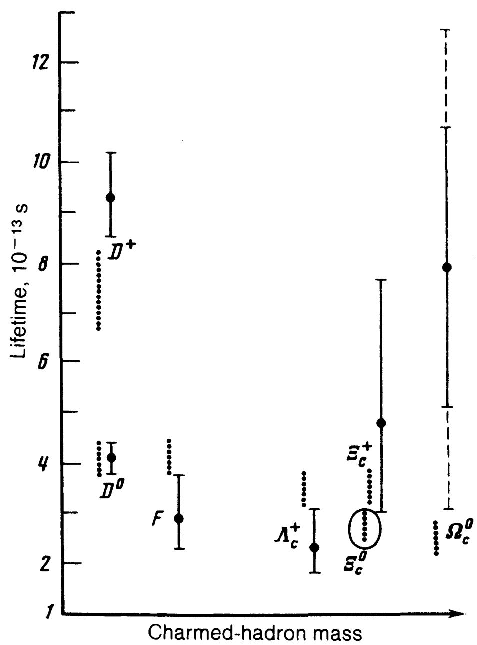

FIG. 7.

lived charmed hadron, but there are as yet no experimental indications for this. (Note, to be sure, that τ_{Ω_c} was measured only for three events.) If our analysis is qualitatively correct, and at present there are no indications to the contrary, experiment should contradict the theory of this question. The result for τ_{Ξ^0} (in the small circle of Fig. 7) is nothing but a prediction, since this baryon has not yet been observed.

In conclusion, we are deeply grateful to A. M. Badalyan for numerical potential-model calculations whose results were cited above. We thank also Ya. I. Azimov, M. V. Danilov, V. A. Khoze, and N. G. Ural'tsev for helpful discussions.

[1)]A potential analysis of the three-particle problem with a linear potential was carried out by A. M. Badalyan, whom we thank for calculating the parameters of interest to us.

[2)]The question of mass splitting for charmed and beautiful baryons is considered, e.g., in the review by Taxil.[13]

[3)]According to the nomenclature adopted in Ref. 11, baryons with $J^P = \frac{1}{2}^+$ are designated as follows: $A_c^+ = cus$, $A_c^0 = cds$, $\Lambda_c^+ = cud$, $T_c^0 = css$. In the current literature a different notation is used: $A_Q \to \Xi_Q^{(A)}$, $T_Q \to \Omega_Q$.

[4)]For a recent discussion see Ref. 11.

[5)]It is just this circumstance that leads to the first relation of (37). Although the Lagrangian (33) does contain terms of type $(\bar{b}b)(\bar{d}d + \bar{s}s)$, it is easy to verify that after averaging these terms over the states B_s^0 or B_s^0 (by factoring or in the nonrelativistic model) we obtain zero if the contribution j^a is neglected.

[6)]Note that Γ_{0b} does not agree exactly with the naive parton result for the b-quark width. The point is that according to the picture developed[7] an overall enhancement of the b-quark nonleptonic decays takes place by a factor $1.07(C_+^2 + C_-^2)/2 \approx 1.3$ in the probability (see Ref. 7 for details). This enhancement improves the agreement between the experimental data for $B(B \to l\nu X)$ and the theory [$B_{\rm exp} \approx 13\%$, Ref. 19, and $B_{\rm theor} \approx 14\%$, whereas in the standard approach $B_{\rm theor} \approx 16\%$, see, e.g., Ref. 8].

[1]B. Guberina, S. Nussinov, R. Peccei, and R. Rückl, Phys. Lett. **89B**, 111 (1979).
[2]M. Bander, D. Silverman, and A. Soni, Phys. Rev. Lett. **44**, 7 (1980); H. Fritsch and P. Minkowski, Phys. Lett. **90B**, 455 (1980); W. Bernreuther, O. Nachtmann, and B. Stech, Z. Phys. **C4**, 257 (1980).
[3]K. Shizuya, Phys. Lett. **100B**, 79 (1981); Y. Igarashi, M. Kuroda, and S. Kitakado, Phys. Lett. **93B**, 125 (1980); M. Deshpande, M. Gronaum, and D. Sutherland, Phys. Lett. **90B**, 431 (1980); M. Gronau and D. G. Sutherland, Nucl. Phys. **B183**, 367 (1981).
[4]B. Block and M. Shifman, Prepring ITEP-37, 1986.
[5]T. Kobayashi and N. Yamazaki, Prog. Theor. Phys. **65**, 775 (1981); R. D. Peccei and R. Rückl, in Proc. Symp. on Special Topics in Gauge Field Theories, Ahersnshoop, DDR, 1981, p. 8; H. Sawayanagi, K. Fujii, T. Okazaki, and S. Okubo, Phys. Rev. **D27**, 2107 (1983); G. Altarelli and L. Maini, Phys. Lett. **118B**, 414 (1982).
[6]N. Bilic, B. Guberina, and J. Trampetic, Nucl. Phys. **B248**, 261 (1984).
[7]N. B. Voloshin and M. A. Shifman, Yad. Fiz. **41**, 187 (1985) [Sov. J. Nucl. Phys. **41**, 120 (1965)].
[8]L. B. Okun', *Leptons and Quarks*, North-Holland, 1981.
[9]M. K. Gaillard and B. W. Lee, Phys. Rev. Lett. **33**, 608 (1974); G. Altarelli and L. Maiani, Phys. Lett. **52B**, 351 (1974).
[10]M. Bauer and B. Stech, Phys. Lett. **152B**, 381 (1985); A. J. Buras, J.-M. Gerard, and R. Ruckl, Nucl. Phys. **B268**, 16 (1986).
[11]M. Shifman and M. Voloshin, Preprint ITEP-54, 1986.
[12]P. E. Close, *An Introduction to Quarks and Partons*, Academic Press, London, 1979.
[13]P. Taxil, in Proc. XX Rencontre de Morion, Jan. 1985. La Plange, France, p. 111.
[14]A. R. Zhitnitskiĭ, I. R. Zhitnitskiĭ, and V. L. Chernyak, Yad. Fiz. **38**, 1277 (1983) [Sov. J. Nucl. Phys. **38**, 775 (1983)]; T. M. Aliev and V. L. Eletskiĭ, *ibid.* p. 1537 [p. 936].
[15]A. I. Vaĭnshtein, V. I. Zakharov, and M. A. Shifman, Zh. Eksp. Teor. Fiz. **72**, 1275 (1977) [Sov. Phys. JETP **45**, 670 (1977)].
[16]M. B. Voloshin and K. A. Ter-Martirosyan, Theory of Gauge Interactions of Elementary Particles [in Russian], Energoatomizdat, 1984.
[17]A. I. Vaĭnshtein and I. B. Khriplovich, Pis'ma Zh. Eksp. Teor. Fiz. **18**, 141 (1973) [JETP Lett. **18**, 83 (1973)]; M. K. Gaillard and B. W. Lee, Phys. Rev. **D10**, 897 (1974).
[18]E. Franco, M. Lisignoli, and A. Pugliese, Nucl. Phys. **B194**, 403 (1982).
[19]E. H. Thorndike, Ann. Rev. Part. Nucl. Sci. **35**, 193 (1985).
[20]C. Caso and M. C. Touboul, Preprint CERN EP/85-176, 1985 (Rev. Nuovo Cimento, in press).

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